



# The CCFM uPDF evolution uPDFevol Version 1.0.00

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# The CCFM uPDF evolution

## uPDFevolv

### Version 1.0.00

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#### Abstract

uPDFevolv is an evolution code for TMD parton densities using the CCFM evolution equation. A description of the underlying theoretical model and technical realisation is given together with a detailed program description, with emphasis on parameters the user may want to change.

#### PROGRAM SUMMARY

*Title of Program:* uPDFevolv 1.0.00

*Computer for which the program is designed and others on which it is operable:* any with standard Fortran 77 (gfortran) and C++, tested on Linux, MAC

*Programming Language used:* FORTRAN 77, C++

*High-speed storage required:* No

*Separate documentation available:* No

*Keywords:* QCD, small  $x$ , high-energy factorization,  $k_t$ -factorization, CCFM, unintegrated PDF (uPDF), transverse momentum dependent PDF (TMD)

*Nature of physical problem:* At high energies collisions of hadrons are described by parton densities dependent on the longitudinal momentum fraction  $x$ , the transverse momentum  $k_t$  and the evolution scale  $p$  (transverse momentum dependent (TMD) or unintegrated parton density functions (uPDF)). The evolution of the parton density with the scale  $p$  valid at both small and moderate  $x$  is given by the CCFM evolution equation

*Method of solution:* Since the CCFM evolution equation cannot be solved analytically, a Monte Carlo approach is applied, simulating at each step of the evolution the full four-momenta of the initial state partonic cascade.

*Restrictions on the complexity of the problem:* None

*Other Program used:* ROOT for plotting the result.

*Download of the program:* <https://updfevolv.hepforge.org>

*Unusual features of the program:* None

# 1 Theoretical Input

## 1.1 CCFM evolution equation and Transverse Momentum Dependent PDFs

QCD calculations of multiple-scale processes and complex final-states require in general transverse-momentum dependent (TMD), or unintegrated, parton density and parton decay functions [1–10]. TMD factorization has been proven recently [1] for inclusive and semi-inclusive deep-inelastic scattering (DIS). For special processes in hadron-hadron scattering, like heavy flavor or heavy boson (including Higgs) production, TMD factorization holds in the high-energy limit (small  $x$ ) [11–13].

In the framework of high-energy factorization [11, 14] the deep-inelastic scattering cross section can be written as a convolution in both longitudinal and transverse momenta of the TMD parton density function  $\mathcal{A}(x, k_t, \mu)$  with off-shell partonic matrix elements, as follows

$$\sigma_j(x, Q^2) = \int_x^1 dz \int d^2 k_t \hat{\sigma}_j(x, Q^2, z, k_t) \mathcal{A}(z, k_t, p), \quad (1)$$

with the DIS cross sections  $\sigma_j$  ( $j = 2, L$ ) related to the structure functions  $F_2$  and  $F_L$  by  $\sigma_j = 4\pi^2 F_j / Q^2$ . The hard-scattering kernels  $\hat{\sigma}_j$  of Eq. (1) are  $k_t$ -dependent and the evolution of the transverse momentum dependent gluon density  $\mathcal{A}$  is obtained by combining the resummation of small- $x$  logarithmic contributions [15–17] with medium- $x$  and large- $x$  contributions to parton splitting [18–20] according to the CCFM evolution equation [21–23].

The factorization formula (1) allows one to resum logarithmically enhanced  $x \rightarrow 0$  contributions to all orders in perturbation theory, both in the hard scattering coefficients and in the parton evolution, taking fully into account the dependence on the factorization scale  $p$  and on the factorization scheme [24, 25].

The CCFM evolution equation [21–23] is an exclusive equation for final state partons and includes finite- $x$  contributions to parton splitting. It incorporates soft gluon coherence for any value of  $x$ .

### 1.1.1 Gluon distribution

The evolution equation for the TMD gluon density  $\mathcal{A}(x, k_t, p)$ , depending on  $x$ ,  $k_t$  and the evolution variable  $p$ , is

$$\begin{aligned} \mathcal{A}(x, k_t, p) &= \mathcal{A}_0(x, k_t, p) + \int \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(p - zq) \\ &\times \Delta_s(p, zq) P(z, q, k_t) \mathcal{A}\left(\frac{x}{z}, k_t + (1-z)q, q\right), \end{aligned} \quad (2)$$

where  $z$  is the longitudinal momentum fraction,  $q$  is the angular variable and the  $\Theta$  function specifies the ordering condition of the evolution [26].

The first term in the right hand side of Eq. (2) is the contribution of the non-resolvable branchings between the starting scale  $q_0$  and the evolution scale  $p$ , and is given by

$$\mathcal{A}_0(x, k_t, p) = \mathcal{A}_0(x, k_t, q_0) \Delta_s(p, q_0), \quad (3)$$

where  $\Delta_s$  is the Sudakov form factor, and  $\mathcal{A}_0(x, k_t, q_0)$  is the starting distribution at scale  $q_0$ . The integral term in the right hand side of Eq. (2) gives the  $k_t$ -dependent branchings in terms of the Sudakov form factor  $\Delta_s$  and unintegrated splitting function  $P$ . The Sudakov form factor  $\Delta_s$  is given by

$$\Delta_s(p, q_0) = \exp \left( - \int_{q_0^2}^{p^2} \frac{dq^2}{q^2} \int_0^{1-q_0/q} dz \frac{\bar{\alpha}_s(q^2(1-z)^2)}{1-z} \right), \quad (4)$$

with  $\bar{\alpha}_s = C_A \alpha_s / \pi = 3\alpha_s / \pi$ .

For application in Monte Carlo event generators, like CASCADE [27,28], it is of advantage to write the CCFM evolution equation in differential form:

$$p^2 \frac{d}{dp^2} \frac{x\mathcal{A}(x, k_t, p)}{\Delta_s(p, q_0)} = \int dz \frac{d\phi}{2\pi} \frac{P(z, p/z, k_t)}{\Delta_s(p, q_0)} x' \mathcal{A}(x', k_t', p/z), \quad (5)$$

where the splitting variable  $x'$  is given by  $x' = x/z$ ,  $k_t' = q_t(1-z)/z + k_t$ , and  $\phi$  is the azimuthal angle of  $q_t$ .

For the evolution of the parton densities, however, a forward evolution approach, starting from the low scale  $q_0$  towards the hard scale  $p$ , is used.

The splitting function  $P_{gg}(z_i, q_i, k_{ti})$  for branching  $i$  is given by [29] (set by `IpGG=1, ns=1` in `uPDFevolv`)

$$\begin{aligned} P_{gg}(z_i, q_i, k_{ti}) &= \bar{\alpha}_s(q_i^2(1-z_i)^2) \left( \frac{1}{1-z_i} - 1 + \frac{z_i(1-z_i)}{2} \right) \\ &+ \bar{\alpha}_s(k_{ti}^2) \left( \frac{1}{z_i} - 1 + \frac{z_i(1-z_i)}{2} \right) \Delta_{ns}(z_i, q_i^2, k_{ti}^2) \end{aligned} \quad (6)$$

where  $\Delta_{ns}$  is the non-Sudakov form factor defined by

$$\log \Delta_{ns} = -\bar{\alpha}_s(k_{ti}^2) \int_0^1 dz' \left( \frac{1}{z'} - 1 + \frac{z'(1-z')}{2} \right) \int \frac{dq^2}{q^2} \Theta(k_{ti} - q) \Theta(q - z'q_{ti}). \quad (7)$$

In addition to the full splitting function, simplified versions are useful in applications and are made available. One uses only the singular parts of the splitting function (set by `IpGG=0, ns=0` in `uPDFevolv`):

$$P_{gg}(z, q, k_t) = \frac{\bar{\alpha}_s(q^2)}{1-z} + \frac{\bar{\alpha}_s(k_t^2)}{z} \Delta_{ns}(z, q^2, k_t) \quad (8)$$

with

$$\log \Delta_{ns} = -\bar{\alpha}_s(k_{ti}^2) \int_0^1 \frac{dz'}{z'} \int \frac{dq^2}{q^2} \Theta(k_{ti} - q) \Theta(q - z'q_{ti}). \quad (9)$$

Another uses  $\alpha_s(q^2)$  also for the small  $z$  part (set by `IpGG=2, ns=2` in `uPDFevolv`):

$$P_{gg}(z, q, k_t) = \frac{\bar{\alpha}_s(q^2)}{1-z} + \frac{\bar{\alpha}_s(q^2)}{z} \Delta_{ns}(z, q^2, k_t) \quad (10)$$

with

$$\log \Delta_{ns} = - \int_0^1 \frac{dz'}{z'} \int \frac{dq^2}{q^2} \bar{\alpha}_s(q^2) \Theta(k_{ti} - q) \Theta(q - z' q_{ti}). \quad (11)$$

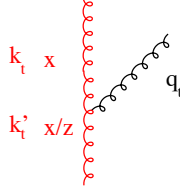


Figure 1: Gluon branching

In general a four-momentum  $\mathbf{a}$  can be written in light-cone variables as  $\mathbf{a} = (a^+, a^-, a_T)$  with  $a^+$  and  $a^-$  being the light-cone components and  $a_T$  being the transverse component. The CCFM (as well as the BFKL) evolution depends only on one of the light-cone components. Assuming that the other one can be neglected, this leads to the condition that the virtuality of the parton propagator  $a^2 = 2a^+a^- - a_T^2$  should be dominated by the transverse component, while the contribution from the longitudinal components is required to be small. The condition that  $a^+a^- = 0$  leads to the so-called consistency constraint (see Fig. 1), which has been implemented in different forms (set by `Ikincut=1, 2, 3` in `uPDFevolv`)

$$q_t^2 < \frac{k_t^2}{z} \quad \text{LDC [21,30]} \quad (12)$$

$$q_t^2 < \frac{(1-z)k_t^2}{z} \quad [31] \quad (13)$$

$$k_t'^2 < \frac{k_t^2}{z} \quad \text{BFKL [31]} \quad (14)$$

### 1.1.2 Valence quarks

Using the method of [32,33] valence quarks are included in the branching evolution at the transverse-momentum dependent level according to

$$\begin{aligned} xQ_v(x, k_t, p) &= xQ_{v0}(x, k_t, p) + \int \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(p - zq) \\ &\times \Delta_s(p, zq) P_{qq}(z, q, k_t) xQ_v\left(\frac{x}{z}, k_t + (1-z)q, q\right), \end{aligned} \quad (15)$$

where  $p$  is the evolution scale. The quark splitting function  $P_{qq}$  is given by

$$P_{qq}(z, q, k_t) = \frac{C_F}{2\pi} \alpha_s (q^2(1-z)^2) \frac{1+z^2}{1-z}. \quad (16)$$

In Eqs. (15),(16) the non-Sudakov form factor is not included, unlike the CCFM kernel given in the appendix B of [22], because we only associate this factor with  $1/z$  terms. The term

$xQ_{v0}$  in Eq. (15) is the contribution of the non-resolvable branchings between starting scale  $q_0$  and evolution scale  $p$ , given by

$$xQ_{v0}(x, k_t, p) = xQ_{v0}(x, k_t, q_0)\Delta_s(p, q_0) , \quad (17)$$

where  $\Delta_s$  is the Sudakov form factor.

### 1.1.3 Sea quarks

For a complete description of the final states also the contribution from sea-quarks needs to be included. We include splitting functions  $P_{ab}$  according to

$$P_{gg}(z) = \bar{\alpha}_s \left( \frac{1}{1-z} - 1 + \frac{z_i(1-z_i)}{2} \right) + \bar{\alpha}_s \left( \frac{1}{z_i} - 1 + \frac{z_i(1-z_i)}{2} \right) \Delta_{ns} \quad (18)$$

$$P_{qg}(z) = \bar{\alpha}_s \frac{1}{4C_A} (z^2 + (1-z)^2) \quad (19)$$

$$P_{gq}(z) = \bar{\alpha}_s \frac{C_F}{2C_A} \left( \frac{1+(1-z)^2}{z} \right) \quad (20)$$

$$P_{qq}(z) = \bar{\alpha}_s \frac{C_F}{2C_A} \left( \frac{1+z^2}{1-z} \right) \quad (21)$$

with  $\bar{\alpha}_s = C_A\alpha_s/\pi$ ,  $C_A = 3$  and  $C_F = 4/3$ .

The  $g \rightarrow q\bar{q}$  splitting has been calculated in a  $k_t$ -factorized form in [24],

$$P_{qg}(z, \tilde{q}, k_t) = \bar{\alpha}_s \frac{1}{4C_A} \left[ \frac{\tilde{q}^2}{\tilde{q}^2 + z(1-z)k_t^2} \right]^2 \left( z^2 + (1-z)^2 + 4z^2(1-z)^2 \frac{k_t^2}{\tilde{q}^2} \right) \quad (22)$$

with  $\tilde{q} = q - zk_t$ , and  $q(k_t)$  being the transverse momentum of the quark (gluon).

The evolution equation for the TMD sea-quark density  $\mathcal{S}(x, k_t, p)$ , depending on  $x$ ,  $k_t$  and the evolution variable  $p$ , is

$$\begin{aligned} \mathcal{S}(x, k_t, p) &= \mathcal{S}_0(x, k_t, p) \\ &+ \int \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(p-zq) \Delta_s(p, zq) P_{qg}(z, q, k_t) \mathcal{A} \left( \frac{x}{z}, k_t + (1-z)q, q \right) \\ &+ \int \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(p-zq) \Delta_s(p, zq) P_{gq}(z, q, k_t) \mathcal{S} \left( \frac{x}{z}, k_t + (1-z)q, q \right), \end{aligned} \quad (23)$$

where  $\mathcal{S}_0(x, k_t, p)$  is the non-resolvable branching probability similar to Eqs. (3),(17).

The evolution of the TMD gluon density including the contribution from quarks is given by

$$\begin{aligned} \mathcal{A}(x, k_t, p) &= \mathcal{A}_0(x, k_t, p) \\ &+ \int \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(p-zq) \Delta_s(p, zq) P_{gg}(z, q, k_t) \mathcal{A} \left( \frac{x}{z}, k_t + (1-z)q, q \right) \\ &+ \int \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(p-zq) \Delta_s(p, zq) P_{gq}(z, q, k_t) \mathcal{S} \left( \frac{x}{z}, k_t + (1-z)q, q \right). \end{aligned} \quad (24)$$



### 1.1.4 Monte Carlo solution of the CCFM evolution equations

The evolution equations Eqs.(23,24) are integral equations of the Fredholm type

$$f(x) = f_0(x) + \lambda \int_a^b K(x, y) f(y) dy$$

and can be solved by iteration as a Neumann series

$$\begin{aligned} f_1(x) &= f_0(x) + \lambda \int_a^b K(x, y) f_0(y) dy \\ f_2(x) &= f_0(x) + \lambda \int_a^b K(x, y_1) f_0(y_1) dy_1 + \lambda^2 \int_a^b \int_a^b K(x, y_1) K(y_1, y_2) f_0(y_2) dy_2 dy_1 \\ &\dots \end{aligned} \tag{25}$$

using the kernel  $K(x, y)$ , with the solution

$$f(x) = \lim_{n \rightarrow \infty} \sum_{i=0}^n f_i(x). \tag{26}$$

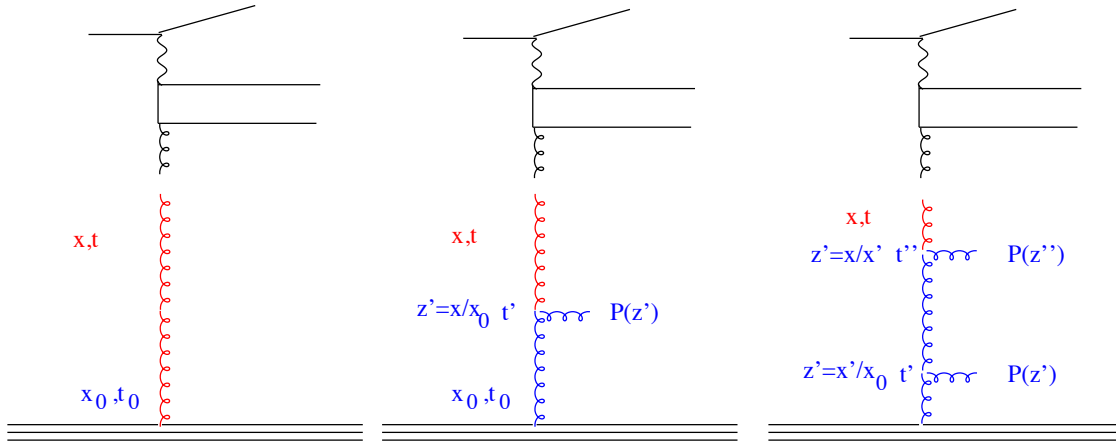


Figure 2: Evolution by iteration

Applying this to the evolution equations Eqs.(23,24), we identify  $f_0$  with the first term in eqs.(24), where we use for simplicity here and in the following  $\Delta_s(p) = \Delta_s(p, q_0)$ :

$$\mathcal{A}_0(x, k_t, p) = \mathcal{A}_0(x, k_t) \Delta_s(p). \tag{27}$$

The first iteration involves one branching:

$$\begin{aligned} \mathcal{A}_1(x, k_t, p) &= \mathcal{A}_0(x, k_t) \Delta_s(p) \\ &+ \int_x^1 \frac{dz'}{z'} \int_{q_0}^p \frac{dq'^2}{q'^2} \Theta(p - z'q') \frac{\Delta_s(p)}{\Delta_s(zq')} \tilde{P}(z') \mathcal{A}_0(x/z', k'_t, q'). \end{aligned} \quad (28)$$

The second iteration involves two branchings,

$$\begin{aligned} \mathcal{A}_2(x, k_t, p) &= \mathcal{A}_0(x, k_t) \Delta(p) \\ &+ \int_x^1 \frac{dz'}{z'} \int_{q_0}^p \frac{dq'^2}{q'^2} \Theta(p - z'q') \frac{\Delta(p)}{\Delta(q')} \tilde{P}(z') \mathcal{A}_1(x/z', k'_t, q') \\ &= \mathcal{A}_0(x, k_t) \Delta(p) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz'}{z'} \int_{q_0}^p \frac{dq'^2}{q'^2} \Theta(p - z'q') \frac{\Delta_s(p)}{\Delta_s(zq')} \tilde{P}(z') \mathcal{A}_0(x/z', k'_t, q') \\ &+ \left(\frac{\alpha_s}{2\pi}\right)^2 \int_x^1 \frac{dz'}{z'} \int_{q_0}^p \frac{dq'^2}{q'^2} \Theta(p - z'q') \frac{\Delta_s(p)}{\Delta_s(z'q')} \tilde{P}(z') \\ &\times \int_x^1 \frac{dz''}{z''} \int_{q_0}^p \frac{dq''}{q''} \Theta(p - z''q'') \frac{\Delta_s(p)}{\Delta_s(z''q'')} \tilde{P}(z'') \mathcal{A}_0(z''/z', k''_t, q''), \end{aligned} \quad (29)$$

$$\mathcal{A}_3(x, k_t, p) = \dots$$

⋮

In a Monte Carlo (MC) solution [34, 35] we evolve from  $q_0$  to a value  $q'$  obtained from the Sudakov factor  $\Delta_s(q', q_0)$  (for a schematic visualisation of the evolution see fig. 2). Note that the Sudakov factor  $\Delta_s(q', q_0)$  gives the probability for evolving from  $q_0$  to  $q'$  without resolvable branching. The value  $q'$  is obtained from solving for  $q'$ :

$$R = \Delta_s(q', q_0), \quad (30)$$

for a random number  $R$  in  $[0, 1]$ .

If  $q' > p$  then the scale  $p$  is reached and the evolution is stopped, and we are left with just the first term without any resolvable branching. If  $q' < p$  then we generate a branching at  $q'$  according to the splitting function  $\tilde{P}(z')$ , as described below, and continue the evolution using the Sudakov factor  $\Delta_s(q'', q')$ . If  $q'' > p$  the evolution is stopped and we are left with just one resolvable branching at  $q'$ . If  $q'' < p$  we continue the evolution as described above. This procedure is repeated until we generate  $q > p$ . By this procedure we sum all kinematically allowed contributions in the series  $\sum f_i(x, p)$  and obtain an MC estimate of the parton distribution function.

With the Sudakov factor  $\Delta_s$  and using

$$\frac{\partial}{\partial q'^2} \Delta_s(p, zq') = \frac{\partial}{\partial q'^2} \frac{\Delta_s(p)}{\Delta_s(zq')} = \frac{\Delta_s(p)}{\Delta_s(zq')} \left[ \frac{1}{q'^2} \right] \int^{z_{max}} dz \tilde{P}(z),$$

we can write the first iteration of the evolution equation as

$$\begin{aligned} \mathcal{A}_1(x, k_t, p) &= \mathcal{A}_0(x, k_t, p) \\ &+ \int_x^1 \frac{dz'}{z'} \int_{q_0}^p d\Delta_s(p, z'q') \tilde{P}(z') \mathcal{A}_0(x/z', k'_t, q') \left[ \int^{z_{max}} dz \tilde{P}(z) \right]^{-1}. \end{aligned} \quad (31)$$

The integrals can be solved by a Monte Carlo method [36]:  $z$  is generated from

$$\int_{z_{min}}^z dz' \tilde{P}(z') = R_1 \int_{z_{min}}^{z_{max}} dz' \tilde{P}(z'), \quad (32)$$

with  $R_1$  being a random number in  $[0, 1]$ , and  $q'$  is generated from

$$\begin{aligned} R_2 &= \int_{-\infty}^x f(x') dx' = F(x) \\ &= \int_{zq}^p \frac{\partial}{\partial q'^2} \left( \frac{\Delta_s(p)}{\Delta_s(zq')} \right) dq'^2 \\ &= \Delta_s(p, zq') \end{aligned} \quad (33)$$

solving for  $q'$ , using  $z$  from above and another random number  $R_2$  in  $[0, 1]$ .

This completes the calculation on the first splitting. This procedure is repeated until  $q' > p$  and the evolution is stopped.

With  $z'$  and  $q'$  selected according to the above the first iteration of the evolution equation yields

$$\begin{aligned} x\mathcal{A}_1(x, k_t, p) &= x\mathcal{A}_0(x, k_t) \Delta_s(p) \\ &+ \sum_i \tilde{P}(z'_i) x'_i \mathcal{A}_0(x'_i, k'_{t_i}, q'_i) \left[ \int^{z_{max}} dz \tilde{P}(z) \right]^{-1}, \end{aligned} \quad (34)$$

with  $x'_i = x/z_i$ .

### 1.1.5 Normalisation of gluon and quark distributions

The valence quark densities are normalised so that they fulfil for every  $p$  the flavor sum rule.

The gluon and sea quark densities are normalised so that for every  $p$

$$\int_0^1 dx \int_0^\infty dk_t^2 x \mathcal{A}(x, k_t, q_0) = \int_0^1 dx \int_0^\infty dk_t^2 (x\mathcal{A}(x, k_t, p) + x\mathcal{S}(x, k_t, p)). \quad (35)$$

## 1.2 Computational Techniques: CCFM Grid

When using the CCFM evolution in a fit program to determine the starting distribution  $\mathcal{A}_0(x)$ , a full MC solution [34, 35] is no longer suitable, since it is time consuming and suffers from numerical fluctuations. Instead a convolution method introduced in [37, 38] is used. The kernel  $\tilde{\mathcal{A}}(x'', k_t, p)$  is determined once from the Monte Carlo solution of the CCFM evolution equation, and then folded with the non-perturbative starting distribution  $\mathcal{A}_0(x)$ ,

$$\begin{aligned} x\mathcal{A}(x, k_t, p) &= x \int dx' \int dx'' \mathcal{A}_0(x') \tilde{\mathcal{A}}(x'', k_t, p) \delta(x'x'' - x) \\ &= \int dx' \mathcal{A}_0(x') \cdot \frac{x}{x'} \tilde{\mathcal{A}}\left(\frac{x}{x'}, k_t, p\right). \end{aligned} \quad (36)$$

The kernel  $\tilde{\mathcal{A}}$  incorporates all of the dynamics of the evolution, including Sudakov form factors and splitting functions. It is determined on a grid of  $50 \otimes 50 \otimes 50$  bins in  $x, k_t, p$ . The binning in the grid is logarithmic, except for the longitudinal variable  $x$  where we use 40 bins in logarithmic spacing below 0.1, and 10 bins in linear spacing above 0.1.

Using this method, the complete coupled evolution of gluon and sea quarks is more complicated, since it is no longer a simple convolution of the kernel with the starting distribution. To simplify the approach, here we allow only for one species of partons at the starting scale, either gluons or sea-quarks. During evolution the other species will be generated. This approach, while convenient for QCD fits, has the feature that sea-quarks, in the case of gluons only at  $q_0$ , are generated with perturbative transverse momenta ( $k_t > k_{t \text{ cut}}$ ), without contribution from the soft (non-perturbative) region.

## 1.3 Functional Forms for starting distribution

### 1.3.1 Standard parametrisation

For the starting distribution  $\mathcal{A}_0$ , at the starting scale  $q_0$ , the following form is used:

$$x\mathcal{A}_0(x, k_t, q_0) = A_1 x^{-A_2} \cdot (1-x)^{A_3} (1 - A_4 x + A_5 \sqrt{x} + A_6 x^2) \exp[-k_t^2/\sigma^2] \quad , \quad (37)$$

with  $\sigma^2 = q_0^2/2$  and free parameters  $A_1, \dots, A_6$ .

Valence quarks are treated using the method of [32, 33, 38] with starting distributions at scale  $q_0$  parameterized using standard collinear pdfs (set by `Ipdf` in `uPDFevolv`) as

$$xQ_{v0}(x, k_t, q_0) = xQ_{v \text{ coll.pdf}}(x, q_0) \exp[-k_t^2/\sigma^2] \quad . \quad (38)$$

with  $\sigma^2 = q_0^2/2$ . At every scale  $p$  the flavor sum rule is fulfilled for valence quarks.

### 1.3.2 Saturation ansatz

A saturation ansatz for the starting distribution  $\mathcal{A}_0$  at scale  $q_0$  is available, following the parameterisation of the saturation model by Eq.(18) of [39],

$$x\mathcal{A}_{sat} = \frac{1}{\alpha_s} \frac{3\sigma_0}{4\pi^2} R_0^2(x) k_t^2 \exp(-R_0^2(x) k_t^2), \quad (39)$$

with  $R_0^2(x) = (x/x_0)^\lambda$ . The free parameters are  $\sigma_0 = A_2$ ,  $\lambda = A_3$ ,  $x_0 = A_4$  and  $\alpha_s = A_5$ . In order to be able to use this type of parameterisation over the full  $x$  range, an additional factor of  $(1-x)^{A_6}$  (see [40]) is applied.

## 1.4 Plotting TMDs

A simple plot program is included in the package. For a graphical web interface use TMD-PLOTTER [41].

## 1.5 Application

The evolution of the TMD gluon density has been used to perform fits to the DIS precision data [42,43], as described in detail in [38].

# 2 Description of the program components

## 2.1 Program history

```
*
*-----
uPDFevolv      Version 10000
*              first public release
*
*-----
```

## 2.2 Subroutines and functions

The source code of uPDFevolv and this manual can be found under:  
<https://updfevolv.hepforge.org/>

sminit	to initialise
sminfn	to generate starting distributions in $x$ and $k_t$
smbran	to simulate perturbative branchings
splittgg	to generate $g \rightarrow gg$ splitting via $P_{gg}$
splittgq	to generate $g \rightarrow qq$ splitting via $P_{gq}$
splittqg	to generate $q \rightarrow gg$ splitting via $P_{qg}$
splittqq	to generate $q \rightarrow qq$ splitting via $P_{qq}$
szvalnew	to calculate $z$ values for $g \rightarrow gg$ splitting
smqtem	to generate $t$ from the corresponding Sudakov factor
updfgrid	to build, fill and normalise the updf grid.
asbmy(k $t$ )	to calculate $\frac{C_A}{\pi}\alpha_s(k_t)$

Utility routines:

evolve tmd	Main routine to perform CCFM evolution
updfread	example program to read and plot the results
gadap	1-dimensional Gauss integration routine
gadap2	2-dimensional Gauss integration routine

divdif            linear interpolation routine (CERNLIB)  
 ranlux            Random number generator RANLUX (CERNLIB)

### 2.3 Parameter in steering files

'updf-grid.dat' name of the grid file  
 oneLoop = 0        to select *all loop* CCFM or *one loop* DGLAP type evolution  
 saturation = 0    to select standard or saturated initial condition  
 Ipdf = 60500      LHApdf set name for collinear valence quark starting distribution  
 Itarget = 2212    hadron target ID (2212=proton)  
 Iglu = 1          for gluon only evolution  
 Ipgg = 1          parameter for  $P_{gg}$  splitting function  
 ns = 1            parameter for treatment of non-sudakov form factor  
 ikincut = 2      flag for consistency constraint  
 Qg = 2.2          starting value  $q_0$  for perturbative evolution  
 QCDlam = 0.20    value for  $\Lambda_{qcd}$   
 A1, ..., A6      values for starting distribution; meaning depends on whether standard  
                   or saturation ansatz is used.

## 3 Example Program

```

Program ccfm_uPDF

  Include 'SMallx.inc'
  Integer Iev
C--- event common block
  Integer NMXHEP,NEVHEP,NHEP,ISTHEP, IDHEP, JMOHEP, JDAHEP
  Double Precision PHEP,VHEP,EVWGT
C---Event weight
  COMMON/HEPWGT/EVWGT
  Integer nobran,ikincut
  Common/myvar/nobran,ikincut
  Double Precision Qbarmy,Qbar_min,Qbar_max
  Common/mglubran/Qbarmy,Qbar_min,Qbar_max
  Integer neve
  Common/myevt/neve
  Integer nloop
  Common/myloop/nloop
  Double Precision x3lmin,x3lmax,x3ldif
  Integer Nbp
  Parameter (Nbp=50)
  Double Precision X3(0:Nbp+1)
  Double Precision X3M(0:Nbp)
  Double Precision x3b(0:Nbp)
  Common/gridtt/x3m

  Integer ng_max
  Integer nrglu
  Common/mynrglu/nrglu
  Integer nmax,i,nx3,kev,ic
  Integer Ipgg,ns_sel
  Double Precision scal
  Common/Pggsel/Ipgg,ns_sel,scal
  Double Precision Qgmin
  Character *72 TXT
  CHARACTER    FILNAME*132,testNAME*132
  Common/gludatf/filename
  Double Precision Xnorm
  Common/snorm/ Xnorm
  Logical pdflib,quark,gluon,photon,saturation

```

```

Common /SMbran2/pdflib,quark,gluon,photon,saturation
Integer Ioneloop,Itarget,Iglu,Isaturation
Integer Ipdf
Common/pdf/Ipdf
Integer iparton
Common /SMquark/iparton
Character *15 char
Double Precision BB
Common /splitting/ BB
Double precision ininorm(-6:6)
Common/smininorm/ininorm

Integer IRR
Couble precision au
Logical first
Common/f2fit/au(50),first

*
Read(5,*) filename
Write(6,*) ' output file ',filename
xnorm = 1.
Read(5,101) TXT
Read(txt,1005) char,Ioneloop
Write(6,*) txt,char,Ioneloop
1005 format(a10,I8)
Read(5,101) TXT
Read(txt,1010) char,Isaturation
1010 format(a14,I8)
Write(6,*) txt,char,Isaturation
Read(5,101) TXT
Read(txt,1006) char,Ipdf
Write(6,*) txt,char,Ipdf
1006 format(a7,I8)
Read(5,101) TXT
Read(txt,1007) char,Itarget
Write(6,*) txt,char,Itarget
1007 format(a10,I8)
Read(5,101) TXT
Read(txt,1008) char,Iglu
Write(6,*) txt,char,Iglu
1008 format(a7,I8)
Read(5,101) TXT
101 Format(A72)
Read(txt,1000) char,Ipgg
Write(6,*) txt,char,Ipgg
1000 format(a7,I8)
Read(5,101) TXT
Read(txt,1001) char,ns_sel
Write(6,*) txt,char,ns_sel
1001 format(a5,I8)
Read(5,101) TXT
Read(txt,1011) char,ikincut
Write(6,*) txt,char,ikincut
1011 format(a10,I8)
Read(5,101) TXT
Read(txt,1002) char,Qg
Write(6,*) txt,char,Qg
1002 format(a5,F16.8)
Read(5,101) TXT
Read(txt,1003) char,Qs
Write(6,*) txt,char,Qs
1003 format(a5,F16.8)
Read(5,101) TXT
Read(txt,1018) char,QCDlam
Write(6,*) txt,char,QCDlam
1018 format(a9,F16.8)
Read(5,101) TXT
Read(txt,1003) char,AU(1)
Read(5,101) TXT
Read(txt,1003) char,AU(2)
Read(5,101) TXT
Read(txt,1003) char,AU(3)
Read(5,101) TXT
Read(txt,1003) char,AU(4)
Read(5,101) TXT
Read(txt,1003) char,AU(5)
Read(5,101) TXT
Read(txt,1003) char,AU(6)

If(iglu.ne.0) then
  gluon = .true.

```

```

        else
            gluon = .false.
        Endif
    If(Ioneloop.eq.1) then
        onel = .true.
        else
            onel=.false.
        Endif
    If(Isaturation.eq.1) then
        saturation = .true.
        else
            saturation=.false.
        Endif
    If(iglu.eq.0) then
        Read(50,101) TXT
        Read(txt,1009) char,Iparton
        Write(6,*) txt,char,Iparton
        Write(6,*) txt
1009 Format(a9,I8)
    Endif
    Close(50)

C---Initialize run
    Call SMinif
    neve = 0
    nmax =nev
    Xini = 0.
    Write(6,*) ' output file ',filename
    Write(6,*) ' selection Ipgg = ',Ipgg,' ns_sel = ',ns_sel
    Write(6,*) ' Qg = ',Qg,' Qs = ',Qs,' Xnorm = ',Xnorm
    Write(6,*) ' LHAPDFLIB for val quark Ipdf = ',Ipdf
    Write(6,*) ' Itarget = ',Itarget
    Write(6,*) ' BB = ',BB

    Qgmin = max(Qg-0.5d0,QCDlam)
    Qgmin = max(Qg,QCDlam)
    x3lmin = log(Qgmin)
    x3lmax = log(qmax)
    x3ldif = (x3lmax-x3lmin)/Real(Nbp)
    Do I=0,Nbp+1
        x3(I) = exp(x3lmin + x3ldif*Real(I))
    Enddo
    Do I=0,Nbp
        x3m(i) = (x3(i) + x3(i+1))/2.
        x3b(i) = x3(i+1) - x3(i)
    Enddo
    Nx3 = -1
C---Initialize analysis
    Xini=0.
    Xfin=0.
    Call updfgrid(1)
    Nx3 = 49
600 Nx3 = Nx3 + 1
    write(6,*) ' ng_max = ',ng_max,' at nx3-1 ',nx3-1
    IF(Nx3.gt.Nbp) Then
        write(6,*) ' evolve_tmd: Nx3 gt Npb -> Program stopped'
        stop
    Endif
    Qbarmy = x3m(Nx3)
    nloop = Nx3
    ng_max = 0

    Write(6,*) ' Qbar_my ',Qbarmy

    Do Iparton=0,2
        Write(6,*) ' evolving parton ID: ',Iparton
        If(iparton.eq.0) then
            gluon=.true.
            else
                gluon=.false.
            Endif
        Xini = 0
        Do I=1,nev
C---Initialize event
            Call SMinfn

C---gluon branching process
            Call smbran
            Xini = Xini + x0
            If(ng_max.le.nrglu) ng_max=nrglu

```



```

        If(wt.NE.0.0) then
            kev=i
            if (kev.gt.0) ic=100000
            if (mod(kev,ic).eq.0) write(6,*) ' event ',kev,nev,' loop P_max ',nloop,Qbarmy
        Endif
    Enddo
Enddo
C---Terminate analysis
call updfgrid(3)
Stop
80 Write(6,*) ' steering file ccfm_updf not found '
stop
End

```

## 4 Program Installation

uPDFevolv follows the standard AUTOMAKE convention. To install the program, do the following

1) Get the source

```

tar xvfz uPDFevolv-XXXX.tar.gz
cd uPDFevolv-XXXX

```

2) Generate the Makefiles (do not use shared libraries)

```

./configure

```

3) Compile the binary

```

make

```

4) Install the executable

```

make install

```

4) The executable is in bin

```

run it with:
bin/updf_evolve < steer_gluon-JH-2013-set2

```

plot the result with:

```

bin/updfread

```

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