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Memorandum

RHEL/M/A26

RATE DEPENDENT MAGNETIZATION IN FLAT TWISTED SUPERCONDUCTING CABLES

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INTRODUCTION

Flat twisted superconducting cables of the type shown in fig. 1 have recently been developed at RHEL for use in pulsed magnets. It has been found that these cables can be compacted to much higher densities than hitherto, with very little damage to the constituent wires.[1] Cables of this type are therefore expected to be widely used in pulsed magnet construction.

The cable consists essentially of a hollow tube of helically twisted wires which has been squashed flat by rolling. It is desirable to have the wires insulated from each other to prevent magnetization currents being induced to flow across the cable by a time varying magnetic field. In practice however it is found that the cable must be heat treated after rolling and this makes it difficult to use conventional insulation on the wires. A layer of oxide on the surface of the wires is not destroyed by heating however and it does provide an impedance to magnetization currents.

Formulae are derived in the following three sections for the rate dependent magnetization of a cable whose strands are electrically linked by a contact resistance. In section (IV) the cable magnetization is combined with the magnetization of its constituent wires, each of which is a filamentary composite. From the combined formula one can judge whether a given transverse resistance between wires in the cable is adequate. Under normal circumstances it turns out that an oxide layer has sufficient resistance to keep the additional magnetization (and hence losses) from the cable down to a small fraction of the basic composite magnetization.

1. FIELD PERPENDICULAR TO FACE - VERTICAL RESISTIVE CURRENTS

When the changing magnetic field is perpendicular to a flat face of the cable, the dominant pattern of induced magnetization currents is as shown in fig. 2(a). Diamond shaped current loops flow horizontally without resistance in the superconducting wires and vertically through the contact resistance at the crossover points. The magnitude of these currents depends on the flux enclosed by the loop and the contact resistance. In an actual cable however there are many overlapping diamond shaped current loops which are electrically linked via the contact resistance. The current flowing in each of the diamonds in 2(a) is not therefore constant but varies continuously from the outer edge to the central cross over point. To calculate the distribution of this current, we must consider the increments of current entering and leaving the main diamond to flow around subsidiary loops as shown in fig. 2(b). Referring to 2(b), let the current in the main loop AEFG be I(x) where x is the longitudinal distance from the edge point A. At the crossover B, an increment ΔI(x) leaves the main loop to circulate around the subsidiary loop ABCD. The magnitude of this current may be calculated by equating the rate of change of enclosed flux to the resistive voltage at the crossover points B and D.

$$\text{Area of ABCD} = 2c\,(2x - x^2/L)$$

$$\text{Contact resistance} = r\cos\theta/(2a\Delta x)$$

(see nomenclature on page 6)

thus

$$\dot{B}_\perp\, 2c(2x - x^2/L) = \Delta I(x)\, r\cos\theta/(2a\Delta x)$$

let $\Delta x \rightarrow 0$

$$\frac{dI}{dx} = \frac{2ac\dot{B}_\perp}{r\cos\theta}\left\{2x - \frac{x^2}{L}\right\}$$

By symmetry, the current I(x) reverses at the centre point E so that the correct boundary condition is I(L) = 0, thus

$$I(x) = \frac{2ac\dot{B}_\perp}{r\cos\theta}\left\{x^2 - \frac{x^3}{3L} - \frac{2L^2}{3}\right\}$$

The total magnetic moment of I(x) flowing around AEFG is:

$$m = \mu_o \sum I \times \text{area enclosed}$$

$$= \mu_o \frac{2ac\dot{B}_\perp}{r\cos\theta} \cdot 2c \int_0^L \left(1 - \frac{x}{L}\right)\left\{x^2 - \frac{x^3}{3L} - \frac{2L^2}{3}\right\} dx$$

$$= \mu_o \frac{\dot{B}_\perp a\, c^2}{r\cos\theta} \cdot \frac{16}{15} L^3$$

The magnetization is the magnetic moment per unit volume. Because the diamonds overlap to completely fill the space available, the effective volume occupied by the loop AEFG is simply the volume occupied by the wire:

$$\text{Volume} = 8\,c\,a^2/\sin\theta$$

$$\text{Magnetization } M_1 = \frac{2\mu_o}{15} \cdot \frac{B_\perp c^2 L^2}{r\,a} \qquad (1)$$

II. FIELD PERPENDICULAR TO FACE - LONGITUDINAL RESISTIVE CURRENTS

Fig. 3 shows a second pattern of shielding currents which can arise with the field perpendicular to the face of the cable. The currents in the top and bottom faces are now quite independent and flow longitudinally across the contact resistance between adjacent wires. The linear current density $J(y)$ is determined simply by the flux enclosed per unit length and the contact resistance of the sloping boundary per unit length ie:

$$2\dot{B}_{\perp}y = J(y)\ \frac{\Delta y . 2r\,\mathrm{Sin}\theta}{2a\Delta y}\ .\ \frac{\mathrm{Sin}\theta}{2a}$$

$$J(y) = 4\ \frac{\dot{B}_{\perp}a^2}{r\,\mathrm{Sin}^2\theta}\ .\ Y$$

The total magnetic moment per unit length is

$$m = 2\mu_o \int_0^c J\,(y)\ y\ dy$$

$$= \frac{8\ \mu_o\ \dot{B}_{\perp}a^2\ c^3}{3\ r\ \mathrm{Sin}^2\theta}$$

hence the magnetization

$$M_2 = \frac{2\mu_o\dot{B}_{\perp}aL^2}{3r\,\mathrm{Cos}^2\theta} \qquad (2)$$

III. FIELD PARALLEL TO FACE - LONGITUDINAL RESISTIVE CURRENTS

If the applied field is parallel to the flat face of the cable, longitudinal currents will be induced to flow through a resistance which is much the same as in (ii). A similar calculation to (II) gives the magnetization

$$M_3 = \frac{2\mu_o\dot{B}_{\parallel}\ a^3\ L^2}{r\ c^2\mathrm{Cos}^2\theta} \qquad (3)$$

IV. TOTAL MAGNETIZATION OF THE CABLE

Provided each effect is well below saturation (and this is usually the case) we may simply add the three separate contributions (1) (2) and (3) to obtain the cable magnetization. In order to obtain the total magnetization of the cable we must also add the filament magnetization M_o and the rate dependent magnetization of the composite[(2)] $M_r = 2\mu_o B\ell^2/3\pi\rho_e$.

$$M \;=\; M_o + M_r + M_1 + M_2 + M_3$$

It is convenient to simplify M_{123} by noting that

$\dfrac{r}{2a} = \rho_a$ the average transverse resistivity of the cable

$\cos\theta \simeq 1$ for a practical twist pitch

$\dfrac{c}{2a} = \alpha$ the aspect ratio of the cable

thus

$$M = M_o + \frac{2\mu_o}{3\pi}\frac{\dot{B}\ell^2}{\rho_e} + \frac{4\mu_o}{15}\frac{\dot{B}_\perp \alpha^2 L^2}{\rho_a} + \frac{\mu_o \dot{B}_\perp L^2}{3\rho_a} + \frac{\mu_o \dot{B}_\parallel L^2}{4\alpha^2 \rho_a}$$

Substituting the filament magnetization

$$M_o = 2\mu_o \lambda J_c d/3\pi$$

$$M = M_o\left\{1 + \frac{1}{\lambda J_c d}\left(\frac{\dot{B}\ell^2}{\rho_e} + \frac{\dot{B}_\perp L^2}{\rho_a}\left[\frac{2\pi\alpha^2}{5} + \frac{\pi}{2}\right] + \frac{\dot{B}_\parallel L^2}{\alpha^2\rho_a}\cdot\frac{3\pi}{8}\right)\right\}$$

The dominant cable term is thus $\dot{B}_\perp$ and it may be seen that the cable magnetization will start to become comparable with the rate dependent composite magnetization when $\alpha^2 L^2/\rho_a \sim \ell^2/\rho_e$. In typical flat cables the twist pitch (4L) is $\sim$ 20 x the wire twist pitch (4ℓ) and the aspect ratio is $\sim$ 4. For an acceptable cable magnetization we must have therefore $\rho_a >> \frac{\alpha^2 L^2}{\ell^2} \rho_e \simeq 6 \times 10^3 \rho_e$.

It is this large ratio between cable resistivity and effective composite resistivity ρ_e which makes soldered cables generally unsuitable for pulsed use.

Typical values of ρ_e for three component composites are $\sim 10^{-9} - 10^{-8}$ ΩM so that we require cable transverse resistivities of $\sim 10^{-4}$ ΩM if the addition losses from the cable are to be small. It has been found that a layer of oxide on the surface of the wires will produce resistivities of about this order.

NOMENCLATURE - SI UNITS THROUGHOUT

α	=	aspect ratio of cable
2a	=	composite wire diameter
B	=	applied field
2c	=	cable width
d	=	superconducting filament diameter
J_c	=	critical current density in filaments
I	=	magnetization current in cable
J	=	linear current density
ℓ	=	1/4 composite twist pitch
L	=	1/4 cable twist pitch
λ	=	filling factor of filaments in composite
m	=	magnetic moment of cable loop
M	=	magnetization
M_o	=	filament magnetization
M_r	=	rate dependent composite magnetization
r	=	contact resistance per unit area between cable strands
ρ_a	=	average transverse resistivity across cable
ρ_e	=	effective transverse resistivity across composite
θ	=	slope angle of wires in cable
X	=	longitudinal distance along cable
Y	=	distance from cable centre line

REFERENCES

1. G E Gallagher Daggitt, RHEL/M/A25

2. M N Wilson, 'Filamentary Composite Superconductors for Pulsed Magnets' 1972 Applied Superconductivity Conference, Annapolis, USA (RHEL preprint RPP A89)

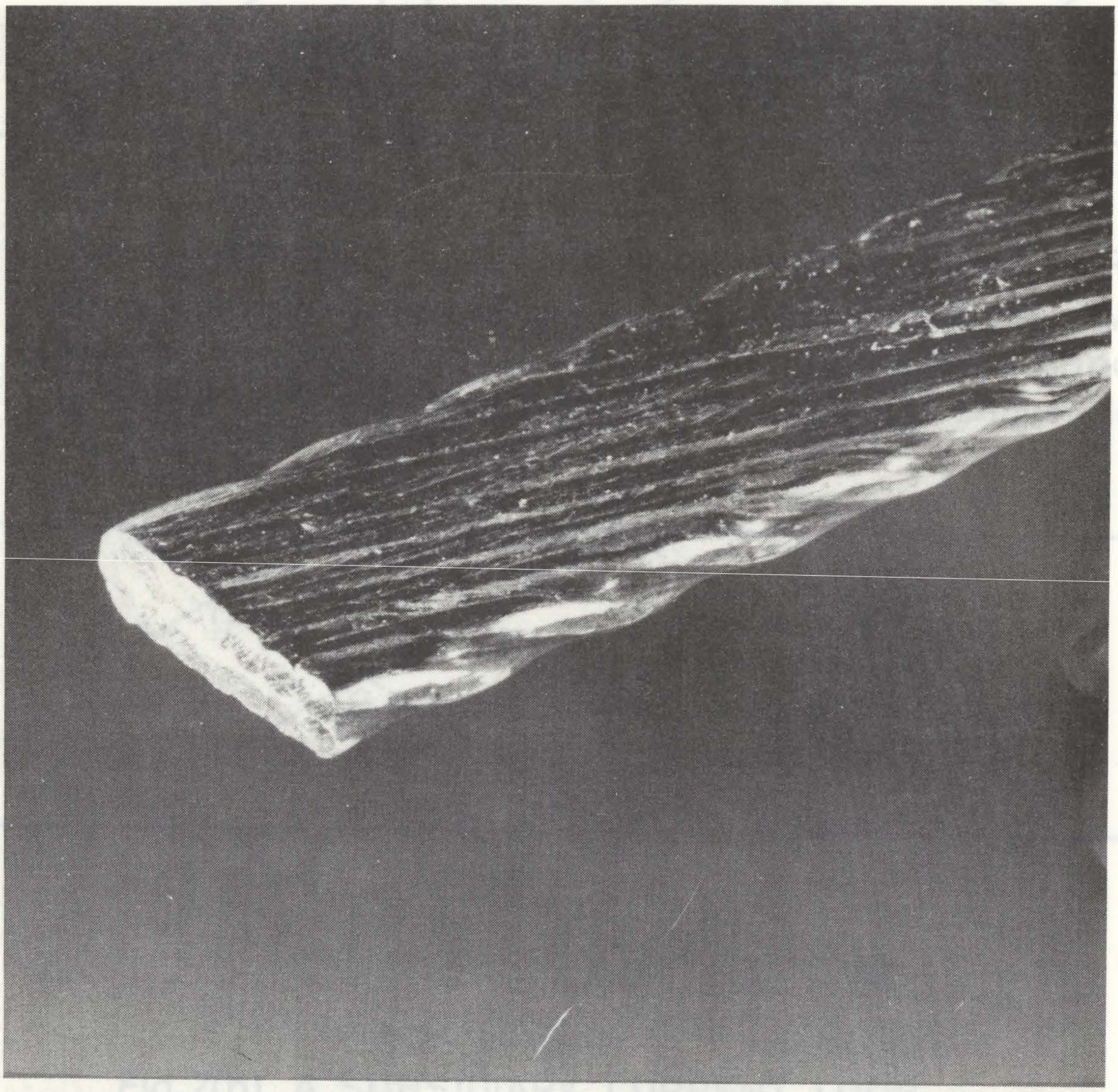

FIG. I. A 15 STRAND FLAT TWISTED CABLE

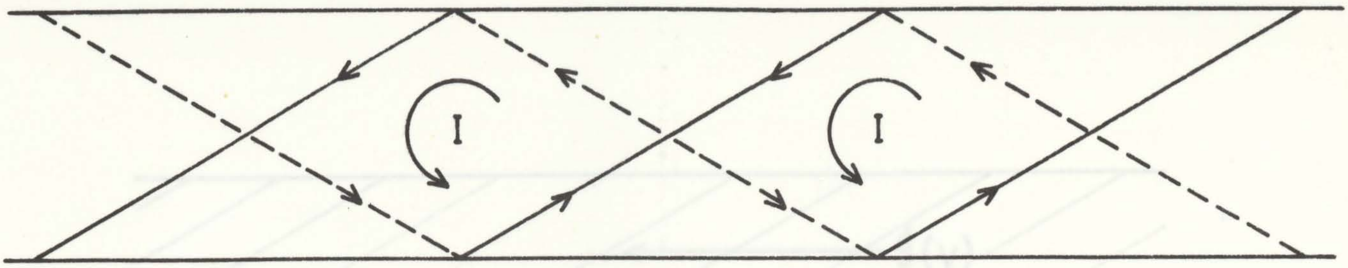

Fig. 2(a). DOMINANT PATTERN OF MAGNETIZATION CURRENTS: field is normal to face of cable (into paper) and resistive currents flow vertically at intersection points (into paper). The wires on the bottom face of the cable are shown dotted.

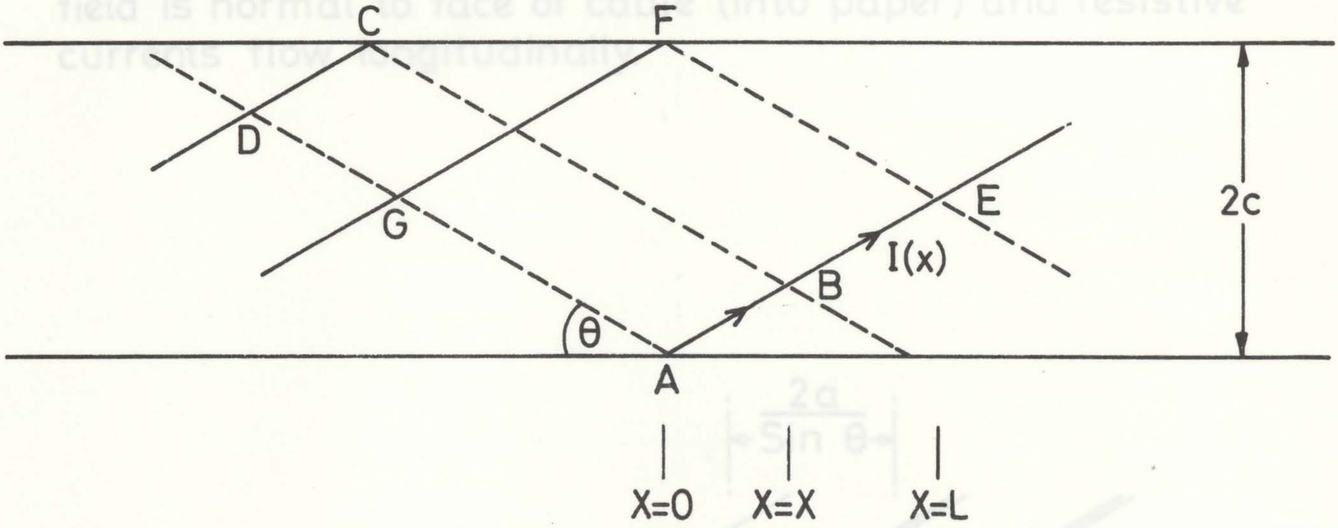

Fig. 2(b). A SUBSIDIARY CURRENT LOOP

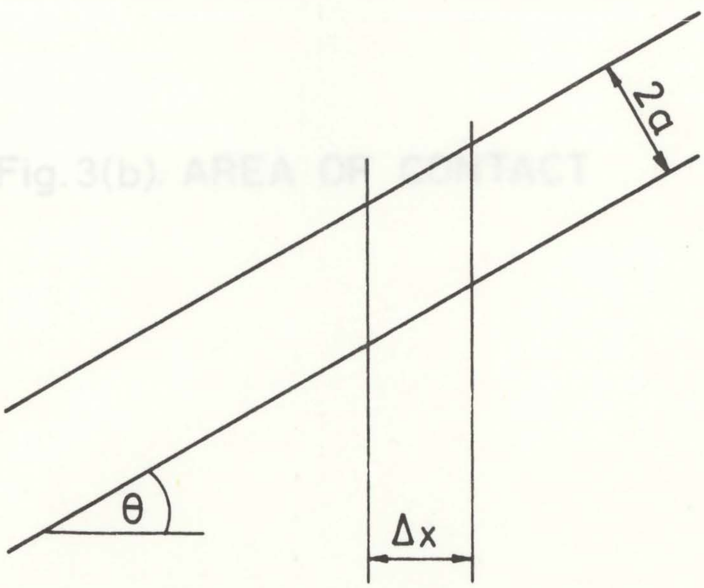

Fig. 2(c). EFFECTIVE CONTACT AREA

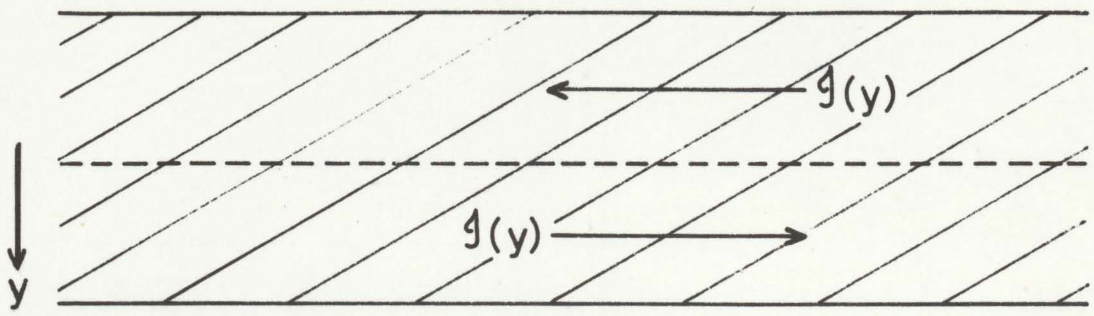

Fig. 3(a). SECONDARY PATTERN OF MAGNETIZATION CURRENTS: field is normal to face of cable (into paper) and resistive currents flow longitudinally.

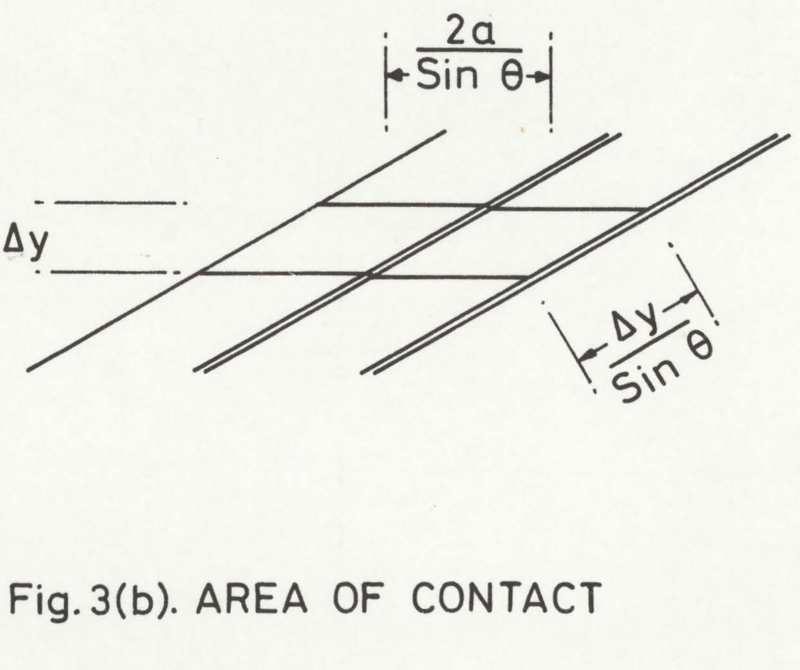

Fig. 3(b). AREA OF CONTACT