

Scheme dependence, leading order and higher twist studies of MRST partons

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Abstract

We extend a recent global analysis of nucleon parton distributions carried out at next-to-leading order (NLO) in the $\overline{\text{MS}}$ scheme to provide distributions in the so-called DIS scheme. We pay particular attention to the translation of the heavy quark distributions in going from the $\overline{\text{MS}}$ to the DIS scheme. We repeat the global analysis at leading order (LO) and comment on the major effects produced when going from LO to NLO. Finally we include in the global analysis a freely parameterised form of possible higher twist contributions to make an exploratory study of the size of these effects as a function of x .

1. Introduction

We recently published a global analysis (MRST)¹ [1] of data in which we extracted quark and gluon distributions for the proton. The aim of the analysis was to constrain the partons by the present data – on deep inelastic scattering (DIS) at HERA H1 [2, 3], from DIS with fixed target experiments [4, 5, 6, 7, 8], and from prompt photon production (PPP) [9, 10]. In addition the Drell-Yan measurements [11, 12] together with the pp/pn asymmetry in Drell-Yan [13, 14] and the asymmetry of the rapidity distributions of the charged lepton from $W^\pm \rightarrow l^\pm \nu$ decays at the Tevatron $p\bar{p}$ collider [15] were used in the analysis. Particular attention was given to the PPP data and the influence they have on the determination of the gluon at large x . To obtain a satisfactory description of the higher energy data [10] some intrinsic transverse momentum of the partons needs to be included and by varying the value of $\langle k_T \rangle$ within an acceptable range we arrived at a range of estimates for the gluon distribution at large x . The parton distributions corresponding to the extremes of this range were labelled MRST($g \uparrow$) and MRST($g \downarrow$) and these together with the set using the ‘central’ gluon, MRST, all used a value of $\alpha_S(M_Z^2) = 0.1175$. In order to reflect the range of uncertainty in $\alpha_S(M_Z^2)$ we also provided sets of partons (with the central gluon choice) where $\alpha_S(M_Z^2)$ was varied by ± 0.005 and these sets were labelled MRST($\alpha_S \uparrow\uparrow$) and MRST($\alpha_S \downarrow\downarrow$).

Our analysis included next-to-leading order (NLO) corrections which were computed in the $\overline{\text{MS}}$ factorisation scheme. A special feature of the analysis was the new treatment of heavy flavour production in DIS, using the procedure developed by two of us [16, 17], which describes the threshold behaviour correctly and which is consistent with the $\overline{\text{MS}}$ scheme. The resulting five sets of parton distributions were therefore appropriate only to processes evaluated in the $\overline{\text{MS}}$ scheme.

In this paper we extend our previous analysis to confront a variety of issues. In particular we present² (i) parton distributions also at NLO but computed in the so-called DIS scheme [18], (ii) parton distributions resulting from simply a LO analysis and (iii) parton distributions resulting from a NLO analysis which in addition allows for an empirical higher twist contribution.

2. Partons in the DIS scheme

The DIS scheme is simply a device for absorbing the one-loop $\overline{\text{MS}}$ coefficients into a re-definition of the parton distributions so as to exactly preserve the NLO value of F_2 but using an apparent LO expression. No new global fitting procedure is involved.

¹In this paper we use ‘MRST’ to denote our previous parton analysis [1] in the $\overline{\text{MS}}$ factorisation scheme.

²The FORTRAN code for all the parton sets described in this paper together with those of MRST can be obtained from <http://durpdg.dur.ac.uk/HEPDATA/PDF>, or by contacting W.J.Stirling@durham.ac.uk. In addition, because of the slightly complicated expressions involving heavy flavours, the packages for computing the structure functions from each set are provided there. A FORTRAN routine to compute the charged current structure functions F_2 and xF_3 from the MRST parton distributions has also recently been included.

The relation between a quark density in the DIS and $\overline{\text{MS}}$ schemes is, at NLO,

$$q_i^{\text{DIS}}(x, Q^2) = q_i^{\overline{\text{MS}}}(x, Q^2) + \left(\frac{\alpha_S}{4\pi}\right) C_{2,q}^{(1)}(z) \otimes q_i^{\overline{\text{MS}}}(x/z, Q^2) + \left(\frac{\alpha_S}{4\pi}\right) C_g^{(1)}(z) \otimes g^{\overline{\text{MS}}}(x/z, Q^2) \quad (1)$$

where $C_{2,q}^{(1)}(z)$ and $C_g^{(1)}(z)$ are the normal $\overline{\text{MS}}$ massless coefficient functions, e.g.

$$C_g^{(1)}(z) = P_{qg}^{(0)}(z) \log((1-z)/z) + 8z(1-z) - 1.$$

The corresponding relation for the gluon is a matter of convention and normally one fixes it to maintain the conservation of momentum.

$$g^{\text{DIS}}(x, Q^2) = g^{\overline{\text{MS}}}(x, Q^2) - \left(\frac{\alpha_S}{4\pi}\right) C_{2,q}^{(1)}(z) \otimes \Sigma^{\overline{\text{MS}}}(x/z, Q^2) - 2n_f \left(\frac{\alpha_S}{4\pi}\right) C_g^{(1)}(z) \otimes g^{\overline{\text{MS}}}(x/z, Q^2), \quad (2)$$

where $\Sigma(x, Q^2) = \sum_{i=1}^{n_f} [q_i(x, Q^2) + \bar{q}_i(x, Q^2)]$, and where n_f is the number of light flavours. The attraction of the DIS scheme, of course, is that if to LO the F_2 structure function is given by

$$F_2(x, Q^2) = \sum_i x [a_{2,i} q_i(x, Q^2) + \bar{a}_{2,i} \bar{q}_i(x, Q^2)] \quad (3)$$

then the same expression holds at NLO provided $q_i = q_i^{\text{DIS}}$. However in this scheme it is only the F_2 structure function that has no explicit higher order perturbative corrections. Thus, if at LO xF_3 is given by

$$xF_3(x, Q^2) = \sum_i x [a_{3,i} q_i(x, Q^2) + \bar{a}_{3,i} \bar{q}_i(x, Q^2)], \quad (4)$$

then at NLO we have

$$xF_3(x, Q^2) = \left[\delta(1-z) - \left(\frac{\alpha_S}{4\pi}\right) \frac{8}{3}(1+z) \right] \otimes \sum_i x [a_{3,i} q_i^{\text{DIS}}(x/z, Q^2) + \bar{a}_{3,i} \bar{q}_i^{\text{DIS}}(x/z, Q^2)]. \quad (5)$$

This is the prescription for light favours. For the case of heavy quarks then in some scheme which uses $\overline{\text{MS}}$ evolution for the parton distributions, such as that in MRST, the coefficient functions for the heavy quarks are mass dependent. Hence we have to decide how best to define a DIS scheme for massive quarks. The approach we have adopted is to make exactly the same change of definitions for the partons as above, i.e. using massless coefficient functions only. Hence the heavy quarks in this DIS scheme evolve exactly like the light quarks in DIS scheme (in the $\overline{\text{MS}}$ scheme the evolution starts from zero at $\mu^2 = m_H^2$, so from Eq. (2) we see that in the DIS scheme it starts from a nonzero value dependent on the gluon distribution). However, the DIS scheme coefficient functions for heavy quarks are nontrivial, unlike the case for the light

quarks. Expressing first the charm structure function F_2^c in terms of the $\overline{\text{MS}}$ (MRST) charm quark we have

$$F_2^c = \frac{8}{9} C_{2,c}^{(0)}\left(z, \frac{m_c^2}{Q^2}\right) \otimes c^{\overline{\text{MS}}}(x/z) + \frac{8}{9} \left(\frac{\alpha_S}{4\pi}\right) \hat{C}_{2,g}^{(1)}\left(z, \frac{m_c^2}{Q^2}\right) \otimes g^{\overline{\text{MS}}}(x/z), \quad (6)$$

and then using Eq. (1) to define the DIS charm quark in terms of the $\overline{\text{MS}}$ (MRST) charm quark distribution we find

$$F_2^c = \frac{8}{9} C_{2,c}^{(0)}\left(z, \frac{m_c^2}{Q^2}\right) \otimes c^{\text{DIS}}(x/z) + \frac{8}{9} \left(\frac{\alpha_S}{4\pi}\right) \left[\hat{C}_{2,g}^{(1)}\left(z, \frac{m_c^2}{Q^2}\right) - C_{2,c}^{(0)}\left(z, \frac{m_c^2}{Q^2}\right) \otimes C_{2,g}^{(1)}(z) \right] \otimes g^{\text{DIS}}(x/z), \quad (7)$$

where both coefficient functions depend on m_c (and where in principle there is a NLO quark coefficient function which we omit due to its insignificance in practice). All the relevant coefficient functions can be found in [16]. The gluon coefficient function is actually discontinuous at $Q^2 = m_c^2$, countering the fact that the charm evolution starts from a nonzero value, to give a continuous structure function. The final term above, involving the double convolution of two coefficient functions and the gluon distribution, is potentially rather complicated, especially since $C_{2,c}^{(0)}$ is defined in terms of a convolution itself. However, in practice we find that using $C_{2,c}^{(0)}(z) = (1 - m_c^2/Q^2)^{1/2} \delta(1 - z)$ in this term alone (making the gluon coefficient function continuous) gives an extremely good approximation to the true result; the contribution from the nonzero charm distribution for Q^2 immediately above m_c^2 being negligible. In the limit $Q^2/m_c^2 \rightarrow \infty$, when $C_{2,c}^{(0)}(z) \rightarrow \delta(1 - z)$ and $\hat{C}_{2,g}^{(1)}(z) \rightarrow C_{2,g}^{(1)}(z)$, the usual trivial relationship between the structure function and parton distribution in the DIS scheme is regained.

We could alternatively have defined the heavy quark DIS scheme by demanding this trivial relationship for coefficient functions. This would lead to very complicated, mass-dependent splitting functions in the DIS scheme. Hence we prefer to keep all the mass dependence in the coefficient functions, as in the $\overline{\text{MS}}$ scheme, and let the partons be the usual DIS partons.

We are therefore able to provide the parton distributions in the DIS scheme analogous to those in MRST in the $\overline{\text{MS}}$ scheme [1], labelled MRSTDIS, MRSTDIS($g \uparrow$), MRSTDIS($g \downarrow$), MRSTDIS($\alpha_S \uparrow\uparrow$) and MRSTDIS($\alpha_S \downarrow\downarrow$).

3. Leading order parton distributions

For some purposes, e.g. Monte-Carlo simulation programs, partons distributions which attempt to describe the data at the leading order level are preferable. To obtain such partons we repeat the global analysis at LO. This means that the partons evolve only via the LO DGLAP equations and each process is expressed in terms of the partons via LO formulas. The starting distributions at $Q_0^2 = 1 \text{ GeV}^2$ have the same functional form as in the NLO analysis [1].

The heavy flavour contributions to the structure function F_2 are computed to LO which means that for $Q^2 < m_c^2$ we have

$$F_2^c(x, Q^2) = \frac{8}{9} \left(\frac{\alpha_S}{4\pi}\right) C_g^{(1)\text{FF}}\left(z, m_c^2/Q^2\right) \otimes g(x/z, Q^2), \quad (8)$$

while for $Q^2 > m_c^2$

$$F_2^c(x, Q^2) = F_2^c(x, Q^2 = m_c^2) + \frac{8}{9} C_c^{(0)}(z, m_c^2/Q^2) \otimes c(x/z, Q^2). \quad (9)$$

Again, the coefficient functions in Eqs. (8,9) can be found in [16].

There is a significant difference in the sizes of the LO and NLO gluon distributions at large x which reflect the importance of the NLO corrections to the prompt photon production process (PPP) in that region. Thus to get the same cross section the LO gluon has to be typically greater than the NLO gluon by about 40-50% for $x = 0.3 - 0.45$. The LO gluon is also larger than the NLO distribution at small x , this being required for a good description of the small- x HERA data, and reflecting the importance of the NLO corrections to the quark evolution at small x . This leads to a quite different prediction for the longitudinal structure function (which has to be taken into account when obtaining the values of $F_2(x, Q^2)$ from measurements of the cross section). For example the LO value of F_L is twice the NLO value at $x = 10^{-4}$ and $Q^2 = 4 \text{ GeV}^2$. This again illustrates the importance of a precise measurement of F_L at small x as a test of higher order perturbative corrections. We note also that the Drell-Yan LO cross section requires a phenomenological K -factor of the order of +30% in order to get acceptable agreement with experiment.

To obtain a reasonable description of the DIS data we require a larger value of $\alpha_S(M_Z^2)$ than in the NLO case, but too large a value spoils the simultaneous description of F_2 , F_2^c and the PPP data. Using the simple scale choice of $\mu^2 = Q^2$ we find a value of $\Lambda_{\text{LO}}(4 \text{ flavours}) = 174 \text{ MeV}$; a satisfactory compromise which implies a value $\alpha_S(M_Z^2) = 0.125$. We define our ‘central’ LO solution – MRSTLO – with this value of α_S and with a gluon constrained by the PPP data as before.³ The MRSTLO($\alpha_S \uparrow\uparrow$) and MRSTLO($\alpha_S \downarrow\downarrow$) solutions again correspond to varying this central value of $\alpha_S(M_Z^2)$ by ± 0.005 .

Overall, the quality of the LO and NLO fits are comparable, see Table 1, with two exceptions. The SLAC data (which cover a region⁴ of large x and relatively low Q^2) strongly prefer the NLO corrections – a fact established a long time ago [19]. The effect of these corrections at large x is equivalent to using a LO description where the value of α_S increases with n , the moment of the structure function [20, 19, 21]. While this effect helps the description of the SLAC data, the BCDMS data actually do not favour this trend. Consequently, despite the relatively large value of α_S , the absence of NLO corrections can approximately mimic at large x a NLO fit with a lower value of α_S . Thus the BCDMS data are surprisingly well described by our LO global fit which can be compared with the MRST(NLO) description in Fig. 1.

4. Global analysis including higher twist terms

In MRST we showed that our NLO fit slightly underestimated the slope $dF_2/d \log Q^2$ for the NMC data since the low Q^2 data tended to lie systematically below the fit. Clearly the inclusion

³The same value $\langle k_T \rangle = 0.4 \text{ GeV} (0.92 \text{ GeV})$ for WA70 (E706–530 GeV) is used as in Ref. [1].

⁴Only data for $x < 0.7$ are included in these fits – see the discussion in the next section.

Data set	No. of data pts	MRST	MRSTLO
H1 ep	221	164	159
ZEUS ep	204	269	288
BCDMS μp	174	248	171
NMC μp	130	141	144
NMC μd	130	101	115
SLAC ep	70	119	188
E665 μp	53	59	52
E665 μd	53	61	62
CCFR $F_2^{\nu N}$	66	93	79
CCFR $F_3^{\nu N}$	66	68	83

Table 1: The χ^2 values for the DIS data included in the (NLO- $\overline{\text{MS}}$) MRST fit of Ref. [1] and the corresponding LO fit.

of a negative $1/Q^2$ contribution is bound to improve the quality of the description in this x range $0.02 - 0.1$. Higher twist terms have always been expected to play an important role at very large x – indeed we found in the past [22] that the SLAC data for $x > 0.7$ were dominated by power corrections and for that reason were excluded in MRS leading twist analyses. In those analyses, including MRST, we also imposed a lower W^2 cut on all data of 10 GeV^2 to reduce the effect of unknown higher twist contributions. Now that we are allowing such terms, in this section we relax the constraint so that only data for $W^2 < 4 \text{ GeV}^2$ are removed. In addition we have lowered the Q^2 cut on data included in the fit from 2 to 1.2 GeV^2 .

At very small x we are also interested in examining whether $1/Q^2$ corrections may be important. Some attention has been paid to the observation that at HERA the slope $dF_2/d \log Q^2$ appears to ‘flatten’ off around $x = 10^{-4}$ [23] in contrast to the naive DGLAP expectation. In MRST this was attributed to a ‘valence-like’ behaviour of the gluon at the starting scale $Q_0^2 = 1 \text{ GeV}^2$. That is for x below 10^{-3} the gluon is suppressed and consequently, since for the HERA data x and Q^2 are strongly correlated, this leads to a leveling off of the slope at very low values of x and Q^2 . However this ‘valence-like’ behaviour of the gluon may be an artifact, reflecting some dynamics other than DGLAP, and one candidate is a positive higher twist contribution which is relevant only at small x .

We assume a very simple parameterisation of the higher twist contribution to the DIS structure function,

$$F_2^{\text{HT}}(x, Q^2) = F_2^{\text{LT}}(x, Q^2) \left(1 + \frac{D_2(x)}{Q^2} \right), \quad (10)$$

where the leading twist NLO structure function F_2^{LT} is treated exactly as in MRST. Applying the same overall parameterisation independent of target or beam is probably an over-

simplification but may have some justification if the higher twist contribution can be derived through renormalon dynamics [24].

We parameterise the coefficient $D_2(x)$ by a constant over different bins in x chosen to emphasize aspects of different datasets. We perform fits using the ‘central’ gluon type of solution. In general we find that for $x < 0.5$ the resulting correction is small and negative but beyond 0.6 large and positive. The values of $D_2(x)$ obtained from the fit in each x bin are shown in Table 2.

x	$D_2(x)$ (GeV ²)
0 – 0.0005	0.0147
0.0005 – 0.005	0.0217
0.005 – 0.01	–0.0299
0.01 – 0.06	–0.0382
0.06 – 0.1	–0.0335
0.1 – 0.2	–0.121
0.2 – 0.3	–0.190
0.3 – 0.4	–0.242
0.4 – 0.5	–0.141
0.5 – 0.6	0.248
0.6 – 0.7	1.458
0.7 – 0.8	4.838
0.8 – 0.9	16.06

Table 2: The values of the higher twist coefficient $D_2(x)$ of Eq. (10) (in GeV²) versus x .

Looking at the overall improvement of the resulting fit we note no real difference in the comparison with the HERA data, indeed the higher twist contributions chosen for $x < 0.01$ are very small. We have tried fits where the starting gluon at $Q_0^2 = 1$ GeV² was forced to be flat, or even singular, as $x \rightarrow 0$ to see if the effect of the valence-like gluon could be described instead by a significant positive higher twist term. In each case the quality of the resulting fit was (far) worse. So there appears to be no preference for a description in terms of a ‘conventional’ gluon at low Q^2 with additional power corrections, at least within our admittedly simple parameterisation.

At intermediate x values ($0.01 < x < 0.5$) there is definitely a preference for some negative $1/Q^2$ contribution as we expect from the NMC data. With the relaxing of the Q^2 cut, the number of NMC F_2^p points increases from 130 to 155 but the χ^2 stays close to 140 with the inclusion of the higher twist term. The most significant improvement in the quality of the fit is for the data for $x > 0.1$ where the higher twist corrections are largest. The most dramatic difference is the description of the SLAC data. In Fig. 2 we illustrate the improvement at very

large x due to the addition of the arbitrary higher twist contribution. From this figure and from the values of the coefficient $D_2(x)$ in Table 2 we see that the very large x SLAC data require a large positive $1/Q^2$ correction. We have not attempted to separate the part arising from target mass corrections but a recent analysis [25] suggests a significant fraction can be accounted for in this way.

We may regard the difference between this new set of partons – MRST(HT) – and MRST as one measure of uncertainty in our parton sets. In Fig. 3 we show the ratio of the u and d partons at $Q^2 = 10 \text{ GeV}^2$ between the two sets – the differences being much smaller for other partons. We see that the differences remain less than about 1-2% except where the individual distributions start to become really quite small. At large x , $x > 0.6$, the large differences for the valence distributions are not surprising in view of the large higher twist correction as $x \rightarrow 1$. As Q^2 increases the ratios remain rather constant so that Fig. 3 is a reliable indication of the uncertainty on the u and d partons at all Q^2 arising from possible higher twist contributions.

5. Conclusions

We have presented an extension of our previous global analysis to provide alternative parton distributions. In particular we have obtained the analogues of the partons of MRST but (a) in the DIS NLO scheme as opposed to the $\overline{\text{MS}}$ scheme and (b) by repeating the global analysis at LO. In this latter case the partons are consistent with all processes considered being evaluated to LO. Finally, we present a set of partons obtained from a global analysis in which an empirical universal higher twist contribution is included, which is freely parameterised as a function of x . This empirical higher twist component is found to be surprisingly small in the HERA small x domain. On the other hand it is interesting to see that the higher twist fit is similar at high x to that expected from a renormalon approach for the nonsinglet structure function [24].

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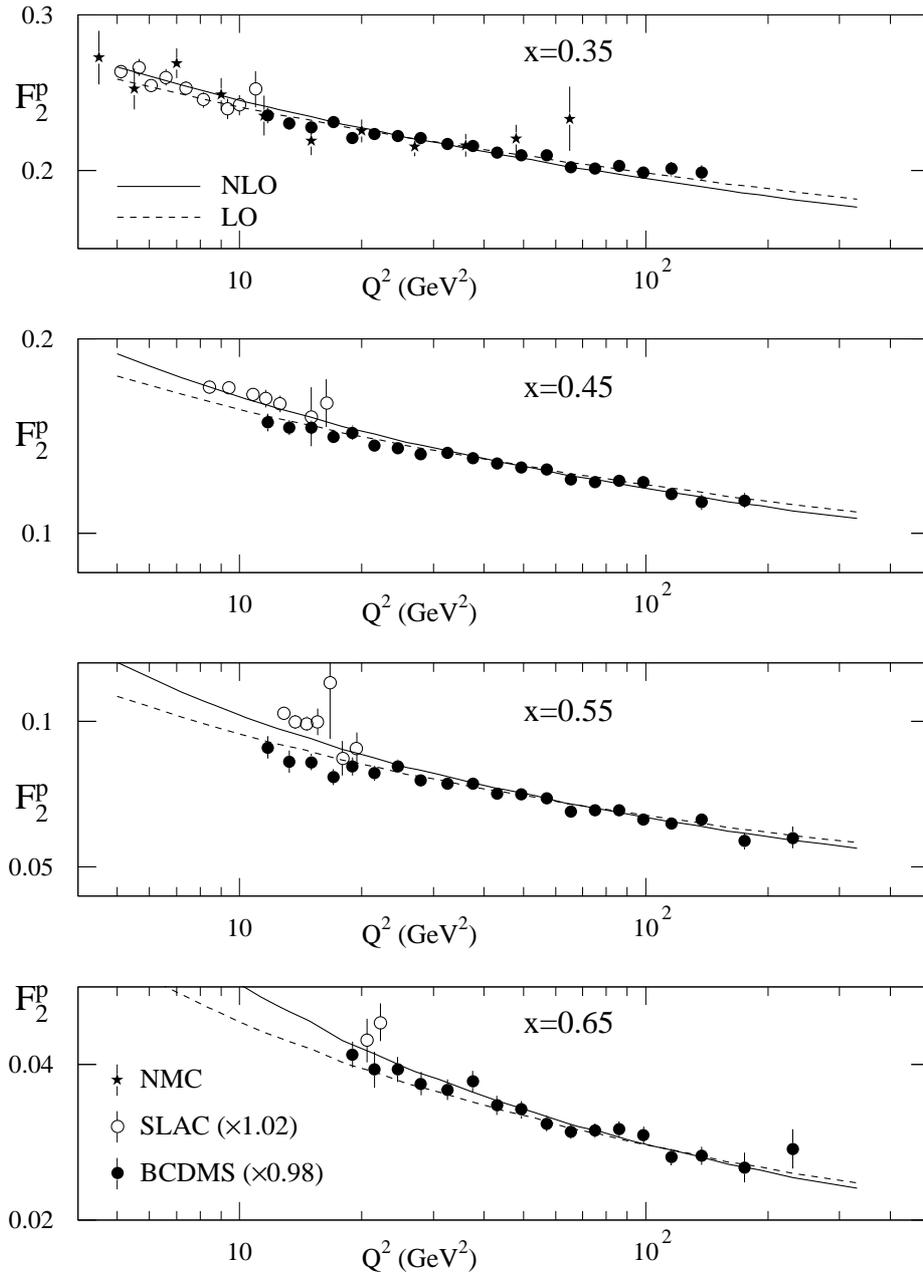


Figure 1: Comparison of large x data from BCDMS, NMC and SLAC with LO and NLO fits of MRST.

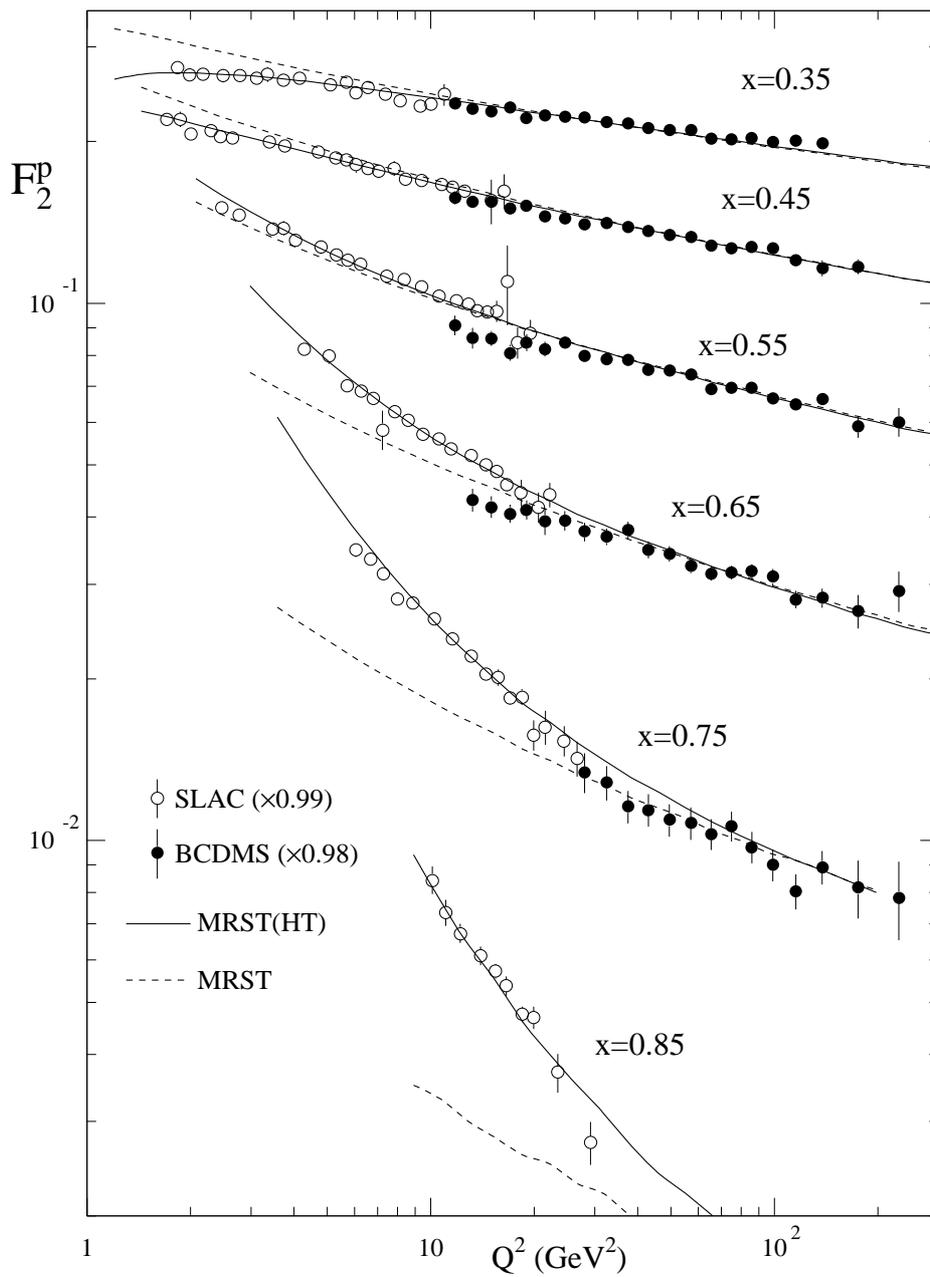


Figure 2: Comparison of very large x data from BCDMS and SLAC with the standard NLO fit of MRST and the higher twist fit MRST(HT).

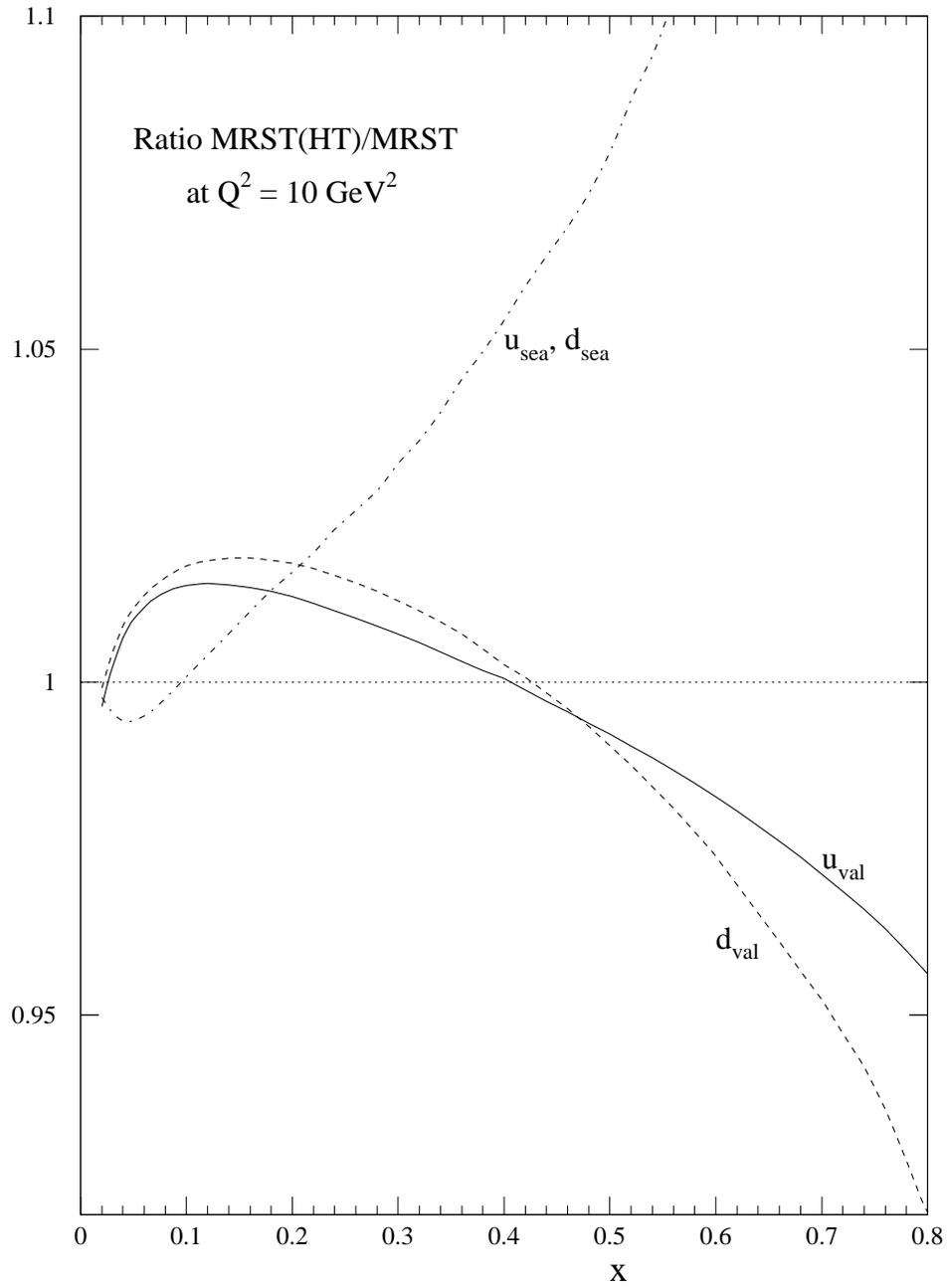


Figure 3: Ratios of u and d partons in the two sets MRST(HT) and MRST.