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COLLECTIVE PLASMA PROCESSES IN THE SOLAR INTERIOR AND THE PROBLEM OF THE SOLAR NEUTRINO DEFICIT

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Abstract

This review presents the results of recent calculations of collective plasma processes of radiation transport in the solar interior. The review introduces a remarkable number of effects previously neglected which are shown to reduce substantially the Rosseland opacity at the center of the Sun (the decrease of opacity is approximately 10% which is greater than previously accepted possible errors in opacity). It is also shown that effects, which were previously treated without taking into account the collective behavior of plasmas, change appreciably when the collective nature of the plasma is included. The analysis is based on the modern concepts of plasma physics in which an essential role is played by photon scattering on ions and the emission by the oscillation cloud of electrons surrounding the ions. The process which contributes most to a decrease in opacity are: the broadening of the Raman resonance (due to both the Doppler effect and binary electron-ion collisions), frequency diffusion of radiation during radiative energy transfer, the processes of stimulated scattering and collective quantum corrections to the scattering. A list of collective plasma effects which influence photon transport in the dense central solar plasma is presented. The results of these new calculations and the developed theory shows that a better agreement between the observed neutrino flux and the theoretical predictions is beginning to appear. New problems are discussed which can be of importance from the point of view of modern plasma physics for solar neutrino production in different energy ranges.

CONTENT

1 INTRODUCTION

There exists a widely held opinion that the measured flux of solar neutrinos is less than that predicted by the Standard Solar Model (SSM). A critical review of solar neutrino experiments and improvements in the SSM's was recently given by Morrison [1] who indicated the need for detailed plasma physics calculations. We concentrate in the present article on the plasma aspects of the problem [2]. Morrison [1] illustrated the tendency of a decrease with time of the discrepancy between the observations and the theoretical models but the plasma aspect of the problem was only briefly mentioned and has not been dealt with in detail. Concerning the discrepancy of the observations and the SSM it should be noted that in the first pioneering experiment this discrepancy was a factor of 8, at the present time there are four experiments going on and on average the discrepancy is a factor of 2 to 3 depending on the experiment. The four experiments are the SAGE (Joint Russian- American experiment) GALLEX (Gran Sasso, Italy) , KAMIOKANDE (Japan) and HOMESTAKE (USA, Davis experiment). The different experiments have different thresholds and are measuring neutrino fluxes within different ranges of energies. The theory predicts the flux for different nuclear reactions and for comparison of theoretical predictions with observations in several cases a subtraction of the results of one experiment from the results of another experiment was performed. The latter is possible only if the absolute calibration of the experiment was performed which is for some experiments questionable. These problems will probably disappear soon but it is believed that the absolute calibration of the HOMESTAKE experiments will be difficult to perform in the near future.

We will leave these problems and concentrate on plasma collective effects in the SSM- the aspect of the problem which is rarely discussed in the current literature dealing with the predictions of the neutrino flux from the Sun.

We will concentrate on the question whether or not in theoretical predictions of the neutrino flux the physics of the processes were treated in a correct manner? This question is more fundamental than the question which is often asked at present, namely is the neutrino deficit due to an incorrect treatment of astrophysics of the solar interior or is it due to neutrino oscillations: MSW effect (named after S.P,Mikheyev; A.Yu.Smirnov and L.Wolfenstein) or other effects of a similar kind)? Literally the problem is stated as "Astrophysics or oscillations?". Such a name was given to the recent workshop held at Gran Sasso where the first results of detailed calculations of collective plasma effects were presented [2].

Why do we want to separate the physics and the astrophysics? The reason is that astrophysics usually uses the known and approved physical processes to construct the models of astrophysical phenomena. But the question is whether or not the physical processes for the conditions in the solar interior are known at a level necessary to predict

the neutrino flux with the accuracy needed for a comparison with observations.

It is necessary to say a few words how the SSM's are calculated. In fact this can be considered as the usual astrophysical treatment. It is assumed that all the physical processes are understood well, the cross-sections of reactions are known and can be corrected if necessary in future laboratory experiments. Also known at the present time are the three main parameters of the Sun: the luminosity of the Sun, its radius and its mass and we know also the abundance of the elements on the surface of the Sun. It is assumed that the initial abundance at the stage of the formation of the Sun corresponds to the observed abundance of elements in the present neighborhood. One follows then the evolution of the initial plasma cloud which forms the Sun (in the literature one often finds the word "gas cloud" which it certainly is not). The evolution is followed up to the present time and determines the present composition of the elements in the solar interior in the way that it corresponds to the observed abundance of elements at the surface of the Sun. The relative abundances of different elements in the solar interior is an important parameter for predictions of the solar neutrino flux in different energy channels. In the center of the Sun hydrogen is burning and the abundance of helium is increasing. The abundance of such elements as C, N, O, Fe is important for predictions of solar luminosity and the neutrino flux in different energy channels. At the present time there exists many solar models which all go by the name SSM, they differ on the composition of different elements and in the dependence of temperature and the abundances as a function of distance from the center of the Sun.

The basic assumption in this "astrophysical "approach is the assumption that all the physical processes are well known and the cross-sections for them at least can be determined from laboratory experiments or can be improved in future laboratory experiments.

The main question arises whether the last statement is correct?

Another question is what else is assumed in the construction of the SSM? One of the assumptions is obvious: in the calculation of the radiation transport in the solar interior it is assumed that the local thermodynamic equilibrium is established with small deviations from it due to the radiation flux which is proportional to the gradient of the temperature. These deviations are described by the first Legendre polynomial with the angle related to the direction of the temperature gradient.

A commonly held opinion among the astrophysics community (which is the basis of the "astrophysical" approach) is that the SSM is based on very well proven statements such that the central regions of the Sun can be calculated with elementary mechanics and statistical physics and that the main processes necessary to determine the neutrino flux are a knowledge of nuclear reaction cross-sections and the photon scattering cross-sections on *free electrons* together with their absorption due to inverse bremsstrahlung (see [3]). However since the central region of the Sun is most definitely a plasma, photon scattering cross-sections can be determined by collective effects where the statement that

the scattering is produced by *free electrons* can be completely wrong since the cross-section of collective scattering depends on the distribution of all other particles. The latter is a main conclusion of modern plasma physics. This means that the cross-sections determined by individual particles have nothing in common with the cross-sections under real plasma conditions where the cross-sections depend on the surrounding plasma density, temperature etc. In the process of scattering the statement that scattering is produced mainly by electrons is valid only for isolated electrons but not from electrons in plasmas. Scattering on single electrons can be found in many astrophysical situations but it is valid only in the limit of very high frequencies when the electrons behave as free particles (for free particles the Thomson cross-section is inversely proportional to the square of the mass and thus is negligible for ions as compared to electrons). We will give the exact criteria when the electrons can be considered as free. The question then is whether or not in the solar interior these conditions are fulfilled and the electrons can be considered as free? In advance we will state that the answer is *no* .

It is worthwhile to mention that the Sun is a "plasma sphere" but not a "gas sphere" and should be treated as a plasma object with all the complications introduced by plasma collective processes. Examination of the collective plasma processes in the center of the Sun will be the main subject of the present review. Recently in some publications there appear the term "plasma processes in the solar interior" which is very strange since the Sun consists only of plasma and the plasma in the Sun should be treated as it is with all known complex collective phenomena. Much experience, knowledge and data etc accumulated during the last decades in the investigation of laboratory plasmas and the plasmas of near space show very definitely that the collective effects determine the plasma properties. This knowledge should not be discarded in the investigations of the solar interior.

Let us clarify why the researcher not well acquainted with the physics of the solar interior but well acquainted with plasma physics can as a first attempt abandon the "astrophysical" approach described above.

It is well known at the present time that plasmas do not want to behave in the way prescribed by simple statistical physics and mechanics.

In the early days of controlled thermonuclear research (CTR) the belief was that plasmas should follow the well established laws of statistical mechanics and should locally have a tendency to form a thermodynamic equilibrium distribution. But after many years of research it is found that the main obstacles to controlled thermonuclear fusion are collective effects.

This term in plasma physics is used to describe two partially independent phenomena. The first phenomena was already discussed and is related to the radical change of cross-sections by collective effects. The second phenomena is related to the development of different types of instabilities which makes the state nonlinear, far from equilibrium and

as a rule such a state could lead to self-organization. Modern plasma physics mainly deals with the second phenomenon. But to find whether or not the collective effects lead to additional nonlinear transport phenomena which are very often observed in experiments, it is necessary to start with understanding the basic state when the instability is absent but taking collective effects in the cross-sections into account. Thus the question what could be the classical transport was the first one to be understood in laboratory plasmas.

The analogy between the CTR research and the construction of SSM's is rather useful for understanding the general situation in plasmas. In both cases in devices for CTR and on the Sun the energy is transferred from the central part to the periphery, but in the drvice for CTR it is related to the thermal conductivity (in the case when instabilities are not developing) by plasma particles, while in the center of the Sun the energy flux if formed by radiation via radiative conductivity. What is indeed known for certain in tokamaks (which is a particular CTR device) is that the energy transport is never classical (more exactly for a toroidal geometry it is called neoclassical). The classical theory of energy transport in tokamaks is constructed in a similar way to the energy transport in SSM. Namely it is supposed that locally a thermodynamic equilibria is established with small deviations due to temperature gradients and proportional to the first Legendre polynomial. In the same way the transport theory of radiation is constructed in SSM. It is assumed that the local thermal distribution is established both for photons and plasma particles and the deviations from the local distributions are due to the temperature gradient. Bearing this analogy in mind we can call the classical theory of energy transport in tokamaks as a Standard Model of Tokamaks (SMT). It is obvious that the experiments do not confirm the SMT. Why then should we rely on SSM to confirm observations from the Sun?

The problem one should start with is : does there exist a SSM which is similar to SMT in the sense that it takes into account all collective changes to the cross-sections? Thus the question is whether there exist the basic starting point from which we will be able to discuss the possible role of instabilities.

We should answer also the question whether there exists at the present time a reasonable explanation of anomalously large energy transport in tokamaks. The answer is unfortunately not. More than three decades of investigation of the anomalous transport in tokamaks has not clarified the nature of anomalous transport although the real progress achieved in CTR is enormous (the maximum temperatures reached are substantially larger than that in the center of the Sun).

The deficit of solar neutrinos in some high energy channel is relative to the Sun's luminosity. The plasma researcher on considering the neutrino deficit will find it a natural phenomenon since he will consider it very probable in the presence of anomalous radiative transfer. The sign of the effect is that expected if the radiative transfer is anomalous. But one can argue that the rate of binary collisions in the Sun is so high that processes

should be considered as classical. But in tokamaks the collision rate is also high otherwise the thermonuclear reactions will not occur. On the other hand even for a high rate of collisions there exists a set of dissipative instabilities. This brings us apparently to another question: Why is the discrepancy between the measured neutrino flux and that calculated by SSM so small?

In the present consideration we will not discuss the problems of instabilities in the solar interior since our intention is only to discuss the *collective phenomena in energy transfer assuming that the instabilities are absent.* This is the problem one needs to start with before any further steps can be made in discussing the possibility of anomalous energy transfer.

Before going to the main subject we make some comments on the possibility of anomalous energy transfer in the Sun due to the development of instabilities. The first question related to this problem is: whether there exists in the Sun a constant source of energy which can drive the instability? The answer will be *yes.* This source is the observed convection and continuous "sunquakes" observed as oscillations of the Sun which can be considered as a continuous source of turbulence. The surface of the Sun is strongly turbulent and this is confirmed by observations. But it is unknown whether the interior of the Sun is also turbulent. But it is very probable that similar to tokamaks the Sun is a self-organized system and what happens inside can not be separated from what happens on the surface. The nonlinear cascades can transfer the energy to small scales important for energy transfer.

These comments are made here intentionally to have a picture of the Sun from a plasma point of view and also to demonstrate there is no hope to find from the solar neutrino deficit some definite conclusion about the neutrino oscillations, since one always can include the effects of turbulence and instabilities.

Below we will deal with more simple problems - namely the classical (in the sense of the absence of anomalous transport phenomena) transport of radiation making a special note on the role of collective plasma phenomena. So we will stay on the conservative position that instabilities are absent and discuss the problem whether the collective plasma effects were taken into account with the necessary accuracy to predict the neutrino flux and whether all the collective effects taken into account are properly included in SSM? The answer to this question is *no* - many effects were missed, some were included improperly with the physics not well understood.

As concerning the collective properties the traditional astrophysical approach is not adequate since it is impossible to use the cross-sections of nuclear and electromagnetic processes measured in laboratory experiments for the conditions in the solar interior. Even by laser compression of materials on very short time intervals it is rather difficult to obtain a conclusive answers. In these conditions one obviously should use the plasma theoretical approach.

There exists the general modern plasma physics approach which should be used for this purpose. Only recently such an investigation was started to consider the properties at the very center part of the Sun [4]. It appears that the theoretical plasma physics approach discussed in this review can provide the predictions of solar neutrino flux about 2 times lower than previous predictions.

We consider this approach as an active approach opposite to the passive astrophysical approach described above. We believe that such an active approach (including the collective plasma phenomena known from laboratory and near space measurements) is needed in many other astrophysical problems.

2 HOW SENSITIVE IS THE NEUTRINO FLUX TO THE PARAMETERS IN THE CENTER OF THE SUN?

When the first SSM was discussed the question arose. Is it possible to construct a SSM within the accuracy of a factor $2-3$? But the fact is the high energy neutrino flux (the only one measured in the first experiments) is very sensitive to small changes of temperatures in the solar interior. For example a change of the central temperature by only $2-3\%$ changes the predictions of the high energy neutrino flux by a factor 2. It is worthwhile to give here an outline of neutrino production from nuclear reactions in the center of the Sun:

$$
\begin{array}{rcll}
p+p & \rightarrow & d+e^{+}+\nu & (1)\\
p+p+e^{-} & \rightarrow & d+\nu & (2)\\
p+d & \rightarrow & {}^{3}He+\nu & (3)\\
85\% \swarrow & & \searrow 15\% & \\
{}^{3}He+{}^{3}He \rightarrow & \alpha+2p; & {}^{3}He+\alpha \rightarrow {}^{7}Be+\gamma & (4)\\
 & & \swarrow & \\
{}^{7}Be+e^{-} \rightarrow & {}^{7}Li+\nu; & {}^{7}Be+p \rightarrow {}^{8}B+\nu & (5)
\end{array}
$$

The last row shows the generation of ${}^{8}B$ neutrinos which gives an 80% contribution in the Chlorine experiments and about 100% in the Kamiokande experiments. In the Chlorine experiments 20% corresponds to the contribution of ${}^{7}Be$ neutrinos and in Sage and Gallex experiments the measured neutrinos correspond to the main process of nuclear synthesis, which is described in the first two rows (1) and (2). The strongest dependence

on temperature is for the most energetic ^8B neutrinos- for the neutrino flux Φ_ν the dependence is $\Phi_\nu^B \propto T^{18}$. For ^7Be neutrinos the dependence on the temperature is also rather strong $\Phi_\nu^{Be} \propto T^8$ and the weakest dependence on temperature exists for the proton-proton reactions $\Phi_\nu^{pp} \propto T^{-1.2}$.

The solar luminosity $L_\odot$ is determined by the relative value of the temperature gradient and is inversely proportional to the Rosseland opacity κ_R (solar opacity) and is proportional to T^4 (i.e., intensity of the blackbody radiation). Therefore a decrease of the Rosseland opacity of 12% for a given luminosity corresponds to a decrease of the temperature by only 3% which means a decrease of the ^8B neutrino flux by two or three times. Therefore it is recognized that the neutrino flux is very sensitive to small changes in the solar opacity.

At present there have been no physical reason to change the opacity by as much as 12% or even 10%. The latter number corresponds to change of the temparuture by 2.5% which is even more appropriate concerning the existing solar seismology data. However, in this paper we present new results of plasma collective processes which can change the opacity by as much as 10%.

The value of the solar opacity (more precisely the coefficient of Rosseland opacity defined below) is determined mainly by scattering of photons, by bremsstrahlung absorption (not in "free-free transitions", as was used previously, since when collective effects are included the plasma particles can not be considered as free particles) and by line absorption. The value of the Rosseland opacity was corrected many times and together with the corrections of nuclear cross-sections the disagreement between predictions and observations was reduced from a factor 10 to a factor $2-3$. The plasma collective effects in the coefficient of the Rosseland opacity have only been taken into account recently [5,6], but many of them where omitted and the necessary change of 12% was not obtained.

The processes we are speaking about are well known in plasma physics e.g. scattering, bremsstrahlung absorption and line absorption for elements which are not fully ionized, we will discuss them in detail in his article. Although all the processes are known, for the Sun contrary to laboratory experiments what is also important are the integrated values over the frequencies, angles and thermal distributions of particles and such values are not given in the plasma literature.

It should be noted that the plasma also effects the nuclear reactions, since the tunneling is rather sensitive to small changes of the potential barrier which can be due to plasma shielding [7]. For this aspect of the problem there are still many unknowns and some effects are not completely clear, for example we have in mind the capture of electrons by ^7Be nuclei and the generation of neutrinos. The laboratory experiments give in this case only the result for the case of a single electron bombardment of the nuclei and it is also needed to extrapolate toward the lower energies [1] . But even if the experiments can be refined they will not give an answer for the electron capturing from a dense Debye shell

which corresponds to the solar conditions and this capturing could be different than that of a single electron. Probably this reaction will be the most sensitive to the collective plasma effects. We restrict ourselves to this comment since this problem is still waiting to be analysed taking the plasma collective effects into account. This problem is actual since some observations indicate that the deficit of beryllium neutrinos is larger than for other solar neutrinos (the above problem of absolute experimental calibration is also important for this problem).

It is necessary here to say a few words concerning the value of the coefficient of the Rosseland opacity κ_R. The possibility of introducing such a coefficient or to obtain its value without solving the transport equation is rarely discussed. The usual treatment in the SSM is to consider the solar opacity to be known and then with the known opacity to solve the transport equations integrated over frequency. Such an approach can be used only if the structure of the radiation transport equation has a certain form - namely the transport equation should not contain the derivatives of the intensity of radiation with respect to the frequency otherwise one should first solve the differential transport equation and then one can introduce the value called opacity, but such an approach will be useless. The solar opacity is defined as an integral with respect to the frequency characteristic of the radiation transport and in the case where the transport equations can not be first integrated with respect to the frequency the concept of opacity is useless. As we will show in the presence of collective plasma effects the latter is always true (some arguments for the estimation of the effect is given below). The value of the Rosseland opacity κ_R is defined as a factor connecting the flux of radiation F (an integral with respect to all frequencies of the spectral flux of radiation) and the temperature gradient:

$$F = \int F_\omega d\omega = -\frac{4\pi c}{3}\frac{1}{\rho\kappa_R}\frac{dB^T}{dr} \tag{6}$$

where B^T is the energy density of radiation of a Planck blackbody of temperature T, and ρ is the mass density of the matter. Naturally the right hand side of equation (6) can be written in terms of the temperature gradient:

$$\frac{dB^T}{dr} = \int \frac{\partial B^T_\omega}{\partial T}\frac{dT}{dr}d\omega \tag{7}$$

where B^T_ω is the spectral density of Planck distribution.

Let us illustrate the existence of the possibility to use the opacity κ_R by using the example of radiation scattering on electrons. Let the scattering cross-section $\sigma(\omega, \omega', x')$ be a function of the frequencies of the scattering and scattered waves respectively (ω, ω') as well as being the function of the angel of scattering x'. The transport equation which takes into account the direct and inverse scattering processes but ignores the processes of stimulated scattering can in its simplest form be written for the photon occupation

number $N(\omega, x)$ as (x-is the cosine of the angle of photon with frequency ω with respect to the direction of the temperature gradient; x''-the same for the photon with frequency ω'),

$$x\frac{\partial N(\omega,x)}{\partial r} = -N(\omega,x)n_e \int \sigma(\omega,\omega',x')d\omega'dx' + \int N(\omega',x'')n_e\sigma(\omega,\omega',x')d\omega'dx' \quad (8)$$

The deviation of the photon distribution from the equilibrium Planck distribution is assumed to be small,

$$N(\omega,x) = N_\omega^T + x\delta N_\omega \quad (9)$$

where N_ω^T corresponds to the Planck distribution (by the well known formulas it is the related to B_ω^T), and δN_ω is related to the spectral density of radiation introduced above (the factor 4π in (6) corresponds to the total solid angle, and the coefficient 1/3 corresponds to the average value of the square of the cosine of the angel. The left hand side of equation (8) is sufficient to take into account the dependence of the Planck distribution on r , while the right hand side of this equation will contain only the deviations from the Planck distribution related to the radiation flux. Due to axial symmetry of (8) we have $x'' = xx'$ and the equation which describes the transport of radiation is an integral equation of the form:

$$\frac{\partial B_\omega^T}{\partial T}\frac{dT}{dr} = -\frac{3n_e}{4\pi c}\int \sigma(\omega,\omega',x')\left(F_\omega - x'F_{\omega'}\frac{\omega^3}{(\omega')^3}\right)d\omega'dx' \quad (10)$$

This equation (10) allows us to introduce and derive such an integral characteristic as the opacity κ_R only in the case where one assumes that a good approximation could be $\delta N_\omega = \delta N_{\omega'}$ and then:

$$\frac{1}{\rho\kappa_R} = \int_0^\infty \frac{1}{n_e\sigma_\omega^{tr}}\frac{\partial B_\omega^T}{\partial T}d\omega \Big/ \int_0^\infty \frac{\partial B_\omega^T}{\partial T}d\omega \quad (11)$$

where σ_ω^{tr} is the transport scattering cross-section

$$\sigma_\omega^{tr} = \int \sigma_{\omega,\omega',x'}(1-x')dx'd\omega' \quad (12)$$

This example was given not only to remind the reader of the definition of κ_R, but also to emphasize the conditions where the introduction of such a quantity is possible and useful. It is clear from a physics point of view that due to the Doppler effect the frequency of radiation is changed in each act of scattering and although such a value as κ_R can be introduced the expression for it can not in the general case be obtained from the equation of radiative transfer. To solve the general equation for radiative transfer and to find the

intensity of radiation as a function of frequency and r is rather difficult and yet no one has performed such calculations for the solar interior. The natural question then is whether such an integral characteristic of energy transfer as the opacity is a good approximation for describing the collective plasma effects in the solar interior. Unfortunately the answer to this is *no*.

3 PHYSICS OF COLLECTIVE EFFECTS IN SCATTERING AND BREMSSTRAHLUNG.

The physics of collective scattering and bremsstrahlung has already been given in many textbooks and monographs on plasma physics. Mainly it was presented for the case of electrostatic plasma oscillations and not so much for electromagnetic waves, i.e. for photons, although even in 1967 all necessary formulas for scattering of photons were given in [8] (see also [9,10]) and in what follows we will use these results. It is worthwhile to recall the physics of collective scattering, which seems to be, at a first glance, rather simple, but indeed is not at all trivial. This can be the only excuse for wrong statements appearing even at the present time in the astrophysical literature such as "the scattering is occurring only on electrons and the ions can influence the scattering only through the correlations in collective processes" (the statement that "the scattering of the radiation is *produced only by electrons* and the ions only indirectly influence the scattering due to correlations with electrons" is missleading). At the present time much understanding of the physics of scattering in plasma has been made and there is no doubt that in the extreme collective regime the electrons and ions interchange their role as compared to the case of isolated particles, i.e. the ions in the collective case are scattering almost as free electrons in a vacuum and the electrons are scattering very weakly and in most cases as ions in the vacuum. The main results were obtained in plasma physics for plasma oscillations for which the scattering is always collective. Most attention in plasma physics concentrated on stimulated scattering since it describes the nonlinear interaction of plasma waves. In the construction of SSM the formulas of non-collective scattering were used and only spontaneous scattering was taken into account neglecting stimulated scattering.

Let us repeat the main principles of collective scattering and let us make it clear why in the collective regime it is impossible to speak about the influence of ions on the scattering on electrons but it is correct to say that the scattering is produced by the ions themselves. The physical picture is at a first glance very simple. The charges in a plasma are screened at distances of the order of the Debye radius. In the case where the wavelength of the scattered wave becomes larger than the Debye radius the scattering becomes collective. Both electrons and ions have screening shells which consist of an excess of electrons and a suppression of ions in the vicinity of ions and an excess of ions and a suppression of

electrons in the vicinity of electrons.

For high frequency waves it is the electrons which oscillate, both the screening and screened electrons, in the wave field. The electrons screen themselves by producing a deficit of electrons (electron hole) around the screened electron which feels a positive charge. The scattering is equal and out of phase for the screened electron and screening electron with a net result of zero scattering. For the ions which do not oscillate in the high frequency field the screening electrons which have equal but opposite sign are responsible for producing the scattered radiation. For the wavelengths larger than the size of the screening cloud, the ions scatter like a point charge electron in vacuum (for the case of singly charged ions).

For such a physical interpretation it is necessary to take away some doubts which the reader may have. Let us concentrate on the statement which some may consider as unusual namely the presence of strong scattering on ions. The first point is that since in the case of ions only their electron shell is scattering, it may be more correct to speak about the scattering on electrons correlated with ions as some physicists would like to interpret this effect. It is easy to show that such a point of view is not correct. A correct statement is that it is the ions which are scattering the radiation and the electrons play an intermediate role for the transfer of the energy and momentum to the ions during the scattering process. This can be proved both mathematically and from a physical point of view. To check this statement mathematically one can use the same theory to calculate the changes of the ion distribution during the scattering process. One can then easily see that the energy and momentum lost in the scattering process by waves is transferred to ions only. This calculation is based on the same fluctuation theory as the calculation for the scattering of waves. It should be noted that for a large system of particles there exist no other more exact approach than the fluctuation theory and all scattering processes have been previously calculated using it. The equations for the change of the ion distribution are obtained by the same procedure of averaging on the fluctuations as in the simpler approach when the distribution of particles which scatter the radiation is assumed (to a first approximation) fixed. Thus there is no doubt from the point of view of the mathematical procedure used in scattering theory that the scattering in the collective regime is due to the ions.

There is no doubt also from a physical point of view about this statement. Let us recall the process of Cerenkov emission by a particle moving with a velocity greater than the light velocity in the medium . In this case there is no doubt that the polarization cloud of particles in the medium plays an important role in the formation of the radiation. But it is well known that the emitted momentum and energy of the wave is taken from the particle itself. The polarization cloud in the case of a plasma is the Debye shielding cloud. In plasma physics the Cerenkov emission of plasma waves is a very common phenomenon and by using the quasilinear theory it was proved that in this case the energy and momentum

of the sum of particles and waves is conserved (this statement has also been checked experimentally). The polarization cloud in both cases -scattering and Cerenkov emission, plays only the intermediate role in transferring the energy and momentum. This last statement was known in the early stages of the investigation of the Vavilov-Cerenkov emission [11].

The other point of doubt appears concerning the statement that the electrons and ions of the plasma are screened also by electrons and ions. The question is how can the plasma particles be at the same time the scattering centers and be able to shield the other scattering particles? To resolve this doubt one should have in mind that, by definition a plasma as a state of matter, the number of plasma particles in the Debye sphere should be large (and this condition is fulfilled in the center of the Sun). On the other hand to treat the scattering correctly the only approach in plasmas is the fluctuation approach. In the presence of fluctuations one should separate the average particle motion and the fluctuating part of their motion. For the average motion of the particle appears as the center of scattering and during the fluctuations they are able to screen the other particles. Since the number of particles in the Debye sphere is large there is no need for large fluctuations to produce the screening. The given picture is an adequate interpretation of the exact results of fluctuation theory. In all processes like particle collisions, scattering and bremsstrahlung the plasma looks more like a collections of "neutral atoms" than free particles. But the screening is a dynamical screening and as soon as the particles move fast enough (their velocity is larger than the mean thermal velocity) they become "undressed". One should also have in mind that the screening shell consists both of electrons and ions and by increasing their velocities the particles first "take off their ion shell". Such a plasma picture is an achievement of a long term development of plasma theory and the first steps toward it were made by Balescu [12] who proved that the binary particle collisions are the collisions of dynamically screened particles. The screening during the collisions is produced by all other plasma particles. A similar picture appears also for processes of bremsstrahlung, this last statement was proved only recently [13].

It seems to be obvious that such a situation should indeed appear for all electromagnetic processes in a plasma since one can use the approach of test particles. It is obvious that any external charge inserted in a plasma is screened . But an "external" charge can be any electron or ion from the plasma. The selfconsistensy of the plasma description does not allow one to distinguish an "external" electron from the plasma electron.

The picture of plasma as a collection of dynamically screened neutral "classical atoms" which seems to be more appropriate than the picture of a collection of free particles can be considered not as a very good analogy since in the atoms the screening is produced by the same bounded electrons while the screening shells of electrons and ions in a plasma are produced statistically by different electrons and ions of plasma. But it should be noted

that the time needed for example for an electron to cross the Debye sphere is very short (of the order of the inverse plasma frequency) and the screening shell for the processes considered (including the scattering) behaves quasistatically in the case where the wave length is much larger than the Debye length.

We now write the criterion for collective effects in scattering to be dominant for electromagnetic waves and show that this criterion is usually fulfilled for frequencies much larger then the plasma frequency. The wave length of electromagnetic waves for the case when their frequency is much larger than the plasma frequency is c/ω, while the size of the Debye screening shell is of the order of v_{Te}/ω_{pe}, where ω_{pe} is the electron plasma frequency. By comparing these expressions we obtain the criteria when the collective effects for photons are strong which is:

$$\omega_{pe} < \omega \ll \omega_{pe}\frac{c}{v_{Te}} \tag{13}$$

Since the factor c/v_{Te} is rather large for a non-relativistic plasma the range given by expression (13) appears to be rather broad. Outside this range one can expect the usual picture of scattering when the scattering on free electrons dominate, while inside the range the scattering is described by the picture given in this section where the ions dominate in scattering and collective plasma effects are dominant. Even in some recent astrophysical publications one can find statements that the criteria for collective effects to dominate is that the frequency of photons should be close to the plasma frequency. This is difficult to understand and such statements are obviously not correct.

Before starting to construct a SSM's one should answer a natural question is , do the photons taking part in radiative energy transfer in the solar interior have frequencies in the collective range or not?

4 PLASMA PARAMETERS IN THE CENTRAL REGION OF THE SUN

A SSM which takes into account all the collective effects (which we start to discuss in the next section) does not yet exist. The best thing we can do is to use the existing SSM to get the plasma parameters inside the Sun bearing in mind that future investigations should correct the SSM. We should also note that it is possible to change the solar opacity in certain limits (15% change of the solar opacity is probably the maximum allowed from solar seismology , but the later statement is somewhat questionable since at the present time solar seismology does not give direct information in the central regions of the Sun). On the other hand it was also demonstrated that a large change of opacity is not needed.

In any case we will take the plasma parameter data for the central solar region using

the existing SSM [3,14]. According to these models thermonuclear burning occurs only in the central part of the Sun up to distances from the center of $0.1R_{\odot}$. It is assumed that the radiation flux is formed at these distances and this flux independently of transformations in the upper turbulent regions is appearing as emission in the visible domain and determines the solar luminosity (the later conclusion is made from the conservation of flux, constancy of solar luminosity in time and domination of the optical radiation flux of the Sun as compared to other types of charged particle and electromagnetic emission from the Sun). This visible flux of radiation is what we measure at the Earth. The central part of the Sun is assumed not to be turbulent. Unfortunately solar seismology does not detect the central regions of the Sun and this statement or assumption is difficult to prove.

According to the present data the temperature in the central region of the Sun is $1.55keV$ (which is less than temperatures obtained at the present time in laboratory CTR experiments).This corresponds to the electron mean thermal velocity $v_{Te} = 1.53\times10^{9}cm/s$ and thus $c/v_{Te} \approx 20$, the plasma density is $142g/cm^3$, which (for the abundance of hydrogen H, equal to 0.36, and abundance of He, equal to 0.62) corresponds to an electron density $n_e \approx 5.74 \times 10^{25}cm^{-3}$, this corresponds to an electron plasma frequency of $\omega_{pe} = 4.27 \times 10^{17}s^{-1}$. For an estimation of the frequency below which the collective effects dominate we should multiple the last value by 20 to obtain $8.54 \times 10^{18}s^{-1}$. This frequency should be compared with the frequency corresponding to the maximum of the blackbody radiation $3T/\hbar \approx 6 \times 10^{18}s^{-1}$. We can also make the comparison with another value- the frequency of the maximum of the weight factor $\partial B^T_\omega/\partial T$ in the Rosseland opacity κ_R, which corresponds to $3.7T/\hbar$ and corresponds to the frequency $7.4 \times 10^{18}s^{-1}$. Both comparisons definitely show that the whole frequency range responsible for the radiative energy transfer in the solar interior corresponds to the range of frequencies in which collective effects dominate.

This is a very important conclusion which was not made in the early investigations of the Sun using SSM's (it was only taken into account by Boerker in 1987 [5])

Another conclusion for the estimate given above is that the ratio of the maximum frequency to the frequency when collective effects start to dominate is neither large nor small, which means that in a theoretical description we can not use a small parameter and the collective effects should be treated strictly without using the asymptotic expressions. This also means that the contribution of electrons and ions to scattering in the solar interior is of the same order of magnitude. For the following it will be useful to define the electron collective parameter δ_e , which characterizes the role of collective effects for scattering on electrons (later on we define also the ion collective parameter). The electron collective parameter is by order of magnitude equal to the square of the the ratio of the wavelength of the scattered radiation to the electron Debye shielding length. The definition is:

$$\delta_e = \frac{\omega_{pe}^2}{2\omega^2}\frac{c^2}{v_{Te}^2} \tag{14}$$

In the extreme collective regime $\delta_e \gg 1$, in the non-collective regime $\delta_e \ll 1$. For the center of the Sun this parameters correspond neither to the first inequality nor to the second inequality but corresponds to δ_e of the order of one.

We can also find another qualitative conclusion concerning the relation between the scattering and bremsstrahlung absorption (the process inverse to bremsstrahlung emission). Let us introduce an effective cross-section σ^{br} for inverse bremsstrahlung damping by using for the bremsstrahlung damping rate of the photon intensity, $2\gamma^{br}$ the following formula $2\gamma^{br} = n_e c\sigma^{br}$. Then one can easily show from the standard formula that for this absorption the ratio of bremsstrahlung cross-section to the scattering cross-section is of the order $\delta_e^{3/2}$. This estimate definitely shows that the contribution to the solar opacity of the scattering and bremsstrahlung are of the same order of magnitude.

The third contribution to the opacity which is of the same order of magnitude is given by line absorption. The relative abundance of all elements neglecting hydrogen and helium in the center of the Sun is only 2% and all atoms except iron atoms are completely ionized. But iron ions have a line exactly in the range of frequencies important for energy transport. Although the relative abundance of iron ions is small they have a large charge and the presence of the resonance line makes their contribution to the opacity almost of the same order of magnitude as scattering and bremsstrahlung. Therefore, for example, a change in scattering of 10% can change the opacity by only 3%.

In evaluation of the Rosseland opacity κ_R in different SSM's all three components were taken into account and the total cross-section (11) is equal to the sum

$$\sigma^{tot} = \sigma^{sc} + \sigma^{br} + \sigma^{L} \tag{15}$$

The notation used for the three contributions are obvious. The bremsstrahlung cross-section is, as is known, proportional to the effective ion charge Z_{eff} given by:

$$Z_{eff} = \frac{\sum_i n_i Z_i^2}{\sum_i n_i Z_i} \tag{16}$$

where n_i is the relative concentration of ions of type i, and Z_i is their charge. Thus Z_{eff} is another parameter which we should take from existing SSM. This parameter does not differ much in different SSM and is close to the value 1.5. The ratio of the total κ_R to the value which takes into account only scattering and bremsstrahlung varies from one model to another but for each model this ratio is known. Therefore it is useful to relate the corrections to the Rosseland opacity to its value which takes into account only scattering and bremsstrahlung. The coefficient for transferring this value to the total

opacity is known for each SSM but on average it can be taken as a rough estimate to be equal to 2/3.

For calculations of the corrections to the Rosseland opacity $\kappa_R^{(0)}$ one can use formula (11) with the total cross-section, assuming that both the cross-sections and the Planck distribution depend on the value

$$z = \frac{\hbar\omega}{T} \tag{17}$$

then

$$\frac{\kappa_R - \kappa_R^{(0)}}{\kappa_R^{(0)}} = \left(\int_0^\infty \frac{z^4 e^z dz}{\sigma_0(z)(e^z-1)^2} - \int_0^\infty \frac{z^4 e^z dz}{\sigma(z)(e^z-1)^2}\right) \Big/ \int_0^\infty \frac{z^4 e^z dz}{\sigma(z)(e^z-1)^2} \tag{18}$$

The integral equation for radiative transfer can not be solved directly for all plasma collective corrections and the opacity can not be calculated directly from the transport equation. In this case it is necessary to find the solution of the transport equation as a function of frequency and r and the direct use of (16) is not possible. This difficulty can be overcome by perturbation theory in the case where the latter can be used in the transport equation and the explicit expressions for the change in the opacity can be obtained. The conditions where the perturbation approach can be used will be discussed below.

5 ZERO APPROXIMATION FOR OPACITY INCLUDING BREMSSTRAHLUNG ABSORPTION AND COLLECTIVE SCATTERING

For $\delta_e \ll 1$ the collective effects are small and the cross-sections for scattering of photons on electrons and ions are well known and are given by

$$\sigma^e \approx \sigma_T = \frac{8\pi e^4}{3m_e^2 c^4}; \tag{19}$$

$$\sigma^i \approx 0$$

where σ_T is the Thomson cross-section. These expressions are written for nonrelativistic particles (i.e. when the relativistic corrections are small), in the classical limit (i.e. where the quantum corrections are also small) to a first approximation in the parameter m_e/m_i:

$$\tau \equiv \frac{v_{Te}^2}{c^2} = \frac{T}{m_e c^2} \ll 1; \frac{\hbar\omega}{m_e c^2} = z\frac{v_{Te}^2}{c^2} \ll 1 \tag{20}$$

Due to the fact that the effective value of z in the Rosseland opacity is of the order of 3.7 the quantum corrections to the equations (19) are somewhat larger than the classical relativistic corrections. One can write the expressions for the cross-sections taking the next order in the parameters (20) and write the first relation (19) in a form which differs from (19) by a factor $G(z,\tau)$, the explicit expression for which we will not write here (in the collective regime it is not important) but noting that it can be obtained by expansion of the general Klein-Nishina formula [15] with subsequent averaging on the thermal distribution. Such a factor was used in calculations of opacity by taking into account the terms up to second order in the parameter $z\tau$. Below we find the correct expressions for such a factor in the collective regime but only up to first order in expansion in the parameters τ and $z\tau$ (even the collective corrections of this order of magnitude is a very cumbersome calculation which have not been performed before). In the collective case a new parameter of expansion occurs $z^2\tau$ and in this parameter also the first order term in expansion in this parameter will be taken into account (all parameters are small in the solar interior, taking into account that effective values of z in the solar opacity are not very large). When considering linear in τ corrections the powers of z higher than 2 do not appear. For the problems of interest the previous use of the factor $G(z,\tau)$ is not correct since it is written for the non-collective regime and it can not be used in SSM's as was previously done in [6,16]. The real corrections, linear in τ, $z\tau$ and $z^2\tau$ in the collective case have nothing in common with that for the non-collective case and the use of the factor $G(z,\tau)$ for the Solar opacity calculations is incorrect.

At this moment we can give the definition of what will be meant by collective scattering in the zero approximation: it is the scattering where the collective effects are taken into account in zero approximation in the parameters (19) and the new parameter $z^2\tau$. In this form the collective effects have already been taken into account in SSM [5,6]. In these papers an expression for the sum of the cross-sections of scattering on electrons and ions was used. We will give these expressions but then consider separately the cross-sections for scattering on electrons and for scattering on ions respectively to show a rapid decrease of scattering on electrons with an increase in the collective parameter and the growth of scattering on ions with an increase of this parameter and then we will show how the cross-sections change, if, for example, the temperatures of electrons and ions are not equal. The zero approximation for the scattering cross-sections can be obtained from the formulas given as early as 1967, in the book of one of the authors [8]. On averaging over the thermal distributions one can use the so called fluctuation - dissipation theorem (see[17,18]) and find an analytical expression for the cross-sections used in [5,6] for any value of the collective parameter δ_e :

$$\sigma_0^{sc} \equiv \sigma_e^{sc(0)} + \sigma_i^{sc(0)} = \sigma_T \left\{ 1 - \frac{3}{8}\delta_e \left[\delta_i(2 + 2\delta_i + \delta_i^2) \ln \frac{\delta_i}{2 + \delta_i} + 2\delta_i + 2\delta_i^2 + \frac{8}{3} \right] \right\} \quad (21)$$

where

$$\delta_i = (1 + Z_{eff})\delta_e \quad (22)$$

is the collective ion parameter and the effective ion charge is given by expression (16). We labelled the corresponding cross-sections with a subscript zero to emphasize that these expressions are given in the zero approximation. In the value of the Rosseland opacity given above for which the corrections will be calculated we also made the label zero. This is performed intentionally since below all the corrections will be counted from that zero approximation. To be complete we should also define the zero approximation for bremsstrahlung (see below). By relating all corrections to this zero approximation allows us to consider only the new collective effects not taken into account previously and to correct the other expressions. We will give below the expression for $\kappa_R^{(0)}$, the corrections will be calculated relative to this value of the Rosseland opacity.

However here we start with a more detailed consideration of the scattering in the zero approximation. We discuss also the corrections in the zero approximation in τ, $z\tau$ and $z^2\tau$ which are not taken into account in expression (22) used earlier in the literature. Some of them are indeed small but we want to be precise in the analysis of all corrections in the same parameter τ. First of all expression (22) is valid only if one neglects for photons the difference of the refractive index from 1 (the value in vacuum), i.e. it is valid for $\omega \gg \omega_{pe}$. Since the frequency range in the transport of radiation in the solar interior consists of only one decade in frequency from ω_{pe} up to $10\omega_{pe}$ it is worthwhile to find these corrections. We will see that literally these corrections contain the same relativistic factor $\tau = v_{Te}^2/c^2$ as other relativistic corrections do.

Secondly it is rather easy to look at the process of scattering separately for electrons and ions in the case where their temperatures T_e and T_e are not equal. It is not quite certain that this case is of interest for the solar interior since in the dense plasma characteristic time of equalizing of the electron and ion temperatures is very short. An estimation shows that this time is still 5 times larger than that of heating of electrons by absorption of the radiation transported. Although the question about the possibility of the existence of the differences between the electron and ion temperatures needs a special investigation it is worthwhile to give general expressions for scattering on electrons and ions in zero approximation in the parameters τ, $z\tau$ and $z^2\tau$ not assuming their temperatures are equal. Third contrary to previously used formula (22) we write separately the expressions for the transport scattering cross-sections on electrons and on ions to show explicitly that the cross-section of scattering on electrons decreases rapidly with an increase of the collective

parameter and the cross-section of scattering on ions increases with an increase of the collective parameter. We write down only the transport cross-sections of scattering which enter in the transport equation.

For the above conditions (arbitrary ratio ω/ω_{pe} but $\omega > \omega_{pe}$ and arbitrary ratio T_e/T_i) the collective parameters are determined by the relations

$$\delta_i = (1 + Z_{eff}\frac{T_e}{T_i})\delta_e \tag{23}$$

and the electron collective parameter should be changed with respect to definition (14) to take into account the effect of the refractive index in:

$$\delta_e = \frac{c^2}{2v_{Te}^2}\frac{\omega_{pe}^2}{\omega^2 - \omega_{pe}^2} \tag{24}$$

and the scattering cross-sections are described by the expressions

$$\sigma_{sc}^e = \sigma_T\sqrt{1 - \frac{z_0^2}{z^2}}\{1 - \delta_e + \\ +\frac{3}{8}\delta_e^2\left[(2 + 2\delta_e + \delta_e^2)\ln\frac{2 + \delta_e}{\delta_e} - 2 - 2\delta_e\right]\Big\} \tag{25}$$

$$\sigma_{sc}^i = \sigma_T\sqrt{1 - \frac{z_0^2}{z^2}}\frac{3}{8}\frac{T_i}{T_e}\times \\ \times\Big\{\Big[\delta_e^2\Big[-(2 + 2\delta_e + \delta_e^2)\ln\frac{2 + \delta_e}{\delta_e} + 2 + 2\delta_e\Big] + \\ +\delta_i\delta_e\Big[(2 + 2\delta_i + \delta_i^2)\ln\frac{2 + \delta_i}{\delta_i} - 2 - 2\delta_i\Big]\Big\} \tag{26}$$

where $z_0 = \hbar\omega_{pe}/T$. The factor $\sqrt{1 - z_0^2/z^2}$ is equal to $\sqrt{1 - \omega_{pe}^2/\omega^2}$ and shows that the cross-sections of scattering tends to zero when the frequency becomes close to the plasma frequency. For the solar interior $z_0 \approx 0.23$. Shown in Fig.1 are the dependencies of the transport cross-section for scattering on electrons and ions separately and the total transport cross-section as a function of the frequency for $T_e = T_i$ but with exact values of the refractive index (not equal to 1) (curves $1, 2, 3$ respectively). One observes a strong decrease of the cross-section for scattering on electrons and an increase of the cross-section for scattering on ions with decrease of the frequency. The curves were calculated for the parameters in the solar interior. An additional decrease of the cross-section close to the electron plasma frequency is related to the refractive index effect. Curve 4 is calculated without taking into account the refractive index effect and corresponds to the cross-section

which will be taken into account in $\kappa_R^{(0)}$. Finally curve 5 describes the weighting factor $z^4 e^z/(e^z-1)^2$, which enters in κ_R.

From these curves it is clear that the most important frequency range corresponds to the collective range. The maximum frequency in the weighting factor is $\omega/\omega_{pe} \approx 18$, but even for $\omega/\omega_{pe} \approx 30$ a decrease of the total scoss-section is $\approx 18\%$, and for $\omega/\omega_{pe} \approx 2$ it is as much as 40%.

Shown in Fig.2 is the dependence of the transport cross-section and the usual cross-section for scattering on electrons and ions as well as the total cross-section of scattering as a function of the collective parameter δ_e. These curves show the role of collective effects in a clear manner as well as showing that the transport cross-sections do not differ substantially from the usual one. Thus the figures given before showing the change of the transport cross-sections of scattering at the center of the Sun also give a good example of the dependencies of the usual cross-sections (which differs from the transport cross-sections not having the factor $1-x$ in the angular integration). Fig.1 also shows that the influence of the effect of refractive index could not be large since the refractive index effects are the largest in the frequency range where the weighting factor is small. Shown in Fig.2 is the dependence of the total transport cross-section on the parameter δ_e for different values of T_e/T_i from which it follows that the increase of the ratio T_e/T_i can decrease the total cross-section by up to 80%.

Let us consider bremsstrahlung absorption and define the zero approximation for it. Collective effects in bremsstrahlung were previously neglected in SSM's, therefore we will not take into account the collective effect in zero approximation in the processes of bremssrahlung absorption. The effect we discuss in exact terms arises as a balance between stimulated emission and stimulated absorption. In the low frequency limit, $\hbar\omega/T = z \ll 1$, it corresponds to classical absorption of electromagnetic radiation in a plasma due to binary electron ion collisions. The classical limit is not quite appropriate for the energy transport problems, since the most important value of z is of the order or larger than 1. Since after an emission of a bremsstrahlung wave the particle energy decreases by $\hbar\omega$, the term describing stimulated absorption by thermal particles will contain the additional factor $exp(-z)$ as compared to the term describing the stimulated bremsstrahlung emission. This leads for z of the order of 1 to a factor $(1-exp(-z))/z$ in the expression for the wave damping (this factor is 1 in the classical limit) and thus one finds the general expression for damping for arbitrary z values. We write the latter as $2\gamma^{br}(\omega) = n_e c\sigma^{br}(\omega)$ which serves as a definition of the already introduced effective cross-section of bremsstrahlung absorption $\sigma^{br}(\omega)$. It is useful to express this cross-section through the collective parameter (14) and through the Thomson cross-section of scattering σ_T such that

$$\sigma_0^{br} = \sigma_T Z_{eff} \frac{2\delta_e^{3/2}(e^z - 1)}{\sqrt{\pi} z_0} \mathcal{F}_0(\omega); z_0 = \frac{\hbar\omega_{pe}}{T} \tag{27}$$

where

$$\mathcal{F}_0(\omega) = \int_{\sqrt{2\hbar\omega/m_e}}^{\infty} \frac{exp(-\frac{v^2}{2v_{Te}^2})v}{v_{Te}^2} \ln \frac{v + \sqrt{v^2 - \frac{2\hbar\omega}{m_e}}}{v - \sqrt{v^2 - \frac{2\hbar\omega}{m_e}}} dv =$$

$$= 2\int_{\sqrt{z}/2}^{\infty} \frac{dx}{x} e^{-(x+\frac{z}{4x})^2} \tag{28}$$

Expressions (21) and (28) for the zero approximations for cross-sections of scattering and bremsstrahlung will be the starting point for all the corrections we will consider further and only they will be included in the zero approximation for the Rosseland opacity $\kappa_R^{(0)}$. The corrections to this value of the opacity will be calculated by using formula (18). Line absorption in the zero approximation will not be taken into account to find the universal result for the corrections (the coefficient for recalculating the ratio calculated in this manner to the ratio to the total opacity which includes line absorption is different for different SSM's and such a recalculation can be easily performed using a particular SSM). Thus $\kappa_R^{(0)}$ will be defined by $\sigma_0^{br} + \sigma_0^{sc}$. To take into account in the zero approximation both effects -scattering and bremsstrahlung- is very important. For example in the case we take into account only bremsstrahlung then the cross-section will rapidly decrease with increasing z (proportional to $1/z^2$ or proportional to $1/z^3$) and then in the numerator of $\kappa_R^{(0)}$ a large factor z^6 (or a factor z^7) will appear and the effective value of z will be rather large about $6 - 7$. If on the other hand we take into account only the scattering the maximum effective z value will be quite different. The values of κ_R are not additive from the contribution of scattering and from the contribution of bremsstrahlung since the sum of the cross-sections enters in the denominator of κ_R. It appears that for the solar interior both cross-sections are equal at z values corresponding to the exponential decrease of the weighting factor and therefore the results can be sensitive to small changes of the cross-sections.

6 EFFECTS OF REFRACTIVE INDEX IN SCATTERING AND BREMSSTRAHLUNG

We start with a consideration of the effect which is rather small but to take it into account will be necessary since firstly we have the intention to search for any collective effect which contains the small parameter described by the relativistic factor v_{Te}^2/c^2, secondly it will

allow us to describe and use further a simplified expression for the corrections of the Rosseland opacity, and thirdly after giving this result of the estimation of the role of refractive index we will be allowed in further considerations (for other collective effects) to neglect the refractive index effects.

We have already written the general expressions for scattering which takes into account the refractive index effect. In calculations of the refractive index corrections in the opacity, we should also take into account an additional factor $\sqrt{1-\omega_{pe}^2/\omega^2}$ in the energy density of blackbody radiation which appears by changing the integration with respect to wave number to the integration with respect to frequency, we should as well take into account the same factor due to the difference of the group velocity of photons from 1 in the expression for κ_R thus the factor $1-\omega_{pe}^2/\omega^2$ will appear in the numerator. But the denominator in the cross-sections for scattering and bremsstrahlung will contain $\sqrt{1-\omega_{pe}^2/\omega^2}$, by dividing on the factor in the denominator we find that the first power of this factor in the denominator appears in the opacity. The change in the expression for the collective parameter and the change in the lower limit of integration on frequency are also essential. The change in the total value of κ_R due to all these changes is denoted as $\delta\kappa_R^{refr}$ and numerical calculations give:

$$\frac{\delta\kappa_R^{refr}}{\kappa_R^{(0)}} = 0.135\% \tag{29}$$

To illustrate the role of the low frequency part of the range of integration (for which the refraction index corrections are large) as compared to the high frequency range of integration in the expression of the opacity we can assume due to result (29) that the main part of the contribution is given by the high frequency range and thus expand the total cross-section κ_R with refractive index effect taken into account in the parameter $\omega_{pe}^2/\omega^2 = 2\delta_e v_{Te}^2/c^2$. Since in the center of the Sun δ_e is of the order of 1 we conclude that the refractive index corrections have the same factor v_{Te}^2/c^2 as the other relativistic corrections and in principle collecting all the relativistic corrections we can not neglect this one. We have the following expression for the corrections to the refraction index entering in the expression for the opacity:

$$\sigma(z) = \sigma_0(z) + 2\frac{v_{Te}^2}{c^2}H_{refr} \tag{30}$$

where

$$H_{refr} = \delta_e(\sigma_0^{br} + \delta\sigma^{sc}) \tag{31}$$

$$\delta\sigma^{sc} = \frac{1}{2}\sigma_T\left\{1 - \frac{3}{8}\delta_e\left[\delta_i(10+14\delta_i+9\delta_i^2)\ln\frac{\delta_i}{2+\delta_i} + 10\delta_i + 14\delta_i^2 + 8\right]\right.$$

$$-\frac{3\delta_e\delta_i}{2(2+\delta_i)}(2+2\delta_i+\delta_i^2)\Big\} \tag{32}$$

We will give here also a simplified expression for the corrections to the opacity for the case where the corrections in the cross-sections are small and we can also expand on these corrections in the opacity (this formula will be used also below in the case where we believe and finally find that the corrections are indeed small)

$$\sigma(z) = \sigma_0(z) + \delta\sigma(z); \delta\sigma(z) \ll \sigma_0(z) \tag{33}$$

and for corrections to κ_R:

$$\frac{\kappa_R - \kappa_R^{(0)}}{\kappa_R^{(0)}} = \int_0^\infty \frac{\delta\sigma(z)}{\sigma_0(z)^2}\frac{z^4e^z dz}{(e^z-1)^2} \Big/ \int_0^\infty \frac{z^4e^z dz}{\sigma_0(z)(e^z-1)^2} \tag{34}$$

The change in κ_R for the case where the formulas (32) and (34) are used appears to be:

$$\frac{\delta\kappa_R^{refr}}{\kappa_R^{(0)}} = 0.134\% \tag{35}$$

This result coincides closely with the exact result (29) which shows that the main contribution in the opacity is due to the high frequency range although the whole range of frequencies is not large (but ω_{pe}^2/ω^2 changes in this range by two orders of magnitude). Further in consideration of other collective corrections we will neglect the refractive index effects.

Formula (34) will also be used below in the case where the corrections are small .

since the refractive index corrections have the factor v_{Te}^2/c^2 they should also be considered together with other relativistic correction.

Some of the relativistic corrections to scattering were already treated in [6,16] in the form of an additional factor in the form $\sigma^{sc} = \sigma_0^{sc}G(z,\tau)$, where σ_0^{sc} corresponds to the zero approximation used above and the factor $G(z,\tau)$ was taken from the expression for scattering on free electrons. In the collective range of frequencies such an approach is not correct and any use of such expressions in the calculation of the opacity can not be accepted for the solar interior. The real corrections as we show are much larger. Not only the relativistic corrections for scattering were treated previously incorrectly but the relativistic corrections and collective effects to bremsstrahlug were ignored as well as relativistic collective effects in the transport equation for radiation. It appears that the collective effects in bremsstrahlung have the same smallness as the other relativistic corrections.

7 COLLECTIVE EFFECTS IN BREMSSTRAHLUNG ABSORPTION.

We start with the collective effects in bremsstrahlung since this effect, being also small, was previously neglected in the calculation of the opacity. In the existing literature the corrections due to Debye screening were considered [19,41] with the conclusion that there contribution is less than 2%. We will show that the correct expressions for collective effects in bremsstrahlung have nothing in common with Debye screening effects of ions during the process of bremsstrahlung and that they are indeed much smaller than that estimated in [19]. We will also show that expanding the opacity on these corrections we again obtain (for δ_e of the order of 1) a factor v_{Te}^2/c^2 in front of these corrections and thus these corrections should be included in the list of relativistic corrections to the opacity.

A correct calculation of collective effects in bremsstrahlung in the opacity appears to be not a simple problem. It was not considered previously in plasma physics in the form, which can be used in opacity calculations, this consideration has recently been treated in [20]. For a long time bremsstrahlung in plasmas was calculated by using the theory of fluctuations and it was believed that the collective effects results in the Debye screening of the field of the ion in the process of emission of a photon in electron-ion collisions. Such expressions were given many years ago and one can find them in the textbooks and monographs [21,22]. The fact that those expressions are not correct both from a physical point of view and as mathematical expressions is explained in detail in the recent monographs [10,23] and in the review [13].

In a few words the physics of emission is the following: apart from the emission due to electron acceleration in the process of electron- ion collision a new type of emission appears due to the dipole moment produced by the displacement of the screening electron cloud; both effects interfere with each other and the resulting emission is not equal to the sum of the emission in the two processes. It was shown that one should add to the matrix element of the usual bremsstrahlung due to electron acceleration in the field of Debye screened ion $M^{br,0}$ the matrix element $M^{br,coll}$, describing the displacement of the screening shell of the ion. The total cross-section of bremsstrahlung is determined by the square of the absolute value of the sum of these two matrix elements:

$$\sigma^{br} \propto |M^{br,0} + M^{br,coll}|^2 \tag{36}$$

The additional matrix element (one can say the "real collective matrix element") $M^{br,coll}$ is of the same order of magnitude as $M^{br,0}$ and moreover some terms in both of them are the same and opposite in sign which means that they partially compensate each other. This compensation is similar to that which leads to a decrease of the cross-section of scattering on electrons in the collective regime. This compensation in bremsstrahlung

appears to be most important for low values of transferred momenta (from electron to ion) in the process of bremsstrahlung. For example for fast electrons (on the tail of Maxwellian distribution) such a compensation leads to cancellation of that part of the usual matrix element which corresponds to the difference between the field of the screened ion and the field of the unscreened ion with the matrix element, describing the oscillation of the polarization shell. Thus for fast electrons the bremsstrahlung appears as if the ion is not screened at all ("stripping" shell effect described in detail in [13,23]). This effect has a simple physical interpretation; the projectile electron collides both with the ions and its shielding electrons and for the fast electrons the shielding electrons can be considered as free electrons and it is known that the bremsstrahlung for particles with an equal charge to mass ratio is in the first approximation zero. Obviously there are not so many fast electrons in the thermal electron distribution in a plasma but the effect is pronounced even if the electron velocity is of the order of the thermal electron velocity. We will show that the bremsstrahlung cross-section can change in certain domains of frequency by collective effects by as much as 39%.

The result of the correct treatment of collective effects in bremsstrahlung is that the expression for the bremsstrahlung contains in the denominator not the square of the static dielectric constant (as it is written in many textbooks) but the dielectric constant the frequency of which is determined by the velocity of the projectile electron (see.[13,23]). This leads to the result that for velocities larger than the thermal velocity the screening totally disappears. On the other hand due to the fact that the electron shell of the ion has the charge equal in value and opposite to the charge of the ion in the correct expression for the collective effect in bremsstrahlung an additional factor appears which depends on the effective ion charge.

The final result can be written in the form by changing the factor $\mathcal{F}(\omega)$ in the expression for the bremsstrahlung absorption see (28) (where this factor is given without the corrections described as ion field screening -which is not a correct expression- and without the collective corrections in a correct form). We denote as $\mathcal{F}_{scr}(\omega)$ the expression for this factor which corresponds to pure Debye screening of the ion field and we denote by $\mathcal{F}_{coll}(\omega)$ the expression for this factor for using the correct treatment of collective effects in bremsstrahlung (taking into account the interference of the two processes of bremsstrahlung, i.e. taking into account the effect of ion "stripping"). It is useful to express these factors through the integrals over the total normalized electron velocity $y = v/\sqrt{2}v_{Te}$ (note that below for the process of scattering we will used the notation y for another value- the normalized component of the electron velocity in the difference of the wave vectors of scattering and scattered waves). The integration in the expression for bremsstrahlung will also be performed over the total transferred momentum q (naturally in units of $\hbar$)

$$\mathcal{F}(\omega) = \int_0^\infty e^{-y^2} 2y dy \int_{q_{min}}^{q_{max}} \frac{dq}{q} \mathcal{H}(\omega, q) \tag{37}$$

and $\mathcal{F}_{scr}(\omega)$ will contain $\mathcal{H}_{scr}(\omega, q)$, while $\mathcal{F}_{coll}(\omega)$ will contain $\mathcal{H}_{coll}(\omega, q)$. The result is then given by the expressions

$$\mathcal{H}_{scr}(\omega, q) = \frac{1}{\left|1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2}\right|^2} \tag{38}$$

and

$$\mathcal{H}_{coll}(\omega, q) = \frac{1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2}}{\left[1 + (1 + Z_{eff}) \frac{\omega_{pe}^2}{q^2 v_{Te}^2}\right] \left|1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2} W\left(\frac{\omega}{\sqrt{2} q v_{Te}}\right)\right|^2} \tag{39}$$

where $W(x)$ is the well known plasma physics function describing the dispersion of plasma oscillations:

$$W(x) = 1 - 2xe^{-x^2} \int_0^x e^{t^2} dt + i\sqrt{\pi} x e^{-x^2} \tag{40}$$

For large values of its argument $W(x)$ is small and $\mathcal{H}$ is still not equal to one ($\mathcal{H} = 1$ in the absence of both screening and the correct collective effects). The presence of Z_{eff} in $\mathcal{H}_{coll}(\omega, q)$ reflects the role of ions in collective effects in bremsstrahlung. In the case of Debye screening the effect does not depend on ions,does not contain Z_{eff} and does not contain the plasma dispersion function $W(x)$ in the denominator. Thus the Debye screening effect is quite different from (39).

Before describing the possible influence of collective effects in bremsstrahlung on the solar opacity it is necessary to say a few words why the correct expression for the collective effects in bremsstrahlung was not obtained earlier. In fact the use of the full fluctuation approach gives a correct result but only previously the calculation was performed in a full manner [13]. The complete calculation needs a nonlinear approach in a nonstationary and inhomogeneous media. One should take into account that in the absence of the wave (the damping of which is investigated) there exist sharp variations of the particle distribution functions both in time and in space. They are due, as usuall, to the discreteness of the system and are described by standard methods of fluctuations in a plasma for a nonthermal particle distribution. In other words in the initial plasma state not perturbed by the wave there exist rapid and short length variations of refractive index. This leads to the processes of emission and absorption which influence wave propagation. The fluctuations of the particle distribution function leads also to the presence of the fluctuating fields. The field of the propagating wave disturbs these fluctuations and to find the damping

of the wave due to these disturbances one needs to go to a nonlinear theory of the field interactions in a sharply nonstationary and sharply inhomogeneous plasmas since the effects linear in the propagating field will be cubic in the total field which is the sum of the field of the propagating wave and the fluctuating fields. Thus the nonlinear response of the plasma being highly nonstationary and inhomogeneous should be used and all together the problem is not a simple one. All the above discussed problems were only recently treated [13].

It appears that independently a new effect called transition bremsstrahlung as an emission from the screening clouds of two colliding particles was investigated in [24,25,10] and a simple recipe was given to calculate the additional matrix element $M^{br,tr}$ which should be added to the usual matrix element for bremsstrahlung in the way we have already performed for the collective matrix element in expression (36). The progress obtained in the latest research in this field is that it was shown that the expressions $M^{br,tr}$ and the $M^{br,coll}$ found from the fluctuation theory are identical. Therefore a possibility appears to use a simple recipe found for transition bremsstrahlung to calculate $M^{br,coll}$ without dealing with cumbersome general expressions from the fluctuation approach which in turn allows us to find an expression for collective effects in bremsstrahlung in a rather general and compact form [13,20]. The previous more simplified approach uses also the fluctuation theory [21,22] but as can be shown neglects the effect of the same order of magnitude which appear from a nonlinear treatment of a highly nonstationary and inhomogeneous initial state.

The nonlinear approach uses the nonlinear response coefficients and their approximate expressions which allows us to show the presence of the discussed compensation in a rather general form.

Thus essential progress has been achieved recently both in the understanding of the physics of collective effects in bremsstrahlung and its analytical description and these results should be used in applications to the solar opacity calculations.

Collective effects in bremsstrahlung are large for frequencies which do not differ significantly from the plasma frequency. For frequencies of this order of that, corresponding to the maximum of the weighting factor in the expression for the Rosseland opacity both the screening approximation and the exact expression for collective effects in bremsstrahlung give small corrections. For frequencies close to the plasma frequency in the central part of the Sun for the case where all collective effects are taken into account the change in bremstrahlung can be as large as 29%, while for the case of the screening approximation it is 24%.

Let us now calculate the role of collective effects in bremsstrahlung for the solar opacity. As in the case of refractive index corrections one can expect that the most important frequencies are those much larger than the plasma frequency. Therefore we expand the result on the parameter ω_{pe}^2/ω^2

$$\mathcal{F}^{coll}(\omega) = \mathcal{F}_0(\omega) - 4\frac{v_{Te}^2}{c^2}\delta_e \int_{\sqrt{z}/2}^{\infty} x dx e^{-(x+\frac{z}{4x})^2}(Z_{eff} + 2\mathrm{Re}W(x)) \tag{41}$$

$$\mathcal{F}^{scr}(\omega) = \mathcal{F}_0(\omega) - 8\frac{v_{Te}^2}{c^2}\delta_e \int_{\sqrt{z}/2}^{\infty} x dx e^{-(x+\frac{z}{4x})^2} \tag{42}$$

For calculation of the contribution of the collective effects in the Rosseland opacity we can use formula (34) to find

$$\frac{\delta\kappa_R^{br,coll}}{\kappa_R^{(0)}} = -0.22\% \tag{43}$$

$$\frac{\delta\kappa_R^{br,scr}}{\kappa_R^{(0)}} = -0.28\% \tag{44}$$

We should mention that the effect is small because the main contribution to the opacity κ_R is given by the high frequencies where the collective effects are small. Again it is necessary to mention that the collective effects in bremsstrahlung are also of the order of relativistic effects since for δ_e of the order of one these are given as the Thomson cross-section times a factor v_{Te}^2/c^2. Therefore we are speaking only about the smallness of the numerical coefficient in front of this expression . We collect all the effects described by the cross-sections of such a structure including those where the numerical coefficient in front of those expressions is not small. But we will not exclude any such expressions and will do a complete search for them. Below we discuss other effects which have a much larger numerical coefficient although they are of the same smallness.

8 BREMSSTRAHLUNG ABSORPTION WITH RELATIVISTIC EFFECTS.

We discuss here the role of non-collective relativistic corrections in bremsstrahlung described by the well known classical Bethe-Heitler formula [27]. In the solar opacity the effective photon energies making the major contribution are rather high since $\hbar\omega \approx 3.7T$ and therefore the energy conservation law in the process of bremsstrahlung allows only certain particles with an energy exceeding some threshould energy to take part in the absorption process. The threshold is found from the condition that the kinetic energy of the electron after emission is positive. Denoting by v the electron velocity before emission and denoting by v' the electron velocity after the emission we can write the threshold condition as $(v')^2 > 0$. It is importamt that the relativistic corrections will lower the

threshold. Indeed by taking into account the first order relativistic corrections in the energy conservation law we get

$$(v')^2 = (v_0')^2 + \frac{3}{4c^2}(v^4 - (v_0')^4); (v_0')^2 = v^2 - \frac{2\hbar\omega}{m_e} \tag{45}$$

where v_0' is the final particle velocity in which the relativistic corrections are not taken into account. According to [27] the relativistic correction on the other hand lowers the intensity of emission of a single particle. By taking into account only the first order relativistic corrections to the intensity of bremsstrahlung I_ω(intensity emitted per second in frequency interval $d\omega$) we find from [27]:

$$I_\omega = \frac{16Z^2e^6n_i}{3m_e^2vc^3}\left\{\ln\frac{v+v'}{v-v'} - \frac{3vv'}{2c^2} - \frac{(v^2-(v')^2)}{2c^2}\ln\frac{v+v'}{v-v'}\right\} \tag{46}$$

In this equation v' is determined by equation (45) with the relativistic corrections taken into account, and thus the main term $\ln((v+v')/(v-v'))$ also contains the relativistic corrections. Due to a change of threshold by relativistic corrections a direct comparison of the curves with and without relativistic corrections is possible only by shifting the curves in energy or velocity in a way that the thresholds will coinside. Then it appears that the curve in which the relativistic corrections are taken into account is located always under the curve in which the relativistic corrections are not taken into account. For the values of frequencies $\hbar\omega \approx 3.7T$, mainly the particles in the tail of the thermal distribution are taking part in absorption and the increase of their number due to the lowering of threshold is larger than the decrease of absorption by each particle [26].

It is also necessary to take into account the relativistic effects in the electron distribution:

$$f^e(v) \approx \frac{e^{-\frac{v^2}{2v_{Te}^2}}}{(2\pi)^{3/2}v_{Te}^3}\left(1 - \frac{3}{8}\frac{v^4}{v_{Te}^2c^2} + \frac{45}{8}\frac{v_{Te}^2}{c^2}\right)$$

We obtain by performing the integration with the threshold determined by (45) the following change in the expression $\mathcal{F}(\omega)$ which determines the cross-section of bremsstrahlung (we neglect here the collective effects)

$$\mathcal{F}(\omega) = \mathcal{F}_0(\omega) - \delta\mathcal{F}^{br,rel}(\omega)$$

$$\delta\mathcal{F}^{br,rel}(\omega) = -\frac{v_{Te}^2}{c^2}\int_{\frac{\sqrt{z}}{2}}^{\infty}\frac{dx}{x}e^{-\left(x+\frac{z}{4x}\right)^2}f(x,z) \tag{47}$$

$$f(x,z) = \frac{3}{16}z^2\left(x+\frac{1}{x}\right)^4 - \frac{3}{4}z^2\left(x+\frac{1}{x}\right)^2 - \left[x^2+\frac{1}{x^2}+\left(\frac{1}{x^2}-x^2\right)\ln\frac{1}{x^2}\right]+\frac{15}{4} \quad (48)$$

To calculate the change in the opacity we use the formula(18):

$$\frac{\kappa_R^{br,rel} - \kappa_R^{(0)}}{\kappa_R^{(0)}} = +0.18\%. \quad (49)$$

Since the collective corrections to the bremsstrahlung are of the same order of magnitude and are opposite in sign it is not possible to restrict the consideration to non-collective relativistic corrections only as it is done in several of the latest papers. The reason for this is that the collective corrections apart from the same factor v_{Te}^2/c^2 the collective corrections contain a collective parameter δ_e which in the solar interior is of the order of 1.

It is also known [38,39] that non-collective bremsstrahlung in electron-electron collisions is of the relative order of v_{Te}/c^2. One can think that those corrections should be added to the effect of noncollective relativistic corrections considered above. But this is not correct since the collective effects suppress very much the bremsstrahlung in electron-electron collisions making it of the order of v_{Te}^4/c^4. Therefore the electron-electron bremsstrahlung in the solar interior should be neglected in calculations of opacity corrections which are of the order of v_{Te}^2/c^2.

9 RAMAN RESONANCE IN COLLECTIVE SCATTERING.

Before discussing relativistic effects in scattering we should consider a problem which can not be reconciled by a superficial look at the collective scattering but which is an important physical problem leading to an essential decrease in the scattering cross-section when necessary additional effects are taken into account. This is the problem of the contribution of the Raman scattering to the total cross-section of collective scattering on electrons. The Raman resonance corresponds to the case where the difference of the frequencies of the scattered wave and the scattering wave is equal to the electron plasma frequency. Plasma Langmuir waves can exist in the solar interior since the number of particles in the Debye sphere N_d for the temperatures and densities in the center of the Sun which were already given above is $N_d = 4\pi v_{Te}^3/\omega_{pe}^3 \approx 11.4$ is larger than one and the relative damping of plasma waves (the damping rate divided by the plasma frequency) due to the binary collisions is approximately equal to 1/20. The necessary conditions for the Raman resonance are

$$\omega - \omega' \approx \pm\omega_{\mathbf{k}_p}; \mathbf{k} - \mathbf{k}' = \mathbf{k}_p \tag{50}$$

where $\mathbf{k}_p$ is the wave vector of the plasma oscillation. For these conditions the longitudinal dielectric permittivity for the frequency equal to the difference of the frequencies of the initial and scattered waves and the difference of wave vectors close to the wave vector of plasma oscillation is close to zero because this is the dispersion equation for plasma waves

$$\epsilon_{\omega-\omega',\mathbf{k}-\mathbf{k}'} \approx 0 \tag{51}$$

But the probability of scattering on electrons contains this dielectric permittivity in the denominator, which means that the scattering is of a resonant nature for the difference of two frequencies close to the plasma frequency. We write here the probability of scattering on electrons $W^e_{\mathbf{k},\mathbf{k}'}$ in the form it was defined in the book [7] (namely the probability of scattering of a single photon in a unit time into the range $d^3k/(2\pi)^3$ from the range $d^3k'/(2\pi)^3$ related to these range intervals) assuming for simplicity that the electron velocity is much less than the average ion thermal velocity (which is a good approximation for averaging of the scattering on the thermal electron distribution):

$$W^e_{\mathbf{k},\mathbf{k}'} = \frac{(2\pi)^3 e^4}{2m_e^2\omega\omega'}(1+x^2)\left|\frac{1}{\epsilon_{\omega-\omega',\mathbf{k}-\mathbf{k}'}}\right|^2 \delta(\omega - \omega' - (\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}) \tag{52}$$

From this expression we see indeed that the Raman resonance can be very pronounced. The question is what is the relative contribution of this resonance to the total cross-section of scattering averaged over the thermal electron distribution and integrated over all frequencies of the scattered wave? And another question: how is this relative contribution changing with the collective parameter? It appears that for $\delta_e \gg 1$ the contribution of the Raman resonance is the largest in the total cross-section and this fact was not even mentioned or realized in the calculations of opacity in SSM's. The final answer is that the width of the Raman resonance for the approximation already used in SSM's can be extremely narrow and many effects not taken into account can broaden the resonance, thus decreasing its role in the total cross-section which then become much smaller and leads to a diminishing of the opacity.

To show this behaviour we should first say a few words about the Doppler effect in scattering. Since the velocity of the photons is close to the light velocity and $k \approx \omega/c$ the Doppler effect gives a correction of the order of the difference of the frequencies which is to a first approximation of the order of v_{Te}/c. More exactly from the δ-function in the probability (52) (describing from a quantum point of view the conservation of energy and momentum in the scattering) we can find by expanding on the difference of the frequencies the terms describing the linear and the quadratic Doppler effect namely

$$\omega' \approx \omega \left[1 - 2\frac{v_{Te}}{c} y\sqrt{1-x} + 2\frac{v_{Te}^2}{c^2}(1-x)y^2\right] \tag{53}$$

where y is the normalized component of the electron velocity with respect to the vector along the difference of the wave vectors of the scattering and scattered waves respectively:

$$y = \frac{(\mathbf{k} - \mathbf{k}') \cdot \mathbf{v}}{\sqrt{2} v_{Te} |\mathbf{k} - \mathbf{k}'|} \tag{54}$$

In the zero approximation where the Doppler effect is neglected the frequencies of the scattered and the scattering waves coincide. Only in this approximation was the cross-section of scattering denoted as scattering in the zero approximation. Due to the symmetry of the electron distribution function in y, the Doppler corrections should be of the order of v_{Te}^2/c^2. This means that we are again considering the terms of the order of v_{Te}^2/c^2 which we are searching for.

A new question nevertheless arises whether it is possible to expand on the Doppler corrections close to the Raman resonance? In the case such an expansion is not possible the contribution of the Doppler effect could be larger than the rough estimation v_{Te}^2/c^2 given above. To answer this question it is necessary to know the role of the resonance and its relative contribution to the total cross-section and to know its width (since in the case its width is less than v_{Te}/c the answer to the question of the possibility of the expansion in v_{Te}/c will be negative and one should treat the resonance with its Doppler broadening exactly). Of course the resonance can also be broadened by other means including binary collisions. At the present stage of consideration we wanted to consider the width of the Raman resonance without taking all those broadening effects in the form that it appears in the zero approximation used as a reference model. Then one should use the expression for the dielectric permittivity which takes into account only kinetic effects including as a damping effect only Landau damping. Then one can uses the expression for the collisionless dielectric permittivity. From the radiation extinction coefficient using the probability (52) we find the following expression for the transport cross-section:

$$\sigma_{(e)}^{sc} = \frac{3}{8}\int_{-1}^{1}(1+x^2)(1-x)dx \int_{-\infty}^{+\infty} \frac{e^{-y^2}}{\sqrt{\pi}} dy \frac{1}{\left|1 + \frac{\delta_e}{1-x} W(y)\right|^2} \tag{55}$$

where $W(y)$ is the plasma dispersion function given by expression (48). Under the integral with respect to y stays the square of the absolute value of the dielectric permittivity. Thus the total cross-section includes integration over the Raman resonance. It appears that the integration over y can be performed in a general case using the dispersion relations which relate the real and imaginary parts of the dielectric permittivity [27,17,18]. (or by using the fluctuation dissipative theorem). Indeed the imaginary part of $W(y)$ in

(48) contains an additional factor y as compared to the expression which enters in the numerator of (55). The expression under the integral (55) can be written as an imaginary part of $1/\omega\epsilon$ and the y integration can be considered as integration with respect to the frequency. Then by integrating in the upper complex plane of ω noting that the function $1/\epsilon$ has no poles, what is left is the integration due to circling the pole $1/\omega$ on the real axis. We then find:

$$\int_{-\infty}^{+\infty} \frac{e^{-y^2}dy}{\sqrt{\pi}\left|1+\frac{\delta_e}{1-x}W(y)\right|^2} = \frac{1}{1+\frac{\delta_e}{1-x}} \tag{56}$$

Using this expression in (55) leads immediately to the expression (25) for the cross-section of collective scattering used above.

In connection with relation (56) a new important question arises immediately concerning the asymptotic behaviour of the cross-section for large values of δ_e. In the case of large δ_e one can think it possible to neglect 1 as compared to the term containing δ_e in the denominator of the left hand side of (56). Then in the case where the integral is converging one finds the asymptotic behaviour proportional to $1/\delta_e^2$. It appears that indeed the integral is converging and the result of the calculation will be:

$$\int_{-\infty}^{+\infty} \frac{e^{-y^2}dy}{\sqrt{\pi}|W(y)|^2} = 3 \tag{57}$$

This means that the left hand side expression (56) has an asymptote $1/\delta_e^2$, while the right hand side of it has the asymptote $1/\delta_e$, therefore there is a contradiction. Where the mistake was made in such calculations? It appears that the mistake is in the assumption that the main contribution in the integral is given by that part of the integration where the y values are of the order of 1 or less than 1. But how can it be different in the case where the function under the integral contains $exp(-y^2)$? It appears that this is possible if there exists a very sharp exponentially narrow resonance this is the Raman resonance. The assumption that the main contribution comes from y of the order of 1 neglects the contribution of the Raman resonance which occurs for $\delta_e \gg 1$ in the range $y \gg 1$. But this will then mean that for large δ_e the value of the cross-section is almost completely determined by the contribution of the Raman resonance. Let us show that this indeed is the case

The resonance corresponds to zero of the real part of the dielectric permittivity. This resonance should be reached at large values of y otherwise we will be in contradiction with the previous considerations. For large values of y the function $W(y)$ has the following asymptote $W(y) \approx -1/2y^2$ which gives two possible values of y for which the resonance condition is satisfied:

$$y = y_r = \pm\sqrt{\frac{\delta_e}{2(1-x)}} \tag{58}$$

The latter expression shows that indeed for large values of δ_e the resonance corresponds to large values of y_r. Then making an expansion close to the resonant points we get the left hand side of (56) in the form :

$$\int_{-\infty}^{+\infty} \frac{(1-x)^2 \mathrm{Im}W(y_r)dy}{\pi y_r \delta_e^2 \left[(y-y_r)^2 \left(\frac{\partial \mathrm{Re}W(y_r)}{\partial y}\right)^2 + (\mathrm{Im}W(y_r))^2\right]} = \frac{1-x}{\delta_e} \tag{59}$$

which corresponds to the right hand side of (56).

Thus in the expressions for zero order scattering in the cross-sections, for values of the collective parameter $\delta_e \gg 1$, the largest contribution is from the Raman resonance, the width of which is exponentially decreasing with increasing of δ_e:

$$\frac{\delta y_r}{y_r} \approx \frac{\mathrm{Im}W(y_r)}{y_r \frac{\partial \mathrm{Re}W(y_r)}{\partial y_r}} \approx \sqrt{\pi} y_r^3 e^{-y_r^2} \tag{60}$$

The exponentiallly small thickness of the resonance makes it very dangerous to use the zero approximation for scattering but this was the only type of consideration of the role of collective scattering that was made at the present time in SSM's.

The broadening of the Raman resonance due both to the Doppler effect and to binary collisions can substantially decrease its contribution to the total cross-section thus making it much smaller than previous accounts. Then the main contribution will be made by thermal particles (y of the order of 1), the scattering cross-section will be proportional to $1/\delta_e^2$ and will be much smaller. But the problem is that this effect is very pronounced for large δ_e while the most important contribution to the opacity occurs for δ_e of the order of 1 when the weight function in the opacity is a maximum. Nevertheless we can see from Fig 1. that the weight function in κ_R is rather broad and thus the part with high values of δ_e can be essential.

10 DOPPLER AND COLLISIONAL BROADENING OF THE RAMAN RESONANCE.

This problem was recently considered in [28]. A simultaneous consideration of both effects of broadening is an important point since it can be proved that in the absence of the Doppler effect the cross-section will be determined by the static dielectric permittivity which, as is known, does not depend on the collisions. This problem of the necessity of simultaneous consideration of Doppler effects and binary collisions is of general importance

in nonlinear interactions [29]. Concerning the solar interior it will be very strange from a general point of view that for such a high collision rate of the order of $2 \times 10^{16} s^{-1}$ one still uses the collisionless approximation for scattering and that the binary collisions have no way of influencing the scattering process but this was the way the scattering was previously considered. The collision frequency should be compared not with the frequency of radiation or even not with the plasma frequency but with the width of the Raman resonance. Then it becomes obvious that the broadening of the Raman resonance can be important for the reduction of the opacity. The contribution of the Doppler effect and the binary collisions can be calculated in the denominator by using the perturbation approach but the expansion of these effects close to the resonance is not possible. In using the perturbation calculation in the dielectric permittivity in the denominator we in fact use only the small parameters v_{Te}/c and ν_{coll}/ω_{pe}, where ν_{coll} is the effective frequency of the binary electron ion collisions. We will use such an expansion in the dielectric constant but will not use it to expand on these parameters close to the point of Raman resonance leaving the corresponding terms quadratic in v_{Te}/c and linear in ν_{coll}/ω_{pe} in the denominator. The quadratic terms in the Doppler effect are necessary to take into account since in the range outside the resonance where the expansion is possible only the quadratic Doppler terms will survive. As a result the cross-section will have the form (55) but with denominators which take into account the Doppler and collisional broadening of the Raman resonance and with an additional factor in the numerator which takes into account the first two terms in the expansion of the parameter v_{Te}/c (in the latter it is necessary to take into account the Doppler corrections outside the resonance where the expansion is possible). We then find :

$$\sigma_e^{sc} = \frac{3}{8}\int_{-1}^{1}(1+x^2)(1-x)dx\int_{-\infty}^{+\infty}\frac{e^{-y^2}}{\sqrt{\pi}}dy\frac{A(x,y)}{|F(x,y)|^2} \tag{61}$$

where previously for (55) we had

$$A(x,y) = A_0(x,y) = 1; F(x,y) = F_0(x,y) = 1 + \frac{\delta_e}{1-x}W(x) \tag{62}$$

and now

$$A(x,y) = 1 - 3\frac{v_{Te}}{c}y\sqrt{1-x} + 2\frac{v_{Te}^2}{c^2}y^2(3-2x) \tag{63}$$

and

$$F(x,y) = 1 + \frac{\delta_e}{1-x}W_R(y)\left[1 + 2\frac{v_{Te}}{c}y\sqrt{1-x} - 2\frac{v_{Te}^2}{c^2}(2-x)\right] +$$

$$i\frac{Z_{eff}\ln\Lambda\omega_{pe}^3 y\delta_e^{3/2}}{24\pi^{3/2}n_e v_{Te}^3(1-x)^{3/2}}\left\{[Ei(y^2)-i\pi]e^{-y^2}-\frac{1}{y^2}\right\} \tag{64}$$

where $\ln\Lambda$ is the Coulomb logarithm, the function

$$W_R(z)=1-z\int_{-\infty}^{+\infty}\frac{e^{-y^2}}{\sqrt{\pi}}R(y)\frac{dy}{z-y}+i\sqrt{\pi}e^{-y^2}R(z) \tag{65}$$

is a generalization of the function $W(z)$ which takes into account the relativistic corrections in the distribution function of electrons

$$R(y)=e^{-\frac{3v_{Te}^2}{2c^2}y^4}\int_0^\infty xe^{-x^2[1+\frac{3v_{Te}^2}{2c^2}(x^2+2y^2)]}dx\Big/\int_0^\infty\frac{4}{\sqrt{\pi}}x^2e^{-x^2-\frac{3v_{Te}^2}{2c^2}x^4}dx \tag{66}$$

and finally,

$$Ei(z^2)=\int_{-\infty}^{z^2}\frac{e^t}{t}dt \tag{67}$$

Numerical calculations of [28] of the corrections to the Rosseland opacity by using these formulas, for the parameters in the center of the Sun given in previous sections results in

$$\frac{\kappa_R^{sc,broad,res}-\kappa_R^{(0)}}{\kappa_R^{(0)}}=-3.0\% \tag{68}$$

In these calculations we use formula (18) since the corrections are not very small. The value given by (68) is a substantial contribution which was previously not taken into account in SSM's.

11 RELATIVISTIC CORRECTIONS TO COLLECTIVE SCATTERING

Previous explainations and further considerations already given makes it even obvious that the relativistic corrections to the scattering can not be found simply by multiplying the zero order cross-sections by a factor taken from the expressions where the collective effects are neglected. Nevertheless it was the only consideration of relativistic effects in cross-sections previously done for the solar opacity problem. There are several reasons why this approach is wrong, the first is that the relativistic corrections are different for electrons and ions and one can not multiply the zero approximation by the same factor for electrons and ions, the second is that inside the Raman resonance the expansion in

the parameter v_{Te}/c is not possible and the third is that the relativistic corrections have another dependence on the collective parameter δ_e from that of the zero approximation.

We can illustrate the latter statement by using the above argument in the interference of the effects occurring in scattering on the charge itself and that of its shielding cloud. Let us denote the matrix element of scattering on an individual "naked" electron by M_T and let us denote the relativistic corrections to it by δM_T^{rel}. Let us then denote the matrix element for scattering on the screening "cloud" by M_{coll} and let us denote the relativistic corrections to it by δM_{coll}^{rel}. The total cross-section for scattering will be proportional to

$$\left|M_T + M_{coll} + \delta M_T^{rel} + \delta M_{coll}^{rel}\right|^2 \tag{69}$$

The corrections which were taken into account in SSM's correspond only to $2\delta M_T^{rel}/M_T$ and they were multiplied by the square of the total zero order matrix element $|M_T+M_{coll}|^2$. But, even in the case one neglects the relativistic corrections to the collective matrix element, the procedure does not give the correct result. After dropping the term δM_{coll}^{rel} we can see from (69) that the corrections δM_T^{rel}, should be multiplied by $M_T + M_{coll}$, but not M_T. Apart of this one can not neglect δM_{col}^{rel} in which not only the effect due to the relativistic corrections in the particle motion should be taken into account, but also the relativistic corrections to the particle distributions which enter in the nonlinear plasma response on which the collective matrix element depends as well as the relativistic corrections in the electron distribution while averaging the cross-sections should be taken into account. Previously in plasma physics such detailed calculations were not performed. These are very cumbersome and one should use general expressions for matrix elements given in [9,10] which take into account in principle relativistic effects exactly. Such calculations were performed in [30] and the final result can be written using the already introduced function $F(x,y)$ (see (68)), which takes into account all the above effects of resonant broadening of the Raman resonance (because the Raman resonance enters also in all relativistic corrections). We will denote the relativistic corrections for scattering on electrons for an arbitrary value of the collective parameter δ_e by $\delta\sigma_e^{rel}$. Naturally when the collective effects are unimportant the expression for $\delta\sigma_e^{rel}$ converts to the known expression. We find:

$$\delta\sigma_e^{rel} = -\frac{3v_{Te}^2}{4c^2}\sigma_T \int_{-\infty}^{+\infty} \frac{dy}{\sqrt{\pi}} e^{-y^2} \int_{-1}^{1} \frac{dx}{|F(x,y)|^2} \times$$
$$\left[(1-x)^{5/2}\mathrm{Re}F(x,y)(y^2 f_1(x) + 2f_2(x)) - \delta_e(1-x)^3 G(x,y)\right] \tag{70}$$

where

$$f_1(x) = 1 + x + x^2 - x^3; f_2(x) = \frac{1}{2}(3 - x + x^2 - x^3) \tag{71}$$

$$G(x,y) = 1 + x + x^2 - x^3 + \frac{1}{2}W(y)(9 + 4x + 9x^2) - 2y^2W(y)(1 + x + x^2 - x^3) \quad (72)$$

The relativistic corrections for scattering on ions is related only to the electron shell of ions (since the other relativistic corrections of the order of v_{Ti}^2/c^2 are here naturally completely neglected) and is determined by δM_{coll}^{rel} and therefore the ion velocity does not enter in these corrections. Therefore for the averaging on the ion distribution the integrals are of the same form as in the zero approximation and the result can be obtained in an analytic form by using relation (56), which is a consequence of the fluctuation-dissipation theorem. The relativistic corrections for ions are denoted by $\delta\sigma_i^{rel}$ and are given by the following expression:

$$\delta\sigma_i^{rel} = \frac{3v_{Te}^2}{4c^2}\delta_e\left[g(\delta_e) - g(\delta_i)\right]\sigma_T \quad (73)$$

where

$$g(z) = \left(2 - z + \frac{7}{2}z^2 + \frac{3}{2}z^3 + z^4\right)\ln\frac{2+z}{z} + \frac{28}{3} + \frac{35}{3}z + 11z^2 + 2z^3 \quad (74)$$

A numerical calculation of the sum of corrections due to electrons and ions by using the formulas given in this section lead to the following change of the opacity in the center of the Sun

$$\frac{\kappa_R^{e,screl} - \kappa_R^{(0)}}{\kappa_R^{(0)}} = -0.2\% \quad (75)$$

12 EFFECTS OF FREQUENCY CHANGE IN THE PROCESSES OF RADIATION TRANSPORT

In addition to the relativistic corrections to the transport cross-sections there appear also the corrections of the same order of magnitude in the equation for the transport of radiation. To clarify this point let us write here the equation for photon transport by taking into account the processes of bremsstrahlung and scattering. Since the starting point in the theory of photon transport is the assumption that the distribution of photons is locally an equilibrium distribution with small deviations from the equilibrium distribution due to the presence of temperature gradients such an equation should include both the processes of spontaneous bremsstrahlung emission and spontaneous scattering and the processes of stimulated bremsstrahlung emission and absorption and the processes of stimulated scattering (in equilibrium the spontaneous and stimulated processes exactly

balance each other giving the Planck distribution). In the theory of radiation transfer it is necessary to take into account both the small deviations from the equilibrium in the spontaneous processes and the small deviations from equilibrium in the stimulated processes. We will write down a general expression describing the propagation of photons for their occupation number $N_{\mathbf{k}}$ assuming the photon distribution does not depend on time (see [31]) and assuming also that the photon frequency is much larger than the electron plasma frequency:

$$\frac{dN_{\mathbf{k}}}{dt} = \mathbf{v}_g \cdot \frac{\partial N_{\mathbf{k}}}{\partial \mathbf{r}} - \frac{\partial \omega}{\partial \mathbf{r}} \cdot \frac{\partial N_{\mathbf{k}}}{\partial \mathbf{k}} \approx$$

$$\cos\theta \left(c\frac{\partial N_\omega^T}{\partial r} - \frac{\omega_{pe}^2}{2\omega^2}\frac{1}{n_e}\frac{\partial n_e}{\partial r}\omega\frac{\partial N_\omega^T}{\partial \omega} \right) =$$

$$= -\int (W_{\mathbf{k},\mathbf{k}'}^e f_{\mathbf{p}}^e + \sum_i W_{\mathbf{k},\mathbf{k}'}^i f_{\mathbf{p}}^i)(N_{\mathbf{k}} - N_{\mathbf{k}'})\frac{d^3p d^3k'}{(2\pi)^6} +$$

$$+N_{\mathbf{k}} \int W_{\mathbf{k},\mathbf{k}'}^e \hbar(\mathbf{k}-\mathbf{k}') \cdot \frac{\partial f_{\mathbf{p}}^e}{\partial \mathbf{p}} N_{\mathbf{k}'} \frac{d^3p d^3k'}{(2\pi)^6} + 2\gamma_{\mathbf{k}}^{br} N_{\mathbf{k}} + q_{\mathbf{k}}^{br} \tag{76}$$

where $W_{\mathbf{k},\mathbf{k}'}^{e,i}$ are the probabilities of scattering on electrons (superscript e) and ions (superscript i) respectively, $\cos\theta$ is the angel between $\mathbf{k}$ and $\mathbf{r}$; in the left hand side of the equation the thermal equilibrium distribution N_ω^T is substituted, while in the right hand side only the deviations from the thermal distribution contribute and the terms linear in these deviations are left. The last two terms describe the spontaneous and stimulated bremsstrahlung ; since in equilibrium they balance each other and since the spontaneous emission has not been perturbed the only term left is the deviation of the stimulated bremsstrahlung related to the flux of radiation, this effect was already considered above and expressed through the effective bremsstrahlung absorption cross-section. In the first term of the right hand side of the transport equation, which describes the scattering the frequencies of the photons before and after scattering do not coincide since due to the Doppler effect the frequency of photons changes in the scattering process. This term leads to a transport cross-section described above only if the deviations from the thermal distribution are proportional to the cosine of the angle between the direction of photon propagation and the direction of the temperature gradient and only if the Doppler effect is neglected in the expressions for the photon occupation number (the latter approximation is very important). Then we have $N_{\mathbf{k}} = N_\omega^T + \cos\theta\delta N_\omega$, where δN_ω is proportional to the flux of radiation. In the case we want to calculate the quadratic corrections in the parameter v_{Te}/c, there appear new terms which are proportional to the derivatives of the flux of radiation with respect to the frequency since we need to use an expansion of the occupation numbers in the powers of the frequency difference:

$$\delta N_{\omega'} \approx \delta N_\omega + (\omega' - \omega)\frac{\partial \delta N_\omega}{\partial \omega} + \frac{1}{2}(\omega' - \omega)^2 \frac{\partial^2 \delta N_\omega}{\partial \omega^2} \tag{77}$$

The physical nature of these effects is obvious: in each act of scattering the frequency of the photons is changing and as a result of many scatterings the distribution of the photons diffuse in frequency.

The next term in the transport equation describes stimulated scattering and since it is in the first approximation odd in velocities it will contain the first derivative of the flux of radiation with respect to the frequency and thus describes a systematic redshift of the photon frequencies in the process of scattering.

A systematic change of the photon frequency due to the density inhomogeneity is described by the second term in the left hand side of the transport equation (76). Its meaning is also obvious, it is to conserve the adiabatic invariant, the occupation number of photons. It is easy to see that the latter term is also the term containing, in front of it, the relativistic parameter v_{Te}^2/c^2. Indeed by introducing the collective parameter δ_e, one finds that the change of the left hand side of the transport equation due to the plasma density gradient is described by an additional factor in the left hand side having the following form:

$$1 + \delta_e \frac{v_{Te}^2}{c^2} \frac{\partial \ln n_e}{\partial \ln T} \tag{78}$$

Since for the solar interior δ_e is of the order of 1 the density inhomogeneity effect is again of the order of the relativistic effects.

Thus the transport equation contains three new effects of the order of that already considered in previous presentations of the present review and all of them are related to the change of frequency of photons in the process of radiative energy transfer.

13 CORRECTIONS DUE TO DENSITY INHOMOGENEITY

The existing SSM can be used to estimate the density gradients in the center of he Sun and find the value of $\partial \ln n_e / \partial \ln T$. To calculate the change of Rosseland opacity due to the density inhomogeneity we will use formula (34), this gives:

$$\frac{\delta \kappa_R^{inh}}{\kappa_R^{(0)}} = -0.14\% \tag{79}$$

Although this effect is rather small we do not exclude it from our consideration to have a complete result of the effects of the relative order of v_{Te}^2/c^2.

14 CORRECTIONS DUE TO PHOTON FREQUENCY DIFFUSION DURING THE RADIATION TRANSFER

This effect leads to a new type of transport equation for photons which raises several problems concerning possible solutions and the question whether the opacity approach can be at all applied to the transport of radiation inside the Sun. The corresponding contributions of effects of frequency diffusion in the transport equation are described by differential operators applied to the functions describing the radiation flux. The structure of the transport equation changes in some sense in a cardinal way since the transport equation became a differential equation in frequency containing the second order derivatives [31]. The terms with higher derivatives have a small parameter in front of them which raises several mathematical problems not yet solved and the presence of the terms with derivatives do not allow us to integrate the transport equations over the frequencies to obtain the opacity directly from the equations. The mathematics reflects the physics and thus several physical problems arise in this context. We will first write the operators describing the effect of frequency diffusion in a form acting on the already introduced above disturbance of the photon occupation numbers δN_ω. We recall that the flux of radiation $\mathcal{F}_\omega$ is proportional to this occupation number perturbation times ω^3 and therefore the operator $\hat{a}$, acting on δN_ω, corresponds in the transport equation to the following operator acting on the radiation flux $\mathcal{F}_\omega$:

$$\omega^3 \hat{a}\left(\frac{\mathcal{F}_\omega}{\omega^3}\right) \tag{80}$$

In treating the frequency diffusion effects we will take into account the broadening of the Raman resonance. In this case the first term in the right hand side of the transport equation(76) leads to a sum of the already considered term describing the transport cross-section (61) and an additional term which can be written in operator form and thus called an operator cross-section $\hat{\sigma}^{fd}$ describing the effect of frequency diffusion (remember that in a transport equation it will appear in accordance with relation (80)):

$$\hat{\sigma}^{fd} = \sigma_1^{fd}(\omega)\omega\frac{\partial}{\partial\omega} + \sigma_2^{fd}(\omega)\omega^2\frac{\partial^2}{\partial\omega^2} \tag{81}$$

and

$$\sigma_1^{fd}(\omega) = \frac{3}{4}\sigma_T\int_{-\infty}^{+\infty}\frac{dy}{\sqrt{\pi}}e^{-y^2}\int_{-1}^{1}\frac{(1+x^2)xdx}{|F(x,y)|^2}\times$$

$$\left[\frac{v_{Te}}{c}y\sqrt{1-x}-4\frac{v_{Te}^2}{c^2}y^2(1-x)\right] \tag{82}$$

$$\sigma_2^{fd}(\omega)=-\frac{3v_{Te}^2}{4c^2}\sigma_T\int_{-\infty}^{+\infty}\frac{dy}{\sqrt{\pi}}e^{-y^2}y^2\int_{-1}^{1}\frac{(1+x^2)x(1-x)dx}{|F(x,y)|^2} \tag{83}$$

Since with these operator terms taken into account in the transport equation the latter becomes a differential equation in frequency with a small parameter in front of the highest derivative it raises some questions already known from mathematical courses and textbooks. There are in this connection at least two questions. The first is whether it is possible to exclude the small parameter from the equation by changing the variables? The second question is what kind of "boundary" conditions in frequency should be taken and how critical the final solution depend on these conditions? Both questions are rather difficult to answer in a general form . But these answers can alter the conclusions of the role of these effects in the solar interior. The question also arises why such a problem does not arise in the energy transfer in a vacuum (more exactly for the case when the collective effects do not play an important role). The answer is that the frequency diffusion terms were not previously obtained even for this simple case.

The pioneering work in this field is the work of Sampson [32] who wrote the integral transport equation for radiative transfer using the Klein-Nishina quantum formula for the cross-section of scattering. The integral equation is of the type we wrote in the previous section and obviously it contains all the same or even a more complicated problem in the case one tries to solve it analytically. By the way the transport equation we wrote in the previous section is exactly the same as in [32] except for the probability used and the equation is written in the classical limit. In equation (76) all collective effects are included while in Sampsons's paper the collective effects were ignored but the probability includes all the quantum effects. The collective quantum effects will be discussed in subsequent sections. But here we discuss why similar problems with frequency diffusion do not appear in Sampson's approach. The answer is that in this paper they were simply neglected by two simplifying conditions which he needed to accept to solve the problem numerically (the assumptions I and II of [32]). Thus the frequency diffusion terms are derived in [31] for the first time even for the simple case of undressed particles, and this means that the problem exists even in this case. To have such an analytical equation with a diffusion term is a substantial progress in the transport theory since there is hope to solve it analytically, at the present time such an equation was solved only numerically. In the subsequent papers [33,34] the authors have excluded the two assumptions of Sampson but still only made numerical computations of the integral equation. Even by excluding the assumptions of [32] the authors of [34] were not able (due to numerical difficulties) to treat the temperature region corresponding to the solar interior (the calculations made in [34] were performed for temperatures greater than those in the solar interior). Therefore the

consideration made in [31] is more consistent within a small parameter v_{Te}/c and takes into account both collective effects and broadening of the Raman resonance. But there still remains the problem in all three papers which make another third assumption that the solution of the transport equation has a definite form - it is proportional to the cosine of the angle with inhomogeneity (as usual in all transport equations) and the derivative of the black body radiation with respect to the coordinate. he latter assumption was not made in [30] and only the assumption that the solution contains the cosine of the angle was used. The form of the solution in [30] is determined by the initial equation and, as it occurs in a form different from that used in [32,33,34], contains the derivatives of the intensity with respect to the frequency. Thus the equations with the frequency diffusion term can not be derived with the assumptions made in all three referred papers. The problem was in ref.[31] even formulated in a more general form for the case of scattering in vacuum in the absence of collective effects. It can be seen that the terms in the form given in the papers [32,33,34] can be obtained from a more general equation written here if only in these terms one can use a perturbation approach namely first calculate the flux without frequency diffusion terms and then substitute this solution into the terms describing the frequency diffusion. But to use the perturbation approach for an equation with a small parameter in front of the highest derivatives is known from mathematics to be very dangerous. The only arguments to help in not resolving this problem but only suggesting the way for a possible solution are the following: in the case of undressed particles the frequency diffusion terms contain only the powers of the operator $\omega\partial/\partial\omega$, which does not allow the exclusion of the small parameter by changing the variables.

But we obtained above a more general result including in the frequency diffusion terms all collective effects which makes the coefficients in front of those operators depend on the collective parameter δ_e and thus depend on frequency. In the collective case the question about the possibility of the use of the perturbation approach is even more serious.

At the present moment we are not able to resolve it and the best we can do is to use the perturbation approach. But the important point is that in using such an approach we should bear in mind the existing uncertainty of the theoretical predictions since as we will see the total effect of the frequency diffusion on the solar opacity is not small (by finding some particular solutions without using the perturbation approach we can demonstrate that these effects can be even larger).

The problem reminds us about the "boundary" conditions in the frequency diffusion terms. In the perturbation approach they do not appear and only in the perturbation approach we will be able to find the corrections to the opacity explicitly. What do all these problems mean physically? Depending on the "boundary" conditions in frequency we can have or can not have the effect of photon accumulation in a certain frequency domain. This effect is well known in plasma physics for Langmuir waves and has the name Langmuir condensation. Probably the condensation of photons will not appear

since they have a large inverse bremsstrahlung absorption but still it is an open question. Depending on "boundary" conditions in frequencies one can expect another possible effect a "runaway"of photons from absorption. All these problems mean that there could occur specific instabilities of photon distributions related directly to photon transport and generated by the transport phenomena. To investigate them is a new problem for the future research in this field.

In any case the possibility of writing down an analytic equation of transport including the frequency diffusion terms seems to have more advantages in future research in this field since up to the present time only the numerical solutions of integral equations was investigated and any numerical results have the disadvantage of needing to be repeated each time for new conditions of interest. The differential transport equation we obtained has only the limitations that the temperature of particles should be nonrelativistic. A preliminary investigation of this equation for the vacuum case was performed in [35] and show that exact solutions are not at all trivial and the spectrum obtained has some peculiarities with rapid frequency variations the averaging which lowers the total transferred flux. We leave this problem as it is at the present moment underlying the existence of uncertainties in the solar opacity arising from these problems. Having nothing more exact at the present moment we will use the perturbation approach to at least estimate the order of magnitude of the possible effect of frequency diffusion on the opacity. One should bear in mind that this estimation can give the lower limit of possible reductions of the opacity since the oscillations in the frequency distribution can only enhance the decrease of the opacity due to this effect. The numerical results were performed together with effects of stimulated scattering since the latter also lead to terms with derivatives with respect to the frequency of the radiation flux. The results are given in the next section.

15 EFFECT OF STIMULATED SCATTERING

This effect was also missed in the consideration of SSM's. This effect does not contain a term describing diffusion on frequency but only the term with a first derivative with respect to the frequency which means it leads to a systematic change in the photon frequency during the transport of radiation.

The corresponding operator we denote as $\hat{\sigma}^{st}$:

$$\hat{\sigma}^{st} = \sigma_0^{st}(\omega) + \sigma_1^{st}(\omega)\frac{\partial}{\partial\omega} \tag{84}$$

where

$$\sigma_0^{st}(\omega) = -\frac{3v_{Te}}{4c}\sigma_T\frac{z}{e^z-1}\int_{-\infty}^{+\infty}\frac{ydy}{\sqrt{\pi}}e^{-y^2}\int_{-1}^{1}dx\frac{(1+x^2)(1+x)\sqrt{1-x}}{|F(x,y)|^2}+$$

$$\frac{3}{2}\sigma_T\frac{v_{Te}^2}{c^2}\frac{z}{e^z-1}\int_{-\infty}^{+\infty}\frac{y^2e^{-y^2}dy}{\sqrt{\pi}}\int_{-1}^{1}\frac{\left[2(1+x)-\frac{ze^z}{e^z-1}\right](1+x^2)(1-x)}{|F(x,y)|^2} \tag{85}$$

$$\sigma_1^{st}(\omega)=\frac{3}{2}\frac{v_{Te}^2}{c^2}\frac{z}{e^z-1}\int_{-\infty}^{+\infty}\frac{y^2e^{-y^2}dy}{\sqrt{\pi}}\int_{-1}^{1}\frac{x(1-x)(1+x^2)}{|F(x,y)|^2} \tag{86}$$

By using the perturbation theory in the transport equation one can consider the effect of the sum of frequency diffusion and stimulated scattering on the value of the solar opacity in the center of the Sun (the opacity can be introduced phenomenologically as a factor between the radiation flux and temperature gradient). Then we find a modification of expression (34) due to the operator character of the corresponding contributions

$$\frac{\kappa_R-\kappa_R^{(0)}}{\kappa_R^{(0)}}=\int_0^{\infty}\frac{z^3dz}{\sigma_0(z)}\left\{\sigma_0^{st}(z)+z\frac{\partial}{\partial z}\left[\sigma_1^{fd}(z)+\sigma_1^{st}(z)\right]\times\right.$$
$$\left.\times z^2\frac{\partial^2}{\partial z^2}\sigma_2^{fd}(z)\right\}\frac{ze^zdz}{(e^z-1)^2}\Bigg/\int_0^{\infty}\frac{z^4e^zdz}{\sigma_0(z)(e^z-1)^2} \tag{87}$$

The result of numerical calculations taking into account the broadening of the Raman resonance is:

$$\frac{(\delta\kappa_R^{fd}+\delta\kappa_R^{st})}{\kappa_R^{(0)}}=-4.5\% \tag{88}$$

This is a rather large effect which can not be neglected further in the construction of SSM's.

16 QUANTUM EFFECTS IN SCATTERING

Quantum effects in scattering can be important only for electrons. In vacuum for "naked " electrons the quantum effects are described by the well known Klein-Nishina formula [15]. For the case of collective scattering those results can not be applied. The collective quantum scattering was considered only recently [36,37] in connection with the problem of scattering in the solar interior and only the first quantum corrections in the parameter $\hbar\omega/m_ec^2$ were obtained. But this is sufficient for our purpose here.

We can start the estimation by not including in the first stage the collective effects. This estimation can be obtained from the first quantum corrections using the Klein-Nishina formula. Even from this estimate we will see that for problems of energy transfer the quantum corrections can be of the order of or larger than the relativistic corrections. Thus summing all the effects which have in front of them an additional factor v_{Te}^2/c^2 we

will need to include also the quantum corrections. Indeed, from the Klein-Nishina formula we get the first quantum corrections for scattering on nonrelativistic electrons in the form:

$$\sigma_e^{sc,q,KL-N} = \sigma_T \left(1 - \frac{2\hbar\omega}{m_e c^2}\right) \tag{89}$$

The additional term $2\hbar\omega/m_e c^2$ can also be written in the form $2zv_{Te}^2/c^2$. By taking into account that the weighting factor in the Rosseland opacity has a maximum at z equal to 3.7 we indeed see that the quantum corrections are of the same order of magnitude as the relativistic corrections

But in the solar interior the scattering is collective and the use of the Klein-Nishina formula is not justified. In building a quantum collective theory of scattering it is necessary to reconsider many problems and solve cumbersome equations. One can use the general theory of fluctuations but to get from the general expressions even a first quantum correction is difficult since one needs first to separate those effects which are related with scattering from other effects. Even a classical theory of fluctuations is not that simple for such a separation. There exists three other approaches known in classical plasma physics to calculate the scattering probability from which in ref.[36] these was used the quantum generalization of the approach related to the momentum diffusion of the electron distribution on the electrostatic beat wave produced by the initial and the scattered waves. This approach allows us to find the expression of the collective part of the scattering matrix element including the first quantum corrections in the parameter $\hbar\omega/m_e c^2$. Then it is possible to show that the matrix element of the Klein-Nishina scattering has no corrections to the matrix element of Thomson scattering to first order in this parameter. Thus the Klein-Nishina corrections appear only from the δ-function describing the quantum conservation law of momentum and energy in the scattering process (they describe only the recoil effect in scattering). Thus the matrix element of individual scattering can be taken as the matrix element of Thomson scattering in the case that we are interested only in the first order quantum corrections in the parameter zv_{Te}^2/c^2. A cumbersome calculation for the collective matrix element then show that it differs from the classical one only by containing the quantum expression for the dielectric permittivity instead of its classical limit in the classical expression. The quantum expression for the dielectric permittivity appears in the probability for the frequency equal to the difference of the frequencies of two waves (the scattered one and the scattering one) and the wave vector equal to the difference of the wave vectors of these waves. We write down here the expression for the quantum dielectric permittivity for the frequency ω and the wave vector $\mathbf{k}$:

$$\epsilon_{\omega,\mathbf{k}} = 1 + \frac{4\pi e^2}{k^2} \int \frac{d^3p}{(2\pi)^3} \frac{\Phi_{\mathbf{p}} - \Phi_{\mathbf{p}-\hbar\mathbf{k}}}{\hbar\omega + \epsilon_{\mathbf{p}-\hbar\mathbf{k}} - \epsilon_{\mathbf{p}} + i0} \tag{90}$$

With this dielectric permittivity the quantum expression for the scattering probability

in which only the terms square in the parameter $\hbar\omega/m_e c^2$ are neglected has the following form:

$$W_{\mathbf{p}}(\mathbf{k},\mathbf{k}') = \frac{(2\pi)^3 e^4}{2m_e^2\omega\omega'}(1+x^2)\left|\frac{1}{\epsilon_{\omega-\omega',\mathbf{k}-\mathbf{k}'}}\right|^2 \times$$
$$\times\delta\left(\omega-\omega'-(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}-\hbar\frac{(\mathbf{k}-\mathbf{k}')^2}{2m_e}\right) \tag{91}$$

It is necessary at this point to stress that in (91) not only the corrections of the order of zv_{Te}^2/c^2 appear (as they appear in the Klein-Nishina formula) but also dielectric permittivity corrections of the order of $\hbar k/p \approx zv_{Te}/c$ and of the order of $z^2v_{Te}^2/c^2$ can appear. The second small parameter is much larger than the parameter in the Klein-Nishina expansion. The second order term in this parameter has in front of it the factor v_{Te}^2/c^2, i.e it is of the same order as the relativistic effects. These new terms of the order of $z^2v_{Te}^2/c^2$ appear only from the expansion of the dielectric constant and thus are essential only in the collective regime of scattering. The expansion contains both terms of the order of zv_{Te}^2/c^2 and the terms of the order of $z^2v_{Te}^2/c^2$.Taking into account the presence of z^2 in the last term we can then find when expanded on it the weighting factor in the opacity will have a maximum close to z equal to 6, but not 3.7 as it was before. This will enlarge all the quantum contributions. Thus although they all have the same factor v_{Te}^2/c^2 as the other relativistic contributions the presence of an additional factor z^2 can make their contribution large enough. There is another problem related to the first order corrections in this parameter zv_{Te}/c. Although in the expansion of all expressions the first order corrections vanishes due to the symmetry of the distribution of electrons, the expansion is not valid inside the Raman resonance. When considering the Raman resonance broadening we will keep all the linear terms in this parameter in the denominator of the Raman resonance as we kept above the linear terms for the Doppler effect.

The quantum dielectric permittivity can also be expressed through the plasma dispersion function $W(s)$, as its classical value. The parameter s will be given by:

$$s = \frac{\omega-\omega'}{|\mathbf{k}-\mathbf{k}'|\sqrt{2}v_{Te}} \tag{92}$$

We will not need this general expression and give only its approximation up to the first order expansion in the parameter $z^2v_{Te}^2/c^2$

$$\epsilon_{\omega-\omega',\mathbf{k}-\mathbf{k}'} \approx 1 + \frac{\omega_{pe}^2}{|\mathbf{k}-\mathbf{k}'|^2 v_{Te}^2}\left(W(s) + \frac{\kappa^2}{6}\frac{\partial^2}{\partial s^2}W(s)\right) \tag{93}$$

where

$$\kappa = \frac{\hbar|\mathbf{k}-\mathbf{k}'|}{2\sqrt{2}m_e v_{Te}} \tag{94}$$

It is sufficient to use the first approximation for $|\mathbf{k}-\mathbf{k}'| = \sqrt{2(1-x)}\omega/c$ and then in the correction term in (93) we have $\kappa^2 = z^2(1-x)v_{Te}^2/4c^2$ which shows that indeed the corrections are of the order of $z^2 v_{Te}^2/c^2$, but not of the order of $z v_{Te}^2/c^2$ as in the case of non-collective scattering.

In the classical limit the conservation law of the momentum and energy in the scattering gives $s = y$, but not in the quantum case where we need to use an expansion up to terms of second order in the parameter v_{Te}/c, this is written below. We mention here another important point which shows that the expansion we wanted to use will be different in different terms of the transport equation. Indeed, the transport cross-section is obtained by balancing the processes of direct and inverse scattering. The inverse process is determined by the electrons the initial momentum of which differs from the initial momentum of electrons in a direct process by the momentum transferred in the scattering process $\hbar(\mathbf{k}-\mathbf{k}')$ .The particle momentum enters in the probability only under the sign of the δ- function describing the quantum conservation law of energy and momentum in the scattering process. By shifting the particle momentum by the amount $\hbar(\mathbf{k}-\mathbf{k}')$ we first convert the particle distribution in the final state to that of the initial one (which means that we should only average over the initial distribution in the transport equation) and secondly, the only change in the probability will be the sign of the quantum corrections in the term describing the process of inverse scattering.

Having in mind the possibility to perform this type of simplification in the transport equation we will write down the expression for s and other quantities such as the photon frequency ω' after scattering with two signs of z corresponding to different signs of the quantum corrections in the expressions for direct and inverse scattering. We denote the corresponding expressions with a subscripts $\pm$ and take into account all terms of expansion up to second order in the parameter v_{Te}/c:

$$\omega'_{\pm} = \omega\left(1 - 2\frac{v_{Te}}{c}y\sqrt{1-x} + 2\frac{v_{Te}^2}{c^2}y^2(1-x) \mp z\frac{v_{Te}^2}{c^2}(1-x)\right) \tag{95}$$

$$(\mathbf{k}-\mathbf{k}')^2_{\pm} = \frac{2\omega^2(1-x)}{c^2}\left\{1 - 2\frac{v_{Te}}{c}y\sqrt{1-x} + \frac{v_{Te}^2}{c^2}[2(2-x)y^2 \mp z(1-x)]\right\} \tag{96}$$

$$s_{\pm} = y \pm \frac{z}{2}\left[\frac{v_{Te}}{c}\sqrt{1-x} - \frac{v_{Te}^2}{c^2}y(1-x)\right] \tag{97}$$

$$A_{\pm}(x,y) = 1 - 3\frac{v_{Te}}{c}y\sqrt{1-x} + 2\frac{v_{Te}^2}{c^2}y^2(3-2x) \mp 2\frac{v_{Te}^2}{c^2}z(1-x) \tag{98}$$

$$F_{\pm}(x,y) = 1 + \frac{\delta_e}{1-x} W(s_{\pm}) \left[1 + 2\frac{v_{Te}}{c} y\sqrt{1-x} - 2\frac{v_{Te}^2}{c^2}(2-x) \pm \frac{v_{Te}^2}{c^2} z(1-x) \right] +$$

$$+\delta_e \frac{z^2 v_{Te}^2}{24c^2} \frac{\partial^2}{\partial y^2} W(y) \tag{99}$$

Then the transport cross-section which takes into account the collective quantum corrections can be written in the form:

$$\sigma_e^{sc,q,coll} = \frac{8}{3}\sigma_T \int_{-\infty}^{+\infty} \frac{e^{-y^2}dy}{\sqrt{\pi}} \int_{-1}^{1} (1+x^2)dx \left\{ \frac{A_+(x,y)}{|F_+(x,y)|^2} - x\frac{A_-(x,y)}{|F_-(x,y)|^2} \right\} \tag{100}$$

The result of numerical computations using the last formula without the collisional broadening of the Raman resonance is

$$\frac{\delta\kappa_R^{sc,q}}{\kappa_R^{(0)}} = -0.7\% \tag{101}$$

By taking into account the collisional broadening of the Raman resonance we get

$$\frac{\delta\kappa_R^{sc,q,br}}{\kappa_R^{(0)}} = -1.0\% \tag{102}$$

17 QUANTUM EFFECTS IN FREQUENCY DIFFUSION AND STIMULATED SCATTERING

The effects of frequency diffusion are related to the inverse process of scattering which means that to take into account the quantum effects in them it is necessary to substitute $A_-(x,y)$ for $A(x,y)$ and substitute $F_-(x,y)$ for $F(x,y)$. For the stimulated scattering the result taking into account the quantum corrections can be written in a more compact form :

$$\sigma_0^{st,q}(z) = -\frac{3}{8}\frac{z}{e^z-1}\sigma_T \int_{-\infty}^{+\infty} \frac{e^{-y^2}dy}{\sqrt{\pi}} \int_{-1}^{1} (1+x^2) \left\{ \frac{A_+(x,y)}{|F_+(x,y)|^2} - \frac{A_-(x,y)}{|F_-(x,y)|^2} \right\} \tag{103}$$

$$\sigma_1^{st,q}(z) = -\frac{3}{8}\frac{z}{e^z-1}\sigma_T \int_{-\infty}^{+\infty} \frac{e^{-y^2}dy}{\sqrt{\pi}} \int_{-1}^{1} (1+x^2) \frac{(\omega'_{\pm}-\omega)}{\omega}$$

$$\left\{ \frac{A_+(x,y)}{|F_+(x,y)|^2} - \frac{A_-(x,y)}{|F_-(x,y)|^2} \right\} \tag{104}$$

The numerical calculations using formula (87) without the collisional broadening of the Raman resonance gives

$$\frac{(\delta\kappa_R^{fd,q} + \delta\kappa_{R,q}^{st})}{\kappa_R^{(0)}} = -3.0\% \tag{105}$$

By taking into account the collisional broadening of the Raman resonance we get

$$\frac{(\delta\kappa_R^{fd,q,br} + \delta\kappa_R^{st,q,br})}{\kappa_R^{(0)}} = -4.5\% \tag{106}$$

The last fugure is preliminary. When sustracted the effect of frequency diffusion in which the quantum corrections are not taken into account we find that the pure quantum corrections both for scattering itself and for stimulated scattering and frequency diffusion is close to -2%.

18 QUANTUM CORRECTIONS TO THE ELECTRON DEGENERACY

The change in solar opacity due to partial electron degeneracy was first considered in [16]. But together with the degeneracy the relativistic corrections were taken into account by using the $G(\tau, z)$ factor derived from the Klein-Nishina formula which can not be used for the conditions in the solar interior. To separate the effect of degeneracy from the incorrect relativistic correction we calculate using the results of [39] only the effect of degeneracy without the relativistic corrections which were already discussed above in detail. The corrections due to electron degeneracy should be added to the results given above for relativistic corrections including the collective effects. The result of our numerical calculation is

$$\frac{\delta\kappa_R^{deg}}{\kappa_R^{(0)}} = -2.0\% \tag{107}$$

19 THE TABLE OF NEW COLLECTIVE EFFECTS IN THE ROSSELAND OPACITY OF THE CENTER OF THE SUN

We given the final table for the ratio of the calculated new effects related to a zero order Rosseland opacity which does not take into account the line absorption and then give an approximate transition coefficient relating the results to the total Rosseland opacity accepted at the present time.

Table

N	The name of the effect	$\delta\kappa_R/\kappa_R^{(0)}$ in%
1.	Doppler and collisional broadening of Raman resonance	−3.0
2.	Relativistic corrections for scattering on electrons and electron polarization cloud of ions	−0.2
3.	Diffusion in frequencies and stimulated scattering	−4.5
4.	Collective effects in bremsstrahlung	−0.2
5.	Relativistic effects in bremsstrahlung	+0.2
6.	Quantum effects in scattering	−2.0
7.	Effects of electron degeneracy	−2.0
8.	Refractive index effects	+0.1
9.	Density inhomogeneity effects	−0.1
	Sum	−11.7
	$\frac{2}{3}$ of the sum	−7.8

Recently [39,40] the corrections due to ion correlations were calculated in [39] and are of the order of −1.5%. In the case one added this value to the −7.8% given in the Table one gets −9.3%. The latest data indicate also the contribution of line absorption in the center of the sun could be less than that obtained previously. and can be as low as 1/4 not 1/3 as it was assumed in the Table. In the case one takes this figure for iron absorption one should introduce not a factor 2/3 but a factor 3/4 which leads to −8.8% instead of −7.8% given in the Table. After adding −1.5% for ion correlations we get −10.3%. All these estlmates are given to show the uncertainties still existing in estimations of the value of solar opacity. Other uncertainties are discussed in the next section.

The total value of the change of the solar opacity due to the new collective effects is large and should be taken into account in future developments of any SSM. The Table given above gives the corrections in the central part of the Sun. For construction of a solar

model it is necessary to have tables of opacities with collective effects taken into account which can be used in all regions of the sun To find such a table is a rather combersome and not a simple mathematical problem as our experience shows.

But one important qualitative effect can be mentioned already here. As soon as the distance from the center of the Sun increases both the temperatures and densities drop. The Debye radius containing the square root of the ratio of temperature to density should change not as rapidly as the frequency of the maximum of the form-factor in the opacity which is $3.7T/\hbar$. So one may think that the collective effects should increase rapily with a distance from the center of the sun. They indeed increase but not so drastically as one can expect looking superficialy at this problem. The point is that one should be interested only in regions of electromagnetic flux formation which corresponds to the region where the thermonuclear reactions take place. Burning ceases already at distances of about $0.25R_{\odot}$. In this region the temperature is decreasing but not as fast as the density due to the nuclear burning. By using the existing SSM one finds the dependence of the collective parameter δ_e on the distance from the center of the Sun. These data provide the proof that there exist a systematic growth of the collective parameter in the whole range of distances up to $0.25R_{\odot}$. At the edge of this region an increase of the collective parameter is 1.7 times. This shows that the collective effects are important in the whole region of formation of the electromagnetic flux and that they are growing towards the edge of ths region. Thus the Table above gives only the lower limit of the collective corrections. These estimates can be improved in future after the collective effects are taken into account for all distances from the center of the Sun. Therefore the complete coverage of the contribution of collective effects as a funtion of distance from the center of the Sun will be an important problem for future investigations.

20 CONCLUSIONS

In conclusion we summarize the results of our analysis. Previously it was accepted that the uncertainty in the value of the Rosseland opacity can not be larger than 3% with the possibility of both sign corrections. In this paper the total effect of collective corrections leads to the conclusion that these corrections are of the same sign and give a *decrease* of Rosseland opacity by approximately 10%. Since the solar luminosity is inversely proportional to the Rosseland opacity and proportional to T^4 this decrease of opacity leads to a decrease of the estimated temperature in the center of the Sun by about 3%. A strong dependence of the high energy neutrino flux on temperature leads to a decrease of the predicted flux by a factor 2 or 2.5. Although this result agrees better with the observations and theoretical predictions it does not solve completely the problem of the neutrino deficit, since for example the deficit of beryllium neutrino's decreases less than that of boron neutrino's, while the observations seem to indicate that the deficit of beryl-

lium neutrinos is larger than that of boron neutrinos. But the beryllium neutrinos are not measured directly in one and the same experiment. The difference of the neutrino deficits in different energy channels can also be due to the influence of collective plasma effects on the nuclear reactions and particularly on the process of capture of protons by beryllium ions in the dense solar plasma. There exist also other problems which are noted in the above analysis. But independently of them the 10% change of the opacity is a rather large effect in the problem of the solar neutrino deficit.

Let us once more emphasize that we present the estimation of *all* collective plasma effects which for δ_e of the order of one have a smallness v_{Te}^2/c^2, a complete search of such effects was performed. We used new analytical results previously not calculated for most of these effects. The numerical results were double checked by standard numerical programs and by a special numerical program developed for this problem in the Institute of Applied Mathematics (Napoli). The difference in the results is about one tenth of one percent. The numerical program developed in theInstitute of Applied mathematic is rather large and it checks the accuracy at each stage of the calculations. The standard numerical program needs more than one week on a P.C. to calculate one point of the corrections. In the Table the results of computations performed in the Institute of Applied Mathematics (Napoli) are given. It takes more time to get a result but it is more accurate.

We will give the limits of the uncertainties of the results. These uncertainties are related to the new problems which arise from the considerations made in this article. We give a list of the problems and some comments to this list.

1.Collective effects in line absorption.

The most important is the Fe ion line the wavelength of which almost exactly coincides with the Debye length. Up to the present time the line absorption on Fe ions was considered as if it were in a vacuum. There exists methods to treat the problem when both the bounded electrons and the Debye shielding electrons contribute to the absorption. Bremsstrahlung absorption in the case where the frequency is close to the resonance has some specific features and the best term is the line absorption [23]. The method is explained in detail in Chapter 6 of the monograph [23]. Since the relative contribution to the opacity of the absorption on iron ions is approximately 1/3 even a relatively small correction to the absorption by iron ions can change substantially the opacity. This problem is not yet solved and should be the subject of future investigations. The collective effects in absorption on iron ions is necessary to bear in mind when estimating the remaining uncertainties in opacity.

2.Solutions of the transport equation of radiation with effects of frequency diffusion and stimulated scattering without using perturbation methods.

The problem is only formulated for the general collective case. For scattering in vacuum some preliminary results are obtained. The observed effect of rapid changes of intensity with frequency probably indicates that a more precise consideration of the frequency diffusion effects and the stimulated scattering effects will result in a further decrease of the value of the predicted opacity.

3.Collective effects in capturing of protons and electrons by ^7Be ions.

The formulation of the problem discussed in the text above; the only comment is that in plasmas the electrons (or protons) simultaneously play a role of the particle which can be captured and the role of the particle which contributes to the Debye screening. This has not yet been taken into account. The proper consideration can be made by developing the theory of fluctuations in a plasma which takes into account the possibility of capturing of electrons by the nucleus of ions.

4.The possibility of the existence of the difference of electron and ion temperatures in the solar interior.

Although this possibility is interesting; to make a definite statement that this possibility can be realized in the solar interior it is necessary to perform additional investigations. In many laboratory plasmas the electron temperature is higher than the ion temperature. An example of this kind of plasmas (which seems in some sense to model the solar conditions) is radio frequency discharges in plasma where the electrons receive the energy from the RF external field faster than they transfer the energy to ions. These experiments of course are opposite to the solar conditions in the sense they correspond to the case of optically thin plasmas. Nevertheless in the solar interior the radiative transfer occurs in a similar manner since the radiation transferred is absorbed on electrons and the time of the conversion of this energy to ions via electron-ion binary collisions is 5 times larger than the characteristic time of receiving the energy by electrons from the radiation. This estimation does not mean that we insist on the existence of a difference of the electron and ion temperatures inside the solar interior by this reason. But this problem we think should be analysed in more detail having in mind also the possibility of the development of instabilities which often make the electron temperature larger than the ion temperature.

Why is this effect, if it exists, of importance? First of all even from the formulas of the zero approximation given in this articles for the case of an arbitrary difference between of electron and ion temperatures it is easy to find that the additional decrease of the opacity taking into account the factor 2/3 will be -2.7%. Secondly the solar seismology shows the presence of sound waves which from a general point of view can form the usual cascade toward smaller wavelengths. For the case of equal temperatures this cascade can not propagate in the collision less range of sound frequencies but it can propagate in the presence of a temperature difference. Also the well known plasma physics effect predicts

a formation of a tail in the proton distribution which can alter the predictions of the neutrino flux. But all these possibilities are at the present time only speculative.

5.A possibility of the development of instabilities generated by the transport process.

This problem is only partially discussed in the text. Note that photons with different frequencies are absorbed over different lengths. The question is whether this effect can produce unstable nonthermal electron distributions?

6.Anomalous transport of radiation due to the possibility of the presence of turbulence in the central regions of the Sun.

There exists some indications that the central region of the Sun can have a larger differential rotational velocity as well as other indications that the central region of the Sun can be convectionally unstable. In the case where these statement have some observational support it will be necessary to analyse the possibility of anomalous radiative transfer of radiation.

7.Energy transport by fast particles generated by turbulence.

This problem still waits to be calculated.

In the construction of future SSM's it will be necessary to take into account not only the new collective plasma effects discussed above but also the problems stated in the conclusion section.

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FIGURE CAPTIONS

Fig.1.Dependence of the cross-sections of scattering of photons on electrons and ions in the solar interior on the photon frequency. Plasma electron density $n_e = 5.4 \times 10^{25} cm^{-3}, T_e = T_i = 1.5 kev, v_{Te} = 1.53 \times 10^9 cm/s, z_0 = \hbar\omega_{pe}/T = 0.21, < Z >= 1.53$. The solid line shows the cross-section of scattering on electrons, the dotted line shows the cross-sections of the sum of scattering on ions (abundance of elements is taken from a standard solar model [3]),the dashed line shows the sum of scattering on electrons and ions, the dashed-dotted line shows the 1/5 of form factor $z^4 \exp(z)/(\exp(z) - 1)^2$ which enters in the expression for opacity.

Fig.2.Dependence of cross-sections of scattering on the collective parameter δ_e. The figure gives both the transport cross-section and the usual cross-sections: the solid and the dotted lines correspond to scattering on electrons, the dashed and the dotted lines correspond to scattering on ions, the solid and dotted lines correspond to the sum of of scattering on electrons and ions; $< Z >= 1.53, T_e = T_i$

Fig.3. Dependence of the total transport cross-section of scattering on the collective parameter for different ratios of electron to ion temperature. The cross-sections decrease continiously with increase of $\tau = T_e/T_i$: the solid line (upper) corresponds to $\tau = 1$, the dotted line (upper) corresponds to $\tau = 2$, the dashed line corresponds to $\tau = 3$, the dashed-dotted line corresponds to $\tau = 4$, the solid line (lower) corresponds to $\tau = 5$, the dotted line (lower) corresponds to $\tau = 6$.

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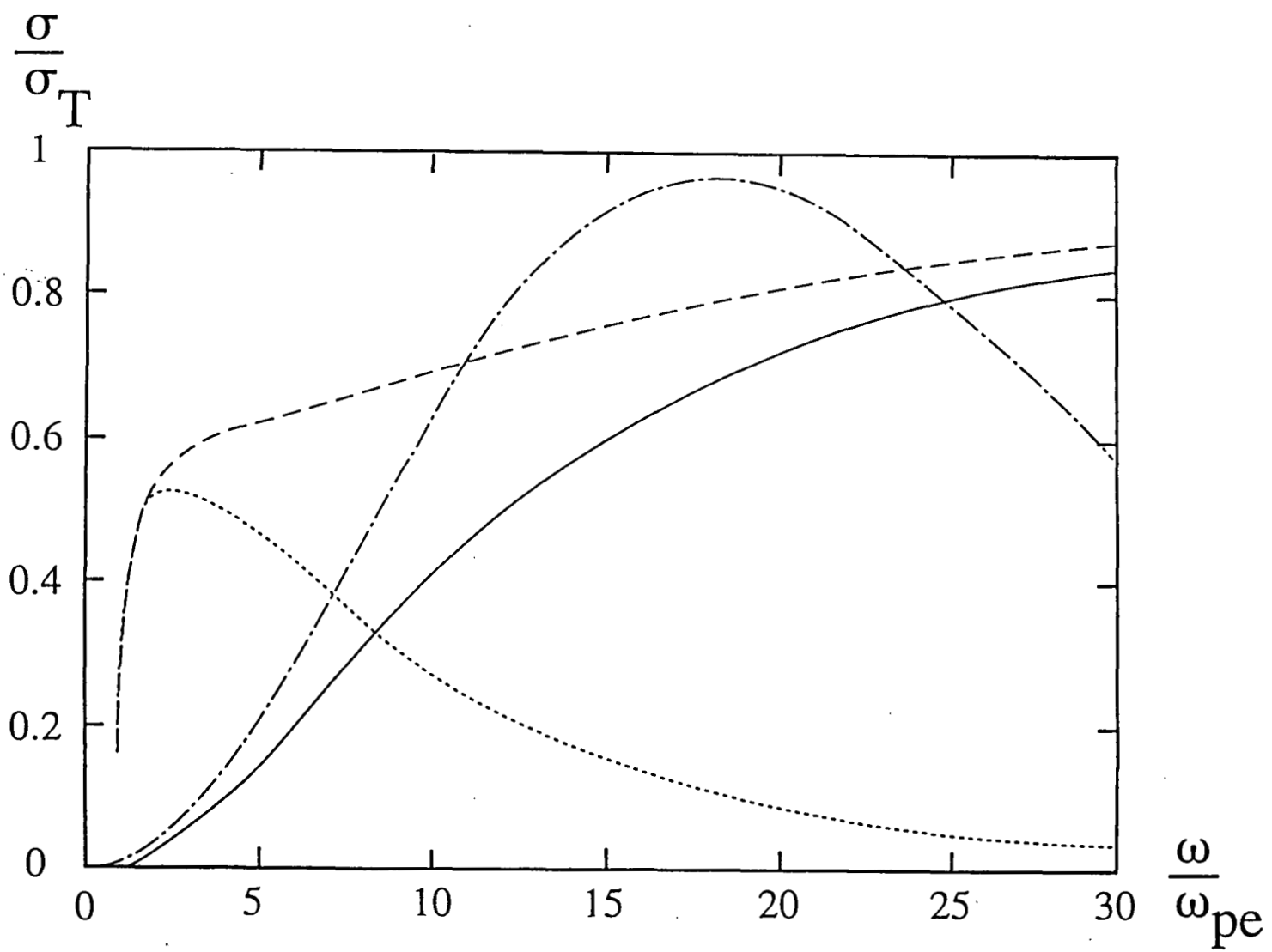

Fig 1

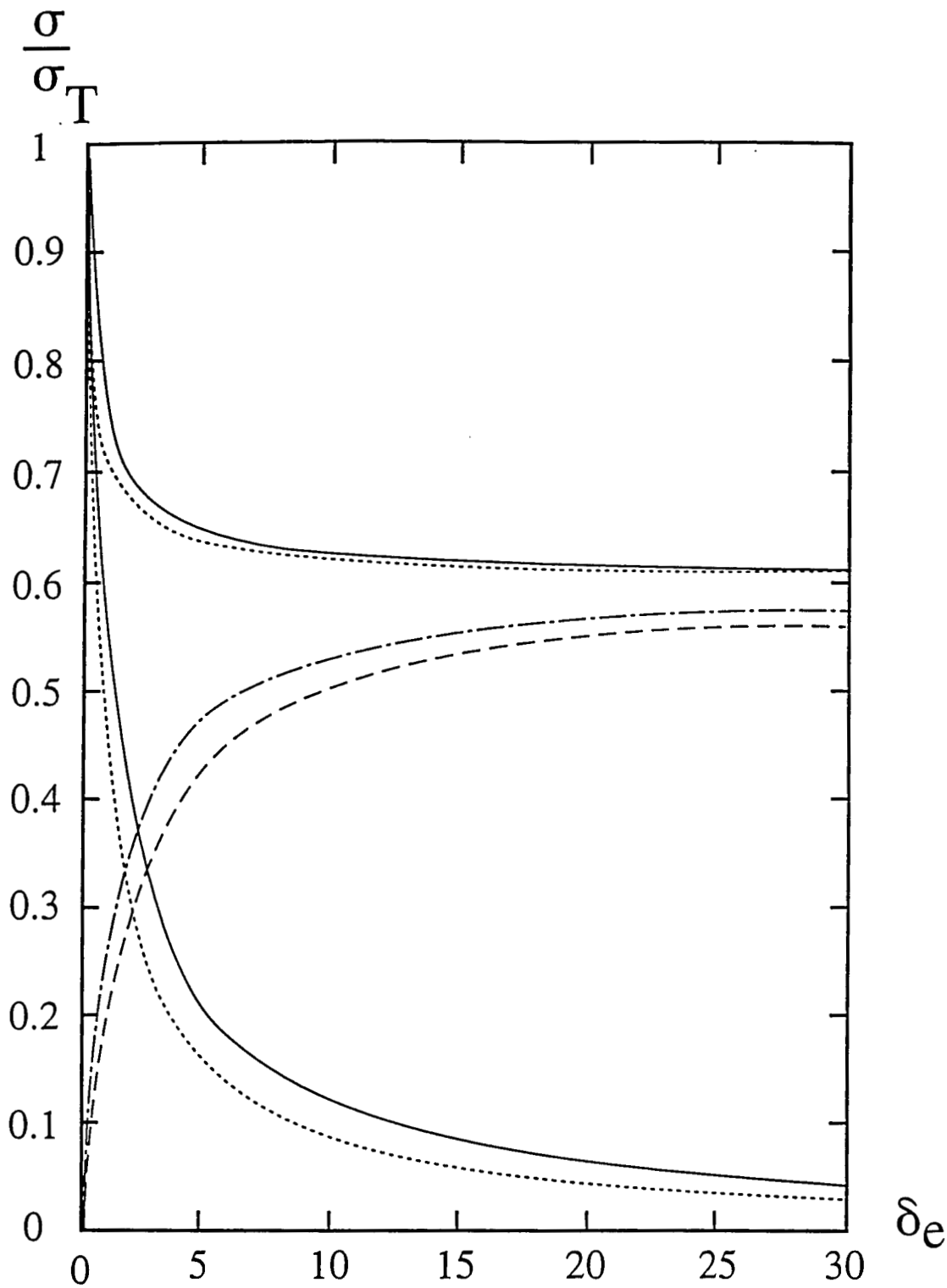

Fig 2

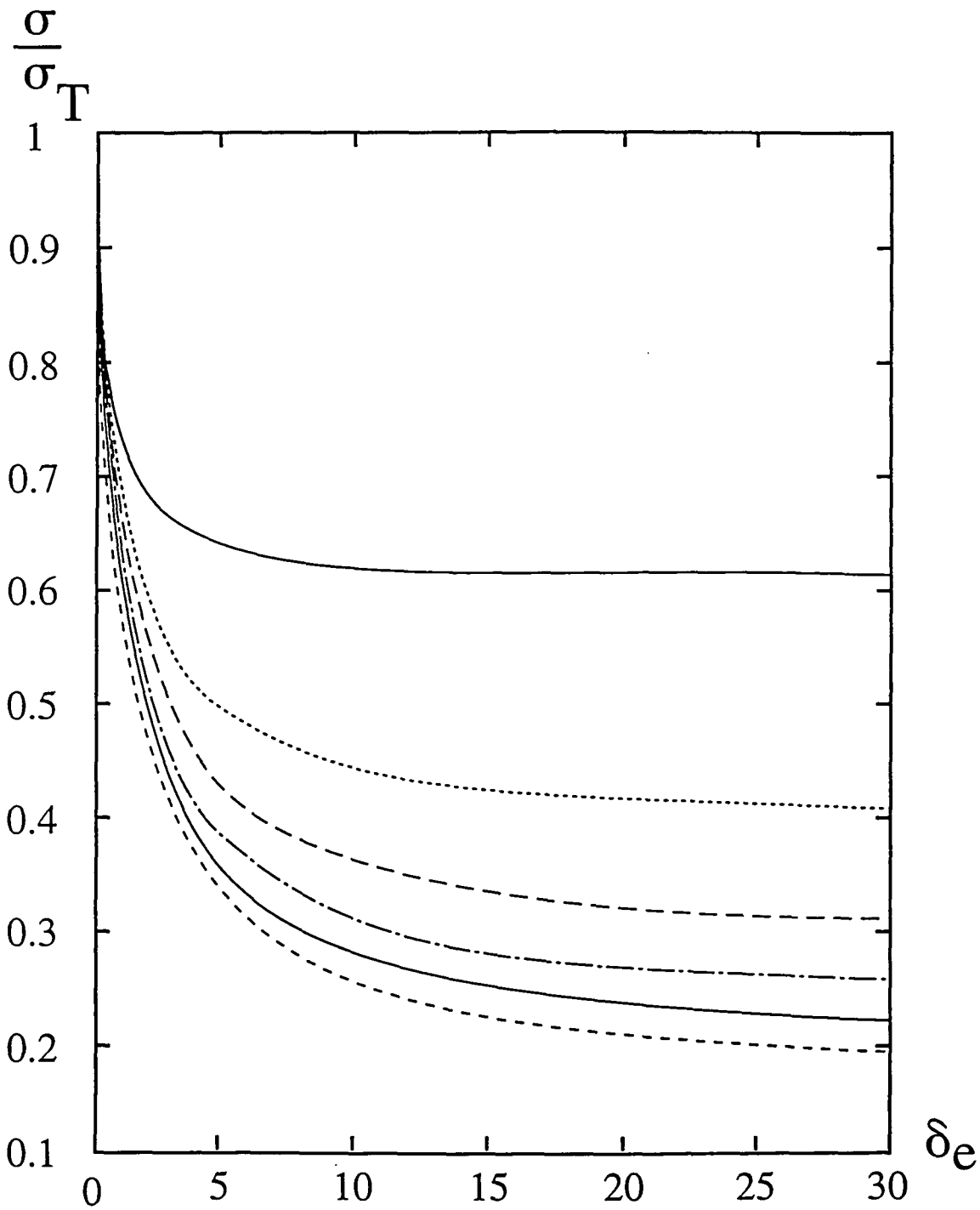

Fig 3