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Infra-red fixed point structure characterising SUSY SU(5) symmetry breaking

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Abstract

We analyze the one-loop renormalisation group equations for the parameters of the Higgs potential of a supersymmetric SU(5) model with first step of symmetry breaking involving an adjoint Higgs. In particular, we investigate the running of the parameters that decide the first step of symmetry breaking in an attempt to establish which symmetry-breaking scenarios would be most likely if the model is the effective low-energy description of some more fundamental theory. An infra-red fixed point is identified analytically. It is located at the boundary between the region of Higgs parameter space corresponding to unbroken SU(5) and the region corresponding to the breaking of SU(5) to the Standard Model, and we elaborate on its implications. We also observe that certain forms of the Higgs potential discussed at tree level in the literature are not renormalisation group invariant.

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One of the non-predictive aspects of GUTs (Grand Unification Theories) is the SSB (Spontaneous Symmetry Breaking) pattern [1]. Even after selecting the matter (Higgs) content, most GUTs may break in several different ways depending on which component of the Higgs field acquires a vacuum expectation value, and this in turn depends on the (free input) parameters in the Higgs potential of the model.

In this paper, we consider the GUT as a low-energy effective description of some more fundamental theory[2, 3], possibly including gravity, and therefore the parameters of the Higgs potential are assumed to become meaningful at some scale M^* (possibly given by the Planck scale $M_P \sim 10^{19} GeV$) higher than the GUT scale M_X (the scale, of order 10^{16} GeV, where the low energy couplings unify). From this viewpoint it makes sense to study the RGEs (Renormalisation Group Equations) describing the running of the parameters of the Higgs potential between M^* and M_X ; in fact, if strong infra-red structures were encountered in these RGEs, it could then be argued that some SSB directions are more *natural* than others. For example, a given direction of SSB would be considered to be natural if a strongly attractive infra-red fixed point was found within the corresponding region of Higgs parameter space, since then values for the parameters of the Higgs potential corresponding to the given direction of SSB could be obtained at the GUT scale from rather generic input values at the scale M^* .

This viewpoint is related to the one adopted in the recent literature[2] in which predictions for the low-energy values of certain quantities are obtained from the infra-red structure of the relevant RGEs. The results of those investigations lead to the observation that the values of the (low-energy) parameters relevant for the description of the known physics are strongly influenced by the infra-red structure of RGEs. This encourages an attempt to “understand” the SSB pattern as a possible result of renormalisation group flow. In this letter, in order to illustrate this idea and test its viability, we analyze the first (GUT-scale) step of SSB in a SUSY (supersymmetric) SU(5) GUT, which involves the Higgs of the 24-dimensional irreducible representation (the adjoint). Besides the $\underline{24}$, the Higgs sector of the minimal SUSY SU(5) model also includes $\underline{5} + \overline{\underline{5}}$ Higgs, which are used in the second SSB step. However, for simplicity in our analysis of the first SSB step we neglect the effects of the $\underline{5} + \overline{\underline{5}}$ Higgs. We therefore limit our analysis to the potentials involving the $\underline{24}$ Higgs. The superpotential is taken to be [4]

$$W = \lambda_1 \text{Tr}(\Sigma^3) + \mu \text{Tr}(\Sigma^2), \quad (1)$$

where Σ denotes the 24-dimensional superfield multiplet. We assume that SUSY

breaking is explicit, via the “soft” SUSY-breaking terms in the potential

$$V_{soft} = \left[\frac{m_3}{6} \text{Tr}(\sigma^3) + m_2^2 \text{Tr}(\sigma^2) + \frac{M}{2} \lambda \lambda + \text{h.c.} \right] + m_{3/2}^2 \text{Tr}(\sigma^\dagger \sigma), \quad (2)$$

where σ represents the scalar component of Σ and λ denotes the SU(5) gaugino. The full Higgs potential relevant for the first step of SSB can be written as

$$V = \left| \frac{\partial W}{\partial \Sigma_i} \right|^2 + V_{soft} + \text{D-terms}. \quad (3)$$

Based on hierarchy arguments [5] we expect $m_3, m_{3/2}, M \sim 1$ TeV and $m_2 \sim 10^{11}$ GeV, while μ is a GUT scale parameter expected to be of order 10^{16} GeV. In the special case in which V_{soft} results from simple models of spontaneously broken supergravity [6] the soft breaking parameters at tree level are constrained by

$$m_3 = 6m_{3/2}A\lambda_1, \quad m_2^2 = (A-1)m_{3/2}\mu, \quad (4)$$

with only one free parameter A for the bi- and trilinear terms. In this hypothesis, the quantity $\delta_{3/2} \equiv m_{3/2}/\mu$ measures the relative strength of SUSY breaking in units of M_{GUT} . For $\delta_{3/2}=0$, the scalar potential has three degenerate minima with invariances SU(5), SU(4) \otimes U(1) and $G_{SM} \equiv \text{SU}(3)\otimes\text{SU}(2)\otimes\text{U}(1)$ (the Standard Model gauge group). For the phenomenologically relevant case $|\delta_{3/2}| \ll 1$, one can simply examine the corrections to the scalar potential of first order in $\delta_{3/2}$ that split the degeneracy [4],

$$V_{soft} = \frac{8\mu^4\delta_{3/2}}{27\lambda_1^2} (A-3)b + O(\delta_{3/2}^2), \quad (5)$$

where $b = 30$ for the G_{SM} -invariant minimum, and $b = 20/9$ for the SU(4) \times U(1)-invariant minimum. (Obviously, $b = 0$ in the minimum preserving the full SU(5) invariance.) The direction of SU(5) breaking determined by the vacuum expectation value $\langle \sigma \rangle$ can then be read off the parameter A . The case $A > 3$ does not reproduce the Standard Model phenomenology since then the absolute minimum corresponds to unbroken SU(5) (and even the SU(4) \otimes U(1)-invariant minimum is energetically lower than the G_{SM} -invariant one). On the other hand, for $A < 3$ the lowest minimum of the potential is G_{SM} -invariant (while the SU(4) \otimes U(1)-invariant minimum is energetically lower than the SU(5)-invariant one), leading to the phenomenologically plausible scenario of SU(5) breaking to G_{SM} at the GUT scale.

This concludes the tree-level analysis. It appears quite satisfactory that the phenomenologically plausible scenario simply requires $A < 3$, which would seem to correspond (assuming a simple-minded measure) to roughly half of the parameter space.

However, from the point of view advocated here, one would like to check whether phenomenologically plausible scenarios follow from rather generic choices of input parameter at the scale (higher than the GUT scale) where the GUT becomes meaningful as an effective low-energy description. Let us therefore consider the running of the parameters of the Higgs potential. The one-loop RGEs may be easily derived¹ following the general prescriptions of Martin and Vaughn [7],

$$16\pi^2 \frac{d\lambda_1}{dt} = 3\lambda_1 \left(\frac{189}{40} \lambda_1^2 - 10g^2 \right) \quad (6)$$

$$16\pi^2 \frac{d\mu}{dt} = 2\mu \left(\frac{189}{40} \lambda_1^2 - 10g^2 \right) \quad (7)$$

$$16\pi^2 \frac{dm_3}{dt} = 3 \left[m_3 \left(\frac{189}{40} \lambda_1^2 - 10g^2 \right) + \frac{189}{20} \lambda_1^2 m_3 + 120M\lambda_1 g^2 \right] \quad (8)$$

$$16\pi^2 \frac{dm_2^2}{dt} = 2 \left[m_2^2 \left(\frac{189}{40} \lambda_1^2 - 10g^2 \right) + \frac{63}{40} \lambda_1 \mu m_3 + 20M\mu g^2 \right] \quad (9)$$

$$16\pi^2 \frac{dm_{3/2}^2}{dt} = \frac{567}{20} \lambda_1^2 m_{3/2}^2 + \frac{21}{80} m_3^2 - 40M^\dagger M g^2 \quad (10)$$

$$16\pi^2 \frac{dg^2}{dt} = \beta g^4 \quad (11)$$

$$16\pi^2 \frac{dM}{dt} = \beta g^2 M, \quad (12)$$

where $t = \ln(q^2/M_X^2)$, q is the \overline{MS} renormalisation scale and the one loop beta function, $\beta = 2(S(R) - 15)$, is determined by the sum over all the Dynkin indices of the fields in the theory, $S(R)$. $\beta = -8$ for our SUSY SU(5) model, which hosts the above mentioned Higgs sector plus $3(\underline{10} \oplus \overline{\underline{5}})$ representations corresponding to 3 Standard Model fermionic families (and superpartners).

The different evolution of m_3 , m_2^2 and $m_{3/2}$ implies that the constrained parameterisation (4) is not renormalisation group invariant and consequently the above tree-level analysis of symmetry breaking is not sufficient. In generalising the analysis of symmetry breaking to the case of running parameters in the full potential (3), it is appropriate to consider the three independent parameters $\delta_2 \equiv m_2^2/\mu^2$, $\delta_3 \equiv m_3/\mu$ and $\delta_{3/2} \equiv m_{3/2}/\mu$. In terms of these parameters, the soft-breaking potential can be

¹In our calculation we take the matrix representation of the chiral superfield to be $\Sigma = T^a \Phi^a$, where the generators T^a of the fundamental representation of SU(5) are normalised by $\text{Tr}(T^a T^b) = \delta^{ab}/2$. Some of the RGEs (6-12) have been previously derived in ref. [8] using a different field normalisation.

written as²

$$V_{soft} = \frac{8\mu^4}{27\lambda_1^2} b F, \quad (13)$$

where

$$F \equiv 3\delta_2 - \frac{1}{3\lambda_1}\delta_3 + \frac{3}{2}\delta_{3/2}^2. \quad (14)$$

Hence, for $F < 0$ the G_{SM} -invariant minimum is the lowest one, while SU(5) will remain unbroken for $F > 0$. The value of F at the GUT scale M_X determines the type of residual symmetry below M_X .

To render the fixed point structure explicit, from (6-12) we form the following RGEs for dimensionless ratios

$$16\pi^2 \frac{d}{dt} \left(\frac{\lambda_1^2}{g^2} \right) = 6g^2 \left(\frac{\lambda_1^2}{g^2} \right) \left[\frac{189}{40} \left(\frac{\lambda_1^2}{g^2} \right) - 10 - \frac{\beta}{6} \right] \quad (15)$$

$$16\pi^2 \frac{d}{dt} \left(\frac{m_3}{M\lambda_1} \right) = 9g^2 \left[\left(\frac{m_3}{M\lambda_1} \right) \left[\left(\frac{\lambda_1^2}{g^2} \right) \frac{63}{20} - \frac{\beta}{9} \right] + 40 \right] \quad (16)$$

$$16\pi^2 \frac{d}{dt} \left(\frac{m_2^2}{M\mu} \right) = g^2 \left[-\beta \left(\frac{m_2^2}{M\mu} \right) + \frac{63}{20} \left(\frac{m_3}{M\lambda_1} \right) \left(\frac{\lambda_1^2}{g^2} \right) + 40 \right]. \quad (17)$$

The right hand side of this system of coupled equations vanishes for

$$\left(\frac{\lambda_1^2}{g^2} \right)^* = \frac{40}{189}(10 + \beta/6), \quad \left(\frac{m_3}{M\lambda_1} \right)^* = -6, \quad \left(\frac{m_2^2}{M\mu} \right)^* = -\frac{2}{3}. \quad (18)$$

By linearising (15-17) around the fixed point one easily finds that it is infra-red stable when $\beta < 0$, as in the case of the SUSY SU(5) model considered here. For $\beta > 0$, which can be achieved by adding more matter to the model, one would have a saddle point. Assuming $\delta_{3/2} \ll 1$, as implied by hierarchy arguments, we may neglect the second order contribution of order $\delta_{3/2}^2$, and F is well approximated by

$$F \approx 3\delta_2 - \frac{1}{3\lambda_1}\delta_3 = \frac{M}{\mu} \left[3 \frac{m_2^2}{M\mu} - \frac{1}{3} \frac{m_3}{M\lambda_1} \right], \quad (19)$$

which is zero at the fixed point. Thus, starting at some scale M^* , *e.g.* the Planck scale, and running to the GUT scale, the Higgs parameters evolve towards values at the boundary ($F = 0$) between the region of parameter space corresponding to unbroken SU(5) and the region of parameter space corresponding to SU(5) breaking to G_{SM} ³.

²An interesting alternative to the conventional scenario that we consider is the one of “radiative breaking” at the GUT scale. In particular, this would require considering in what follows the possibility $\mu = 0$, which is stable under the one-loop RGEs. In the present work we shall ignore this possibility. Its analysis would require a generalisation of our study of the Higgs potential, not relying on the simplifications we achieved by assuming $|m_i/\mu| \ll 1$.

³If there are significant contributions from $\delta_{3/2}^2$ the flow to the unbroken-SU(5) region is favoured.

We have also studied our RGEs numerically for the parameter values $M^* = 10^{19}$ GeV, $\mu = M_X = 10^{16}$ GeV, $M(M^*) = m_{3/2}(M^*) = 10^{-13}M_X$ as advocated in most phenomenological soft SUSY-breaking scenarios. The gauge coupling is fixed by $g^2(M_X) = 8\pi/5$ to ensure consistency of SUSY SU(5) unification of the Standard Model couplings with the low-energy values of the Standard Model couplings. The running parameters M and μ evolve slowly, *i.e.* they decrease by a factor of 1/2 between the Planck and the GUT scale; consequently their ratio in (19) does not change sign. Hence, once the initial conditions are fixed, the sign of the function F depends on the relative magnitude of the combinations of parameters $m_3/(3M\lambda_1)$ and $3m_2^2/(M\mu)$. The flow of these is depicted in Fig. 1 for a small and a large initial value of $\lambda_1(M^*)$. The dashed line marks $3m_2^2/(M\mu) = m_3/(3M\lambda_1)$ where $F = 0$. The region to the left of this line corresponds to the breaking of SU(5) to G_{SM} while the region to the right corresponds to unbroken SU(5). For all the chosen initial values

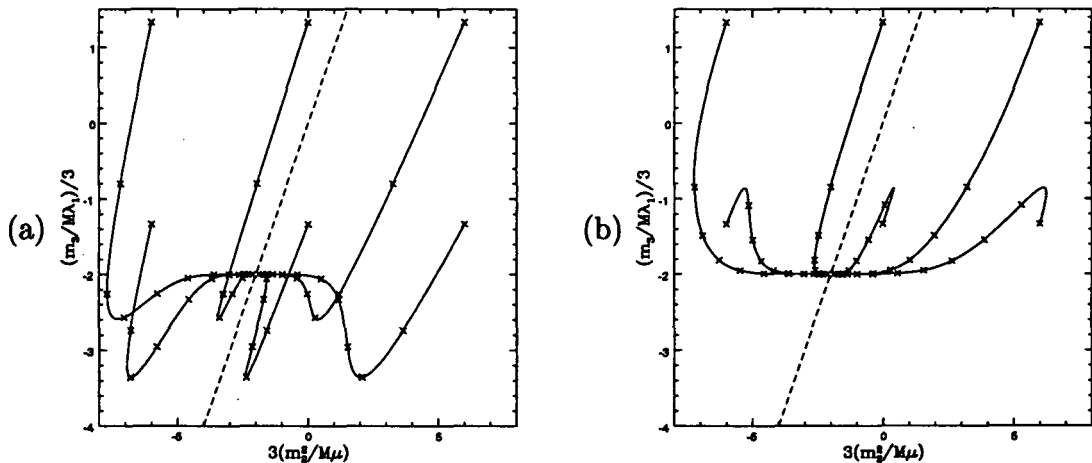


Figure 1: *RG flow of the soft SUSY-breaking parameters in SUSY SU(5) with $\beta = -8$ for initial conditions with a) $\lambda_1(M^*) = 0.3$ and b) $\lambda_1(M^*) = 2.0$. Every decrease of the scale by a factor $10^{3/2}$ is marked on the flow.*

we have checked numerically that the contribution of $\delta_{3/2}^2$ is indeed negligible over the whole range of the running. The figure clearly displays the attracting fixed point; however, the attraction is typically rather weak between the Planck scale (first mark on the flow), and the GUT scale (third mark on the flow). Interestingly, flows starting on the left (right) of the dashed line stay on the left (right); therefore the flows never cross the boundary between the region of parameter space corresponding to unbroken SU(5) and the one corresponding to SU(5) breaking to G_{SM} . This behaviour is also present if the coefficient of the beta function is positive. For example, Fig.2 shows

the flow diagram for the same initial conditions as in Fig.1, but now taking $\beta = 2$ in the RGEs (15)-(17). From Fig.2 it is clear that for $\beta > 0$ a saddle point, rather than a fixed point, is present, and the trajectories flow away from the dashed line. This general property implies that the running does not affect the amount of tuning

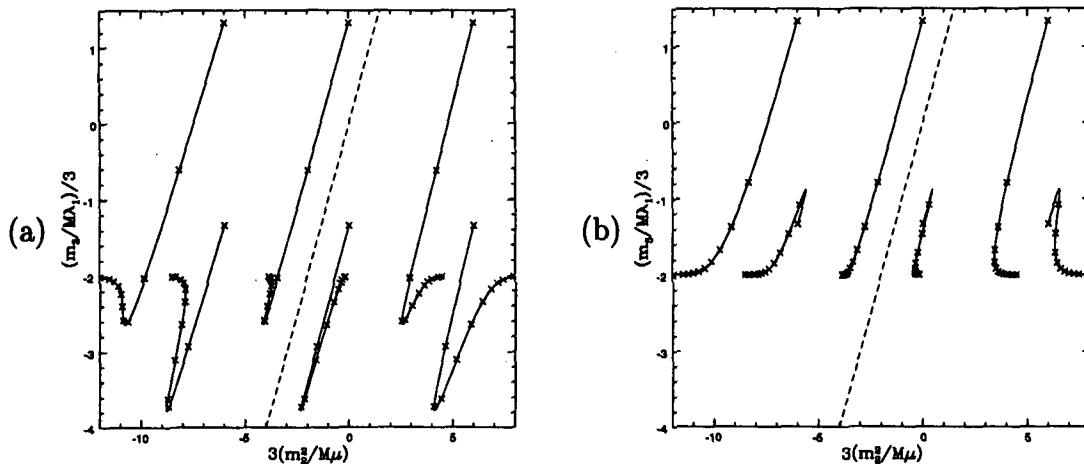


Figure 2: Same as Fig.1, but now for a model with $\beta = 2$.

needed for the phenomenologically desirable scenario of SU(5) breaking to G_{SM} , in the sense that the region of parameter space supporting this scenario is mapped into itself by the RG flow. We conclude that, while it does not require any fine tuning, the scenario with SU(5) breaking to the Standard Model is not a compelling prediction of the infra-red RG structure of SUSY SU(5).

We have limited ourselves to a zero-temperature analysis. However, an important constraint on GUTs is the consistency with a working cosmological scenario, and checking this consistency requires in general a finite-temperature analysis. While we postpone this type of study to future work, we would like to make some statements concerning the possibility of cosmological implications of *Renormalisation Group Naturalness* analyses of the type here reported.

An important factor affecting *supercosmology* [9, 10, 11] is the near degeneracy (up to SUSY breaking terms) of several minima, which we mentioned above. The free-energy difference between the absolute minimum and the other minima is of order SUSY breaking terms, and therefore much smaller than the GUT scale. In such cases one finds, at least within a perturbative analysis, that even when the temperature becomes low enough for the features of the zero-temperature effective potential to be relevant, the universe does not rapidly reach the vacuum corresponding to the absolute

minimum of the zero-temperature effective potential [11]. Actually, estimates within ordinary perturbative approaches suggest that the time needed for the transition to the true vacuum should be longer than the lifetime of the universe [11].

One way to obtain working supercosmology scenarios is to advocate [10] thermal strong-coupling effects, which are indeed at work in SUSY GUTs [12]. The investigation of these issues requires a careful (and very delicate) thermal analysis which goes beyond the scope of this paper. However, it should be noticed that the type of analysis given here is not very relevant to this type of supercosmological scenarios.

A more conventional, but *ad hoc*, way to obtain working supercosmological scenarios is based [13] on fine tuning of the parameters of the Higgs potential. One scales down the entire superpotential, so that the height of the potential barrier between competing vacua is of the same order as their energy difference, while keeping fixed the mass of the gauge bosons mediating proton decay. For example in the SUSY SU(5) GUT one would divide [13] both λ_1 and μ by a common large factor of order 10^{12} , so that the ratio μ/λ_1 giving mass to the gauge bosons mediating proton decay remains unchanged

$$\lambda'_1 \sim 10^{-12} \lambda_1, \quad \mu' \sim 10^{-12} \mu, \quad (20)$$

$$M'_X \sim \frac{\lambda'_1}{\mu'} \sim \frac{\lambda_1}{\mu} \sim M_X. \quad (21)$$

Analyses of the type advocated in the present paper could be relevant for this supercosmology scenario; one can in fact check the level of fine tuning at the Planck scale needed to have, say, a 10^{-12} fine tuning at the GUT scale. We find that the fine-tuned values of λ and μ are so far from the region of attraction of the fixed point that the RG running between M_P and M_X is not substantial; *e.g.*, a fine tuning of 10^{-13} is required at the Planck scale in order to obtain a 10^{-12} fine tuning at the GUT scale.

The SUSY SU(5) GUT examined here is a toy model because, *e.g.*, it does not break electroweak symmetry. We believe that it would be interesting to investigate whether some of the issues exposed here affect the analysis of phenomenologically relevant models. If a non-trivial fixed point structure was found also in those more complicated models it could have important implications for the associated SSB physics. Similarly, there might be important implications if it was found that even in phenomenologically relevant models certain forms of the Higgs potential discussed at tree level in the recent literature are not renormalisation group invariant.

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