Science & Technology Facilities Council

Experience of Linear Solvers in a Nonlinear Interior Point Method

Jonathan Hogg
(joint work with Jennifer Scott)

STFC Rutherford Appleton Laboratory

Software

IPOPT[1]

Interior point optimization code for nonlinear problems
Filter line-search approach
Written by Wächter and Biegler
Part of COIN-OR project
Open Source Eclipse Licence

Probably most widely used open-source IPM solver
Interfaces to a number of linear solvers

[1] A. Wächter and L. T. Biegler, *On the Implementation of a Primal-Dual Interior Point Filter Line Search Algorithm for Large-Scale Nonlinear Programming*, Mathematical Programming 106(1), pp. 25-57, 2006

What's happening

Very simplistically...

while not converged **do**
  Find a descent direction.
  Conduct a line search for next trial point.
    (Repeat with second-order corrections).
  Take the step; update parameters.
end while

Find a descent direction

Solve

$$Ax = b$$

where

$$A = \begin{pmatrix} W + \Sigma_k & J \\ J^T & -\delta_c I \end{pmatrix}$$

By sparse direct method i.e. factorize with pivoting

$$A = LDL^T$$

Find a descent direction

Solve

$$Ax = b$$

where

$$A = \begin{pmatrix} W + \Sigma_k + \delta_w I & J \\ J^T & -\delta_c I \end{pmatrix}$$

By sparse direct method i.e. factorize with pivoting

$$A = LDL^T$$

Requirements and options

- Get the "right" answer. Inaccuracy $\Rightarrow$ more IPM iterations.
- Report correct inertia — required for filter line search to work

Two main options:

1. Static pivoting — if a pivot is too small, add something to it.
   Faster, less accurate
2. Threshold pivoting — if a pivot is small, delay until later.
   Slower, more accurate

HSL_MA86

Recently developed a new multicore code (OpenMP)
Originally aimed at large finite element and general indefinite problems ($> 10^5$ variables)

Problem	n	MA57	HSL_MA86		
		1	1	8	Speedup
Boeing/bcsstk38	8032	0.152	0.087	0.144	0.60
Oberwolfach/t3dh	79171	20.2	12.1	2.17	5.58
GHS_psdef/s3dkq4m2	90449	2.94	1.94	0.440	4.40
ND/nd12k	36000	164	88.5	15.2	5.83
Oberwolfach/bone010	986703	877	590	88.3	6.68

(MA57 is popular choice for IPOPT; it is also used in MATLAB)

Does HSL_MA86 work well if dropped into IPOPT?

Science & Technology Facilities Council

Comparison details

Use 913 solvable problems from CUTEr test set.

529 Trivial $<$ 0.1s to optimum (with MA57)

353 Easy $<$ 0.5s per iteration (with MA57), but not trivial

31 Difficult $>$ 0.5s per iteration (with MA57)

Pitfalls

- 100% accurate inertia not possible
- Different paths to optimum
- Different optima
- Different matrices

Does scaling help?

Good scalings speed up factorizations of poorly scaled matrices.

- MC64 Find weighted maximum matching.
  Good; can be slow.
- MC77 Several matrix-vector multiplies.
  Fast; can be insufficient.
- None Free?
  Fastest; can cause many delayed pivots.

Best approach varies by problem.

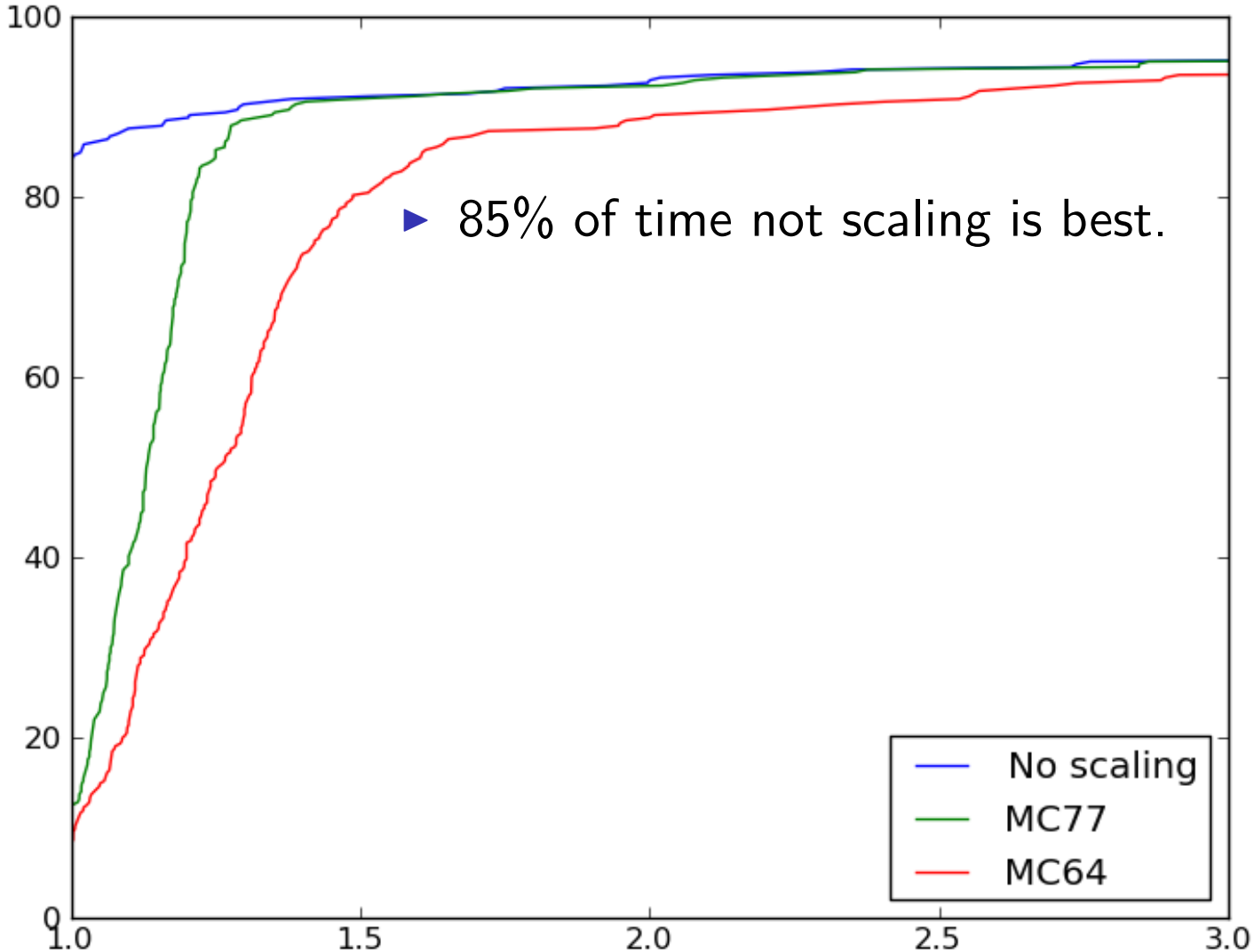
Does scaling help?

Good scalings speed up factorizations of poorly scaled matrices.

MC64	Find weighted maximum matching.	Not parallel
	Good; can be slow.	
MC77	Several matrix-vector multiplies.	Parallelisable
	Fast; can be insufficient.	
None	Free?	Trivial
	Fastest; can cause many delayed pivots.	

Best approach varies by problem.

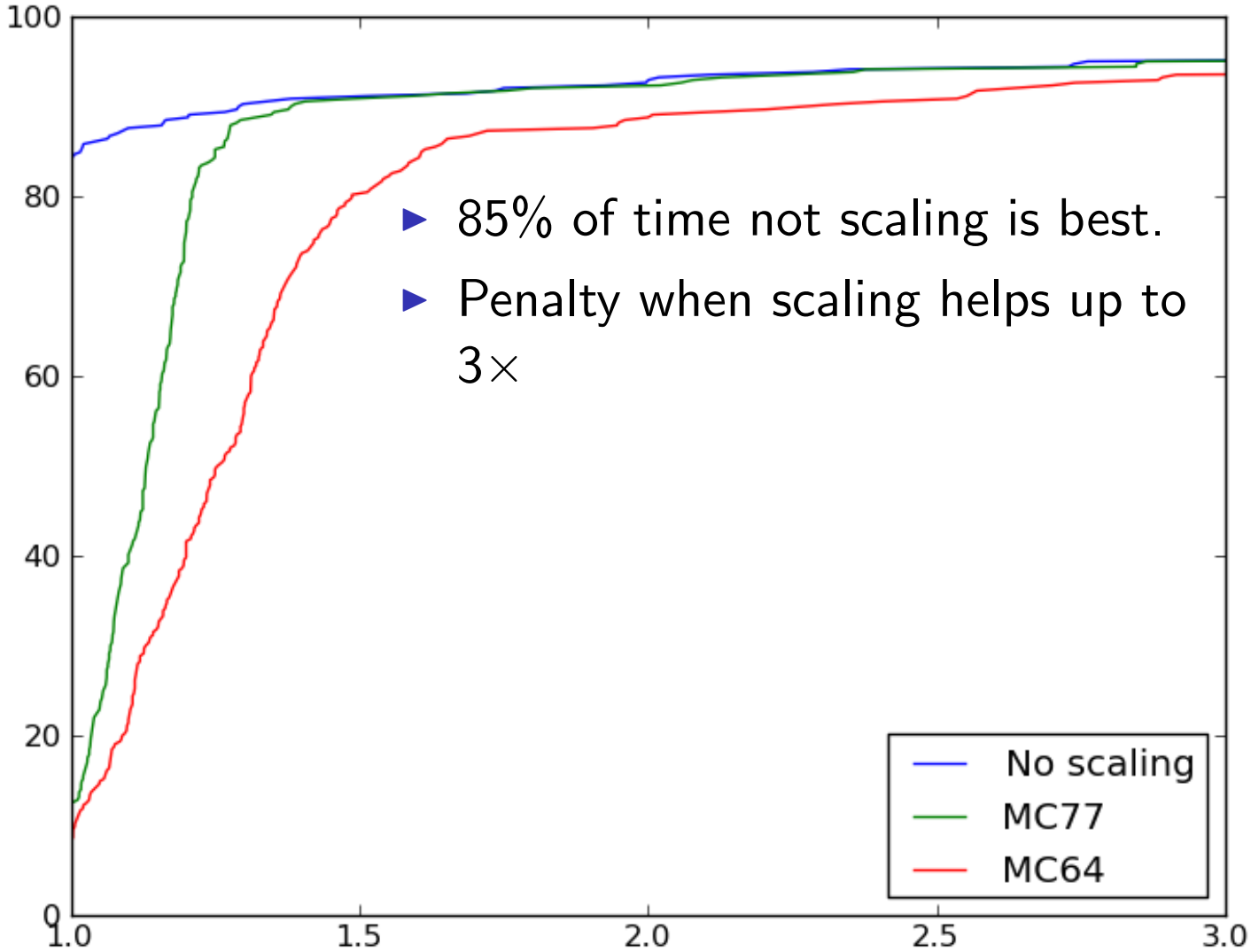
Scaling helps... sometimes



- 85% of time not scaling is best.

Science & Technology Facilities Council

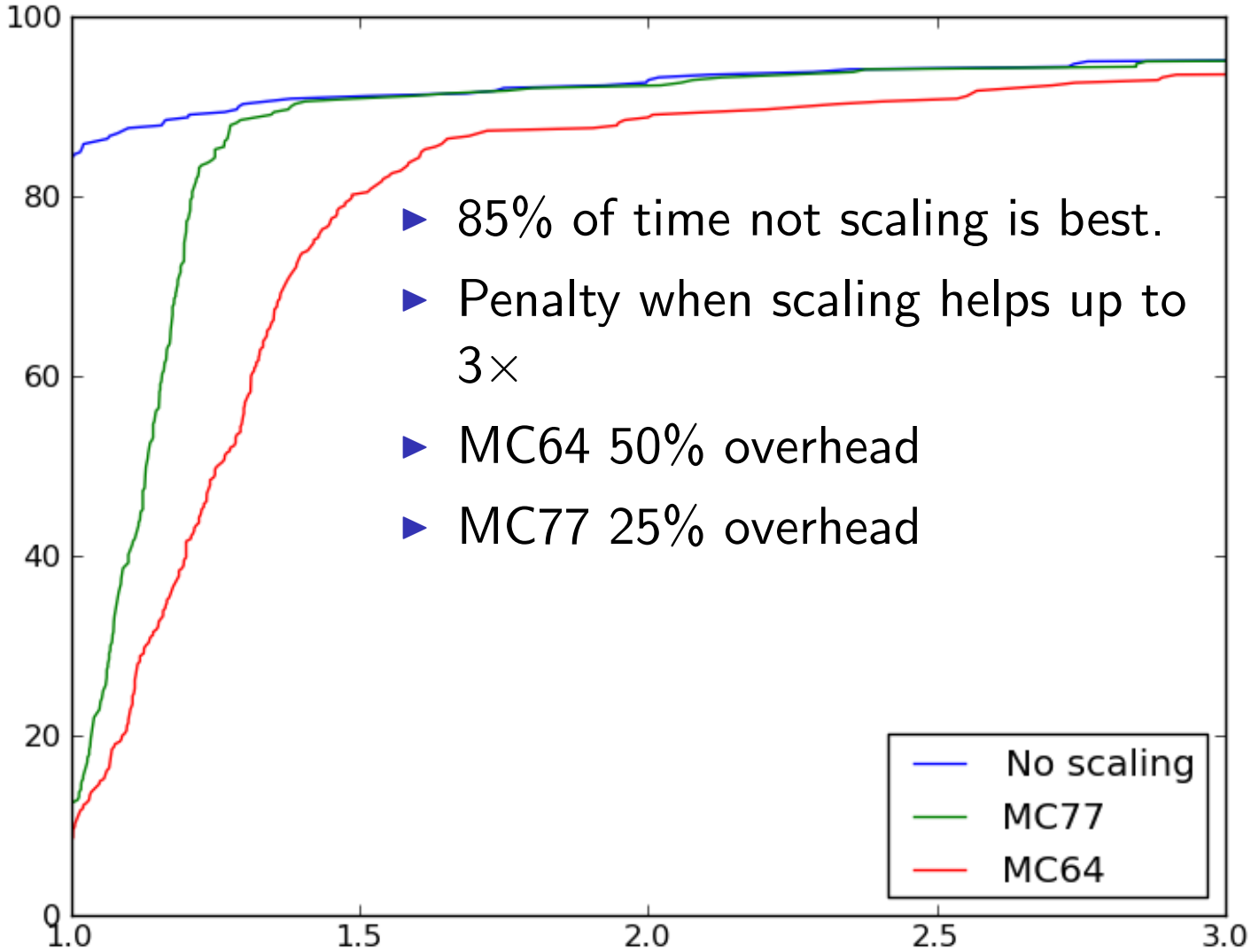
Scaling helps... sometimes



- 85% of time not scaling is best.
- Penalty when scaling helps up to 3×

Science & Technology Facilities Council

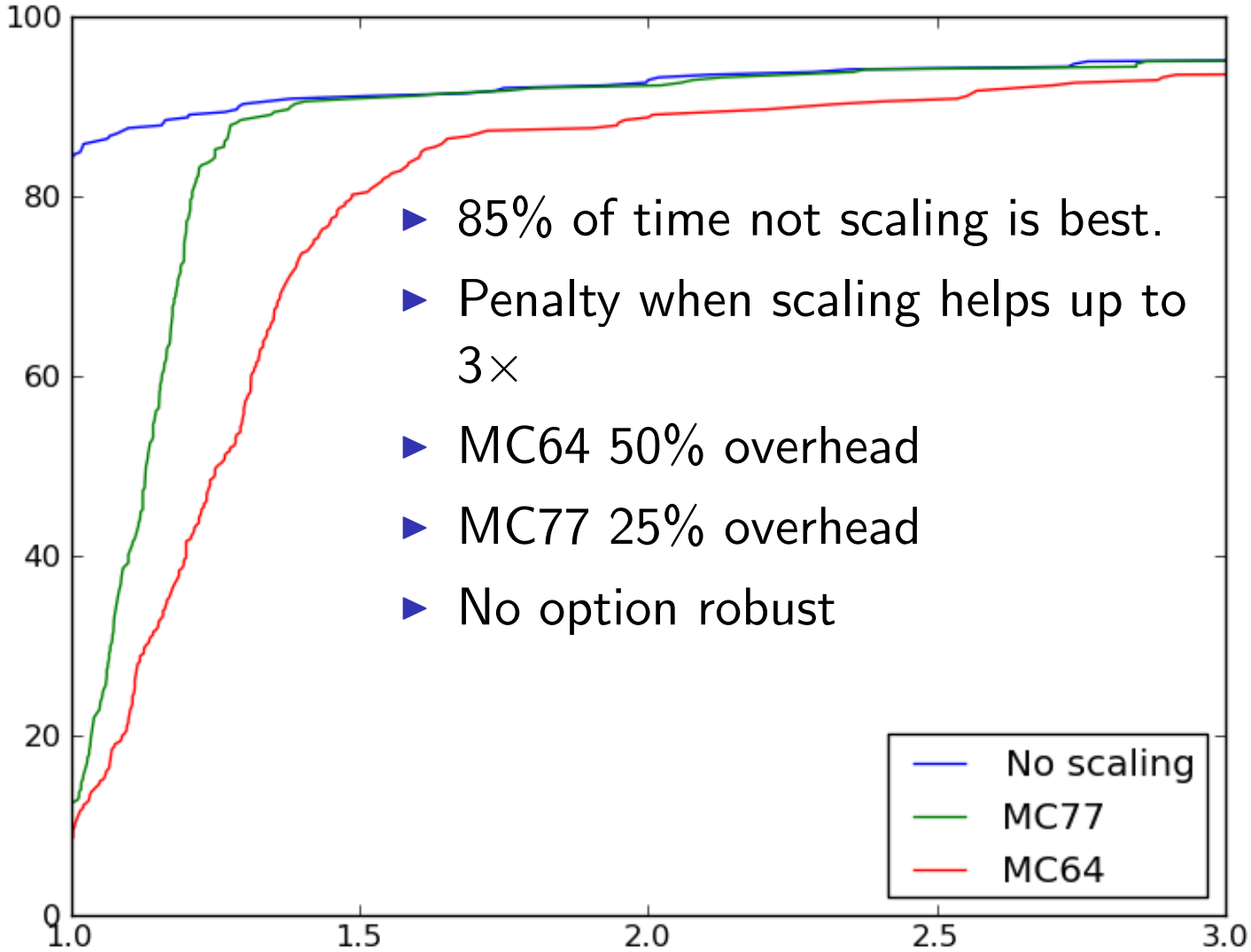
Scaling helps... sometimes



- 85% of time not scaling is best.
- Penalty when scaling helps up to 3×
- MC64 50% overhead
- MC77 25% overhead

Science & Technology Facilities Council

Scaling helps... sometimes



- 85% of time not scaling is best.
- Penalty when scaling helps up to 3×
- MC64 50% overhead
- MC77 25% overhead
- No option robust

Science & Technology Facilities Council

Scaling examples

Times

Problem	None	MC64	MC77
ELEC	82.7	**61.8**	74.4
HADAMALS	**16.6**	24.2	24.7
HVYCRASH	25.0	**11.4**	42.1
LUKVLE12	6.4	5.8	**4.9**

Iterations

Problem	None	MC64	MC77
ELEC	324	185	262
HADAMALS	311	307	386
HVYCRASH	748	298	1427
LUKVLE12	30	30	30

Explanations

- Different paths (Inertia!)
- More work (Delayed Pivots!)

Science & Technology Facilities Council

What should we do?

Based on problem A2ENSNDL

$$\begin{pmatrix} 1e\text{-}22 & & & & & \\ & 1e\text{-}22 & & & & \\ & & 1e\text{-}22 & & & \\ & & & \ddots & & \\ 0.3 & 0.2 & 0.4 & \dots & 1e10 & \\ 0.2 & 0.3 & 0.5 & \dots & 0.5 & 1e10 \end{pmatrix}$$

What is the inertia?

No scaling: Inertia (2,2,996), Maximum front 1000×1000

What should we do?

Based on problem A2ENSNDL

$$\begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & \ddots & & \\ 3e\text{-}11 & 2e\text{-}11 & 4e\text{-}11 & \dots & 1 & \\ 2e\text{-}11 & 3e\text{-}11 & 5e\text{-}11 & \dots & 5e\text{-}11 & 1 \end{pmatrix}$$

What is the inertia?

No scaling: Inertia (2,2,996), Maximum front 1000×1000

MC64: Inertia (500,500,0), Maximum front 2×2

Science & Technology Facilities Council

What should we do?

Based on problem A2ENSNDL

$$\begin{pmatrix} 1e\text{-}19 & & & & & \\ & 1e\text{-}19 & & & & \\ & & 1e\text{-}19 & & & \\ & & & \ddots & & \\ 3e\text{-}11 & 2e\text{-}11 & 4e\text{-}11 & \dots & 1 & \\ 2e\text{-}11 & 3e\text{-}11 & 5e\text{-}11 & \dots & 5e\text{-}11 & 1 \end{pmatrix}$$

What is the inertia?

No scaling: Inertia (2,2,996), Maximum front 1000×1000

MC64: Inertia (500,500,0), Maximum front 2×2

Scaling 3: Inertia (500,500,0), Maximum front 1000×1000

Science & Technology Facilities Council

Parallel?

Modern Intel machine: 2 quad-cores — eight threads

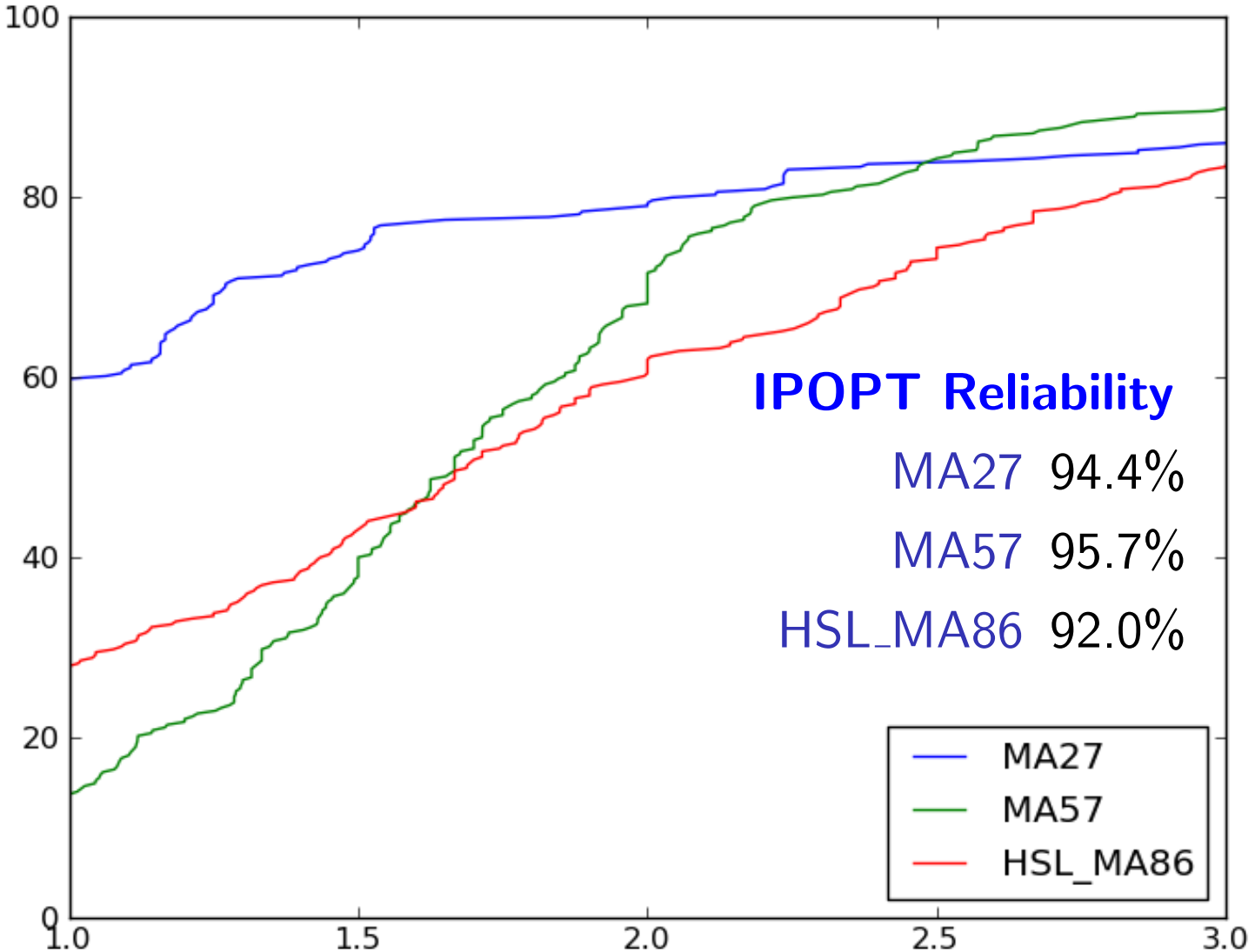
Easy

208 slower
142 faster
3 failed
Min/max/avg speedup = 0.22 / 30.6 / 1.30

Difficult

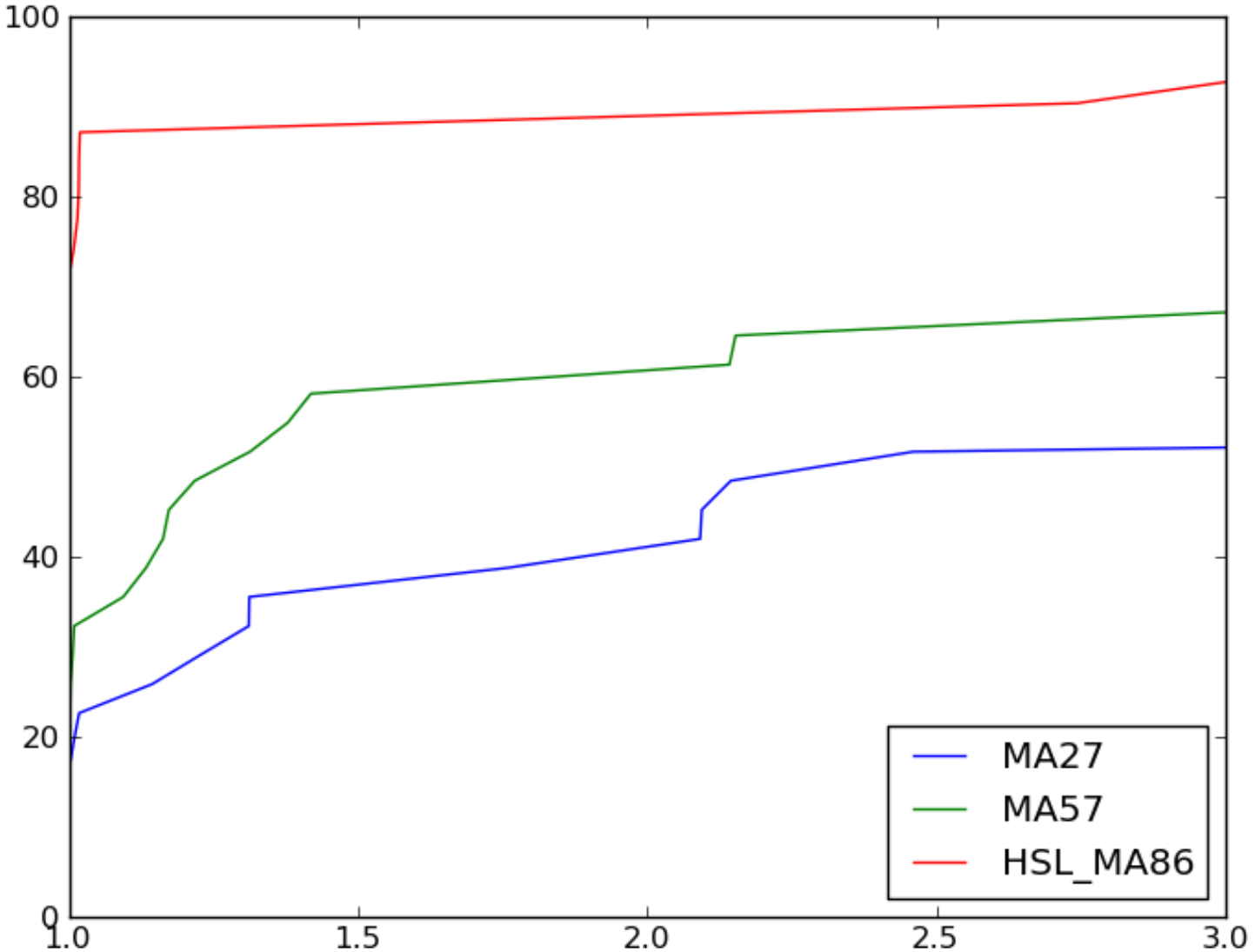
3 slower
24 faster
4 failed
Min/max/avg speedup = 0.27 / 5.35 / 1.76

Easy problems



Science & Technology Facilities Council

Difficult problems



Science & Technology Facilities Council

Open questions?

- Can a code perform well on both small and large problems?
- How should inertia detection be handled with respect to scaling? What is zero?
- Can we get better parallel performance by driving parallelism up into the IPM somehow?

General HSL: http://www.hsl.rl.ac.uk

HSL IPOPT: http://www.hsl.rl.ac.uk/ipopt

HSL is freely available to academics

Questions?

General HSL: http://www.hsl.rl.ac.uk

HSL IPOPT: http://www.hsl.rl.ac.uk/ipopt

HSL is freely available to academics