

# Secondary Heavy Quark Pair Production in $e^+e^-$ Annihilation

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## Abstract

The multiplicity of heavy quarks from gluon splitting in  $e^+e^-$  annihilation has now been theoretically calculated and experimentally measured at LEP. However, the experimental measurement requires theoretical input for the shape of the multiplicity with respect to an event shape. In this paper we calculate the multiplicity of heavy quarks from gluon splitting in  $e^+e^-$  annihilation as a function of the heavy jet mass of the event, to next-to-leading logarithmic accuracy. We compare our result with Monte Carlo predictions.

# Introduction

Heavy quark production in  $e^+e^-$  annihilation can come from two sources: from the hard interaction itself,  $e^+e^- \rightarrow Q\bar{Q}$ , and from the splitting of perturbatively produced gluons,  $e^+e^- \rightarrow q\bar{q}g \rightarrow q\bar{q}Q\bar{Q}$ . We call the latter *secondary* heavy quarks. Their total rate is an infrared-safe quantity, so can be calculated as an order-by-order perturbative expansion in  $\alpha_S$ , starting at  $\mathcal{O}(\alpha_S^2)$ . The leading order term in this expansion was calculated in Refs. [1,2]. At higher orders in  $\alpha_S$ , large logarithms arise,  $\alpha_S^n \log^{2n-1}(s/m_Q^2)$ , potentially spoiling the convergence of the perturbative series at high energies,  $s \gg m_Q^2$ . In Ref. [3] these (leading) logarithms were summed to all orders in  $\alpha_S$ . The next-to-leading logarithmic terms,  $\alpha_S^n \log^{2n-2}(s/m_Q^2)$ , were resummed in Ref. [1] yielding a result that is uniformly reliable for all  $s$ . However, only the total multiplicity was calculated, retaining no dependence on the jet kinematics. In this paper we perform a more differential calculation, allowing the multiplicity to be calculated for various event shapes.

Several experimental measurements of the secondary heavy quark production rate have now been made. In Ref. [4], it was extracted for charm quarks from a measurement of the  $D^*$  fragmentation function, and found to be more than a factor of two above the expectation of Ref. [1], although with large systematic errors coming from uncertainty in the fragmentation function of primary charm quarks. References [5–7] made less model dependent measurements by selecting hard three-jet events, which enhances the fraction of heavy quarks produced by the gluon splitting mechanism. In general the measurements have been above the prediction of Ref. [1], although within the range allowed by variations in  $\alpha_S$  and the quark mass.

For the parameters

$$\alpha_S = 0.118, \tag{1}$$

$$m_c = 1.2 \text{ GeV}, \tag{2}$$

$$m_b = 5.0 \text{ GeV}, \tag{3}$$

we obtain results for the fractions of  $Z^0$  decays that contain a secondary charm or bottom quark pair, of

$$f_c = 2.007\%, \quad f_b = 0.175\%. \tag{4}$$

Notice that these are somewhat higher than those of Ref. [1], principally because we have used a different value for  $\alpha_S^*$ . These should be compared with the LEP values<sup>†</sup> of

$$f_c = (2.44 \pm 0.43)\%, \quad f_b = (0.22 \pm 0.13)\%. \tag{5}$$

It should be noted that the summation of the large logarithms is essential for this agreement. For example, the leading order result alone for  $f_c$  is only 0.878 for the same parameters.

However, these measurements are still somewhat model dependent, since they rely on Monte Carlo event generators [8–10] to correct from the rate seen within the selected three-jet region, to the total rate. Since in this paper we calculate this dependence explicitly, it

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\*For our parameters, the calculation of Ref. [1] gives  $f_c = 2.200\%$  and  $f_b = 0.207\%$ . The remaining difference is due to next-to-next-to-leading logarithms.

†For the bottom quark there has been only one measurement[6], while for the charm quark, we have averaged the results of Refs. [4,5] and [7], assuming that the systematic errors are uncorrelated.

will become possible to directly compare experiment with theory in the measured region, without the need to extrapolate to the unmeasured region.

We begin by outlining the calculation of the resummed multiplicity, retaining the dependence on the jet kinematics, and thereby allowing the calculation of any event shape variable to be performed numerically. We then present numerical results for a few quantities of interest. Finally, we make some comparisons with the Monte Carlo event generators that have been used previously, and draw some conclusions.

## Calculation of the Resummed Multiplicity

### Fixed Order

In order to separate the processes of primary and secondary heavy quark production, it is necessary that the interference between them be zero, or at least very small. Fortunately this is the case[1]. For non-identical quarks coupled to a vector current, the interference term will vanish by Furry's theorem if we assume that the charges of the quarks are not measured (i.e. we do not distinguish quarks from antiquarks). For an axial current, cancellations will occur between up- and down-type quarks, leaving only the case where the "light" quark is a bottom quark. This will only provide an effect of the order of 0.2% of the secondary heavy quark rate[11]. The contribution for identical quarks is only slightly larger. In this case Furry's theorem no longer applies but necessarily the quarks will be of the same flavour – either bottom or charm, depending on the case in question, and can be neglected.

Then, the leading order differential cross section for the production of secondary heavy quarks,  $\gamma^* \rightarrow q\bar{q}Q\bar{Q}$ , is easily calculated. In Ref. [1], this expression is integrated over the heavy quark momenta, yielding the simple result given in Eq. (2) of Ref. [1]. In our case, since we must retain the jet kinematics, we cannot do this integration since we would not correctly account for events in which the heavy quarks fall into different hemispheres or different jets. Instead we must retain the full unintegrated amplitude, which does not have a compact form. This expression is then integrated over the appropriate phase space by Monte Carlo methods in which we can apply any event shape definition desired.

We choose to calculate the resummed multiplicity of secondary heavy quarks as a function of the heavy jet mass. Here we use the thrust definition of heavy jet mass, i.e. we separate each event into two hemispheres via the thrust axis and take the heavy jet mass to be the larger of the two hemisphere masses. Our calculation is therefore easily converted to give the differential distribution with respect to any *thrust-like* event shape.

### Resummed Calculation

There are two requirements to which we must conform in our calculation of the logarithmic contribution. Firstly we must retain the exact kinematics of the  $q\bar{q}g$  production in order to be able to accurately obtain the heavy jet mass for each event. We need not worry about the exact kinematics of the heavy quarks, since the large logarithms arise from the parts

of phase space where they become collinear and only the kinematics of the gluon need be considered (although the exact kinematics of the heavy quarks are, of course, included in the fixed order contribution). Secondly, we must include all soft gluon emission from the light quarks and virtual gluon. These emissions contribute to the heavy jet mass (by making the light quarks massive) through large logarithms, which must be summed to all orders using the coherent branching formalism.

Bearing these considerations in mind, we take the differential multiplicity to be,

$$n_{e^+e^-}^{Q\bar{Q}}(M_H^2, Q^2; Q_0^2) = \int dx_1 dx_2 dk_1^2 dk_2^2 dk_g^2 \frac{x_1^2 + x_2^2}{k_\perp^2/Q^2} f_q(k_1^2, k_{1max}^2) f_q(k_2^2, k_{2max}^2) n_g^{Q\bar{Q}}(k_g^2, k_\perp^2; Q_0^2) \Theta(k_\perp^2 - Q_0^2) \delta(M_H^2 - h(x_1, x_2, k_1^2, k_2^2, k_g^2)). \quad (6)$$

The above notation requires some clarification. As usual,  $x_1$  and  $x_2$  are the energy fractions of the light quark and antiquark respectively, and  $k_1$ ,  $k_2$  and  $k_g$  are the four-momenta of the quarks and gluon. The maximum value of  $k_i^2$ ,  $i = 1, 2$ , as constrained by the phase space limits is  $k_{imax}^2$ . Also,  $k_\perp^2$  is the transverse momentum (squared) of the virtual gluon, given by,

$$k_\perp^2 = (1 - x_1 + \epsilon_1 - \epsilon_2)(1 - x_2 + \epsilon_2 - \epsilon_1)Q^2, \quad (7)$$

where  $Q^2$  is the centre-of-mass energy squared and

$$\epsilon_i = \frac{m_i^2}{Q^2}, \quad i = 1, 2 \quad (8)$$

are the rescaled (primary) quark masses (squared), which result from the soft gluon emission. Here,  $h(x_1, x_2, k_1^2, k_2^2, k_g^2)$  is the heavy jet mass as a function of the exact kinematics of the quark, antiquark and gluon.

Summation to all orders of the leading and next-to-leading logarithms is included in the functions  $f_q$  and  $n_g^{Q\bar{Q}}$ . The function  $f_q$  is the quark jet mass distribution which has been calculated to next-to-leading logarithmic accuracy in Ref. [12]. More explicitly,  $f_q(k^2, Q^2) dk^2$  is the probability that a quark created at a scale  $Q^2$  gives rise to a jet with mass squared between  $k^2$  and  $k^2 + dk^2$ . This function includes all soft gluon emission from the light quarks and sums to all orders leading and next-to-leading logarithms of  $k_{imax}^2/k_i^2$ ,  $i = 1, 2$ .

The function  $n_g^{Q\bar{Q}}$  is the gluon jet mass distribution weighted by the heavy quark pair multiplicity, and has not been calculated elsewhere. In other words,  $n_g^{Q\bar{Q}}(k_g^2, k_\perp^2; Q_0^2) dk_g^2$  is the number of heavy quark pairs within a gluon jet that was formed at a scale  $k_\perp^2$  and has a mass between  $k_g^2$  and  $k_g^2 + dk_g^2$ . In calculating this quantity we must be sure to include leading and next-to-leading logarithms of both  $k_\perp^2/k_g^2$  and  $k_\perp^2/Q_0^2$ .

Notice that the resolution scale at which the heavy quarks are resolved is,

$$Q_0 = 2 m_Q^* = m_Q e^{5/6}. \quad (9)$$

Here,  $m_Q^*$  is the heavy quark effective mass. It has been shown in Ref. [1] that by using this effective mass we can neglect the heavy quark mass in the decay of the gluon while maintaining the correct leading and next-to-leading logarithms<sup>‡</sup>.

<sup>‡</sup>This follows from the comparison of Eqs. (16) and (17) of Ref. [1].

## The Multiplicity Weighted Mass Distribution

As in the case of the jet mass distribution, it is more convenient to calculate the integrated distribution,

$$N_g^{Q\bar{Q}}(k^2, Q^2; Q_0^2) = \int_0^{k^2} dq^2 n_g^{Q\bar{Q}}(q^2, Q^2; Q_0^2). \quad (10)$$

Physically this is the number of  $Q\bar{Q}$  pairs resolved in gluon jets of mass squared *less than*  $k^2$ . It can be derived from  $N_g^g(k^2, Q^2; Q_0^2)$ , the multiplicity of gluons within gluon jets of mass squared less than  $k^2$ , by integrating over the kernel for the splitting  $g \rightarrow Q\bar{Q}$ , i.e.  $P_{qg}$ . Therefore, we have,

$$\begin{aligned} N_g^{Q\bar{Q}}(k^2, Q^2; Q_0^2) &= \int_{Q_0^2}^{k^2} \frac{dq^2}{q^2} \int_0^1 dz \frac{\alpha_S(q)}{2\pi} P_{qg}(z) N_g^g(k^2, Q^2; q^2) \\ &= \frac{1}{3} \int_{Q_0^2}^{k^2} \frac{dq^2}{q^2} \frac{\alpha_S(q)}{2\pi} N_g^g(k^2, Q^2; q^2). \end{aligned} \quad (11)$$

$N_g^g$  has been derived in Ref. [13], and is given by,

$$N_g^g(k^2, Q^2; Q_0^2) = F_g(k^2, Q^2) \left\{ \mathcal{N}_g^g(k^2; Q_0^2) + C_A \left( I^g(k^2; Q_0^2) - I^g(k^4/Q^2; Q_0^2) \right) \right\} \quad (12)$$

where,

$$\begin{aligned} I^g(k^2; Q_0^2) &= \Theta(z_k - z_0) \frac{1}{C_A} \left[ \mathcal{N}^+(z_0, z_k) - 1 + 2B\tilde{\mathcal{N}}(z_0, z_k) \right. \\ &\quad \left. - \frac{1}{2}C \left( 2\tilde{\mathcal{N}}(z_0, z_k) - z_k^2/z_0^2 + 1 \right) \right], \end{aligned} \quad (13)$$

and  $\mathcal{N}_g^g$  is the usual multiplicity of gluons within a gluon[14],

$$\mathcal{N}_g^g(k^2; Q_0^2) = \mathcal{N}^+(z_0, z_k) - C\tilde{\mathcal{N}}(z_0, z_k). \quad (14)$$

Here, we have used the variables introduced by Catani et al.[14], where,

$$z_0^2 = \frac{32\pi C_A}{b^2 \alpha_S(Q_0)}, \quad z_k^2 = \frac{32\pi C_A}{b^2 \alpha_S(k)}. \quad (15)$$

The functions,  $\mathcal{N}^+$  and  $\tilde{\mathcal{N}}$ , are defined in terms of Bessel functions by,

$$\begin{aligned} \mathcal{N}^+(z_0, z_k) &= z_k \left( \frac{z_0}{z_k} \right)^B [I_{B+1}(z_k)K_B(z_0) + K_{B+1}(z_k)I_B(z_0)], \\ \tilde{\mathcal{N}}(z_0, z_k) &= \left( \frac{z_0}{z_k} \right)^B [I_B(z_k)K_B(z_0) - K_B(z_k)I_B(z_0)], \end{aligned} \quad (16)$$

and the parameters  $B$  and  $C$  are given by,

$$B = \frac{1}{b} \left( \frac{11}{3}C_A + \frac{2N_f}{3} - \frac{4C_F N_F}{3C_A} \right), \quad C = \frac{8}{3} \frac{N_f}{b} \frac{C_F}{C_A}, \quad (17)$$

where  $b$  is the first coefficient of the  $\beta$ -function,

$$b = \frac{11}{3}C_A - \frac{2}{3}N_f. \quad (18)$$

The gluon jet mass fraction,  $F_g(k^2, Q^2)$ , is the probability that a jet formed at scale  $Q^2$  will have a mass of less than  $k^2$ . This is the integrated version of  $f_g(k^2, Q^2)$  and is given, in Ref. [12], by,

$$\begin{aligned} F_g(k^2, Q^2) &= \int_0^{k^2} dq^2 f_g(q^2, Q^2) \\ &= \exp \left\{ C_A \log \left( \frac{Q^2}{k^2} \right) f_1 \left( \frac{\alpha_S(Q)}{4\pi} b \log \left( \frac{Q^2}{k^2} \right) \right) + \frac{b}{2} f_2 \left( \frac{\alpha_S(Q)}{4\pi} b \log \left( \frac{Q^2}{k^2} \right) \right) \right\}, \end{aligned} \quad (19)$$

where,

$$f_1(\lambda) = \frac{2}{b\lambda} \left[ (1-2\lambda) \log \left( \frac{1}{1-2\lambda} \right) - 2(1-\lambda) \log \left( \frac{1}{1-\lambda} \right) \right], \quad (20)$$

$$f_2(\lambda) = \frac{2}{b} \log \left( \frac{1}{1-\lambda} \right). \quad (21)$$

Notice that the result of Ref. [12] includes leading and next-to-leading logarithms within  $\log F_g$ , whereas the expression above is accurate only to leading and next-to-leading logarithms within  $F_g$  itself.

The integrals required for the evaluation Eq. (11) have been derived in Ref. [1] and are, to next-to-leading logarithmic accuracy,

$$\int_{z_0}^{z_k} \frac{dz}{z} \mathcal{N}^+(z, z_k) = \widetilde{\mathcal{N}}_1(z_0, z_k) - \frac{2}{z_0^2} \left( 1 - B \left( 1 - \frac{z_0}{z_k} \right) \right) \mathcal{N}^+(z_0, z_k) + \frac{2}{z_k^2}, \quad (22)$$

$$\int_{z_0}^{z_k} \frac{dz}{z} \widetilde{\mathcal{N}}(z, z_k) = \frac{1}{z_0 z_1} \mathcal{N}^+(z_0, z_k) - \frac{1}{z_k^2}. \quad (23)$$

This gives,

$$N_g^{Q\bar{Q}}(k^2, Q^2; Q_0^2) = F_g(k^2, Q^2) \left\{ N_g^{Q\bar{Q}}(k^2; Q_0^2) + C_A \left( I^{Q\bar{Q}}(k^2; Q_0^2) - I^{Q\bar{Q}}(k^4/Q^2; Q_0^2) \right) \right\}, \quad (24)$$

with,

$$\begin{aligned} I^{Q\bar{Q}}(k^2; Q_0^2) &= \Theta(z_k - z_0) \frac{4}{3b C_A} \left\{ \widetilde{\mathcal{N}}_1(z_0, z_k) + \left( 2(B-1) \frac{1}{z_0^2} - C \frac{1}{z_0 z_k} \right) \mathcal{N}^+(z_0, z_k) \right. \\ &\quad \left. - \frac{1}{z_k^2} (2B - 2 - C) - \frac{1}{4} (C + 2) \log \left( \frac{z_k^2}{z_0^2} \right) - \frac{C}{4} \left( 1 - \frac{z_k^2}{z_0^2} \right) \right\}, \end{aligned} \quad (25)$$

and,

$$\mathcal{N}_g^{Q\bar{Q}}(k^2; Q_0^2) = \frac{4}{3b} \left\{ \widetilde{\mathcal{N}}_1(z_0, z_k) + \left( \frac{2}{z_0^2} (B-1) - \frac{1}{z_0 z_k} (2B+C) \right) \mathcal{N}^+(z_0, z_k) + \frac{1}{z_k^2} (C+2) \right\}. \quad (26)$$

In the above and following  $\mathcal{N}_1$  is defined as  $\mathcal{N}$  but with  $B$  replaced by  $B-1$ .

Finally  $n_g^{Q\bar{Q}}$  is obtained by differentiation with respect to the jet mass, yielding the result,

$$\begin{aligned} n_g^{Q\bar{Q}}(k^2, Q^2; Q_0^2) &= \frac{1}{k^2} F_g(k^2, Q^2) \left\{ \mathcal{N}_g^{Q\bar{Q}}(k^2; Q_0^2) + C_A \left( I^{Q\bar{Q}}(k^2; Q_0^2) - 2 I^{Q\bar{Q}}(k^4/Q^2; Q_0^2) \right) \right\} \\ &\quad + \frac{d}{dk^2} \left( \log F_g(k^2, Q^2) \right) N_g^{Q\bar{Q}}(k^2, Q^2; Q_0^2), \end{aligned} \quad (27)$$

with,

$$\begin{aligned}
\mathcal{N}_g^{Q\bar{Q}}(k^2; Q_0^2) &= \frac{16C_A}{3b^2} \left\{ \frac{1}{z_k^2} \mathcal{N}_1^+(z_0, z_k) - \left( \frac{2}{z_0^2} (B-1) - \frac{1}{z_0 z_1} (2B+C) \right) \tilde{\mathcal{N}}(z_0, z_k) \right\}, \\
I^{Q\bar{Q}}(k^2; Q_0^2) &= \frac{16}{3b^2} \left\{ \frac{1}{z_k^2} \left( \mathcal{N}_1^+(z_0, z_k) - 1 \right) + \left( \frac{2}{z_0^2} (B-1) - C \frac{1}{z_0 z_k} \right) \tilde{\mathcal{N}}(z_0, z_k) \right. \\
&\quad \left. - \frac{C}{2} \frac{1}{z_k^2} \left( 1 - z_k^2/z_0^2 \right) \right\}, \tag{28}
\end{aligned}$$

and,

$$k^2 \frac{d}{dk^2} \left( \log F_g(k^2, Q^2) \right) = \frac{4C_A}{b} \log \left( \frac{1-\lambda}{1-2\lambda} \right) - \left( \frac{\alpha_S}{2\pi} \right) \frac{b}{2} \frac{1}{1-\lambda}, \tag{29}$$

where,

$$\lambda = \frac{\alpha_S(Q)}{2\pi} b \log(Q/k). \tag{30}$$

Notice that the matching to tree level is particularly simple. We only require the expansion of  $n_g^{Q\bar{Q}}$  to order  $\alpha_S$ , which is,

$$n_g^{Q\bar{Q}}(k^2, Q^2; Q_0^2) = \frac{1}{3k^2} \frac{\alpha_S}{2\pi} + \mathcal{O}(\alpha_S^2). \tag{31}$$

## Calculation of the Background

The background to secondary heavy quark production in  $e^+e^-$  annihilation, i.e. primary heavy quark production, can be estimated by standard three-jet production, since the mass effects will be small. For the analysis of the heavy jet mass, the fixed order contribution to this background is given by,

$$\begin{aligned}
\frac{1}{\sigma_0} \frac{d\sigma^{(1)}(\tau)}{dM_H^2} &= \frac{1}{M_H^2} C_F \frac{\alpha_S(Q)}{2\pi} \left\{ -4 \log \tau - 3 + 6\tau \log \tau - 4 \frac{\tau^2}{1-\tau} \log(1-\tau) \right. \\
&\quad \left. + \frac{8}{1-\tau} \log(1-\tau) - 6\tau \log(1-2\tau) - \frac{4}{1-\tau} \log(1-2\tau) + 9\tau^2 \right\} \tag{32}
\end{aligned}$$

where  $\sigma^{(1)}$  is the  $\mathcal{O}(\alpha_S)$  contribution to the cross section,  $\sigma_0$  is the Born cross section,  $\tau = M_H^2/Q^2$  and  $M_H$  is the heavy jet mass.

Of course, the large leading and next-to-leading logarithms must also be included and are given by,

$$\frac{1}{\sigma_0} \frac{d\sigma^{(\log)}(\tau)}{dM_H^2} = 2f_q(M_H^2, Q^2) F_q(M_H^2, Q^2). \tag{33}$$

Expanding this to  $\mathcal{O}(\alpha_S)$  gives the first two terms of the fixed order piece,

$$\frac{1}{\sigma_0} \frac{d\sigma^{(\log)}(\tau)}{dM_H^2} = \frac{1}{M_H^2} C_F \frac{\alpha_S(Q)}{2\pi} \{ -4 \log \tau - 3 \} + \mathcal{O}(\alpha_S^2). \tag{34}$$

Therefore it is clear that matching with the fixed order will result in the full answer,

$$\begin{aligned}
\frac{1}{\sigma_0} \frac{d\sigma(\tau)}{dM_H^2} &= 2f_q(M_H^2, Q^2) F_q(M_H^2, Q^2) + \frac{1}{M_H^2} C_F \frac{\alpha_S(Q)}{2\pi} \left\{ 6\tau \log \tau - 4 \frac{\tau^2}{1-\tau} \log(1-\tau) \right. \\
&\quad \left. + \frac{8}{1-\tau} \log(1-\tau) - 6\tau \log(1-2\tau) - \frac{4}{1-\tau} \log(1-2\tau) + 9\tau^2 \right\}. \tag{35}
\end{aligned}$$

This can be seen plotted in Fig. 1.

## Numerical Results

For all the distributions we show, we concentrate on their shape, normalised to the number of secondary heavy quarks, rather than on the total rate. We use the  $\alpha_s$  and quark mass values quoted earlier.

We present the heavy jet mass distribution for  $\sqrt{s} = m_Z$  in Fig. 1. This is closely related to the jet mass difference,  $M_H - M_L$ , which was the event shape used to fit  $f_c$  in Ref. [7]. We see that the heavy jet mass provides a good discriminator of events with secondary heavy quarks from the three-jet background.

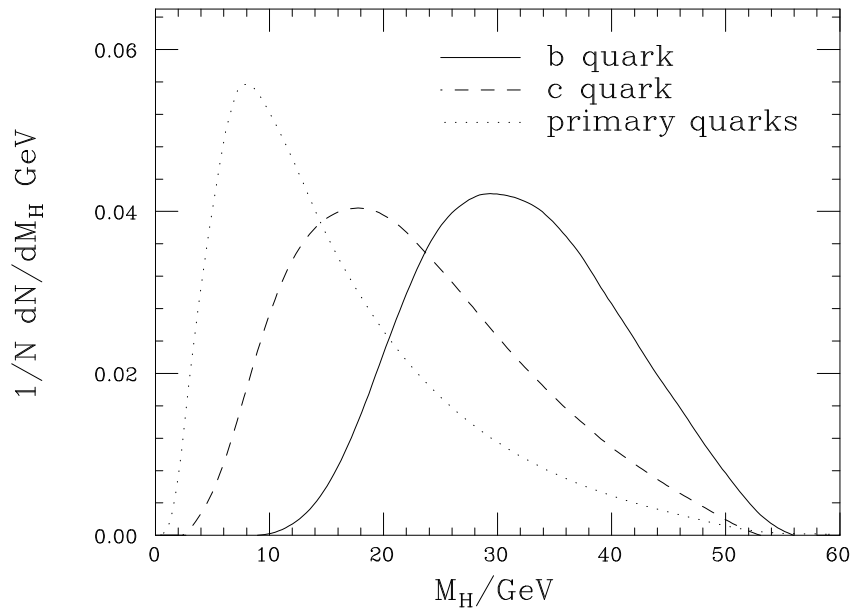


Figure 1: The multiplicity of heavy quark pairs in  $Z^0$  decays as a function of the heavy jet mass, normalised to the number of secondary heavy quarks. The shapes for b quark (solid curve), and c quark (dashed) pairs are compared to the three-jet background (dotted). It is clear that secondary heavy quark production can be distinguished from the background by the shape of the heavy jet mass distribution.

Of course, these shapes are dependent on the values chosen for the parameters,  $\Lambda_{\text{QCD}}$  and  $m_Q$ . The effect of varying these parameters is seen in Fig. 2. Also shown is the contribution from the fixed order term alone, demonstrating the importance of resumming large logarithms.

## Event Generators

Monte Carlo event generators predict quite a wide range for the rate of secondary heavy quark production. While JETSET [8] and HERWIG [9], which are both based on the parton shower formalism, are in quite good agreement, ARIADNE [10], based on the dipole cascade



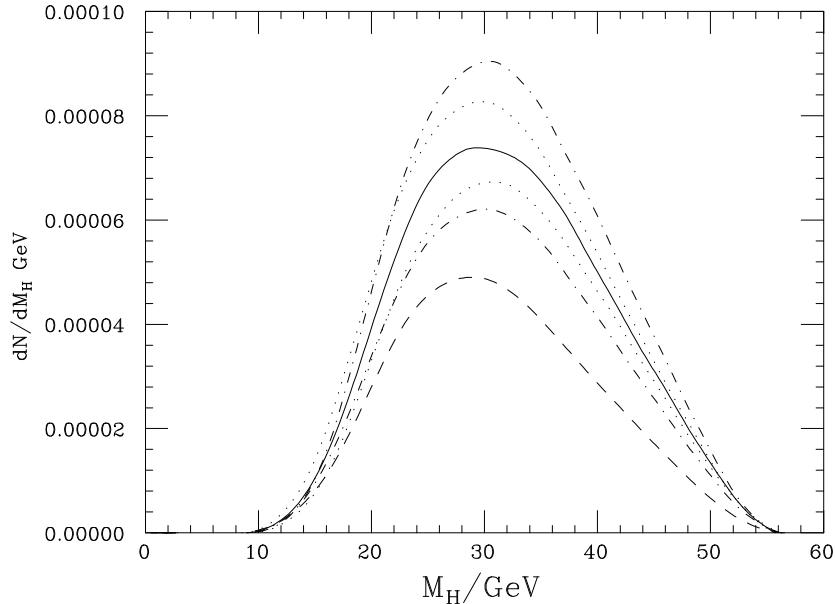


Figure 2: The multiplicity of bottom quark pairs as a function of the heavy jet mass, normalised to the number of  $Z^0$  decays. The dashed curve shows the fixed order result, whereas the solid curve includes the resummation of the large logarithms. Also shown are the results of varying the quark mass by 5% (dotted) and  $\Lambda_{\text{QCD}}$  by a factor of two (i.e.  $\alpha_S$  by 10%) (dash-dotted).

model, lies well above them. At present the data for the total rate lies between the two, in agreement with both, although in somewhat better agreement with ARIADNE.

In Ref. [1] it was argued that the discrepancy between the models is actually due to a specific problem with ARIADNE – the fact that it allows very low transverse momentum gluons to be very massive. If that is the case, it should show up in the distributions calculated in the previous section. Following the suggestion of Ref. [1], later versions of ARIADNE have had an option<sup>§</sup> to veto gluon splitting with  $m_g > k_{\perp g}$ , which should fix this problem.

The parton shower and dipole cascade models are only formally accurate to leading logarithm and do not include the exact matrix elements for  $q\bar{q}Q\bar{Q}$  production. Therefore our calculation is more accurate than them and can be used to check them.

In Fig. 3 we compare our results with the predictions from the event generators for  $\sqrt{s} = m_Z$ . With the exception of the gluon splitting option in ARIADNE, we keep all model parameters at their default values. We see that HERWIG and JETSET give similar predictions for the distribution as well as for the rate and that the unmodified ARIADNE peaks at somewhat lower heavy jet mass than them. Adding the new modification, ARIADNE's distribution is more like the other models', but still somewhat different, particularly at low jet masses. Our results lie between ARIADNE and the other models.

Increasing the centre-of-mass energy, the relative importance of the fixed order term

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<sup>§</sup>This option is switched on by setting `MSTA(28)=1`.

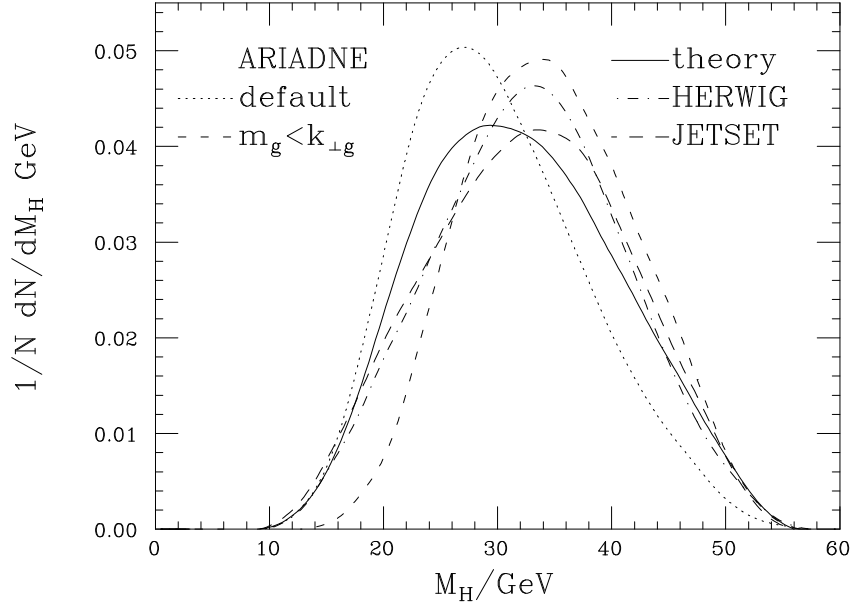


Figure 3: The multiplicity of bottom quark pairs as a function of the heavy jet mass in  $Z^0$  decays from our calculation (solid) and from various Monte Carlo models.

is reduced and one gets a cleaner probe of the parton evolution. In Fig. 4 we show the comparison again at higher energy. The modified version of ARIADNE is in even better

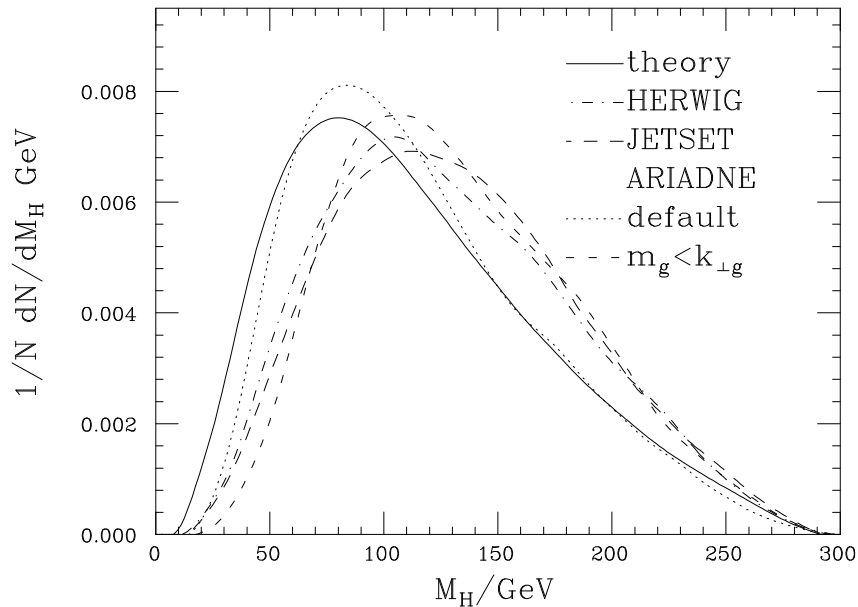


Figure 4: As Fig. 3 but at  $\sqrt{s} = 500$  GeV.

agreement with the other two models, while the unmodified version is in good agreement with our calculation. We therefore see no evidence to support the claim of Ref. [1] that there is a problem with ARIADNE.

## Summary

We have calculated the multiplicity of heavy quarks from gluon splitting in  $e^+e^-$  annihilation, as a function of the heavy jet mass. Our result is exact to leading order in  $\alpha_s$ , and sums leading and next-to-leading logarithms to all orders in  $\alpha_s$ . We find that the fractions of  $Z^0$  decays that contain a secondary charm or bottom quark pair respectively, are

$$f_c = 2.007\%, \quad f_b = 0.175\%. \quad (36)$$

The shape of our result is similar to that predicted by Monte Carlo event generators at the  $Z^0$ , lying between the different models, but in better agreement with ARIADNE at higher energy.

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