Bounds on anomalous gauge couplings from past and near future experiments: the role of the different measurements

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Abstract
We analyze the bounds on the set of four "non-blind" anomalous gauge couplings that will be derived, in absence of deviations from the Standard Model predictions, from near future measurements at LEP2 and at the TEVATRON. The bounds are obtained by combining these negative results with those already available from LEP1 and from atomic parity violation experiments. In this process, the information coming from LEP2 is treated using a recently proposed ("Z-peak subtracted") theoretical approach. This makes it easier to identify the specific role that the different experiments play in this determination, in the spirit of a recent previous investigation for the set of three "blind" couplings.

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1 Introduction.

The possibility that anomalous gauge couplings (AGC) exist, that essentially respect the same gauge symmetry of the Standard Model, has been considered in recent years by several authors [1]. In particular, a classification scheme that differentiates the so-called "blind" operators from the "non-blind" ones has been proposed [2]. This was reformulated in a general form by Hagiwara et al in a work [3] whose notations we shall follow in the present paper.

In a previous publication [4], we have analyzed the overall bounds that can be obtained on the "blind" set by a combination of (supposedly negative) experimental results, obtained in the two $W$ channel at LEP2 [5], with those available from the LEP1 measurements of the $Z$ partial width into $b\bar{b}$ pairs [6] and from the hypothetical future improved measurement of the muon $g-2$ at BNL [7]. A nice feature of that analysis, in our opinion, was the fact that it pointed out in a clear way the fact that the three different experiments were complementary, each one allowing an improved determination of a different subset of parameters.

The aim of this short paper is that of showing that an essentially similar situation can be obtained for the case of the "non-blind" set. Here the available experimental information will be derived from LEP1 measurements, from Atomic Parity Violation (APV) results, from an assumed high precision measurement of the $W$ mass at LEP2 and at the TEVATRON and from three measurements that are being performed at LEP2 in the final two fermion channel (those of the final muons cross section and forward-backward asymmetry and that of the final hadronic states cross section). For what concerns the latter information from LEP2, we shall use a theoretical description based on the so-called "$Z$-peak subtracted" approach [8]. This will allow to treat the AGC contribution in a particularly simple way, thus making the "genuine" role of the separate experiments in this procedure relatively simple to identify.

We now proceed to illustrate our approach. The relevant "non-blind" part of the anomalous Lagrangian can be written in the following way [3] (only dimension six operators are retained):

$$\mathcal{L}^{(NB)} = \frac{f_{DW}}{\Lambda^2} \mathcal{O}_{DW} + \frac{f_{DB}}{\Lambda^2} \mathcal{O}_{DB} + \frac{f_{BW}}{\Lambda^2} \mathcal{O}_{BW} + \frac{f_{\Phi_1}}{\Lambda^2} \mathcal{O}_{\Phi_1},$$

(1)

$$\mathcal{O}_{DW} = Tr([D_\mu, \bar{W}^{\mu\nu}][D_\nu, \bar{W}^{2\mu\nu}]),$$

(2)

$$\mathcal{O}_{DB} = -\frac{g_2^2}{2} (\partial_\mu B_{\nu\rho})(\partial^\mu B^{\nu\rho}),$$

(3)

$$\mathcal{O}_{BW} = \Phi^\dagger B_{\mu\nu} \rightarrow \bar{W}^{2\mu\nu} \Phi,$$

(4)

$$\mathcal{O}_{\Phi_1} = (D_\mu \Phi^\dagger \Phi)(\Phi^\dagger D^\mu \Phi).$$

(5)

The contribution of this Lagrangian to the LEP1 observables can be easily computed at the tree level. At the one loop level in which we shall be interested, it has been shown in Ref.[3] that for massless fermions all the relevant AGC effects can be simply written by formally replacing the four parameters that enter eq.(1) by corresponding "renormalized" quantities. These are four combinations of each one of the parameters of eq.(1) with
contributions coming from the "blind" set, and their expressions can be found in Ref.[3]. In terms of these four renormalized "non-blind" parameters, denoted as \( f_{DW} \), \( f_{DB} \), \( f_{BW} \), \( f_{\Phi,1} \), the expressions of the various observables at one loop can be easily derived.

The previous treatment has one important situation where it does not apply. In the theoretical expression of the partial \( Z \) width in \( b\bar{b} \) pairs, \( \Gamma_b \), the dominant \( \simeq m_t^2 \) contribution (where the top mass cannot be ignored) comes from parameters of the "blind" set, as exhaustively discussed in a previous paper [9]. This fact was actually exploited in Ref.[4] to compute bounds on the corresponding parameter sector. For the aims of this paper, that are orthogonal to those of Ref.[4], the experimental value of \( \Gamma_b \) will consequently not be exploitable. For the identical reason, the value of the full \( Z \) hadronic width \( \Gamma_h \) will also not be considered.

In practice, therefore, we shall be entitled to use the values of the two independent purely leptonic observables, the \( Z \) partial width into (charged) leptons \( \Gamma_l \) and the leptonic forward-backward asymmetry \( A_{FB,l} \). The contributions to these quantities from anomalous gauge couplings have the following theoretical expressions:

\[
\frac{\Gamma_l^{AGC}}{M_Z} = \frac{M_Z^2}{\Lambda^2} \left[ \frac{s_W^2(1-s_W^2)}{2\pi \alpha} \right. \\
\times \left( 1 + \frac{8s_W^2(1-s_W^2)(1-4s_W^2)}{1-2s_W^2} \right) f_{\Phi,1} + \\
+ \frac{8s_W^2(1-s_W^2)}{1-2s_W^2} f_{BW} + \\
+ \frac{8\pi \alpha(1-s_W^2)}{s_W^2} \left( \frac{8s_W^4(1-4s_W^2)}{1-2s_W^2} - 1 \right) f_{DW} + \\
+ \frac{8\pi \alpha s_W^2}{1-s_W^2} \left( \frac{8(1-s_W^2)^2(1-4s_W^2)}{1-2s_W^2} - 1 \right) f_{DB} \right] \tag{6}
\]

\[
A_{FB,l}^{AGC} = \frac{24(1-4s_W^2)(1-(1-4s_W^2)^2)s_W^2 M_Z^2}{(1+(1-4s_W^2)^2)(1-2s_W^2)} \left[ \frac{(1-s_W^2)^2 s_W^2}{2\pi \alpha} f_{\Phi,1} + \\
+ \left( 1-s_W^2 \right) f_{BW} + 8\pi \alpha(1-s_W^2)(f_{DW} + f_{DB}) \right] \tag{7}
\]

where \( s_W^2 = \sin^2 \theta_{W,eff} \) is the "effective" weak mixing angle in the commonly used (LEP1, SLC) definition.

Eqs.(6,7) will be the LEP1 content of our analysis. The next experimental result that we shall use is that coming from the measurement of Atomic Parity Violation [10]. This is usually expressed in terms of the "weak charge" \( Q_W \). The contribution to this quantity from anomalous gauge couplings is:

\[
Q_W^{AGC} = 0.80 \frac{M_Z^2}{\Lambda^2} \frac{4s_W^2(1-s_W^2)}{\alpha} f_{BW} \tag{8}
\]
Next, we have added the experimental information coming from future measurements of the $W$ mass, for which the AGC effect reads:

$$M_W^{AGC} = \left[16.18 GeV\right] \left[8\pi \alpha M_Z^2 \frac{1-s_W^2}{\Lambda^2} \left(1-s_W^2 f_{DW} + f_{DB}\right)\right]$$

$$+ \frac{(1-s_W^2)(1-4s_W^2)^2}{2s_W^2\Lambda^2} f_{\phi,1} - \frac{2M_W^2}{\Lambda^2} f_{BW}$$

(9)

The final experimental input is that derived from the final two fermion channel at LEP2. We shall use three measurements, i.e. those of the muon cross section $\sigma_\mu$ and forward-backward asymmetry $A_{FB,\mu}$ and that of the full final hadronic cross section $\sigma_3$. This last choice might seem in contradiction with our previous remark concerning $\Gamma_b, \Gamma_h$ but in fact it is not so. The reason is that the theoretical expressions that we shall use have been derived in the so-called "Z-peak subtracted" approach, exhaustively illustrated in Ref[8]. This consists, essentially, of replacing the input parameter $G_a$ by quantities directly measured on top of $Z$ resonance. As an immediate byproduct, the theoretical expressions of those one loop corrections that modify the simple $Z$ propagator become subtracted at $q^2 = (p^-_0 + p^+_0)^2 = M_Z^2$ and loose those terms that are energy independent. This happens, in particular, to the dominant part of the $\simeq m_t^2$ "blind" contribution carried from $\sigma_b$ to $\sigma_3$ at LEP2 which is, so to say, "reabsorbed" by the input parameters $\Gamma_5$, and for a full and detailed discussion of this point we defer to Ref[8].

The elimination of the "blind" parameters in $\sigma_3$ is not the only bonus of the use of the "Z-peak subtracted" approach. For the same reason that we have just mentioned, those "non blind" renormalized parameters whose relevant contribution would be energy independent disappear in the subtracted expressions (this is also valid for the contributions to the photon propagator, that are by definition subtracted at $q^2 = 0$). As one can guess, this applies to the two parameters $f_{BW}, f_{\phi,1}$; these are multiplied by the smaller number of derivatives in the associated operators and consequently are reabsorbed, in our approach, in the LEP1 experimental measurements used as new theoretical inputs. In conclusion, the AGC "genuine LEP2" contribution, that is, we repeat, the one that cannot be reabsorbed by the use of the LEP1 measurements, reads:

$$\sigma_\mu(q^2) = \sigma_\mu^{Born}(q^2) \left\{ 1 + \frac{2}{\kappa^2(q^2-M_Z^2)^2+q^4} \left[ \kappa^2(q^2-M_Z^2)^2 \tilde{\Delta}_\alpha(q^2) - q^4 (R(q^2) + 0.5 V(q^2)) \right] \right\}$$

(10)

where $\kappa \equiv \frac{\alpha M_Z}{3\pi l} \simeq 2.64$ and

$$\sigma_\mu^{Born}(q^2) = \frac{4\pi \alpha^2}{3q^2} \left[ q^4 + \frac{\kappa^2(q^2-M_Z^2)^2}{\kappa^2(q^2-M_Z^2)^2} \right]$$

(11)
\[ A_{FB,\mu}(q^2) = A_{FB,\mu}^{Born}(q^2) \{ 1 + \frac{q^4 - \kappa^2(q^2 - M_Z^2)^2}{\kappa^2(q^2 - M_Z^2)^2 + q^4} \left[ \tilde{\Delta}_\alpha(q^2) + R(q^2) \right] + \frac{q^4}{\kappa^2(q^2 - M_Z^2)^2 + q^4} V(q^2) \} \] (12)

where

\[ A_{FB,\mu}^{Born}(q^2) = \frac{3q^2 \kappa(q^2 - M_Z^2)}{2[q^4 + \kappa^2(q^2 - M_Z^2)^2]} \] (13)

\[ \sigma_5(q^2) = \sigma_5^{Born}(q^2) \{ 1 + \left[ \frac{2(q^2 - M_Z^2)^2}{0.81q^4 + 0.06q^2(q^2 - M_Z^2) + (q^2 - M_Z^2)^2} \tilde{\Delta}_\alpha(q^2) \right] \]

\[ - \left[ \frac{0.81q^4 + 0.06q^2(q^2 - M_Z^2) + (q^2 - M_Z^2)^2}{0.81q^4 + 0.06q^2(q^2 - M_Z^2) + (q^2 - M_Z^2)^2} \tilde{\Delta}_\alpha(q^2) - R(q^2) - 24.39V(q^2) \} \] (14)

where

\[ \sigma_5^{Born}(q^2) \simeq \frac{44\pi a_\pi^2}{9q^2} \left[ 1 + 0.81 \frac{q^4}{(q^2 - M_Z^2)^2} + 0.06 \frac{q^2}{q^2 - M_Z^2} \right] \] (15)

and

\[ \tilde{\Delta}_\alpha^{(AGC)}(q^2) = -8\pi \frac{q^2}{\Lambda^2} \left[ f_{DW}^r + f_{DB}^r \right] \] (16)

\[ R^{(AGC)}(q^2) = 8\pi \frac{a_\pi (q^2 - M_Z^2)}{\Lambda^2} \left[ \frac{1 - s_W^2}{s_W^2} f_{DW}^r + \frac{s_W^2}{1 - s_W^2} f_{DB}^r \right] \] (17)

\[ V^{(AGC)}(q^2) = 8\pi \frac{a_\pi (q^2 - M_Z^2)}{\Lambda^2} \left[ \frac{\sqrt{1 - s_W^2}}{s_W} f_{DW}^r - \frac{s_W}{\sqrt{1 - s_W^2}} f_{DB}^r \right] \] (18)

With Eqs.(6)-(9) and (16)-(18) at our disposal, we have moved to the practical task of deriving bounds for the four involved parameters in the (conventional) hypothesis that the new physics scale \( \Lambda \) is 1 TeV, by following the same attitude adopted in Ref.[4]. With this purpose, we have divided our input data into two sets. The first one consists of the two LEP1 and of the APV measurements. For these, we have used the experimental results quoted in Refs.[6, 10] with the related error and the SM predictions corresponding to \( m_t = 173.8 \) GeV (according to the latest combined CDF/D0 result [11]), \( m_H = 300 \) GeV,
\[ \alpha_{QED}(M_Z^2) = 128.923 \text{ [12] and } \alpha_s(M_Z^2) = 0.118 \text{ (by following the electroweak working group choice [6]). To be more precise, the inputs of our analysis are:} \]

\[
\begin{align*}
\Gamma_i^{\text{exp}} &= 83.91 \pm 0.10 \text{ MeV} \quad (83.91 \text{ MeV}) \\
A_{FB,i}^{\text{exp}} &= 0.0171 \pm 0.0010 \quad (0.0151) \\
Q_W^{\text{exp}} &= -72.11 \pm 0.27 \quad (73.11 \pm 0.89)
\end{align*}
\]

where the values in brackets are the SM predictions. The second set of experiments consists of the \(W\) mass determination and of the measurements of the three LEP2 observables (\(\sigma_\mu\), \(A_{FB,\mu}\) and \(\sigma_5\)). Here we have assumed that the final experimental values will agree with the SM predictions; to compute the latter quantities we have used the semianalytical program PALM, that was illustrated in a previous paper [13], where the “Z-peak subtracted” approach is systematically adopted. Concerning the errors, we have used as final precision on the \(W\) mass 30 MeV and we have defined the experimental uncertainties on the measurements at LEP2 as the statistical errors achieved with an overall integrated luminosity of about 500 \(pb^{-1}\) collected by four experiments. Since the sensitivity of \(\sigma_\mu\), \(A_{FB,\mu}\) and \(\sigma_5\) depends on the centre of mass energy, \(E_{cm}\), we worked in the realistic scenario of several measurements performed in the energy range between 130 GeV and 200 GeV with a statistical significance determined, for each data sample, according to the on going LEP operation and to the possible developments in the next two years of run. Namely, we assumed the following centre of mass energy scan:

\[
\begin{align*}
E_{cm} &= 133 \quad 161 \quad 172 \quad 183 \quad 190 \quad 200 \text{ (GeV)} \\
\mathcal{L}_{int} &= 10 \quad 10 \quad 10 \quad 50 \quad 200 \quad 250 \text{ (pb\(^{-1}\))}
\end{align*}
\]

Technically, the results of our analysis have been obtained by minimising, in a conventional minimisation program, the \(\chi^2\) variable

\[
\chi^2 = \sum_{j=1}^{4} \left( \frac{O_j^{th} - O_j^{exp}}{\delta O_j} \right)^2 + \sum_{j=1}^{3} \sum_{k=1}^{6} \left( \frac{O_{jk}^{th} - O_{jk}^{exp}}{\delta O_{jk}} \right)^2
\]

where the index \(j\) runs over the seven observables, while the index \(k\) runs over the six samples of data collected at LEP2. In each term of the \(\chi^2\), the theoretical expression \(O_j^{th}\) consists of the sum of the SM prediction and of the shift induced by the AGC parameters.

For the seek of our study, which aims to estimate the ultimate constraints achievable in the case of negative experiment on the assumed set of non-blind AGC, what is relevant is both the experimental accuracy of each measurement and the inherent sensitivity of the observables to every anomalous coupling. These two ingredients of the analysis affect the shape of the \(\chi^2\) around the minimum but not the location of the minimum itself. On the other hand, the most probable values of the AGC parameters, which can be determined only when the final experimental results will be available, will depend on \(m_t\) and \(m_H\), as will be briefly discussed later. In the following we assume the latest combined CDF/D0 measurement of the top mass \((173.8 \pm 3.2 \pm 3.9 \text{ [11]})\) and \(m_H = 300\text{ GeV}\). Although one expects that the bounds that we derive should be rather stable (unless unexpected strong
variations from the predicted accuracies will occur), clearly, the experimental inputs can (and will) be easily modified as soon as the real final data will be announced. This would allow to exclude (or detect) anomalies in the gauge self-interaction sector by properly taking into account also the uncertainty arising from $m_t$ and $m_H$, which hopefully will have reached a negligible level.

The results of the overall analysis, made using the seven experimental "data" and errors and minimizing the $\chi^2$ with respect to the four AGC parameters at a time, are shown in Table 1.

To be more precise, in the first row of Table 1 we have listed the bounds that would be obtained by only using the four LEP1, APV, $M_W$ results. To shorten our notations, we shall call these data "low energy" data. In the second row, we give the results that would be derivable by adding to the previous "low energy" information that coming from the three LEP2 measurements. In Table 2 and 3 we report the error correlation matrices that correspond to the two cases. Here we have defined $\delta f_{DB}$, $\delta f_{BW}$, $\delta f_{DB}$ and $\delta f_{\phi,1}$ as the distance from the minimum of the hyperplane corresponding to $\chi^2 = \chi^2_{\text{min}} + 1$.

As one sees from inspection of Table 1, the addition of LEP2 data systematically ameliorates the general bounds. In particular, a strong improvement is obtained for $f_{DB}$ and $f_{BW}$ (a factor 4-5). For the remaining parameters, a smaller but still remarkable (a factor 2) reduction of the error bound is derived. To get a more specific feeling of the role of the different measurements, we have first plotted in Fig. ?? the contours, in the six two-dimensional planes, corresponding to the bounds given in Table 1, that is the projections in each plane of the $\chi^2 = \chi^2_{\text{min}} + 1$ hyperplane. Here, as well as in all the plots presented, the artificial central values of the fits have been shifted to zero "by hand" in order to concentrate the attention on the significance of the result obtained with different sets of experimental inputs. The plot shows again, in a more immediate way, the relevance of the addition of the three LEP2 measurements on all the four parameters, including the two ones that in our approach do not appear in the related theoretical expressions (i.e. $f_{BW}, f_{\phi,1}$). This happens as a consequence of the correlation among the parameters in the theoretical expressions of the "low energy" observables entering the $\chi^2$. In particular, from Table 2, one can conclude that $f_{DB}$, which is strongly correlated to the two energy insensitive parameters, drives the overall improvement.

We considered also the occurrence that one, two or three AGC parameters are zero; the bounds achievable for the surviving parameters are listed in Table 4. Although there isn’t any specific theoretical argument in favour of these scenarios, the results of this study clarify the interplay between the four parameters which results in the final correlations (Tables 2 and 3).

Finally, in Table 5 we give the 68.3% C.L. bounds for the four-free parameter fit. Since our $\chi^2$ is a quadratic function of the anomalous couplings, the shape of the region in the parameter space around the minimum with a specific probability content does not depend on the C.L. chosen. A comparison with Table 1 shows that the region of the AGC parameter space allowed with a C.L. of 68.3% corresponds to a scaling by a factor 2.2 of the contours (Fig. ??) previously determined with our "work definition" of the bounds.

The next question that we have addressed is that of understanding which ones of the
three LEP2 measurements that we have considered are more relevant. To answer this point, we have plotted in Fig. ?? the bounds that would be obtained by releasing one out of the three LEP2 measurements in the overall (“low-energy” + LEP2) bound derivation. One sees in fact from those exclusion plots that the bulk of the information is provided by the addition to the low energy data of the two LEP2 cross sections; on the contrary, the role of the muon asymmetry appears to be, in this specific context, marginal, even at the most optimistic level of experimental accuracy.

Having stressed the relevance of the three LEP2 measurements for the derivation of meaningful bounds, we have then studied the relative relevance of the four remaining “low energy” data. In other words, we have considered the four different results that would be obtained by neglecting, each time, one of the four low energy informations. The results are shown in Fig. ??, again showing the projections on the six parameter planes.

As one sees from Fig. ??, the relevance of the three measurements of $\Gamma_l$, $A_{FB,l}(M_2^Z)$ and $M_W$ is essentially similar on all the six pairs: neglecting one of these three measurements introduces in the bounds different (appreciable) comparable shifts. On the contrary, the role of the APV measurement seems in this respect quite negligible, at the present level of overall (experimental and theoretical) accuracy Eq.(21). In fact, a consistent reduction of the present uncertainty on $\Delta q_W$ would be required for an effective impact of this observable in the analysis. Actually, a decrease of the error by a factor of 2 would still only marginally improve the bound on $f_{BW}$ (7%), while a suppression by a factor of 5 is needed in order to achieve a 33% improvement of the bound on $f_{BW}$ and, as a result of the correlation, a 28% decrease of the allowed interval for $f_{DB}$.

A final comment concerns the effect of the top and Higgs masses uncertainties in the result of our analysis. The values of $m_t$ and $m_H$ enter the theoretical expression of the two LEP1 observables and of $M_W$, and consequently affect the minimum of the $\chi^2$ and, therefore, the central values of the allowed intervals for the four anomalous couplings. At the present level of experimental precision on $A_{FB,l}$ and $\Gamma_l$ and of the foreseen final error on $M_W$, the shifts induced in the corresponding SM predictions by the error on $m_t$ (5 GeV) and by the uncertainty on $m_H$ are sizable. For example, when $m_H=300$ GeV, moving the value of $m_t$ from the central measured value by one sigma produces a shift in $\sin^2 \theta_{W_{eff}}(SM)$ equal to 30% of the experimental error arising from the measurement of $A_{FB,l}$; the corresponding shift of $\Gamma_l(SM)$ amounts to 50% of the experimental error and the forecast for $M_W(SM)$ is moved slightly more than 30 MeV. On the other hand, at a fixed value of the top mass, as $m_H$ ranges between 90 GeV and 1 TeV, the prediction on both $\sin^2 \theta_{W_{eff}}$ and $\Gamma_l$ varies by roughly twice the present experimental errors and the variation of the $M_W$ value is about 150 MeV. Those uncertainties reflect on a not negligible shift of the $\chi^2$ minimum in the purely low energy analysis. Namely, we observe a linear drift of the central values of $f_{DW}$, $f_{DB}$ and $f_{B_1}$ with $m_t$. A one sigma shift of $m_t$ moves them respectively by 10%, 2% and 5% of the corresponding 68% C.L. errors. One can observe, therefore, that the future precision in the determination of $m_t$ at the run II of TEVATRON, which is planned to be $\delta m_t < 2$ GeV per experiment [14], will play a very significant role in the definite bounds we will be able to derive on the AGC parameters. Of course, far less predictable is the impact of the outcomes on the Higgs mass of future
experiments. Nevertheless, it’s worth to point out that in our formulation the LEP2 observables are essentially free of $m_t$, $M_H$ whose dominant contributions are reabsorbed in the theoretical input (as discussed in [8] this does not introduce any appreciable theoretical error). Since the major contributions to the bounds comes from LEP2 data, although they bring direct information only on $f_{DW}$ and $f_{DB}$, we expect that the role of $m_t$, $M_H$ will be strongly weakened in the final analysis to be performed.

In conclusion, the most accurate determination of the bounds on the four “non-blind” parameters Eq.(1) appears to be that derivable from an analysis of LEP1, LEP2 data combined with the experimental value of $M_W$. As soon as the final LEP2 results will be established, our analysis will be straightforwardly adapted to provide the final central values to be used in the conclusive formulation.
References


Table 1: Bounds on the anomalous gauge couplings obtained with a combined fit of present and future experimental data. The definition of the parameter uncertainties adopted here is the 1 $\sigma$ error in the $\chi^2$ minimization. **Low** refers to the results from LEP1, APV and from the measurement of $M_W$, **High** to the cross-section and asymmetry measurements at LEP2.

<table>
<thead>
<tr>
<th></th>
<th>$\delta f_{DW}$</th>
<th>$\delta f_{BW}$</th>
<th>$\delta f_{DB}$</th>
<th>$\delta f_{\Phi,1}$</th>
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<td>0.035</td>
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Table 2: Correlation matrix from low energy data.

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<th>$f_{DB}$</th>
<th>$f_{\Phi,1}$</th>
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Table 3: Correlation matrix from low+high energy data.

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<th>$f_{BW}$</th>
<th>$f_{DB}$</th>
<th>$f_{\Phi,1}$</th>
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Table 4: Bounds on the AGC parameters achieved in case of a reduced number of free parameters.

<table>
<thead>
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<th>1 free p.</th>
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<th>( I_{DW} = I_{DB} = f_{\Phi,1} = 0 )</th>
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</thead>
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<td>( \delta f_{BW} = 0.067 )</td>
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<tr>
<td>(low+high)</td>
<td>( \delta f_{DW} = 0.0094 )</td>
<td>( \delta f_{BW} = 0.067 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1 free p.</th>
<th>( I_{DW} = I_{BW} = f_{\Phi,1} = 0 )</th>
<th>( I_{DW} = I_{BW} = f_{DB} = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(low)</td>
<td>( \delta f_{DB} = 0.53 )</td>
<td>( \delta f_{\Phi,1} = 0.008 )</td>
</tr>
<tr>
<td>(low+high)</td>
<td>( \delta f_{DB} = 0.42 )</td>
<td>( \delta f_{\Phi,1} = 0.008 )</td>
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</table>

<table>
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<tr>
<th>2 free p.</th>
<th>( I_{DB} = f_{\Phi,1} = 0 )</th>
<th>( I_{BW} = f_{\Phi,1} = 0 )</th>
<th>( I_{BW} = f_{DB} = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(low)</td>
<td>( \delta f_{DW} = 0.22 )</td>
<td>( \delta f_{BW} = 0.10 )</td>
<td>( \delta f_{DB} = 0.25 )</td>
</tr>
<tr>
<td>(low+high)</td>
<td>( \delta f_{DW} = 0.11 )</td>
<td>( \delta f_{BW} = 0.077 )</td>
<td>( \delta f_{DW} = 0.16 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2 free p.</th>
<th>( I_{DW} = f_{\Phi,1} = 0 )</th>
<th>( I_{BW} = f_{DB} = 0 )</th>
<th>( I_{DW} = I_{BW} = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(low)</td>
<td>( \delta f_{BW} = 0.30 )</td>
<td>( \delta f_{DB} = 2.36 )</td>
<td>( \delta f_{\Phi,1} = 0.034 )</td>
</tr>
<tr>
<td>(low+high)</td>
<td>( \delta f_{BW} = 0.11 )</td>
<td>( \delta f_{DB} = 0.69 )</td>
<td>( \delta f_{BW} = 0.28 )</td>
</tr>
</tbody>
</table>

| 3 free p. | \( I_{DW} = 0 \) | \( I_{BW} = 0 \) |
|---|---|
| (low) | \( \delta f_{BW} = 1.44 \) | \( \delta f_{DB} = 6.14 \) | \( \delta f_{\Phi,1} = 0.087 \) | \( \delta f_{DW} = 0.28 \) | \( \delta f_{DB} = 1.75 \) | \( \delta f_{\Phi,1} = 0.020 \) |
| (low+high) | \( \delta f_{BW} = 0.32 \) | \( \delta f_{DB} = 0.71 \) | \( \delta f_{\Phi,1} = 0.035 \) | \( \delta f_{DW} = 0.18 \) | \( \delta f_{DB} = 1.08 \) | \( \delta f_{\Phi,1} = 0.013 \) |

| 3 free p. | \( I_{DB} = 0 \) | \( I_{\Phi,1} = 0 \) |
|---|---|
| (low) | \( \delta f_{DW} = 0.27 \) | \( \delta f_{BW} = 0.40 \) | \( \delta f_{\Phi,1} = 0.041 \) | \( \delta f_{DW} = 0.27 \) | \( \delta f_{BW} = 0.32 \) | \( \delta f_{DB} = 2.95 \) |
| (low+high) | \( \delta f_{DW} = 0.11 \) | \( \delta f_{BW} = 0.30 \) | \( \delta f_{\Phi,1} = 0.035 \) | \( \delta f_{DW} = 0.18 \) | \( \delta f_{BW} = 0.12 \) | \( \delta f_{DB} = 1.15 \) |
Table 5: 68.3% Confidence Level bounds on the anomalous gauge couplings. As before, Low refers to the LEP1, APV and $M_W$ data, High to the LEP2 measurements.

<table>
<thead>
<tr>
<th></th>
<th>$\delta f_{DW}$</th>
<th>$\delta f_{BW}$</th>
<th>$\delta f_{DB}$</th>
<th>$\delta f_{\Phi,1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>0.58</td>
<td>3.00</td>
<td>13.12</td>
<td>0.184</td>
</tr>
<tr>
<td>Low+High</td>
<td>0.38</td>
<td>0.68</td>
<td>2.42</td>
<td>0.074</td>
</tr>
</tbody>
</table>
**Figure captions**

**Fig. 1:** Projection of the $\chi^2 < \chi^2_{\text{min}} + 1$ region in the space of the four anomalous gauge couplings onto the six possible coordinate planes. The outer ellipses are obtained from low energy data only; the inner ones, by including the LEP2 data.

**Fig. 2:** Projected ellipses obtained by a global fit to the full set of measurements (LEP1, LEP2, APV and $M_W$) releasing one of the **high** energy constraints. The three curves correspond respectively to the exclusion of $\sigma_\mu$ (dashed), $\sigma_5$ (dotted), $A_{FB,\mu}$ (solid). The most internal region (shaded) is the overall result.

**Fig. 3:** Projected ellipses obtained by a global fit to the full set of measurements (LEP1, LEP2, APV and $M_W$) releasing one of the **low** energy constraints. The four curves correspond respectively to the exclusion of $A_{FB,\ell}$ (dashed), $\Gamma_1$ (dotted), $Q_W$ (solid) and $M_W$ (dot-dash). The most internal region (shaded) is the overall result.
Figure 2
Figure 3