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A Abdurrahman

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Improved Lagrangian for the Fermionic String

A. Abdurrahman[1], J. Bordes[2]

Department of Particle Physics
Rutherford Appleton Laboratory
Chilton DIDCOT, Oxon OX11 0QX, U.K.
U.K.

Abstract

The principle of a "world sheet" local gauge invariance is applied to the free Lagrangian for the fermionic string. It is seen that the requirement that the theory be invariant under a world sheet local gauge transformation leads to the introduction of a world sheet gauge field interacting with the fermion field in a definite way. The improved expression, for the fermionic piece of the Lagrangian, could be interpreted as the Lagrangian for a "world sheet" quantum electrodynamics.

Let us consider the Lagrangian for the fermionic string[3] (say eq. (4.1.2) in [1]):

$$S = -\frac{1}{2\pi}\int d\sigma^2(\partial_\alpha X^\mu \partial^\alpha X_\mu - i\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu) \tag{1}$$

This lagrangian and its symmetries have been studied in detail. However, there is a new symmetry of the above Lagrangian that have not been looked at before. So far no one has studied phase transformation, of the first and second kind (global and local gauge transformation) involving the "*world sheet*" of the string and not the "*space time*" of the string, of the fermionic field in (1). We will see below that an improved Lagrangian, with much richer structure than the

[1]E-Mail: abdurrahman@v2.rl.ac.uk
[2]Departament de Física Teórica, Universitat de Valencia, Dr. Moliner 50, E-46100, Burjassot, Spain and IFIC, Centro Mixto Universitat de Valencia-CSIC, Spain.
[3]For detailed references see [1].

original one, will emerge as a result of the proposed gauge transformation. The improved Lagrangian will bee seen to give rise to a new way for the superstring interaction.

Consider the fermionic piece in (1):

$$S_F = \frac{i}{2\pi}\int d\sigma^2 \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu \tag{2}$$

It is clear that this Lagrangian is invariant under the global gauge transformation

$$\psi^\mu(\sigma,\tau) \rightarrow e^{i\theta}\psi^\mu(\sigma,\tau) \tag{3}$$

Now if we demand that the Lagrangian is also invariant under local gauge transformation, that is, $\theta = \theta(\sigma,\tau)$, then we are forced to modify the above Lagrangian to restore gauge invariance. This may look very familiar ; it is like when one imposes local gauge invariance on the free particle Dirac Lagrangian to get all of electrodynamics. However, the implementation is not the same; this is a new symmetry for the fermionic string [4], since the transformation involves the *world sheet* of the fermionic string and not the *space time* of the fermionic string as it is the case in ordinary electrodynamics. Now the local invariant lagrangian is simply [5]

$$S_F = \frac{i}{2\pi}\int d\sigma^2 (\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu + iq\bar{\psi}^\mu \rho^\alpha \psi_\mu A_\alpha) \tag{4}$$

where A_α is new vector field (gauge field) that transform under local gauge transformation according to the rule

$$A_\alpha \rightarrow A_\alpha + \partial_\alpha \lambda \tag{5}$$

and

$$\lambda(\sigma^\alpha) = -\frac{1}{q}\theta(\sigma^\alpha) \tag{6}$$

The above form of the Lagrangian is not the whole story; we must include a free term for the gauge field. A good guess is the Proca type Lagrangian

$$S_P = -\frac{1}{2\pi}\int d\sigma^2 (\frac{1}{2}F^{\alpha\beta}F_{\alpha\beta} - m_A^2 A^\alpha A_\alpha) \tag{7}$$

[4] As far as we know this is the first time local gauge invariance for the fermionic string Lagrangian is considered in the sense implemented here.

[5] For the bosonic part, $X^\mu(\sigma,\tau)$, we assume that $\theta = 0$. This is the same statement as saying that the charge (coupling) for the bosonic part is zero and that the charge of the string is carried only by the fermionic part.

The mass term is not invariant under the above rule therefore for the whole Lagrangian to be invariant the gauge field must be massless. Hence, the complete lagrangian is

$$S_F = \int d\sigma^2[\frac{i}{2\pi}\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu - \frac{1}{2\pi} q\bar{\psi}^\mu \rho^\alpha \psi_\mu A_\alpha - \frac{1}{4\pi} F^{\alpha\beta} F_{\alpha\beta}] \tag{8}$$

and the new improved superstring Lagrangian ($S = S_B + S_F$) is now

$$S = \int d\sigma^2\{-\frac{1}{2\pi}(\partial_\alpha X^\mu \partial^\alpha X_\mu - i\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu) - \jmath^\alpha A_\alpha - \frac{1}{4\pi} F^{\alpha\beta} F_{\alpha\beta}\} \tag{9}$$

At this point it is worth mentioning that A^α is a world sheet potential analogous to the electromagnetic potential. The world sheet current density

$$\jmath^\alpha = \frac{1}{2\pi} q\bar{\psi}^\mu \rho^\alpha \psi_\mu \tag{10}$$

acts as a source for the gauge field (A^α). Note this is a new quantity different from the supercurrent of the original Lagrangian eq. (1),

$$J_\alpha = \frac{1}{2}\rho^\beta \rho_\alpha \psi^\mu \partial_\beta X_\mu \tag{11}$$

(see eq. (4.1.13) in [1]). At this point there are few remarks worth making. The first is that as a result of the extra terms due to the requirement of local gauge invariance the above supercurrent as well as the energy momentum tensor (eq. (4.1.14) in [1]) will certainly change[6]. Thus one may hope that the anomaly in the *super − Virasoro* algebra changes which could modify the critical dimension. The second; if we recall that Witten's interaction does not work for the superstring; here superstrings can interact through the exchange of the gauge particle. The third, we can easily generalize the abelian case to a non-abelian gauge theory thus leading to a "*world sheet*" chromodynamics. Other aspects of the new Lagrangian, eq. (9), may be explored to have a better understanding of this symmetry [2]. Finally if we regard local gauge invariance on the *world sheet* of the string as fundamental, then this new symmetry is likely to influence not only superstring theory but it may be of consequences to other 2D theories.

References

[1] M.B. Green, J.H. Schwarz, And E. Witten, Superstring Theory, Vol. 1, (Cambridge University Press, 1986) contains detailed references.

[2] Work in progress.

[6]Note that the equations of motion (which are easily) derived from the new Lagrangian are different from those derived from the old one.