

THE MULTIPLICITY OF HEAVY QUARK PAIRS FROM GLUON SPLITTING IN e^+e^- ANNIHILATION

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I will briefly review the progress which has been made in the investigation of secondary heavy quark production at LEP. I will show why a calculation of the secondary heavy quark multiplicity, keeping the dependence on an event shape variable, is essential for better exploitation of data, and I will present the results of such a calculation. This will be compared to Monte Carlo studies.

1 Introduction

Heavy quark production in e^+e^- annihilation can come from two possible sources: from the hard interaction itself, $e^+e^- \rightarrow Q\bar{Q}$, and from the splitting of perturbatively produced gluons, $e^+e^- \rightarrow q\bar{q}g \rightarrow q\bar{q}Q\bar{Q}$. I shall refer to the former as *primary* heavy quarks and the latter as *secondary* heavy quarks.

Of course, in order to be able to regard primary and secondary heavy quark production as separate processes, the interference between them must be zero, or at least very small. Fortunately this is the case[1]. For non-identical quarks coupled to a vector current, the interference between them will vanish by Furry's theorem if we assume that the charges of the quarks are not measured. For an axial current, cancellations will occur between up and down type quarks, leaving only the case where the "light" quark is a bottom quark. This will only provide an effect of the order of 0.2% of the secondary heavy quark rate[2]. The contribution for identical quarks is only slightly larger. In this case Furry's theorem no longer applies but necessarily the quarks will be of the same flavour (bottom or charm) and subsequently give only a small contribution. Therefore one can regard secondary heavy quark production as a separate process and investigate it both experimentally and theoretically.

In this talk, I will briefly review the progress which has been made in this investigation. I will show why a calculation of the secondary heavy quark multiplicity, keeping the dependence

on an event shape variable, is essential for better exploitation of data, and I will present the results of such a calculation. This will be compared to Monte Carlo studies.

1.1 Theoretical Progress

The heavy quark mass provides a natural infrared cut-off for the theoretical calculation of the total rate of secondary heavy quark production. It is therefore an infrared-safe quantity, and can be calculated as an order-by-order perturbative expansion in α_S , starting at $\mathcal{O}(\alpha_S^2)$. At higher orders in α_S , large logarithms of s/m_Q^2 arise, potentially spoiling the convergence of the perturbative series at high energies, $s \gg m_Q^2$. It is interesting to note that these are exactly the same logarithms which occur in the resummation of jet multiplicities, where they appear as logarithms of the jet resolution scale, y_{cut} . Here we are able to study their effect without the arbitrariness of jet algorithms.

In Ref. [1] the leading and next-to-leading logarithms were summed to all orders in α_S yielding a result that is uniformly reliable for all s . This gave the fraction of Z^0 decays that contain a secondary charm or bottom quark pair, of

$$f_c = 1.349\%, \quad f_b = 0.177\%. \quad (1)$$

1.2 Experimental Progress

Several experimental measurements of the secondary heavy quark production rate have now been made. In Ref. [3], it was extracted for charm quarks from a measurement of the D^* fragmentation function, and found to be more than a factor of two above the expectation of Ref. [1], although with large systematic errors coming from uncertainty in the fragmentation function of primary charm quarks. Refs. [4–6] made less model dependent measurements by selecting hard three-jet events, which enhances the fraction of heavy quarks produced by the gluon splitting mechanism. In general the measurements have been above the predictions of Eq. (1), although within the range allowed by variations in α_S and the quark mass.

The combined LEP values^a obtained are,

$$f_c = (2.44 \pm 0.43)\%, \quad f_b = (0.22 \pm 0.13)\%. \quad (2)$$

Of course, in order to extract these results one must use some theoretical input in order to separate the primary and secondary heavy quark contributions. In the above experiments this theoretical input was the shape of the secondary heavy quark multiplicity distribution with respect to some event shape variable. For example, Ref. [6] uses the shape of the multiplicity distribution with respect to the jet mass difference. This is fitted to data, extracting a value for the overall normalisation and therefore the total rate. Since a full perturbative QCD calculation of this shape was not available, Monte Carlo event generators[7–9] were used for this theoretical input.

1.3 A New Calculation

Clearly, it is desirable to calculate the resummed multiplicity, retaining the dependence on the jet kinematics, and thereby allowing the calculation of any event shape variable to be performed numerically. This perturbative QCD calculation can be made accurate up to next-to-leading logarithms whereas the Monte Carlo event generators are only accurate to leading logarithms. Furthermore, the leading order, $\mathcal{O}(\alpha_S^2)$, can be included exactly, and the exact kinematics can be used when calculating the required event shape.

^aFor the bottom quark there has been only one measurement[5], while for the charm quark, we have averaged the results of Refs. [3,4] and [6], assuming that the systematic errors are uncorrelated.

Here I present the multiplicity of secondary heavy quarks as a function of the *heavy jet mass*. I employ the ‘thrust-like’ definition of heavy jet mass, where the thrust axis is used to divide each event into two hemispheres and the mass of the heavier hemisphere is defined as the heavy jet mass. Clearly, the calculation can be easily modified to any thrust-like event variable.

2 The Secondary Heavy Quark Multiplicity as a Function of Heavy Jet Mass

The leading order differential cross section for the production of secondary heavy quarks, $\gamma^* \rightarrow q\bar{q}Q\bar{Q}$, is easily calculated, and will not be discussed further here. For the calculation of the logarithmic contribution, there are two requirements to which we must conform. Firstly we must retain the exact kinematics of the $q\bar{q}g$ production in order to be able to accurately obtain the heavy jet mass for each event. We need not worry about the exact kinematics of the heavy quarks, since the large logarithms arise from the parts of phase space where they become collinear and only the kinematics of the gluon need be considered (although the exact kinematics of the heavy quarks are, of course, included in the fixed order contribution). Secondly, we must include all soft gluon emission from the light quarks and virtual gluon. These emissions contribute to the heavy jet mass (by taking the light quarks off mass-shell) and also provide large logarithms which must be summed to all orders using the coherent branching formalism.

Bearing these considerations in mind, the differential multiplicity is taken to be,

$$n_{e^+e^-}^{Q\bar{Q}}(M_H^2, Q^2; Q_0^2) = \int dx_1 dx_2 dk_1^2 dk_2^2 dk_g^2 \frac{x_1^2 + x_2^2}{k_\perp^2/Q^2} f_q(k_1^2, k_{1max}^2) f_q(k_2^2, k_{2max}^2) n_g^{Q\bar{Q}}(k_g^2, k_\perp^2; Q_0^2) \Theta(k_\perp^2 - Q_0^2) \delta(M_H^2 - h(x_1, x_2, k_1^2, k_2^2, k_g^2)). \quad (3)$$

As usual, x_1 and x_2 are the energy fractions of the light quark and antiquark respectively, and k_1 , k_2 and k_g are the four-momenta of the quarks and gluon. The maximum value of k_i^2 , $i = 1, 2$, as constrained by the phase space limits, is k_{imax}^2 . Also, k_\perp^2 is the transverse momentum (squared) of the virtual gluon, given by,

$$k_\perp^2 = (1 - x_1 + \epsilon_1 - \epsilon_2)(1 - x_2 + \epsilon_2 - \epsilon_1)Q^2, \quad (4)$$

where Q^2 is the centre-of-mass energy squared and ϵ_i , ($i = 1, 2$) are the rescaled (primary) quark masses (squared) which result from the soft gluon emission. By retaining the exact kinematics of the quark, antiquark and gluon, we are able to calculate the heavy jet mass $h(x_1, x_2, k_1^2, k_2^2, k_g^2)$ exactly, thereby satisfying the first of our requirements.

The second requirement, that we should include all soft gluon emission, has been achieved by the inclusion of the functions f_q and $n_g^{Q\bar{Q}}$. The function f_q is the quark jet mass distribution which has been calculated to next-to-leading logarithmic accuracy in Ref. [10]. More explicitly, $f_q(k^2, Q^2) dk^2$ is the probability that a quark, created at a scale Q^2 gives rise to a jet with mass squared between k^2 and $k^2 + dk^2$. This function includes all soft gluon emission from the light quarks and sums, to all orders, leading and next-to-leading logarithms of k_{imax}^2/k_i^2 , $i = 1, 2$.

The function $n_g^{Q\bar{Q}}$ is the gluon jet mass distribution weighted by the heavy quark pair multiplicity. In other words, $n_g^{Q\bar{Q}}(k_g^2, k_\perp^2; Q_0^2) dk_g^2$ is the number of heavy quark pairs within a gluon jet which was formed at a scale k_\perp^2 and has a mass between k_g^2 and $k_g^2 + dk_g^2$. To the accuracy required here this quantity must include leading and next-to-leading logarithms of both k_\perp^2/k_g^2 and k_\perp^2/Q_0^2 .

One important simplification of the calculation is allowed by the introduction of a heavy quark *effective* mass[1],

$$m_Q^* = \frac{1}{2}m_Q e^{5/6}. \quad (5)$$

Since the bottom quark mass cannot be neglected we should include all mass effects in the $g \rightarrow b\bar{b}$ splitting. This would lead to integrals of the form,

$$\lim_{x^2 \rightarrow 0} \int_{x^2}^a \frac{dz}{z} \sqrt{1 - \frac{x^2}{z}} \left(1 + \frac{x^2}{2z}\right) \log^{n-1} z = -\frac{1}{n} \log^n x^{*2} + \mathcal{O}(\log^{n-2}), \quad (6)$$

where x is the rescaled heavy quark mass, m_Q/\sqrt{s} , and x^* is the rescaled effective mass. However, exactly the same result is obtained (to next-to-leading logarithmic accuracy) if the *massless* splitting function and the effective mass are used.

$$\lim_{x^{*2} \rightarrow 0} \int_{x^{*2}}^a \frac{dz}{z} \log^{n-1} z = -\frac{1}{n} \log^n x^{*2}. \quad (7)$$

Therefore by using this effective mass one can neglect the heavy quark mass in the decay of the gluon while maintaining the correct leading and next-to-leading logarithms. In Eq. (3) this is manifest as the resolution scale at which the heavy quarks are resolved, $Q_0 = 2m_Q^*$.

2.1 The Multiplicity Weighted Mass Distribution

It is more convenient to calculate the integrated distribution,

$$N_g^{Q\bar{Q}}(k^2, Q^2; Q_0^2) = \int_0^{k^2} dq^2 n_g^{Q\bar{Q}}(q^2, Q^2; Q_0^2). \quad (8)$$

Physically this is the number of $Q\bar{Q}$ pairs resolved in gluon jets of mass squared *less than* k^2 . It can be derived from $N_g^g(k^2, Q^2; Q_0^2)$, the multiplicity of gluons within gluon jets of mass squared less than k^2 , by integrating over the kernel for the splitting $g \rightarrow Q\bar{Q}$. Since we have introduced the effective mass m_Q^* , the appropriate kernel is the massless splitting kernel P_{gq} . N_g^g has been derived in Ref. [11]. The integration yields,

$$N_g^{Q\bar{Q}}(k^2, Q^2; Q_0^2) = F_g(k^2, Q^2) \left\{ \mathcal{N}_g^{Q\bar{Q}}(k^2; Q_0^2) + C_A \left(I^{Q\bar{Q}}(k^2; Q_0^2) - I^{Q\bar{Q}}(k^4/Q^2; Q_0^2) \right) \right\}, \quad (9)$$

with,

$$I^{Q\bar{Q}}(k^2; Q_0^2) = \Theta(z_k - z_0) \frac{4}{3b C_A} \left\{ \tilde{\mathcal{N}}_1(z_0, z_k) + \left(2(B-1) \frac{1}{z_0^2} - C \frac{1}{z_0 z_k} \right) \mathcal{N}^+(z_0, z_k) - \frac{1}{z_k^2} (2B - 2 - C) - \frac{1}{4} (C + 2) \log \left(\frac{z_k^2}{z_0^2} \right) - \frac{C}{4} \left(1 - \frac{z_k^2}{z_0^2} \right) \right\}. \quad (10)$$

In the above, $\mathcal{N}_g^{Q\bar{Q}}(k^2; Q_0^2)$ is the multiplicity of heavy quarks pairs found in a gluon jet, regardless of the jet mass, and $F_g(k^2, Q^2)$ is the probability that a gluon jet produced at a scale Q will have a mass squared below k^2 . The notation used above is standard and can be found in Ref. [12]. \mathcal{N}_1 is defined as \mathcal{N} but with B replaced by $B - 1$.

Notice that Eq. (9) conforms with naïve expectations. One might expect that the number of heavy quark pairs found within a gluon jet created at a scale Q^2 and of mass below k^2 would be simply the probability of finding a gluon jet of mass below k^2 multiplied by the number of heavy quark pairs within. Indeed, this is the first term of our expression for $N_g^{Q\bar{Q}}$, and the naïve expectation requires only the addition of next-to-leading logarithmic corrections.

The expression for $n_g^{Q\bar{Q}}$ can now be trivially obtained from the differentiation of $N_g^{Q\bar{Q}}$ with respect to the jet mass. This result is then matched to the fixed order result to avoid double counting.

2.2 Calculation of the Background

The background to secondary heavy quark production in e^+e^- annihilation, i.e. primary heavy quark production, is estimated by standard three-jet production, since the mass effects will be small. Care has been taken to include both the fixed order contribution and all large leading and next-to-leading logarithms. As for the secondary heavy quarks, the logarithmic contribution is matched to fixed order to avoid double counting. The full result can be seen plotted in Fig. 1.

3 Numerical Results

For all the distributions we show, we concentrate on their shape, normalised to the number of secondary heavy quarks, rather than on the total rate. We use $\alpha_S = 0.118$ and values of 1.2 and 5 GeV for the charm and bottom quark masses respectively.

We present the heavy jet mass distribution for $\sqrt{s} = m_Z$ in Fig. 1. This is closely related to the jet mass difference, $M_H - M_L$, which was the event shape used to fit f_c in Ref. [6]. We see that the heavy jet mass provides a good discriminator of events with secondary heavy quarks from the three-jet background.

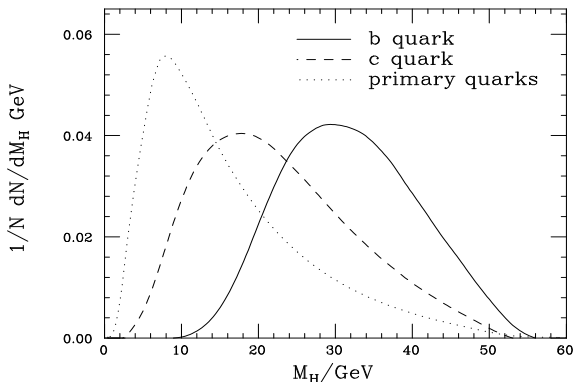


Figure 1: The multiplicity of primary and secondary heavy quark pairs as a function of the heavy jet mass, normalised to the number of heavy quarks.

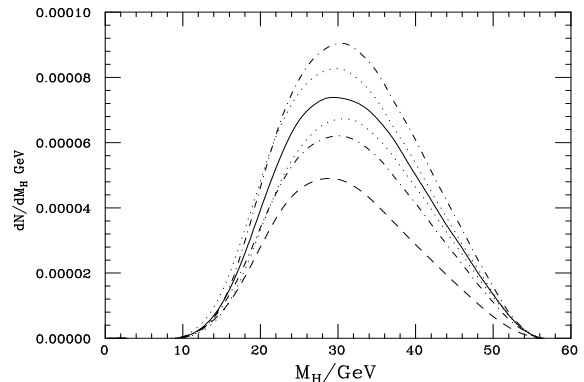


Figure 2: The multiplicity of secondary bottom quark pairs as a function of the heavy jet mass, normalised to the number of Z^0 decays.

Of course, these shapes are dependent on the values chosen for the parameters, Λ_{QCD} and m_Q . The effect of varying the quark mass by 5% (dotted) and Λ_{QCD} by a factor of two (i.e. α_S by 10%) (dash-dotted) is compared to the result with $\alpha_S = 0.118$ and $m_b = 5$ GeV (solid). Also shown is the contribution from the fixed order term alone (dashed), demonstrating the importance of resumming large logarithms.

4 Event Generators

The predictions from Monte Carlo event generators for $\sqrt{s} = m_Z$ can be seen in Fig. 3. Since these models are only formally accurate to leading logarithms and do not include the exact matrix elements for $q\bar{q}Q\bar{Q}$ production, our calculation is more accurate and can be used to check them. We see that HERWIG and JETSET give similar predictions for the distribution as well as for the rate and that ARIADNE peaks at a somewhat lower heavy jet mass. Following a suggestion of Ref. [1], later versions of ARIADNE contain an option to veto gluon splitting with $m_g > k_{\perp g}$. Adding this modification, ARIADNE's distribution is more like the other models', but still somewhat different, particularly at low jet masses. Our results lie between the unmodified ARIADNE and the other models.

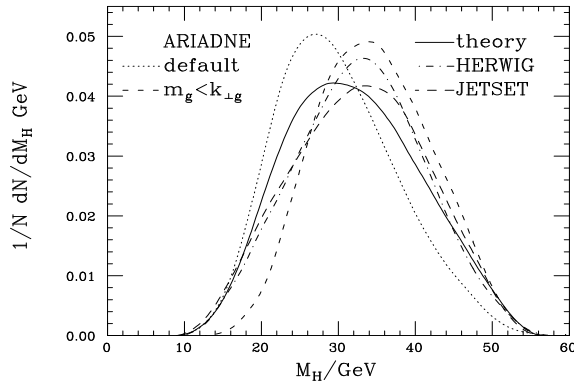


Figure 3: The multiplicity of bottom quark pairs as a function of the heavy jet mass at $\sqrt{s} = m_Z$.

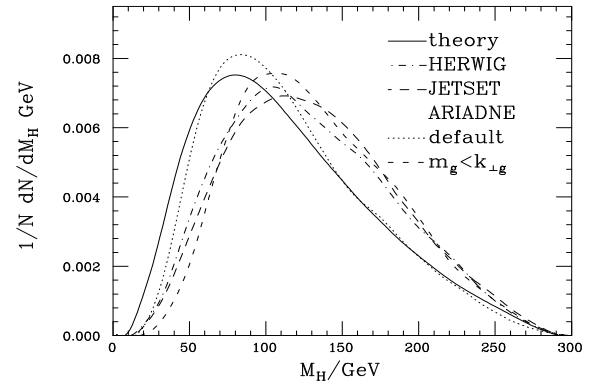


Figure 4: The multiplicity of bottom quark pairs as a function of the heavy jet mass at $\sqrt{s} = 500$ GeV.

Increasing the centre-of-mass energy, the relative importance of the fixed order term is reduced and one gets a cleaner probe of the parton evolution. In Fig. 4 we show the comparison at $\sqrt{s} = 500$ GeV. The modified version of ARIADNE is in even better agreement with the other two models, while the unmodified version is in good agreement with our calculation. We therefore see no evidence to support the claim of Ref. [1] that there is a problem with ARIADNE.

5 Summary

A calculation of the multiplicity of heavy quarks from gluon splitting in e^+e^- annihilation, as a function of the heavy jet mass, has been presented. The shape of the result is similar to that predicted by Monte Carlo event generators at the Z^0 , lying between the different models, but in better agreement with ARIADNE at higher energy.

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