Further Evidence for Threefold Maximal Mixing

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1. MOTIVATION

Threefold maximal mixing has a long history [1-5]. Our interest was triggered by the realisation that such mixing was consistent with a cyclic symmetry among the generations. In fact a complete S_3 permutation symmetry of the mass matrix for a given fermion species would require two of the generations to be degenerate. The hamiltonian matrix for a classical system of three equal weights joined by three equal springs (see Fig. 1) is S_3 symmetric (it is 'circulant' with real off-diagonal elements). The energy spectrum for this system is $E_2 = E_1$, $E_3 = \sqrt{2}E_2$, with some similarity to the measured fermion mass spectra. Experimentally, of course, the first two fermion generations are not perfectly degenerate.

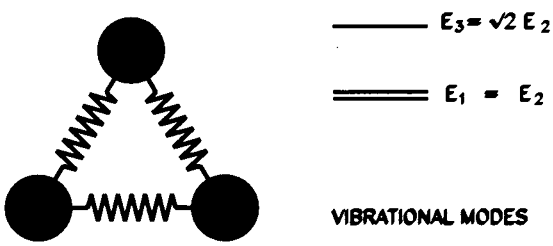

Figure 1. A classical system with S_3 symmetry; the two lowest frequency modes are degenerate.

A cyclic (C_3) symmetry however is consistent with an arbitrary spectrum (the mass matrix is circulant but with complex off-diagonal elements). If we suppose that the neutrino mass matrix (squared) is C_3 symmetric, in a basis where the rows and columns are labelled ν_e, ν_μ, ν_τ, we are led to threefold maximal mixing for leptons:

$$\begin{array}{c} \\ \nu_1 \\ \nu_2 \\ \nu_3 \end{array} \begin{array}{c} \begin{array}{ccc} \nu_e & \nu_\mu & \nu_\tau \end{array} \\ \left(\begin{array}{ccc} \omega/\sqrt{3} & \omega^2/\sqrt{3} & 1/\sqrt{3} \\ \omega^2/\sqrt{3} & \omega/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{array} \right) \end{array}$$

where ω a complex cube-root of unity. We have verified that evolution of the lepton mixing matrix through SM/MSSM loop-corrections is negligible (in contrast to the case of the quarks).

2. THE SCENARIO

It is sometimes useful to think of oscillation effects as similar to diffraction/interference effects involving 'slits' (see Fig. 2). The slits

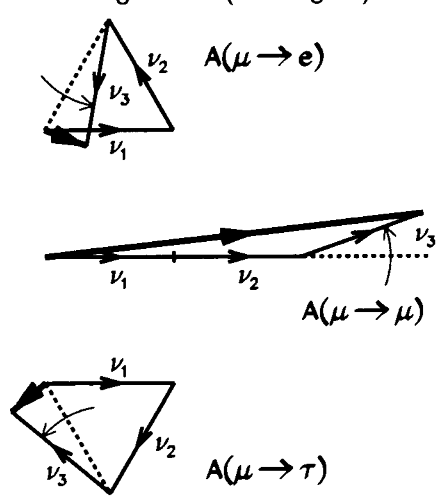

Figure 2. Amplitudes for neutrino oscillation; similar to the diffraction/interference pattern of a 3-slit aperture (see text).

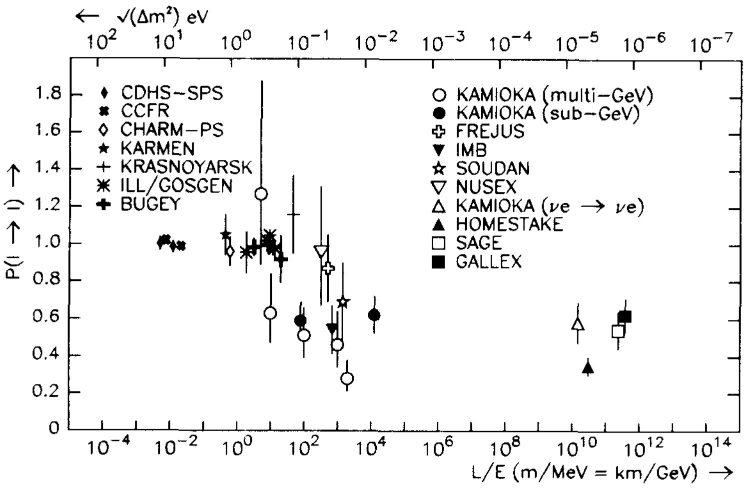

Figure 3. The data on neutrino oscillations as it was in 1994 (disappearance experiments only were plotted, appearance experiments reporting no effect). The data suggest a step at $L/E \simeq 10^2$ km/GeV.

here are in one-to-one correspondence with the neutrino mass eigenstates. Any relative phase changes occuring 'en-route' through the slits (due to mass differences between the neutrinos) leads to a change in the flavour content. In effect the sides of the the unitarity triangles rotate (at different rates). Clearly if only one side of the unitarity triangle moves the $i \longrightarrow f$ and $f \longrightarrow i$ amplitudes are always equal in length and there are no asymmetries.

All 'paths' contribute equally in threefold maximal mixing and the sides of the unitarity triangle are all of length 1/3. With only one neutrino 'active' the total $i \longrightarrow i$ amplitude starts at 1 and falls to 1/3 when the ν_3 amplitude has turned through 180^o. The survival probability oscillates between 1 and 1/9 with mean value of 5/9. At the same time the $i \longrightarrow f$ amplitude starts at 0 and grows to 2/3 so that the appearance probabilities oscillate between 0 and 4/9 with mean value 2/9.

Since all effects are flavour independent you have the right to put all the data on one plot. Fig. 3 shows the raw data on neutrinos oscillations as it was in 1994 on an L/E plot (disappearance experiments only were plotted; appearance experiments were reporting no effect). Even without any theoretical curves superposed the data suggest a step in the region $L/E \sim 10^2$ km/GeV, followed by a plateau corresponding to a survival probability which could be consistent with 5/9. In the perspective of this plot the one or two 'odd-ball' points do not look too much to worry about (taking a deliberately 'broad-brush' view).

3. ATMOSPHERIC NEUTRINOS

For atmospheric neutrinos in the above plot we just plotted the raw atmospheric neutrino ratio $R \equiv (\mu/e)_{DATA}/(\mu/e)_{MC}$. Why doesn't the universal factor of 5/9 'cancel-out' in the ratio? With an initial ν_μ/ν_e ratio of 2/1 the loss of ν_e in threefold maximal mixing is exactly compensated by the production of ν_e from ν_μ ($\frac{5}{9} + 2 \times \frac{2}{9} = 1$). On the other hand the loss of ν_μ is only partially compensated by ν_μ from ν_e ($\frac{5}{9} + \frac{1}{2} \times \frac{2}{9} = \frac{2}{3}$) so that, up to a relatively small correction, the atmospheric neutrino ratio measures the survival probability for ν_μ. For the KAMIOKA multi-GeV data the ν_μ/ν_e ratio is even larger (at least for zenith angles close to 0^o or 180^o) which accounts for the 'odd-ball' (low) KAMIOKA point in Fig. 3.

The new SUPER-K data on atmospheric neutrinos (8 kton yr) were presented at this meeting by Nakmura [6]. Statistically independent of the old KAMIOKA results they confirm the anomaly with the result $R = 0.64 \pm 0.04 \pm 0.06$ for the sub-GeV data. The new result from SOUDAN

(3 kton yr) presented by Gallagher [7] may be stated: $R = 0.61 \pm 0.14 \pm 0.06$. Both these results have been plotted in Fig. 4 together with the older data. There is no question that the results from the water-cerenkov and tracking detectors are consistent with each other and also with the threefold maximal mixing $R = 2/3$.

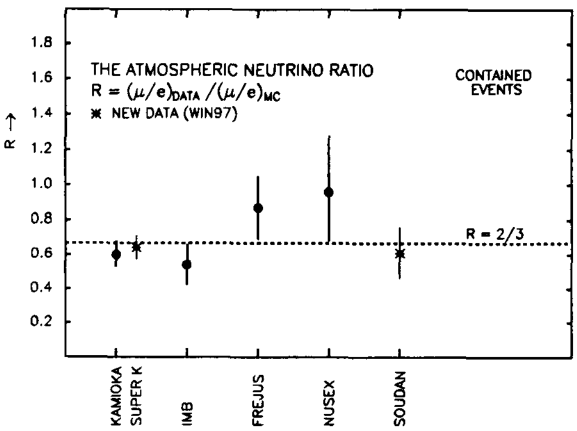

Figure 4. The atmospheric neutrino ratio for contained events from the various experiments.

In our paper [4] we pointed out that as long as two neutrinos remain effectively degenerate, any mixing matrix with the ν_3 maximally mixed:

$$\begin{array}{cc} & \begin{array}{ccc} \nu_e & \nu_\mu & \nu_\tau \end{array} \\ \begin{array}{c} \nu_1 \\ \nu_2 \\ \nu_3 \end{array} & \left(\begin{array}{ccc} * & * & * \\ * & * & * \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{array} \right) \end{array}$$

has identical phenomenology to threefold maximal mixing. Using the zenith angle dependence measured in the KAMIOKA experiment, we went on to show that a range of models with the ν_τ maximally mixed:

$$\begin{array}{cc} & \begin{array}{ccc} \nu_e & \nu_\mu & \nu_\tau \end{array} \\ \begin{array}{c} \nu_1 \\ \nu_2 \\ \nu_3 \end{array} & \left(\begin{array}{ccc} * & * & 1/\sqrt{3} \\ * & * & 1/\sqrt{3} \\ * & * & 1/\sqrt{3} \end{array} \right) \end{array}$$

were excluded (in particular the model of Fritzsch and Xing [8]). The new SUPER-K data on the zenith-angle dependence are consistent with the old KAMIOKA data and do not contradict this conclusion.

4. THE SOLAR DATA

The new SUPER-K result for the total ^{8}B flux (measured with a threshold $E > 6.5$ MeV) was presented by Suoboda [9] at this meeting: $(2.65 \pm 0.09 \pm {}^{0.14}_{0.10}) \times 10^6$ cm$^{-2}s^{-1}$. More recently [10] the SUPER-K result has been quoted as: $(2.44 \pm 0.06 \pm {}^{0.25}_{0.09}) \times 10^6$ cm$^{-2}s^{-1}$. The new results are fully consistent with the old KAMIOKA result: $(2.80 \pm 0.19 \pm 0.33) \times 10^6$ cm$^{-2}s^{-1}$, but do reduce the KAMIOKA/HOMESTAKE difference.

Fig. 5 shows the various [4,11,12] solutions to the solar neutrino problem including the most recent SUPER-K data-point [10]. The ^{8}B flux is treated as an adjustable parameter [13] in these fits, being individually optimised for each solution. Assuming threefold maximal mixing

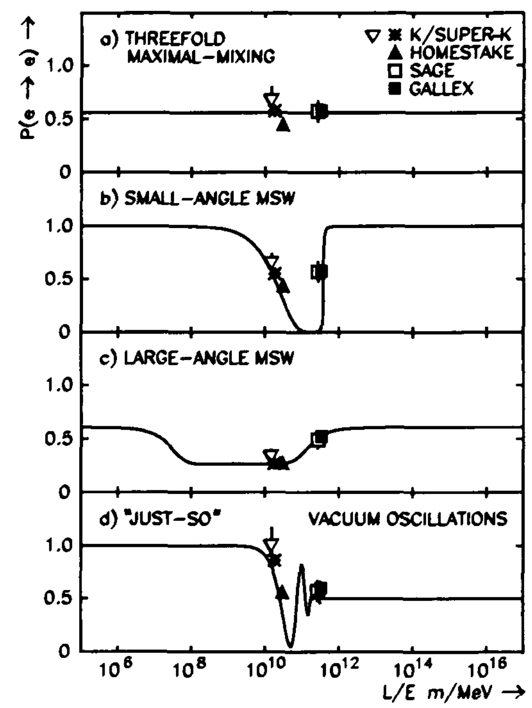

Figure 5. Solutions to the solar neutrino problem. The energy-dependent solutions (b-d) are inherently unconvincing (see text).

the best-fit value for the total ^{8}B flux is now: 3.84×10^6 cm$^{-2}s^{-1}$.

Having compared the various solutions (Fig. 5) I have to say that I personally find the energy-dependent solutions (b-d) wholly unconvincing: all that interesting and informative energy-dependence conveniently localised in just the little 'window' ($<$ 2 decades in E) where our experiments are sensitive (see Fig. 5). Why should we be so lucky? A priori, energy independent solutions (like the threefold maximal mixing solution) are much more plausible.

The new SUPER-K data has reduced the apparent energy dependence. But for further progress I believe we have to look elsewhere. The K/SUPER-K results are based on ν-e elastic scattering and are presumably rather reliable therefore (the response of the SUPER-K detector to electrons has been directly measured). The two gallium experiments have been famously calibrated [14] with neutrinos from ^{51}Cr. In this sense, HOMESTAKE is now the only 'uncalibrated' solar neutrino experiment. Could there be a problem with the cross-section on ^{37}Cl? There was no presentation on HOMESTAKE at this meeting.

5. APPEARANCE EXPERIMENTS

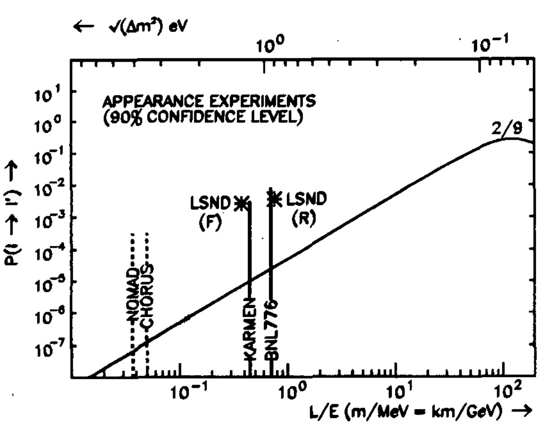

Figure 6. The latest LSND results (R = 'at rest', F = 'in flight') and limits from other appearance experiments compared to threefold maximal mixing with $\Delta m^2 = 0.72 \times 10^{-2}$ eV^2.

Fig. 6 shows that if the LSND results [15] are correct then threefold maximal mixing with $\Delta m^2 \simeq 10^{-2}$ eV^2 is excluded. You could consider increasing $\Delta m^2 \rightarrow 10^{-1}$ eV^2 to fit the LSND 'at-rest' result but then you are in trouble with the reactor data in threefold maximal mixing. You would also have a problem with the new LSND 'in-flight' result: $P(\mu \rightarrow e) = (0.26 \pm 0.10 \pm 0.05)\%$. In the 'leakage' region (small phase changes) the appearance probability is proportional to $(L/E)^2$, independent of the mixing model. The data suggest an L/E-independent appearance probability corresponding to 'saturated' oscillations with even larger Δm^2.

While the LSND results deserve to be taken seriously, it is fair to say that they are in need of independent confirmation. The KARMEN [16] experiment at RAL is expected to be decisive.

Acknowledgement

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