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QED Structure Functions of the Photon

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Abstract

In deep inelastic electron-photon scattering in leading order QED, $\mathrm{e}\gamma \to \mathrm{e}\gamma^{\star}\gamma \to \mathrm{e}\mathrm{f}\bar{\mathrm{f}}$, there are four non-zero structure functions. We calculate them for real photons retaining the full dependence on the fermion mass, and show numerical results of its effect.

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Leading order QED structure functions have long been calculated and can be found in the literature, see for example Ref. [1]. However, to our knowledge the full set has never been written down in one place retaining full dependence on the mass of the produced fermion. In this paper we do that, and show numerical results.

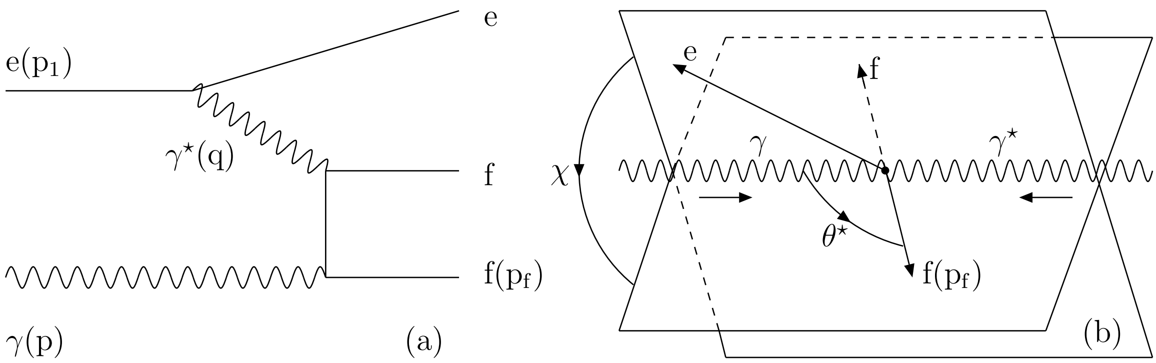

Figure 1: The definition of variables used in the calculation. Shown are (a) a diagram of the reaction $\mathrm{e}\gamma \to \mathrm{e}\gamma^\star\gamma \to \mathrm{ef\bar{f}}$ and (b) an illustration of the azimuthal angle χ for the reaction $\mathrm{e}\gamma \to \mathrm{e}\gamma^\star\gamma \to \mathrm{ef\bar{f}}$, in the $\gamma^\star\gamma$ centre-of-mass system.

The relevant variables are $Q^2 = -q^2$, $x = Q^2/2p\cdot q$, $y = p\cdot q/p_1\cdot p$, the invariant mass squared of the fermion pair, $W^2 = (p+q)^2$ and $z = p\cdot p_\mathrm{f}/p\cdot q$. They are defined from the four-vectors in Figure 1(a). The variable z is related to the fermion scattering angle $\theta^\star$ in the $\gamma^\star\gamma$ centre-of-mass frame, via $z = \frac{1}{2}(1+\beta\cos\theta^\star)$, with $\beta = \sqrt{1 - 4m_\mathrm{f}^2/W^2}$, where m_f denotes the mass of the fermion. The azimuthal angle χ between the electron scattering plane and the $\mathrm{f\bar{f}}$ plane in the $\gamma^\star\gamma$ centre-of-mass frame is defined in Figure 1(b).

In this paper we only consider real target photons $P^2 = 0$ GeV^2. The full differential cross section is given by:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}Q^2\,\mathrm{d}z\,\mathrm{d}\chi/2\pi} = \frac{2\pi\alpha^2}{xQ^4}\left[1+(1-y)^2\right]\left\{\left(2x\widetilde{F}_\mathrm{T}^\gamma + \epsilon(y)\widetilde{F}_\mathrm{L}^\gamma\right) - \rho(y)\widetilde{F}_\mathrm{A}^\gamma\cos\chi + \tfrac{1}{2}\epsilon(y)\widetilde{F}_\mathrm{B}^\gamma\cos 2\chi\right\}, \tag{1}$$

where the functions $\epsilon(y)$ and $\rho(y)$ which can be found in Ref. [1], are both $1-\mathcal{O}(y^2)$:

$$\epsilon(y) = \frac{2(1-y)}{1+(1-y)^2}, \qquad \rho(y) = \frac{(2-y)\sqrt{1-y}}{1+(1-y)^2} \tag{2}$$

and the unintegrated structure functions $\widetilde{F}_\mathrm{T}^\gamma$, $\widetilde{F}_\mathrm{L}^\gamma$, $\widetilde{F}_\mathrm{A}^\gamma$ and $\widetilde{F}_\mathrm{B}^\gamma$ are functions only of x, β and z (i.e. not χ):

$$\begin{aligned}\widetilde{F}_\mathrm{T}^\gamma(x,\beta,z) &= \frac{\mathrm{e_f}^4\alpha}{2\pi}\frac{1}{2z(1-z)}\left\{\left[x^2+(1-x)^2\right]\left[z^2+(1-z)^2\right]\right.\\ &\left.+\frac{1}{2}\left(1-\beta^2\right)\frac{(1-x)\left[x(1-2z)^2+2z(1-z)\right]}{z(1-z)} - \frac{1}{4}\left(1-\beta^2\right)^2\frac{(1-x)^2}{z(1-z)}\right\} \end{aligned}\tag{3}$$

$$\widetilde{F}_{\rm L}^{\gamma}(x,\beta,z) = \frac{4{e_{\rm f}}^4\alpha}{\pi} x^2 (1-x) \left[1 - \frac{1}{4}\left(1-\beta^2\right)\frac{1}{z(1-z)}\right] \tag{4}$$

$$\widetilde{F}_{\rm A}^{\gamma}(x,\beta,z) = \frac{4{e_{\rm f}}^4\alpha}{\pi} x(1-2z)(1-2x)\left[1 - \frac{1}{2}\left(1-\beta^2\right)\frac{1-x}{z(1-z)(1-2x)}\right] \cdot \sqrt{\left[1 - \frac{1}{4}(1-\beta^2)\frac{1}{z(1-z)}\right]\frac{x(1-x)}{4z(1-z)}} \tag{5}$$

$$\widetilde{F}_{\rm B}^{\gamma}(x,\beta,z) = \frac{4{e_{\rm f}}^4\alpha}{\pi} x^2 (1-x) \cdot \left\{1 + \frac{1}{4}\left(1-\beta^2\right)\frac{1-2x}{xz(1-z)} - \frac{1}{16}\left(1-\beta^2\right)^2 \frac{1-x}{xz^2(1-z)^2}\right\} \tag{6}$$

Here $e_{\rm f}$ is the charge (in units of the electron charge) of the produced fermion. These structure functions are proportional to the cross sections for the target photon to interact with different polarisation states of the virtual photon: transverse (T), longitudinal (L), transverse–longitudinal interference (A) and interference between the two transverse polarisations (B). Other combinations of structure functions also appear in the literature, most notably $\widetilde{F}_2^{\gamma}$:

$$\widetilde{F}_2^{\gamma} \equiv 2x\widetilde{F}_{\rm T}^{\gamma} + \widetilde{F}_{\rm L}^{\gamma}. \tag{7}$$

In the above formulae, z and χ always refer to the produced fermion. However, we prefer to define χ slightly differently, as the azimuth of whichever produced particle (fermion or antifermion) has the smaller z value, see Figure 1(b). This definition leaves all the structure functions unchanged except that in $\widetilde{F}_{\rm A}^{\gamma}$, the factor of $(1-2z)$ is replaced by $|1-2z|$. The more well-known integrated structure functions are given by integrating over the kinematically allowed range in z, namely $(1-\beta)/2$ to $(1+\beta)/2$, giving:

$$F_{\rm T}^{\gamma}(x,\beta) = \frac{{e_{\rm f}}^4\alpha}{2\pi}\left\{\left[x^2+(1-x)^2\right]\log\left(\frac{1+\beta}{1-\beta}\right) - \beta + 4\beta x(1-x) + -\left(1-\beta^2\right)(1-x)^2\left(\beta - \left[1-\frac{1}{2}\left(1-\beta^2\right)\right]\log\left(\frac{1+\beta}{1-\beta}\right)\right)\right\} \tag{8}$$

$$F_{\rm L}^{\gamma}(x,\beta) = \frac{4{e_{\rm f}}^4\alpha}{\pi} x^2(1-x)\left[\beta - \frac{1}{2}\left(1-\beta^2\right)\log\left(\frac{1+\beta}{1-\beta}\right)\right] \tag{9}$$

$$F_{\rm A}^{\gamma}(x,\beta) = \frac{4{e_{\rm f}}^4\alpha}{\pi} x\sqrt{x(1-x)}(1-2x)\left\{\beta\left[1+\left(1-\beta^2\right)\frac{1-x}{1-2x}\right] + \frac{3x-2}{1-2x}\sqrt{1-\beta^2}\arccos\left(\sqrt{1-\beta^2}\right)\right\}, \tag{10}$$

$$F_{\rm B}^{\gamma}(x,\beta) = \frac{4{e_{\rm f}}^4\alpha}{\pi} x^2(1-x)\left\{\beta\left[1-\left(1-\beta^2\right)\frac{1-x}{2x}\right] + \frac{1}{2}\left(1-\beta^2\right)\left[\frac{1-2x}{x} - \frac{1-x}{2x}\left(1-\beta^2\right)\right]\log\left(\frac{1+\beta}{1-\beta}\right)\right\} \tag{11}$$

$$
\begin{aligned}
F_2^\gamma(x,\beta) &= \frac{e_f{}^4\alpha}{\pi}x\left\{\left[x^2+(1-x)^2\right]\log\left(\frac{1+\beta}{1-\beta}\right)-\beta+8\beta x\,(1-x)\right.\\
&-\beta\left(1-\beta^2\right)(1-x)^2\\
&\left.+\left(1-\beta^2\right)(1-x)\left[\frac{1}{2}(1-x)\left(1+\beta^2\right)-2x\right]\log\left(\frac{1+\beta}{1-\beta}\right)\right\}
\end{aligned}
\tag{12}
$$

Note that if we had not redefined χ, F_A would have been zero. In most previous analyses, for example Ref. [2], the small mass limit has been taken, neglecting terms of order m_f/W and higher leading to:

$$F_T^\gamma\left(x,\frac{m_f^2}{W^2}\right) = \frac{e_f{}^4\alpha}{2\pi}\left\{\left[x^2+(1-x)^2\right]\log\frac{W^2}{m_f^2}-1+4x\,(1-x)\right\}+\mathcal{O}\left(\frac{m_f^2}{W^2}\right) \tag{13}$$

$$F_L^\gamma\left(x,\frac{m_f^2}{W^2}\right) = \frac{4e_f{}^4\alpha}{\pi}\left\{x^2\,(1-x)\right\}+\mathcal{O}\left(\frac{m_f^2}{W^2}\right) \tag{14}$$

$$F_A^\gamma\left(x,\frac{m_f^2}{W^2}\right) = \frac{4e_f{}^4\alpha}{\pi}\left\{x\,(1-2x)\sqrt{x\,(1-x)}\right\}+\mathcal{O}\left(\frac{m_f}{W}\right) \tag{15}$$

$$F_B^\gamma\left(x,\frac{m_f^2}{W^2}\right) = \frac{4e_f{}^4\alpha}{\pi}\left\{x^2\,(1-x)\right\}+\mathcal{O}\left(\frac{m_f^2}{W^2}\right) \tag{16}$$

$$F_2^\gamma\left(x,\frac{m_f^2}{W^2}\right) = \frac{e_f{}^4\alpha}{\pi}x\left\{\left[x^2+(1-x)^2\right]\log\frac{W^2}{m_f^2}-1+8x(1-x)\right\}+\mathcal{O}\left(\frac{m_f^2}{W^2}\right) \tag{17}$$

Note however that F_A has parametrically larger mass corrections than the other structure functions*.

The recent experimental analysis of F_A^γ and F_B^γ from Refs. [3,4] are based mainly on data in the approximate range in Q^2 from 1.5 GeV^2 to 10 GeV^2. In Figure 2 we show results for the four structure functions for muon final states at $Q^2 = 1$ GeV^2, where the mass corrections are extremely important. Even at $Q^2 = 5.4$ GeV^2, Figure 3, the mass corrections for F_A are sizeable but at $Q^2 = 100$ GeV^2 they are small, Figure 4.

*Note also a common source of confusion: if one cuts off the z integration by introducing a cutoff in transverse momentum $p_{t,\min}$ one obtains instead of (17),

$$F_2 = \frac{e_f{}^4\alpha}{\pi}x\left\{\left[x^2+(1-x)^2\right]\log\frac{W^2}{m_f^2+p_{t,\min}^2}-1+\left(6+2\frac{m_f^2}{m_f^2+p_{t,\min}^2}\right)x(1-x)\right\}$$

and hence in the massless limit an identical form to (17) but with $8x(1-x)$ replaced by $6x(1-x)$,

$$F_2 = \frac{e_f{}^4\alpha}{\pi}x\left\{\left[x^2+(1-x)^2\right]\log\frac{W^2}{p_{t,\min}^2}-1+6x(1-x)\right\}$$

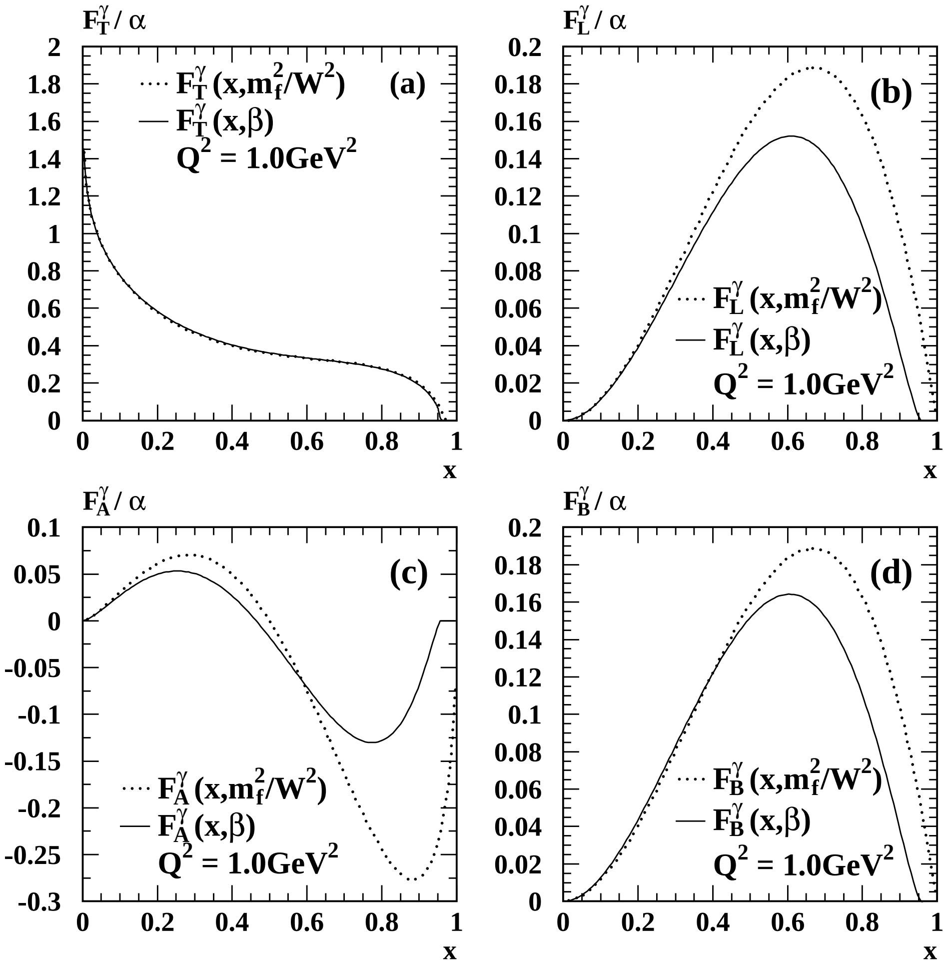

Figure 2: The structure functions for $Q^2 = 1$ GeV^2 with the full mass dependence, $F_i^\gamma(x,\beta)$, and in the small-mass limit, $F_i^\gamma(x, m_\mathrm{f}^2/W^2)$.

References

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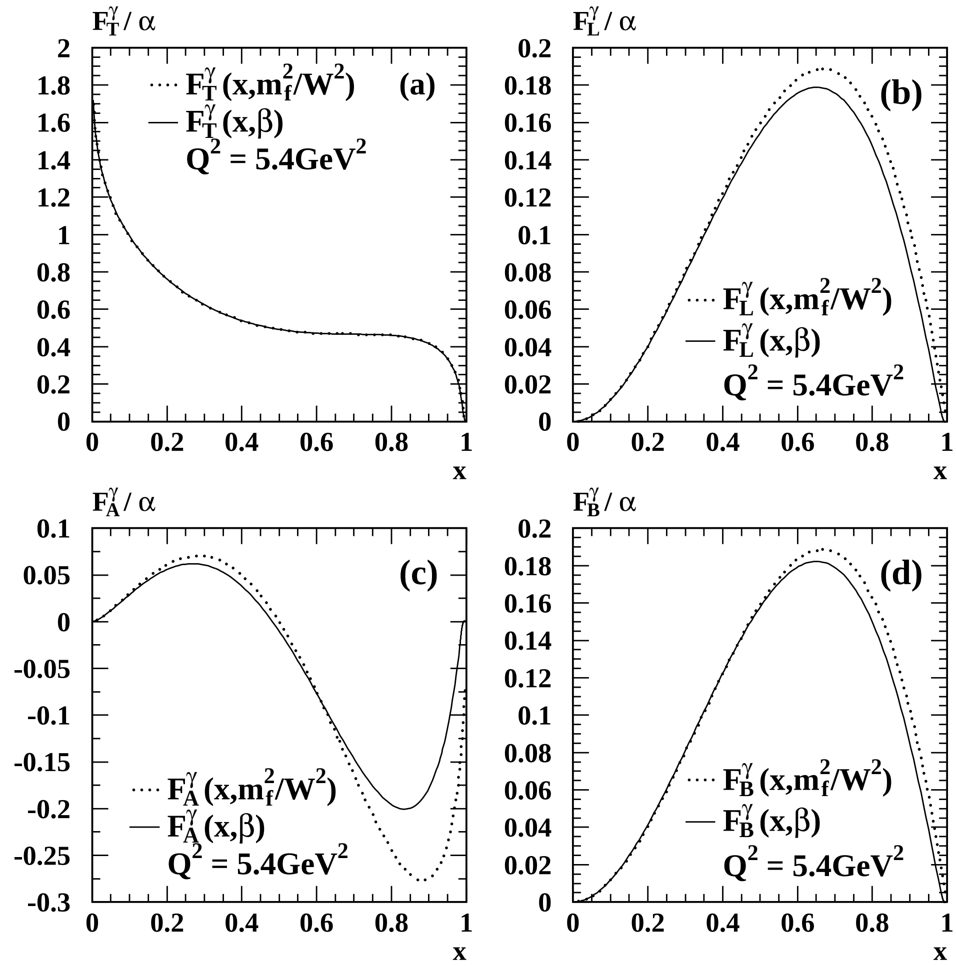

Figure 3: The structure functions for $Q^2 = 5.4$ GeV^2 with the full mass dependence, $F_i^\gamma(x,\beta)$, and in the small-mass limit, $F_i^\gamma(x,m_{\rm f}^2/W^2)$.

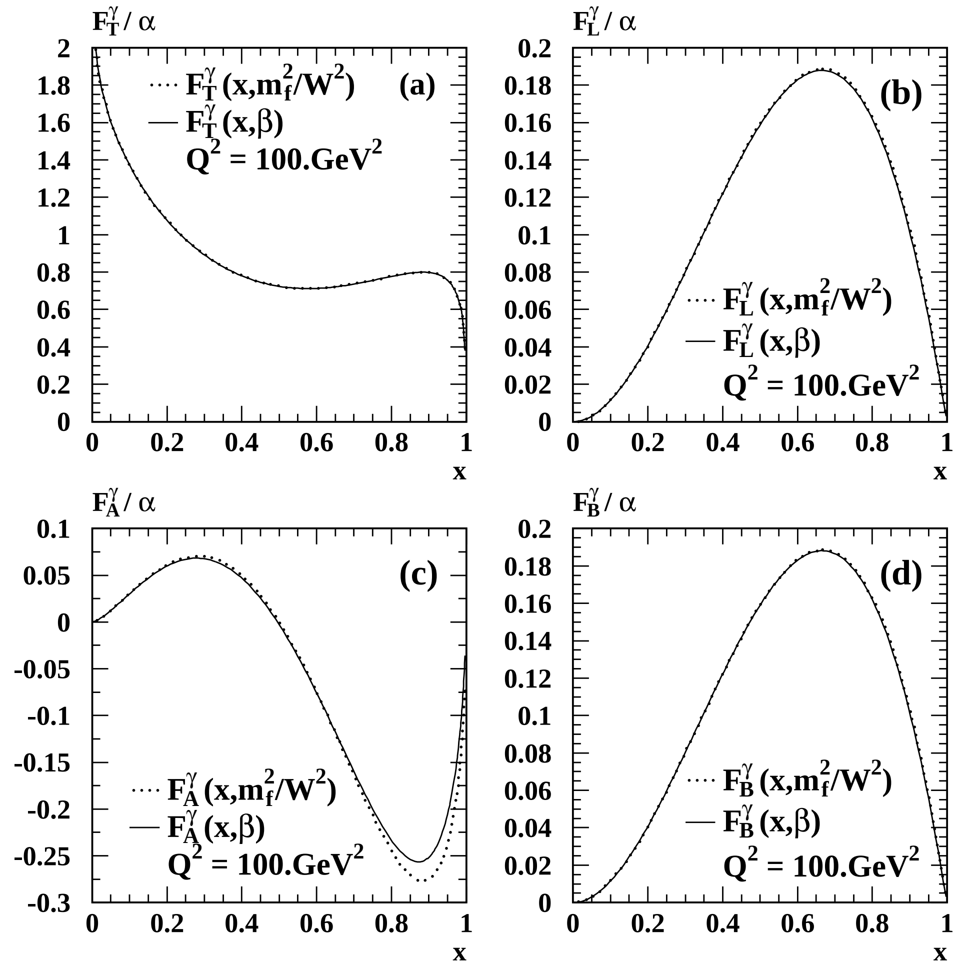

Figure 4: The structure functions for $Q^2 = 100$ GeV^2 with the full mass dependence, $F_i^\gamma(x,\beta)$, and in the small-mass limit, $F_i^\gamma(x, m_{\rm f}^2/W^2)$.