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# TMDlib and TMDplotter: library and plotting tools for Transverse-momentum-dependent parton distributions Version 1.0.0

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**Abstract**

Transverse-momentum-dependent distributions (TMDs) are central in high-energy physics from both theoretical and phenomenological points of view. In this manual, we introduce the library, TMDlib, of fits and parameterisations for transverse-momentum-dependent parton distribution functions (TMD PDFs) and fragmentation functions (TMD FFs) together with an online plotting tool, TMDplotter. We provide a description of the program components and of the different physical frameworks the user can access via the available parameterisations.

## PROGRAM SUMMARY

*Title of Program:* TMDlib 1.0.0

*Computer for which the program is designed and others on which it is operable:* any with standard C++, tested on SGI, HP-UX, SUN, PC, MAC

*Programming Language used:* C++

*High-speed storage required:* No

*Separate documentation available:* No

*Keywords:* QCD, TMD factorisation, high-energy factorisation, TMD PDFs, TMD FFs, unintegrated PDFs, small-$x$ physics.

*Other Program used:* ROOT for plotting the result.

*Download of the program:* `http://tmdlib.hepforge.org`

*Unusual features of the program:* None

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*Citation policy:* please cite the current version of the manual and the paper(s) related to the parameterisation(s).

# 1 Introduction

The Quantum Chromodynamics (QCD) interpretation of high-energy particle reactions requires a simultaneous treatment of processes at different energy scales. Factorisation theorems provide the mathematical framework to properly separate the physical regimes. For instance, when two protons collide in a Drell-Yan (DY) event, one describes the high-energy partonic cross section with a fixed-order perturbative QCD expansion and the soft physics underlying the structure of the hadrons with parton distribution functions (PDFs) and fragmentation functions (FFs), supplemented by evolution. The same picture applies to other (semi-)inclusive processes, including Deep-Inelastic lepton-hadron Scattering (DIS), Semi-Inclusive DIS (SIDIS), and $e^+e^-$ annihilation into hadrons. A PDF describes the likelihood for finding a parton of a particular momentum inside an incoming hadron. In processes with observed hadrons in the final state, FFs enter to describe the transition from a partonic state to an observed final-state hadron.

For sufficiently inclusive processes, only the component of parton momentum collinear to the momentum of its parent hadron is relevant at leading power (*leading twist*) in the hard scale. Factorisation theorems for such processes are traditionally called *collinear* factorisation theorems. In less inclusive processes, however, sensitivity to the intrinsic partonic motion transverse to the direction of the parent hadron can become important. In such cases, the PDFs and FFs must carry information about intrinsic transverse parton momentum in addition to the collinear momentum. One must introduce Transverse-momentum-dependent (TMD) PDFs and FFs and use them in the context of new factorisation theorems, called TMD factorisation theorems. TMD factorisation has been formulated for a number of semi-inclusive processes including SIDIS, DY and $e^+e^-$ annihilation [1–13]. For particular processes in hadronic collisions, like heavy flavour or heavy boson (including Higgs) production, TMD factorisation has also been formulated in the high-energy (small-$x$) limit [14–17]. In this context, the functions encoding the hadronic structure are more often referred to as *unintegrated* parton distribution functions (uPDFs), see *e.g.* Refs. [18–26].

The presence of a large variety of TMD factorisation and evolution frameworks complicates efforts to compare different TMD PDF/FF and uPDF parameterisations. In this paper, we describe a new tool for collecting different fits and parameterisations into a single library, TMDlib, and the online plotter tool, TMDplotter. Provided that the user takes into account all the possible differences between formalisms, collecting parameterisations for both the objects in TMDlib and TMDplotter will also make phenomenological comparisons easier. The paper is organised as follows: In Sec. 2, we briefly introduce the theoretical framework for both TMD and high-energy factorisation and evolution. In Sec. 3, we present a concise documentation of the TMDlib library and TMDplotter tool, discussing the basic procedure to readily use them.

## 2 Theoretical framework

In this section, we briefly describe two different commonly-used frameworks for factorisation and evolution of parton distributions. Specifically, we discuss TMD and high-energy factorisation theorems and evolution equations.

### 2.1 TMD factorisation and evolution

When one hard scale enters a high-energy process (like the invariant mass of the exchanged virtual photon in DIS) and the relevant transverse momenta are integrated over, one applies *collinear* factorisation to separate the hard partonic physics from the soft hadronic physics. When sensitivity to intrinsic transverse momentum is important, one must go beyond the collinear framework to factorise perturbative and non-perturbative dynamics. For example, this is the case in processes with observed transverse momenta in the final states, like SIDIS and DY lepton pair production at low transverse momentum. In these cases the low transverse momentum provides greater access to novel QCD dynamics as compared to the collinear case. If the observable transverse momenta are much larger than $\Lambda_{\rm QCD}$, then often the cross section may be expressed entirely in collinear factorisation, though supplemented by transverse momentum resummation.

Feynman rules allow for a decomposition of the cross section into a contraction of hadronic and leptonic tensors. Where applicable, factorisation theorems separate non-perturbative and hard contributions within the hadronic tensor. In the TMD case, distribution and fragmentation functions are introduced, whose properties depend on the polarisations of the target and/or produced hadrons, the partonic polarisations, and the twist order. For example, in fully unpolarised SIDIS at leading twist the hadronic tensor is factorised into a convolution of one unpolarised TMD PDF (for the incoming target hadron) and one unpolarised TMD FF (for the final state hadron):

$$W^{\mu\nu} \sim \mathcal{H}^{\mu\nu}(Q;\mu) \sum_a \int d^2\mathbf{b}_\perp e^{-i\mathbf{q}_\perp\cdot\mathbf{b}_\perp} f^{a,T}(x,\mathbf{b}_\perp;\zeta_f,\mu)\, D^{a\to h}(z,\mathbf{b}_\perp;\zeta_D,\mu) + \\ + Y_{\rm SIDIS}(\mathbf{q}_\perp, Q) + \mathcal{O}((\Lambda_{\rm QCD}/Q)^p)\,, \quad (1)$$

where $\mathcal{H}$ is the hard part, $a$ is the flavour of the struck parton, $T$ is the target hadron, $h$ is the detected hadron, $x$ and $z$ are the light-cone momentum fractions, and $\mathbf{b}_\perp$ is Fourier-conjugated of the transverse momentum $\mathbf{q}_\perp$. The term $Y_{\rm SIDIS}(\mathbf{q}_\perp, Q)$ is a correction for the region of $q_T \sim Q$ where a separation into TMD PDFs is not valid, and all transverse momentum is generated inside the hard scattering. This so-called *$Y$-term* is calculable in collinear factorisation. With it included, the errors in the factorisation theorem are suppressed by powers of $\Lambda_{\rm QCD}/Q$, point-by-point in $\mathbf{q}_T$, as indicated by the last term, where $p > 0$. Taking into account all the possible combinations of polarisation (parton, target and detected hadron), there are nine TMD PDFs and eight TMD FFs at leading-twist [27,28] and the expression of the hadronic tensor modifies accordingly [29].

TMD parton distributions or fragmentation functions depend on two auxiliary scales, $\zeta_{f,D}$ an $\mu$, and they satisfy evolution equations with respect to both of them. The evolution with respect to $\zeta_f$ and $\zeta_D$ corresponds to Collins-Soper (CS) evolution, and these two scales must satisfy the constraint, $\zeta_f \zeta_D = Q^4$. The evolution in $\mu$ is determined by standard renormalisation group methods, while the evolution with respect to the energy variable $\zeta_{f,D}$ is determined by a process-independent soft factor [6,12,30–37].

When the energy range covered by the experimental data is not large (see, e.g., Ref. [38, 39]) fits of TMD PDFs and FFs can be performed without taking into account effects induced by evolution. These fits rely essentialy on a simple parton model approach and are oriented towards investigations of hadron structure at a relatively low-energy scale. Recent examples are Ref. [40,41]. In order to explore the evolution of hadron structure with the energy scale, these fixed scale fits can be incorporated into a Collins-Soper-Sterman (CSS) style of factorisation theorem like Eq. (1), as described in Refs. [42,43]. There, fixed scale fits from [44–48] are combined with traditional CSS style fits from Refs. [32,49].

## 2.2 High-energy factorisation and evolution

A form of TMD factorisation holds at high energy [14,50,51] and has been applied to several processes in photon-hadron, lepton-hadron and hadron-hadron collisions. For instance, the high-energy factorisation expresses the heavy-quark leptoproduction cross section in terms of the TMD gluon density via well-prescribed, calculable perturbative coefficients [50]. This framework is extended to deep-inelastic structure functions in [52,53]. Perturbative applications of the method include the resummation of small-$x$ logarithmic corrections to DIS to all orders in $\alpha_s$ at leading and next-to-leading $\ln x$ level [52–55]. In hadron-hadron scattering, high-energy factorisation has been applied to processes such as heavy flavour and Higgs boson production [17,50].

In the framework of high-energy factorisation [14, 50, 51] the DIS cross section can be written as a convolution in both longitudinal and transverse momenta of the unintegrated parton density function $\mathcal{A}(x, k_t, \mu)$ with off-shell partonic matrix elements

$$\sigma_j(x, Q^2) = \int_x^1 dz \int d^2k_t \, \hat{\sigma}_j(x, Q^2, z, k_t) \, \mathcal{A}(z, k_t, \mu) \, , \tag{2}$$

where the DIS cross sections $\sigma_j$, ($j = 2, L$) are related to the structure functions $F_2$ and $F_L$ by $\sigma_j = 4\pi^2 F_j/Q^2$, and the hard-scattering kernels $\hat{\sigma}_j$ of Eq. (2) are $k_t$-dependent.

The factorisation formula, Eq. (2), allows for resummation of logarithmically enhanced $x \to 0$ contributions to all orders in perturbation theory, both in the hard-scattering coefficients and in the parton evolution, taking into account the full dependence on the factorisation scale $\mu$ and on the factorisation scheme [52,53].

Realistic applications of this approach at collider energies require matching of $x \to 0$ contributions with finite-$x$ contributions. To this end, the evolution of the gluon uPDF $\mathcal{A}$ is obtained by combining the resummation of small-$x$ logarithmic contributions [56–58] with medium- and large-$x$ contributions to parton splitting [59–61], according to the CCFM evolution equations [62–65].

The cross section $\sigma_j$, ($j = 2, L$) is usually computed in a Fixed Flavour Number (FFN) scheme, where the photon-gluon fusion process ($\gamma^* g^* \to q\bar{q}$) is included. The masses of the quarks are explicitly included with the light and heavy quark masses being free parameters. In addition to $\gamma^* g^* \to q\bar{q}$, the contribution from valence quarks is included via $\gamma^* q \to q$ by using CCFM evolution of valence quarks [66–68]. A fit of CCFM uPDFs to the combined DIS precision data [69, 70] has been recently presented in Ref. [68] using the evolution given in Ref. [71]. Earlier CCFM fits to DIS were presented in Ref. [72]. In Ref. [73] the unintegrated gluon distribution has been obtained by means of a saturation ansatz.

# 3 TMDlib documentation

In this section, we provide the user with a concise documentation of the TMDlib library, discussing the basic procedure to readily install and use it.

The source code of TMDlib is available from the following web page:

**`http://tmdlib.hepforge.org/`**

and can be installed using the *standard* `autotools` sequence

```
./configure
make
make install
```

By default and if the user has root privileges, the installation path is `/usr/local/` and the files with the available TMD/uPDF distributions will also be installed there. A different installation path can be specified as

```
./configure --prefix=/path/to/the/installation/folder
```

but, in this case, it should be added to the environmental variable `LD_LIBRARY_PATH`.

The TMDlib library requires the `LHAPDF` PDF library and the `ROOT` data analysis framework library to be installed.[1] The path to the `LHAPDF` library must be specified during the installation as an option of the `./configure` command as follows

```
./configure --with-lhapdf=/path/to/lhapdf
```

Useful information about the installation and the determination of compiler flags in custom `makefiles` is provided by executing the `TMDlib-config` script.

[1]The current release of TMDlib assumes that `LHAPDF5.9.0` or a more recent version has been previously installed. For details about the installation of `LHAPDF`, please see the corresponding hepforge web page [74].

Once TMDlib has been properly compiled and installed, the user has at her/his disposal a set of functions that can be called from a main program. We refrain from giving here a detailed description of all the functions available in TMDlib, since the library is supposed to be frequently updated as new features will be developed. The up-to-date list of these functions and their description are provided at

**http://tmdlib.hepforge.org/pdfset.html**.

Here, we restrict ourselves to discussing the basic steps required to handle the available uPDF/TMD distributions within TMDlib.

1. **INITIALISATION.** The first step consists in initialising the desired uPDF/TMD set. This will assign the chosen uPDF/TMD set, specified by its name, an identifying number proper to that set. This number is stored into memory and called every time the identification of the uPDF/TMD set is needed by any TMDlib internal function.[2] If available, Monte Carlo replica or uncertainty sets can be initialised. Also at this stage, the user can specify whether grid or parameterisation (if available) should be used for uPDF/TMD distributions. The complete list of uPDF/TMD sets available in TMDlib 1.0.0 , with the corresponding name, identifying number, kinematic coverage, and reference, is given in Tab. 1.

2. **CALL TO THE DISTRIBUTION.** The second step consists in calling the desired function. At this stage, the user must specify some input variables, typically the light-cone momentum fractions $x^+$ and $x^-$, carried by the parton, the parton transverse momentum $k_t$ (in GeV), the energy scale $\mu$ (in GeV) and the flavour code identifying the target (*e.g.* proton or antiproton). The value for $x$ times the uPDF/TMD is then returned, either for individual flavours, when available, or for a combination of them.

Both the initialisation and the call to the uPDF/TMD parton set are performed via function overloading of, respectively

```
TMDinit;
TMDpdf;
```

where the several methods and input/output variables for each of these functions are summarised in Tabs. 2-3. Additional useful code, including collection of methods and examples, can be found in the `src` folder. In particular:

- `TMDhandler.cc`: The code to handle the call for TMDlib, according to the two steps described above and the methods summarised in Tabs. 2-3;
- `TMDutils.cc`: A collection of methods used in TMDlib, including functions to get details about the initialised uPDF/TMD set (like $\alpha_s$, $\Lambda_{\rm QCD}$, number of flavours) as listed in Tab. 4;

[2]Note that so far only one set of uPDF/TMD at a time can be called.

| parton | uPDF/TMD set | iset | $\Lambda_{qcd}^{(4)}$ | $k_t^{cut}$ [GeV] | $Q_0$ [GeV] | Ref. |
|---|---|---|---|---|---|---|
| gluon | ccfm-JS-2001 | 101000 | 0.25 | 0.25 | 1.4 | [72] |
| | ccfm-setA0 | 101010 | 0.25 | 1.3 | 1.3 | [72] |
| | ccfm-setA0+ | 101011 | 0.25 | 1.3 | 1.3 | [72] |
| | ccfm-setA0- | 101012 | 0.25 | 1.3 | 1.3 | [72] |
| | ccfm-setA1 | 101013 | 0.25 | 1.3 | 1.3 | [72] |
| | ccfm-setB0 | 101020 | 0.25 | 0.25 | 1.3 | [72] |
| | ccfm-setB0+ | 101021 | 0.25 | 0.25 | 1.3 | [72] |
| | ccfm-setB0- | 101022 | 0.25 | 0.25 | 1.3 | [72] |
| | ccfm-setB1 | 101023 | 0.25 | 0.25 | 1.3 | [72] |
| | ccfm-JH-set 1 | 101001 | 0.25 | 1.33 | 1.33 | [75] |
| | ccfm-JH-set 2 | 101002 | 0.25 | 1.18 | 1.18 | [75] |
| | ccfm-JH-set 3 | 101003 | 0.25 | 1.35 | 1.35 | [75] |
| | ccfm-JH-2013-set1 | 101201 | 0.2 | 2.2 | 2.2 | [68] |
| | ccfm-JH-2013-set2 | 101301 | 0.2 | 2.2 | 2.2 | [68] |
| | GBWlight | 200005 | – | – | – | [73] |
| | GBWcharm | 200006 | – | – | – | [73] |
| quark | ccfm-setA0 | – | 0.25 | 1.3 | 1.3 | |
| | ccfm-JH-2013-set1 | – | 0.2 | 2.2 | 2.2 | [68] |
| | ccfm-JH-2013-set2 | – | 0.2 | 2.2 | 2.2 | [68] |
| | SBRS-2013-TMDPDFs | 300005 | – | – | 1.55 | [40] |

Table 1: Available uPDF/TMD parton sets in TMDlib.

- `TMD_test.cc`: An example program to handle uPDF/TMD distributions;
- `TMDplotter.cc`: A `ROOT`-based script to plot uPDF/TMD distributions as obtained from TMDlib.

The TMDlib library is released together with the online plotter platform TMDplotter, also available at

**`http://tmdplotter.desy.de/`**

This includes the same uPDF/TMD sets available in TMDlib (for details see Tab. 1) and allows for online plotting of distributions. In particular, they can be displayed at a given energy scale either as functions of the momentum fraction $x$ or the transverse momentum $k_t$, respectively at fixed $k_t$ or $x$ values. Distributions integrated over $k_t$ can also be plotted. Two snapshots from a typical usage of TMDplotter are shown in Fig. 1: the gluon from the `ccfm-JH-2013-set1` set is compared to the `GBW` as a function of $k_t$ and $x$.

## 4 Conclusions and feedback

We have presented TMDlib and TMDplotter; they are, respectively, a `C++` library for handling different parameterisations of uPDFs/TMDs and a corresponding online plotting tool. Our aim is to update these tools with more uPDF/TMD parton sets and new features, as

| Method | Usage |
|---|---|
| `TMDinit(name)` | To initialise the uPDF/TMD set specified by its name `name`. A complete list of uPDF/TMD sets available in the current version of TMDlib with the corresponding name is provided in Tab. 1. |
| `TMDinit(name,irep)` | To initialise a given `irep` replica in a Monte Carlo uPDF/TMD set specified by its name `name`. |
| `TMDinit(name,irep,imode)` | To initialise the uncertainty sets with `irep` or to initialise a given `irep` replica in a Monte Carlo uPDF/TMD set specified by its name `name` and `imode`:<br>• `imode`=0: the value obtained from the analytic form of the distribution is returned<br>• `imode`=1: the value obtained as a polynomial interpolation on a numerical grid is returned<br>• `imode`=2: the value obtained from the analytic form of the Fourier transform (b-space distribution) |

Table 2: The function overload for `TMDinit` used to initialise uPDF/TMD parton sets.

| Method | Usage |
|---|---|
| `TMDpdf(x,xbar,kt,mu, uval,dval,s,c,b,glu)` | To return the value of $xF(x, \bar{x}, k_t, \mu)$ ($F$ is the initialised uPDF/TMD) for valence u-quarks `uval`, valence d-quarks `dval`, light sea-quarks `s`, charm-quarks `c`, bottom-quarks `b`, and gluons `glu` inside a proton. The input variables `x` and `xbar` are the light-come momentum fractions $x^+$ and $x^-$ (in some parameterisations the latter is set to zero), `kt` is the parton transverse momentum $k_t$, and `mu` is the energy scale $\mu$ (in GeV). |
| `TMDpdf(kf,x,xbar,kt,mu, uval,dval,s,c,b,glu)` | As the function above, but for hadron with flavour code `kf` (`kf` = 2212 for proton and `kf` =-2212 for antiproton) |
| `TMDpdf(x,xbar,kt,mu)` | Void-type function returning the value of $xF(x, \bar{x}, k_t, \mu)$ ($F$ is the initialised uPDF/TMD) as an array `xpq[13]`: at index $0, \dots, 6$ is $\bar{t}, \dots, \bar{d}$, at index 7 is the gluon, and at index $8, \dots, 13$ is $d, \dots, t$ densities for a proton target. |
| `TMDpdf(kf,x,xbar,kt,mu)` | As the function above, but for hadron with flavour code `kf` (`kf` = 2212 for proton and `kf` =-2212 for antiproton) |
| `TMDpdf(x,xbar,kt,mu,xpq)` | Double-type function returning the value of $xF(x, \bar{x}, k_t, \mu)$ ($F$ is the initialised uPDF/TMD) as an array `xpq[13]`: at index $0, \dots, 6$ is $\bar{t}, \dots, \bar{d}$, at index 7 is the gluon, and at index $8, \dots, 13$ is $d, \dots, t$ densities for a proton target. |
| `TMDpdf(kf,x,xbar,kt,mu, xpq)` | As the function above, but for hadron with flavour code `kf` (`kf` = 2212 for proton and `kf` =-2212 for antiproton) |

Table 3: The function overload for `TMDpdf` used to call uPDF/TMD parton sets.

| Method | Usage |
|---|---|
| `TMDalphas(mu)` | Return $\alpha_s$ used in the set initialised by `TMDinit(name)`. |
| `TMDgetLam4( )` | Return the value of $\Lambda_{QCD}$ at $N_f = 4$. |
| `TMDgetNf( )` | Return the number of flavours, $N_f$, used for the computation of $\Lambda_{QCD}$. |
| `TMDgetOrderAlphaS( )` | Return the perturbative order of $\alpha_s$ used in the evolution of the TMD/uPDF set initialised by `TMDinit(name)`. |
| `TMDgetOrderPDF( )` | Return the perturbative order of the evolution of the TMD/uPDF set initialised by `TMDinit(name)`. |
| `TMDgetXmin()` | Return the minimum value of the momentum fraction $x$ for which the TMD/uPDF set initialised by `TMDinit(name)` was determined. |
| `TMDgetXmax()` | Return the maximum value of the momentum fraction $x$ for which the TMD/uPDF set initialised by `TMDinit(name)` was determined. |
| `TMDgetQ2min()` | Return the minimum value of the energy scale $\mu$ (in GeV) for which the TMD/uPDF set initialised by `TMDinit(name)` was determined. |
| `TMDgetQ2max()` | Return the maximum value of the energy scale $\mu$ (in GeV) for which the TMD/uPDF set initialised by `TMDinit(name)` was determined. |
| `TMDnumberPDF(name)` | Return the number `iset` associated with the TMD/uPDF set initialised by `TMDinit(name)`. |

Table 4: The list of methods included in the `TMDutils.cc` file.

they become available and are developed. Future releases will also contain parameterisations of TMD FFs. We encourage discussions, feedback and comments about the TMDlib and TMDplotter projects, which can be addressed through the mailing list of the "TMDlib" project

**http://tmdlib.hepforge.org**

or directly to the contact authors.


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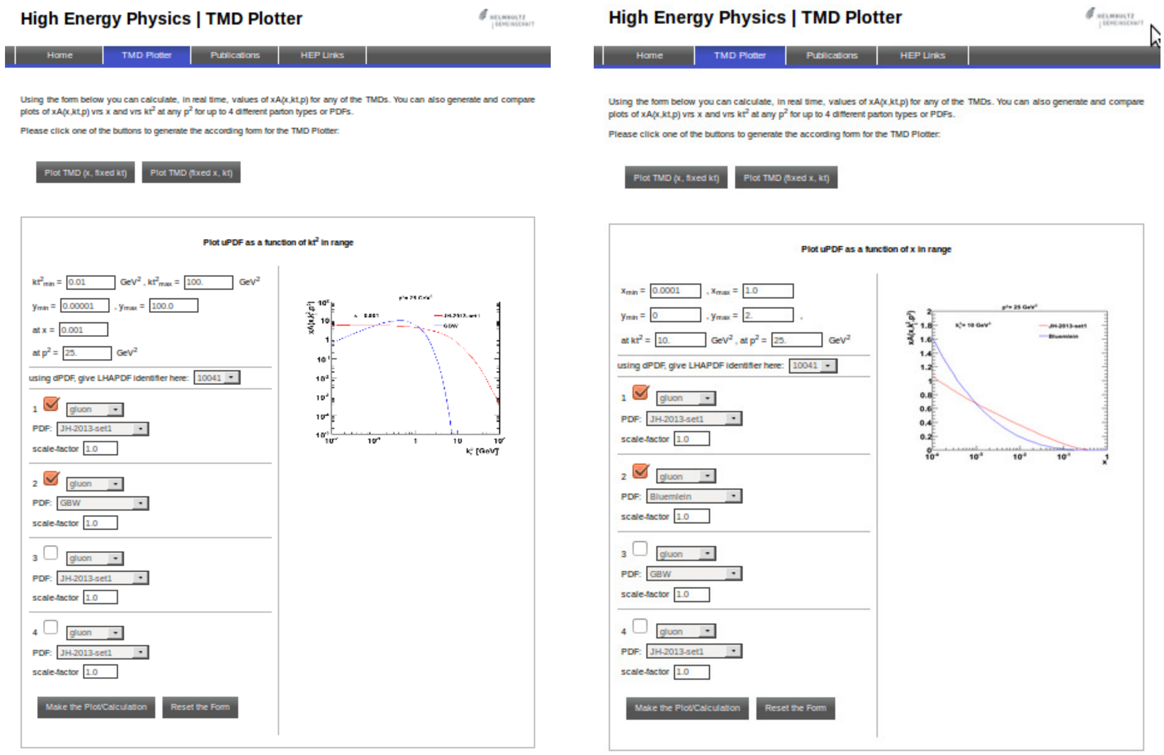


Figure 1: Two snapshots from the online portal TMDplotter for plotting uPDF/TMD distributions: the gluon from the `ccfm-JH-2013-set1` set compared to the `GBW` as a function of $k_t$ (left) and $x$ (right).


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