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ABSTRACT

A $\mu^+\mu^-$ collider is an appealing machine to probe resonant CP-violating transitions between a CP-even Higgs particle and the Z boson or between Higgs scalars with different CP quantum numbers. The size of the CP-violating effects is estimated by using an observable of CP asymmetry based on longitudinally polarized muons. These phenomena are studied within a manifestly gauge-invariant approach implemented by the pinch technique, which respects the discrete symmetries of the classical Lagrangian and is therefore free from CP-odd gauge artifacts. The CP invariance of an extended Higgs sector motivated by E_6 supersymmetric models is assumed to be broken by the presence of heavy Majorana fermions. CP violation originating from Higgs-Z mixing is found to be very modest, whereas CP-number violating transitions involving Higgs scalars only can be resonantly enhanced up to order of unity.

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Recently, much research and theoretical effort have been put to the design and the physics capabilities of a muon collider, which could potentially serve as an important Higgs factory [1]. Given the technical facilities of such a collider, it has been argued [1] that one could exploit the resonant enhancement of an s-channel interaction to copiously produce Standard Model (SM) Higgs-bosons, H, in the mass range $100 \leq M_H \leq 200$ GeV, despite the smallness of the $H\mu\mu$ coupling. On the other hand, a muon collider may be the most ideal place to search for large Higgs-Z-boson mixing effects or for transitions of Higgs scalars with opposite CP quantum numbers within extensions of the SM, which may have an underlying supersymmetric (SUSY) origin. In fact, if quantum effects allow for a CP-even Higgs scalar, H, to go into the CP-odd Z boson or another CP-odd scalar, A say, such a transition alone would signify CP/T violation in a CPT-invariant theory [2]. In the SM, there is no HZ mixing up to two-loop electroweak order. The reason is that a non-trivial CP-odd rephasing invariant combination of Cabbibo-Kobayashi-Maskawa (CKM) matrix elements is required inside the fermionic loops. However, in natural extensions of the SM, involving Majorana fermions [3] or more than one Higgs doublet [4], a HZ-mixing effect may occur in the decays of the H into top-quark pairs.

In this paper, we study the possibility of CP-violating HZ and/or HA transitions in a model, in which the CP invariance of the Higgs sector is broken by the presence of heavy Majorana fermions. Such a model may be the minimal SUSY model, in which Majorana fermions could be identified with the heavy neutralinos, or other scenarios inspired by E_6 theories, which can predict heavy Majorana neutrinos [5,6] at the TeV mass scale. Here, we will work on the latter realization. To be specific, we adopt the CP-violating scenario of [3] for the neutrino sector, which may resemble the model discussed by the authors in Ref. [6] at the electroweak scale. The model contains three heavy Majorana neutrinos, denoted here by N_1, N_2, and N_3, from which N_1 is predominantly a sequential isodoublet, whereas N_2 and N_3 are mainly singlets under $SU(2)_L$. Furthermore, we consider that the Lagrangians governing the interactions between N_i (with $i = 1, 2, 3$) and H, A, and the would-be Goldstone boson G^0, have the following generic form [7]:

$$\mathcal{L}_A = \frac{ig}{4M_W} A\chi_A^u \sum_{i,j=1}^{3} \bar{N}_i\Big[\gamma_5 (m_i + m_j)\Re eC_{ij} + i(m_j - m_i)\Im mC_{ij}\Big]N_j , \qquad (1)$$

$$\mathcal{L}_H = -\frac{g}{4M_W} H\chi_H^u \sum_{i,j=1}^{3} \bar{N}_i\Big[(m_i + m_j)\Re eC_{ij} + i\gamma_5(m_j - m_i)\Im mC_{ij}\Big]N_j , \qquad (2)$$

where the parameters $\chi_{A,H}^u$ are related with the vacuum expectation values (VEVs) of the Higgs fields in an extended Higgs sector and C_{ij} are mixing matrices defined in [7]. The coupling $G^0N_iN_j$ can be recovered from Eq. (1), if we set $\chi_H^u = 1$ and $A \equiv G^0$. This model has a non-trivial CP-violating phase contained in the rephasing invariant quantity $\Im mC^2_{N_1N_2} = \sin\delta_{CP}|C_{N_1N_2}|^2$ [3], which is taken to be maximum of order one.

To analyze CP violation originating from HZ and/or HA mixing, we have to find an observable sensitive to this kind of effects. Assuming that the facility of having longitudinally polarized muon beams will be available without much loss of luminosity, we can define the CP asymmetry

$$\mathcal{A}_{CP} = \frac{\sigma(\mu_L^-\mu_L^+ \to f\bar{f}) - \sigma(\mu_R^-\mu_R^+ \to f\bar{f})}{\sigma(\mu_L^-\mu_L^+ \to f\bar{f}) + \sigma(\mu_R^-\mu_R^+ \to f\bar{f})} . \qquad (3)$$

We must emphasize that $\mathcal{A}_{CP}$ is a genuine observable of CP violation if one is able to tag on the final fermion pair $f\bar{f}$ (*e.g.*, $\tau^+\tau^-$, $b\bar{b}$, or $t\bar{t}$), since the helicity states $\mu_L^-\mu_L^+$ transform into $\mu_R^-\mu_R^+$ under CP in the centre-of-mass system (CMS). Similar CP/T-violating observables based on T-odd aplanarities at e^+e^- machines were considered by the authors of Ref. [8], who suggested to look for CP violation in vector and axial-vector currents. Here, we require, however, that both muons are left-handed or right-handed polarized. Similar ideas have been applied to study CP violation in the top-pair production at LHC and TeV-e^+e^- colliders [9,4,3] and, more recently, to muon colliders [10] as well.

In our analysis, we consider a manifestly gauge-invariant approach for resonant transitions [11], which is implemented by the pinch technique (PT) [12]. This approach is free from CP-odd gauge artifacts; it reassures the absence of a HZ mixing in a CP-invariant and anomaly-free theory, thus preserving the discrete symmetries of the classical action after quantization. Furthermore, we make use of a mechanism for resonant CP violation induced by particle widths in scattering processes, which was discussed in [13] some time ago. This mechanism of CP violation gives rise to resonant enhancement for certain CP-violating observables, such as $\mathcal{A}_{CP}$ in Eq. (3), and so yields measurable effects for a wide range of heavy Higgs masses as we will see below.

We now discuss in short how resummation involving ZH mixing takes place within our approach. There are PT identities that can be employed to convert ZH and ZZ strings into G^0H and G^0G^0 ones [14] before resummation occurs. These identities are

$$p^\mu \widehat{\Pi}^{ZH}_\mu + iM^0_Z \widehat{\Pi}^{G^0H} = 0\,, \quad p^\mu p^\nu \widehat{\Pi}^{ZZ}_{\mu\nu} - (M^0_Z)^2 \widehat{\Pi}^{G^0G^0} = 0\,, \text{and} \quad p^\mu \Gamma^{Zf\bar{f}}_\mu = -iM^0_Z \Gamma^{G^0f\bar{f}}\,, \tag{4}$$

where p^μ of the Z boson always flows into the fermionic vertex and M^0_Z is the bare Z-boson mass. If $\widehat{\Pi}^{ZH}_\mu(p) = p_\mu \widehat{\Pi}^{ZH}(p^2)$ and $\widehat{\Pi}^{ZZ}_{\mu\nu}(p) = t_{\mu\nu}(p)\widehat{\Pi}^{ZZ}_T(p^2) + \ell_{\mu\nu}(p)\widehat{\Pi}^{ZZ}_L$, with $t_{\mu\nu}(p) = -g_{\mu\nu} + p_\mu p_\nu/p^2$ and $\ell(p) = p_\mu p_\nu/p^2$, then $\widehat{\Pi}^{ZH}(p^2) = -iM^0_Z \widehat{\Pi}^{G^0H}(p^2)/p^2$ and $\widehat{\Pi}^{ZZ}_L(p^2) = (M^0_Z)^2 \widehat{\Pi}^{G^0G^0}(p^2)/p^2$. In this context, it worth stressing the fact that $p^\mu \widehat{\Pi}^{\gamma G^0}_\mu(p) = p^\mu \widehat{\Pi}^{\gamma H}_\mu(p) = 0$, within the PT framework, which implies the absence of γH and γG^0 mixing, *i.e.*, $\widehat{\Pi}^{\gamma H}_\mu = \widehat{\Pi}^{\gamma G^0}_\mu = 0$, independently of whether CP-violating interactions are present in the theory. As a result, one is left with solving the simple coupled Dyson-Schwinger equation system, in which only H and G^0 mix, *i.e.*,

$$[\hat{\Delta}(p^2)]^{-1} = \left[\begin{array}{cc} p^2 + \widehat{\Pi}^{G^0G^0}(p^2) & \widehat{\Pi}^{G^0H}(p^2) \\ \widehat{\Pi}^{HG^0}(p^2) & p^2 - (M^0_H)^2 + \widehat{\Pi}^{HH}(p^2) \end{array} \right], \tag{5}$$

where the convention $i\Pi(p^2)$ is used. Inverting this matrix, we find

$$\begin{aligned} \hat{\Delta}_{G^0}(p^2) &= \left\{ p^2 + \widehat{\Pi}^{G^0G^0}(p^2) - [\widehat{\Pi}^{G^0H}(p^2)]^2/[p^2 - (M^0_H)^2 + \widehat{\Pi}^{HH}(p^2)] \right\}^{-1}, \\ \hat{\Delta}_{H}(p^2) &= \left\{ p^2 - (M^0_H)^2 + \widehat{\Pi}^{HH}(p^2) - [\widehat{\Pi}^{G^0H}(p^2)]^2/[p^2 + \widehat{\Pi}^{G^0G^0}(p^2)] \right\}^{-1}, \\ \hat{\Delta}_{G^0H}(p^2) &= -\widehat{\Pi}^{G^0H}(p^2) \Big\{ [p^2 - (M^0_H)^2 + \widehat{\Pi}^{HH}(p^2)][p^2 + \widehat{\Pi}^{G^0G^0}(p^2)] \\ &\quad - [\widehat{\Pi}^{G^0H}(p^2)]^2 \Big\}^{-1}. \end{aligned} \tag{6}$$

The above considerations can be extended to include additional Higgs scalars, such as the physical CP-odd scalar, A. In such a case, the inverse propagator in Eq. (5) becomes a 3×3 matrix.

Within the PT, it is known [15] that the analytic expression of $\Im m\widehat{\Pi}^{HH}(s)$ coincides, to one loop, with that of the background field gauge for $\xi_Q = 1$. More explicitly, we obtain for the different channels

$$\Im m\widehat{\Pi}^{HH}_{(f\bar{f})}(s) = \frac{\alpha_w N_c^f}{8}(\chi_H^f)^2\, s\, \frac{m_f^2}{M_W^2}\left(1 - \frac{4m_f^2}{s}\right)^{3/2} \theta(s - 4m_f^2)\,, \tag{7}$$

$$\begin{aligned}\Im m\widehat{\Pi}^{HH}_{(VV)}(s) &= \frac{n_V \alpha_w}{32}(\chi_H^V)^2 \frac{M_H^4}{M_W^2}\left(1 - \frac{4M_V^2}{s}\right)^{1/2} \\ &\quad \times \left[1 + 4\frac{M_V^2}{M_H^2} - 4\frac{M_V^2}{M_H^4}(2s - 3M_V^2)\right] \theta(s - 4M_V^2)\,, \end{aligned} \tag{8}$$

where $\alpha_w = g^2/4\pi$, $n_V = 2,\ 1$ for $V \equiv W,\ Z$, respectively, and $N_c^f = 1$ for leptons and 3 for quarks. In Eqs. (7) and (8), we have parametrized fermionic and bosonic channels by the model-dependent factors χ_H^f and $\chi_H^{W,Z}$. By analogy, calculation of $\Im m\widehat{\Pi}^{AA}(s)$ gives $\chi_A^V = 0$, while χ_H^f should then be replaced by $(1 - 4m_f^2/s)^{-1}\chi_A^f$ in Eq. (7). There may also be other channels involving the HZA vertex, which are, however, considered to be phase-space suppressed in the kinematic region $M_H \simeq M_A$, relevant for resonant CP violation. The complete treatment including these effects as well as other refinements due to SUSY may be given elsewhere.

Considering the Lagrangians (1) and (2), it is straightforward to calculate the CP-violating HG^0 and/or HA mixing in our model, viz.

$$\frac{\widehat{\Pi}^{AH}(s)}{s} = -\frac{\alpha_w}{4\pi}\, \chi_A^u \chi_H^u \sum_{j>i}^{3} \Im m C^2_{N_iN_j} \sqrt{\lambda_i\lambda_j}\left[B_0(s/M_W^2, \lambda_i, \lambda_j) + 2B_1(s/M_W^2, \lambda_i, \lambda_j)\right], \tag{9}$$

where $\widehat{\Pi}^{AH}(s) = \widehat{\Pi}^{HA}(s)$, $\lambda_i = m_{N_i}^2/M_W^2$, and B_0 and B_1 are the usual Veltman-Passarino loop functions, expressed in the convention of Ref. [16]. Again, the HG^0 transition is recovered from Eq. (9) by setting $\chi_A^u = 1$.

From Fig. 1, it is not difficult to see that in the case of HZ mixing, only two diagrams can contribute constructively to $\mathcal{A}_{CP}$ through the interference of the G^0-exchange graph with the amplitude depending on $\hat{\Delta}_{HG^0}$. The reason is that contraction of a scalar current with a pseudoscalar one vanishes identically. In this way, we obtain

$$\mathcal{A}_{CP}(s) \;=\; -2\,\frac{\widehat{\Pi}^{G^0H}(s)}{s}\;\frac{\Im m\widehat{\Pi}^{HH}(s)}{s}\,. \tag{10}$$

From Eq. (10), we find that CP asymmetries can be large only for heavy Higgs boson masses far above the two-real W-boson production threshold, since the Higgs width is then comparable to M_H. To give an example, we find $\mathcal{A}_{CP} \simeq 2.\ 10^{-2}$ for $M_H = 500$ GeV and $m_{N_1,N_2,N_3} = 0.5,\ 1.5,\ 3$ TeV, while the production cross-section is $\sigma \simeq 1$ fb. It seems that one is unlikely to observe HZ-mixing effects, even if assuming a high integrated luminosity of 50 fb^{-1} for the muon collider [1].

The situation changes drastically if the heaviest CP-even H mixes with a CP-odd Higgs scalar, A, with $M_A > 2M_Z$. In a SUSY model, H may couple predominantly to fermions, *i.e.*, $\chi_H^W \simeq 0$, and naturally be degenerate with A, $M_H \simeq M_A$ [17]. Moreover, the coupling parameters of the model are $\chi_H^d = \chi_A^d = 1/\chi_H^u = 1/\chi_A^u = \tan\beta$. Taking these into account, we have $\mathcal{A}_{CP}$ behaving at $s \simeq M_H^2$ as

$$\mathcal{A}_{CP} \;\sim\; -\,\frac{2\widehat{\Pi}^{AH}(s)\,[\Im m\widehat{\Pi}^{HH}(M_H^2) + \Im m\widehat{\Pi}^{AA}(M_H^2)]}{(M_H^2 - M_A^2)^2 + [\Im m\widehat{\Pi}^{AA}(M_H^2)]^2 + [\Im m\widehat{\Pi}^{HH}(M_H^2)]^2}\,. \tag{11}$$

In Fig. 2, we have presented cross sections (solid lines) and CP asymmetries (dotted lines) as a function of the CMS energy, $\sqrt{s}$, in two different scenarios. We also assume that tuning the collider energy to the mass of H is feasible, *i.e.*, $\sqrt{s} = M_H$. In our estimates, we take $\tan\beta = 2$, $m_{N_1,N_2,N_3} = 0.4,\ 0.7,\ 1$ TeV, and $m_t = 170$ GeV. We analyze two reactions: (a) $\mu_L^+\mu_L^- \to b\bar{b}$, for $M_A = 170$ GeV, and (b) $\mu_L^+\mu_L^- \to t\bar{t}$, for $M_A = 400$ GeV. In reaction (a), one can have a significant CP-violating signal if $M_H = 170 \pm 8$ GeV. We also observe the resonant enhancement of CP violation when $M_H = M_A$. More promising is the reaction (b), in which CP violation may be observed for a wider range of Higgs-boson masses, *i.e.*, for $M_H = 350$ GeV $-$ 430 GeV. Again, the mechanism of resonant CP violation is very important to render the effect measurable.

In conclusion, we have analyzed the possibility of CP-violating HZ or HA transitions at a proposed collider [1] with the technical facility of longitudinally polarized muons [1,10]. Although HZ-mixing effects may be elusive in such a collider, the mixing of a CP-even Higgs scalar, H, with a CP-odd Higgs, A, induced by Majorana fermions can be large in E_6 inspired SUSY models [5,6]. Note that this mixing may also be generated in the minimal SUSY SM, where the rôle of Majorana fermions is assumed by neutralinos and charginos. In fact, we have found that the interference between H and A can lead to resonant enhancement of the CP-violating observable, $\mathcal{A}_{CP}$, as shown in Fig. 2, thus making CP violation measurable. Finally, we must stress that, even though building a muon machine with high degree of polarization may become a difficult task, our analysis will, however, carry over to searches for resonant CP-violating effects in the decay products of the final states, *i.e.*, in the angular-momentum distributions and energy asymmetries of the produced charged leptons and jets [9].

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Figure Captions

Fig. 1: CP-violating HA and HZ (for $A \equiv G^0$) transitions in $\mu^+\mu^-$ collisions.

Fig. 2: Numerical estimates of production cross-sections and CP violation for $\mu^-_{L,R}\,\mu^+_{L,R} \to (H,\ A) \to f\bar{f}$ in two different SUSY scenarios with heavy Majorana neutrinos (see also text).

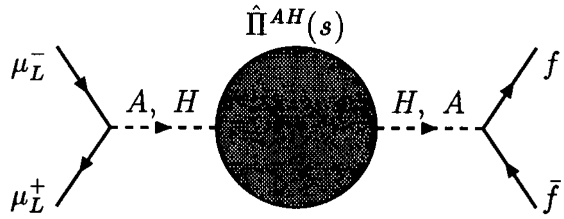
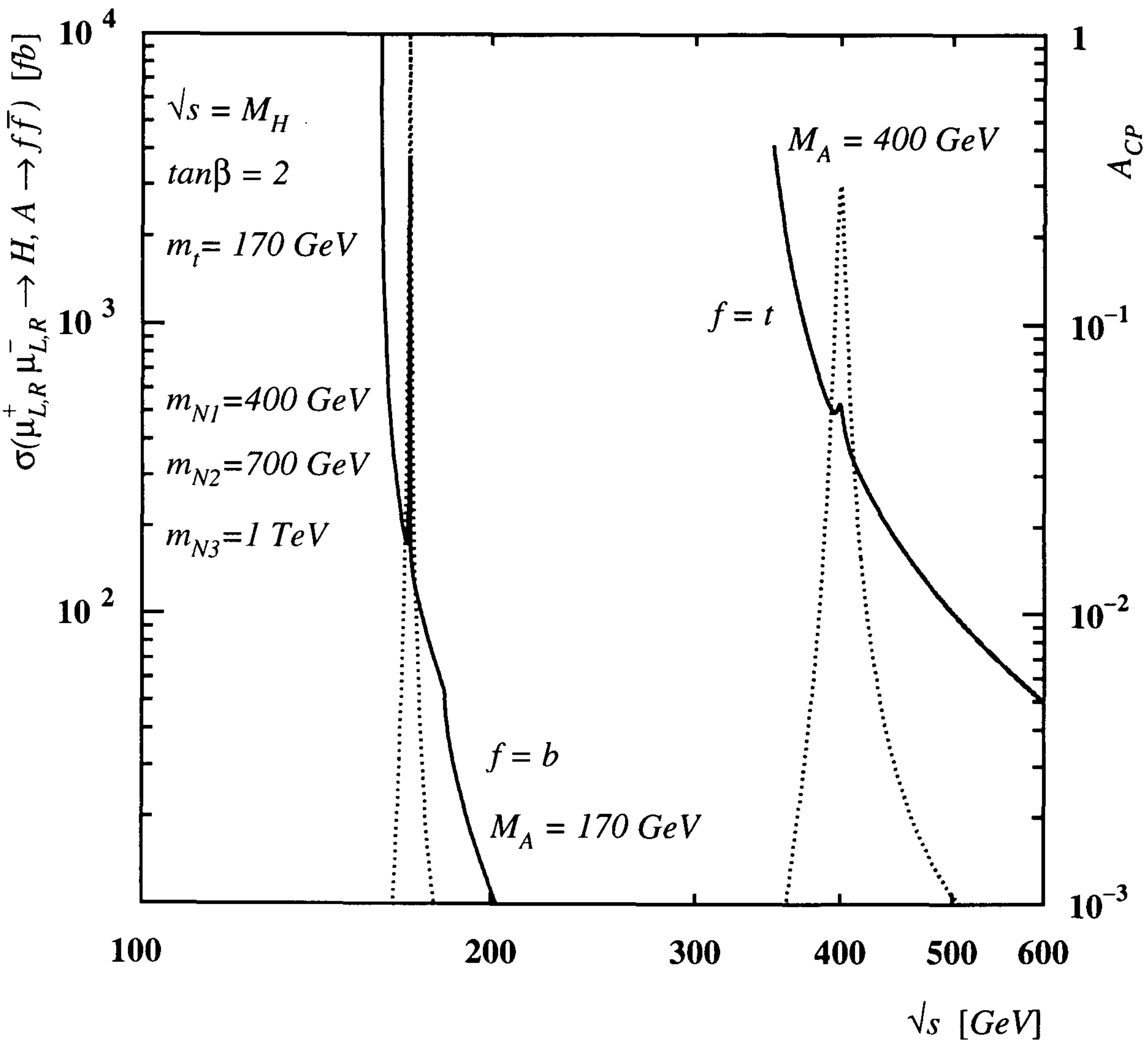


Fig. 1



10^4
10^3
10^2
1
10^{-1}
10^{-2}
10^{-3}
$\sigma(\mu^+_{L,R}\mu^-_{L,R} \to H, A \to f\bar{f})$ [fb]
A_{CP}
$\sqrt{s} = M_H$
$tan\beta = 2$
m_t= 170 GeV
m_{N1}=400 GeV
m_{N2}=700 GeV
m_{N3}=1 TeV
M_A = 400 GeV
$f = t$
$f = b$
M_A = 170 GeV
100
200
300
400
500
600
$\sqrt{s}$ [GeV]

Fig. 2