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Some Noise Calculations for time invariant filters.

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This paper presents the noise calculations for some detector readout circuits following the method outlined in Ref 1 and Ref 2. The results are summarised in Ref 3. The circuits considered are a charge amplifier combined with the following filters :-

RC
RC-CR
$CR\text{-}RC^2$
$CR\text{-}RC^n$
$CR^2\text{-}RC$ (bipolar)
Indefinite Cusp
Trapezoidal
CR followed by Correlated Double Sampling
Triple Sampled Deconvolution.

The voltage-to-voltage transfer functions for the filters are shown in Appendix A.. The calculations of the thermal, shot and flicker noise for each filter are given in Appendix B.. The calculations are performed in the time domain and then repeated in the frequency domain. The calculation of flicker noise is not done in the time domain.
The current-to-voltage transfer function and impulse response from the noise current source in the front end device of the charge amplifier to the output, are given by

$$H_r(s) = \frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)} H_v(s)$$

$$h_r(t_{m1} - t_p) = \frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)} h_v(t_{m1} - t_p)$$

respectively. The voltage-to-voltage transfer function of the filters are given by $H_v(s)$. The voltage-to-voltage impulse response of the filters are given by $h_v(t_{m1}\text{-}t_p)$.
The current-to-voltage transfer function and impulse response from the noise current source of the detector leakage current to the output, are given by

$$H_i(s) = \frac{A}{\left(c_{tot} + A\, c_f\right)} G_v(s)$$

$$h_i(t_{m1} - t_p) = \frac{A}{\left(c_{tot} + A\, c_f\right)} g_v(t_{m1} - t_p)$$

respectively. The voltage-to-voltage step response function of the filters are given by $G_v(s)$. The voltage-to-voltage step response of the filters are given by $g_v(t_{m1}\text{-}t_p)$.

References.

[1] P. Seller, Noise analysis in linear electronic circuits. Nucl. Inst. and Meth. A 376 (1996) 229-241.
[2] P. Seller, Erratum to "Noise analysis in linear electronic circuits". Nucl. Inst. and Meth. A 408 (1998) 603-604.
[3] P. Seller, Summary of thermal, shot and flicker noise in detectors and readout circuits. Nucl. Inst. and Meth. (1996) To be published.

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A. The voltage-to-voltage responses.

RC $H_v(s)=\dfrac{1}{1+s\tau}$

$$h_v\left(t_{m1}-t_p\right)=\frac{1}{\tau}e^{-\frac{\left(t_{m1}-t_p\right)}{\tau}}$$

$$G_v(s)=\frac{1}{s(1+s\tau)}$$

$$g_v\left(t_{m1}-t_p\right)=1-e^{-\frac{\left(t_{m1}-t_p\right)}{\tau}}$$ peak value $=1$ at $\left(t_{m1}-t_p\right)=\infty$

CR $H_v(s)=\dfrac{s\tau}{1+s\tau}$

$$h_v\left(t_{m1}-t_p\right)=\delta(0)-\frac{1}{\tau}e^{-\frac{\left(t_{m1}-t_p\right)}{\tau}}$$

$$G_v(s)=\frac{\tau}{1+s\tau}$$

$$g_v\left(t_{m1}-t_p\right)=e^{-\frac{\left(t_{m1}-t_p\right)}{\tau}}$$ peak value $=$ 1 at $\left(t_{m1}-t_p\right)=0$

CR - RC $H_v(s)=\dfrac{s\tau}{(1+s\tau)^2}$

$$h_v\left(t_{m1}t_p\right)=\left(\frac{1}{\tau}-\frac{\left(t_{m1}-t_p\right)}{(\tau)^2}\right)e^{-\frac{\left(t_{m1}-t_p\right)}{\tau}}$$

$$G_v(s)=\frac{\tau}{(1+s\tau)^2}$$

$$g_v\left(t_{m1}-t_p\right)=\frac{\left(t_{m1}-t_p\right)}{\tau}e^{-\frac{\left(t_{m1}-t_p\right)}{\tau}}$$ peak value $=\dfrac{1}{e}$ at $\left(t_{m1}-t_p\right)=\tau$

$CR-RC^2$ $H_v(s)=\dfrac{s\tau}{(1+s\tau)^3}$

$$h_v(t)=\left(\frac{t_{m1}-t_p}{(\tau)^2}-\frac{\left(t_{m1}-t_p\right)^2}{2(\tau)^3}\right)e^{-\frac{\left(t_{m1}-t_p\right)}{\tau}}$$

$$G_v(s)=\frac{\tau}{(1+s\tau)^3}$$

$$g_v(t)=\frac{\left(t_{m1}-t_p\right)^2}{2(\tau)^2}e^{-\frac{\left(t_{m1}-t_p\right)}{\tau}}$$ peak value $=\dfrac{2}{e^2}$ at $\left(t_{m1}-t_p\right)=2\tau$

A. The voltage-to-voltage responses.

$CR - RC^n \quad H_v(s) = \frac{s\tau}{(1+s\tau)^{n+1}}$

$$h_v(t) = \left(\frac{\left(t_{m1} - t_p\right)^{n-1}}{(\tau)^n (n-1)!} - \frac{\left(t_{m1} - t_p\right)^n}{(\tau)^{n+1} n!} \right) e^{-\frac{(t_{m1}-t_p)}{\tau}}$$

$$G_v(s) = \frac{\tau}{(1+s\tau)^{n+1}}$$

$$g_v\left(t_{m1} - t_p\right) = \frac{\left(t_{m1} - t_p\right)^n}{(\tau)^n n!} e^{-\frac{(t_{m1}-t_p)}{\tau}}$$

peak value $= \frac{n^n}{n! e^n}$ at $\left(t_{m1} - t_p\right) = n\tau$

$CR^2 - RC \quad H_v(s) = \frac{(s\tau)^2}{(1+s\tau)^3}$

$$h_v\left(t_{m1} - t_p\right) = \left(\frac{1}{\tau}\right) - \frac{2\left(t_{m1} - t_p\right)}{(\tau)^2} + \left(\frac{\left(t_{m1} - t_p\right)^2}{2(\tau)^3}\right) e^{-\frac{(t_{m1}-t_p)}{\tau}}$$

$$G_v(s) = \frac{s(\tau)^2}{(1+s\tau)^3}$$

$$g_v\left(t_{m1} - t_p\right) = \left(\frac{\left(t_{m1} - t_p\right)}{(\tau)} - \frac{\left(t_{m1} - t_p\right)^2}{2(\tau)^2} \right) e^{-\frac{(t_{m1}-t_p)}{\tau}}$$

peak at $\left(t_{m1} - t_p\right) = \left(2 \pm \sqrt{2}\right)\tau$ crosses zero at $\left(t_{m1} - t_p\right) = 2\tau$

positive peak $= \left(-1 + \sqrt{2}\right) e^{-2+\sqrt{2}} = 0.23 = \frac{1}{4.3}$ negative peak $= -0.079$

A. The voltage-to-voltage responses.

Cusp $\qquad H_v(jw)=\dfrac{2w\tau}{1+(w\tau)^2}$

For time invariant filter peaking at $t_{m1}-t_s$ (t_s will tend to ∞ later in noise calculations)

$$h_v\left(t_{m1}-t_p\right)=\frac{1}{\tau}\left(1-e^{-\frac{t_s}{\tau}}\right)e^{-\frac{\left(t_{m1}-\left(t_p+t_s\right)\right)}{\tau}} \qquad t_0<t_p<t_{m1}-t_s$$

$$h_v\left(t_{m1}-t_p\right)=-\frac{1}{\tau}\left(e^{\frac{\left(t_p+t_s-t_{m1}\right)}{\tau}}-e^{-\frac{t_s}{\tau}}\right) \qquad t_{m1}-t_s<t_p<t_{m1}$$

$$G_v(jw)=\frac{2\tau}{1+(w\tau)^2}$$

$$g_v\left(t_{m1}-t_p\right)=\left(1-e^{-\frac{t_s}{\tau}}\right)e^{-\frac{\left(t_{m1}-\left(t_p+t_s\right)\right)}{\tau}} \qquad t_0<t_p<t_{m1}-t_s$$

$$g_v\left(t_{m1}-t_p\right)=\left(e^{\frac{\left(t_p+t_s-t_{m1}\right)}{\tau}}-e^{-\frac{t_s}{\tau}}\right) \qquad t_{m1}-t_s<t_p<t_{m1}$$

peak value $=\left(1-e^{-t_s}\right)$ at $t_p=t_{m1}-t_s$

Trapezoidal $\quad |H(jw)|=\dfrac{4}{wt_{rise}}\sin\dfrac{wt_{rise}}{2}\sin\left(\dfrac{t_{rise}+t_{flat}}{2}\right) \qquad \left(\text{if } t_{rise}=t_{fall}\right)$

$$h_v\left(t_{m1}-t_p\right)=0 \qquad t_0<t_p<t_{m1}-\left(t_{rise}+t_{flat}+t_{fall}\right)$$

$$h_v\left(t_{m1}-t_p\right)=\frac{1}{t_{fall}} \qquad t_{m1}-\left(t_{rise}+t_{flat}+t_{fall}\right)<t_p<t_{m1}-\left(t_{rise}+t_{flat}\right)$$

$$h_v\left(t_{m1}-t_p\right)=0 \qquad t_{m1}-\left(t_{rise}+t_{flat}\right)<t_p<t_{m1}-t_{rise}$$

$$h_v\left(t_{m1}-t_p\right)=\frac{-1}{t_{rise}} \qquad t_{m1}-t_{rise}<t_p<t_{m1}$$

$$|G(jw)|=\frac{4}{w^2t_{rise}}\sin\frac{wt_{rise}}{2}\sin\left(\frac{t_{rise}+t_{flat}}{2}\right) \qquad \left(\text{if } t_{rise}=t_{fall}\right)$$

$$g_v\left(t_{m1}-t_p\right)=0 \qquad t_0<t_p<t_{m1}-\left(t_{rise}+t_{flat}+t_{fall}\right)$$

$$g_v\left(t_{m1}-t_p\right)=\frac{\left(t_p+t_{rise}+t_{flat}+t_{fall}\right)-t_{m1}}{t_{fall}} \qquad t_{m1}-\left(t_{rise}+t_{flat}+t_{fall}\right)<t_p<t_{m1}-\left(t_{rise}+t_{flat}\right)$$

$$g_v\left(t_{m1}-t_p\right)=1 \qquad t_{m1}-\left(t_{rise}+t_{flat}\right)<t_p<t_{m1}-t_{rise}$$

$$g_v\left(t_{m1}-t_p\right)=\frac{t_{m1}-t_p}{t_{rise}} \qquad t_{m1}-t_{rise}<t_p<t_{m1}$$

peak value $=1$

A. The voltage-to-voltage responses.

RC followed by Correlated Double Sampler $\left(\text{samples spaced at } \Delta = t_{m2} - t_{m1}\right)$

$$b_1 = -1,\ b_2 = 1$$

$$H_v(s) = \frac{1}{1+s\tau}\left(1-e^{-s\Delta}\right) \qquad \left|H_v(jw)\right|^2 = \frac{4\sin^2\left(\frac{w\Delta}{2}\right)}{\left(1+w^2\tau^2\right)} = \frac{2(1-\cos w\Delta)}{\left(1+w^2\tau^2\right)}$$

$$G_v(s) = \frac{1}{s(1+s\tau)}\left(1-e^{-s\Delta}\right) \quad \left|H_v(jw)\right|^2 = \frac{4\sin^2\left(\frac{w\Delta}{2}\right)}{w^2\left(1+w^2\tau^2\right)} = \frac{2(1-\cos w\Delta)}{w^2\left(1+w^2\tau^2\right)}$$

if input signal is at t_{m1} then the output grows as $\left(1-e^{-\frac{t_{m1}}{\tau}}\right)$ and peak value $=1$

RC - CR - Triple sample deconvolution. Samples spaced at Δ and second sample on start of signal.

$$b_1 = \frac{\tau}{\Delta}e^{-\frac{\tau+\Delta}{\tau}}, b_2 = -\frac{2\tau}{\Delta}e^{-\frac{\tau}{\tau}}, b_3 = \frac{\tau}{\Delta}e^{-\frac{\tau-\Delta}{\tau}}$$

$$\left|H_v(jw)\right|^2 = \frac{w^2\tau^2}{\left(1+w^2\tau^2\right)^2}\left(\frac{\tau}{e\Delta}\right)^2\left(e^{\left(\frac{\Delta}{\tau}\right)} + e^{-\left(\frac{\Delta}{\tau}\right)} - 2\cos w\Delta\right)^2$$

$$\text{signal gain} = 0b_1 + 0b_2 + \frac{\Delta}{\tau}e^{-\left(\frac{\Delta}{\tau}\right)}b_3 = e^{-1}$$

Zero ohm resistor

$$H_v(s) = 1 \qquad h_v\left(t_{m1}-t_p\right) = \delta\left(t_{m1}-t_p\right)$$

$$G_v(s) = \frac{1}{s} \qquad g_v\left(t_{m1}-t_p\right) = u\left(t_{m1}-t_p\right)$$

B. Thermal noise through a charge amp and RC filter in time domain.

$$\sigma_{Vout}^2(t_{01}) = \int_{t=t_0}^{t=t_{m1}} \pi Q_t h_r^{\ 2}(t_m - t_p) dt_p$$

$$h_r(t_m - t_p) = \left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)} \right) \frac{1}{\tau} e^{-\frac{t_m - t_p}{\tau}}$$

$$\sigma_{Vout}^2(t_{01}) = \int_{t_p=t_0}^{t_p=t_{m1}} \pi Q_t \left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)} \right)^2 \frac{1}{\tau^2} e^{-\frac{2(t_{m1} - t_p)}{\tau}} dt_p$$

$$\sigma_{Vout}^2(t_{01}) = \pi Q_t \left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)} \right)^2 \frac{1}{\tau^2} \frac{\tau}{2} \left[e^{-\frac{2(t_{m1} - t_p)}{\tau}} \right]_{t_0}^{t_{m1}}$$

$$\sigma_{Vout}^2(t_{01}) = \frac{\pi Q_t}{2\tau} \left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)} \right)^2 \left(1 - e^{-\frac{2(t_{m1} - t_0)}{\tau}} \right)$$

$$\sigma_{Vout}^2(t_\infty) = \frac{kTr}{\tau} \left(\frac{Ac_{tot}}{c_{tot} + Ac_f} \right)^2$$

B. Thermal noise through a charge amp and RC filter in the frequency domain.

$$\sigma_{Vout}^2(t_\infty) = \int_0^\infty Q_t \left|H_r(jw)\right|^2 dw$$

$$H_r(s) = \frac{Ac_{tot}}{gm\left(c_{tot} + Ac_f\right)} \frac{1}{(1+s\tau)}$$

$$\sigma_{Vout}^2(t_\infty) = \int_0^\infty Q_t \left(\frac{Ac_{tot}}{gm\left(c_{tot} + Ac_f\right)}\right)^2 \frac{1}{1+(w\tau)^2} dw$$

$$\sigma_{Vout}^2(t_\infty) = Q_t \left(\frac{Ac_{tot}}{gm\left(c_{tot} + Ac_f\right)}\right)^2 \frac{1}{\tau}\left[\tan^{-1}(w\tau)\right]_0^\infty$$

$$\sigma_{Vout}^2(t_\infty) = \left(\frac{Ac_{tot}}{gm\left(c_{tot} + Ac_f\right)}\right)^2 \frac{Q_t}{\tau}\frac{\pi}{2}$$

$$\sigma_{Vout}^2(t_\infty) = \left(\frac{Ac_{tot}}{\left(c_{tot} + Ac_f\right)}\right)^2 \frac{kTr}{\tau}$$

B. Shot noise through a charge amp and RC filter in the time domain.

$$\sigma^2_{Vout}(t_{01}) = \int_{t_p=t_0}^{t_p=t_{m1}} \pi Q_s h_i^2(t_m - t_p)dt$$

$$h_i(t_m - t_p) = \frac{A}{c_{tot} + A\,c_f}\left(1 - e^{-\frac{t_m - t_p}{\tau}}\right)$$

$$\sigma^2_{Vout}(t_{01}) = \int_{t_p=t_0}^{t_p=t_{m1}} \pi Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \left(1 - e^{-\frac{t_{m1} - t_p}{\tau}}\right)^2 dt_p$$

$$\sigma^2_{Vout}(t_{01}) = \pi Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \int_{t_p=t_0}^{t_p=t_{m1}} \left(1 - 2e^{-\frac{(t_{m1} - t_p)}{\tau}} + e^{-\frac{2(t_{m1} - t_p)}{\tau}}\right) dt_p$$

$$\sigma^2_{Vout}(t_{01}) = \pi Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \left[t_p - 2\tau e^{-\frac{(t_{m1} - t_p)}{\tau}} + \frac{\tau}{2} e^{-\frac{2(t_{m1} - t_p)}{\tau}}\right]_{t_0}^{t_{m1}}$$

$$\sigma^2_{Vout}(t_{01}) = \pi Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \left(t_{m1} - 2\tau + \frac{\tau}{2} - t_0 + 2\tau e^{-\frac{t_{m1} - t_0}{\tau}} - \frac{\tau}{2} e^{-\frac{2(t_{m1} - t_0)}{\tau}}\right)$$

calling $t_{01} = t_{m1} - t_0$

$$\sigma^2_{Vout}(t_{01}) = \pi Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \left(t_{01} - \frac{3\tau}{2} + 2\tau^{-\frac{t_{01}}{\tau}} - \frac{\tau}{2} e^{-\frac{2(t_{01})}{\tau}}\right)$$

If t_{01} is large then this tends to

$$\sigma^2_{Vout}(t_{01}) = iq \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 (t_{01})$$

B. Flicker noise through a charge amp and RC filter in the frequency domain.

$$\sigma_{Vout}^2(t_\infty) = \int_0^\infty \frac{Q_f}{w} \left|H_r(jw)\right|^2 dw$$

$$H_r(s) = \frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)} \frac{1}{(1+s\tau)}$$

$$\sigma_{Vout}^2(t_\infty) = \int_0^\infty \frac{Q_f}{w} \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)}\right)^2 \frac{1}{1+(w\tau)^2} dw$$

$$\sigma_{Vout}^2(t_\infty) = Q_f \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)}\right)^2 \int_0^\infty \frac{1}{w\left(1+(w\tau)^2\right)} dw$$

$$\sigma_{Vout}^2(t_\infty) = Q_f \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)}\right)^2 \left[\frac{1}{2}\log\left(\frac{w^2}{1+(w\tau)^2}\right)\right]_0^\infty$$

$$\sigma_{Vout}^2(t_\infty) = Q_f \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)}\right)^2 \left[\frac{1}{2}\log\left(\frac{w^2}{1+(w\tau)^2}\right)\right]_0^\infty$$

for a FET

$$\sigma_{Vout}^2(t_\infty) = \frac{Kf_2}{WLC_{ox}} \left(\frac{A\ c_{tot}}{\left(c_{tot} + A\ c_f\right)}\right)^2 \left[\frac{1}{2}\log\left(\frac{w^2}{1+(w\tau)^2}\right)\right]_0^\infty$$

So at infinite time after switch-on the noise is infinite.

B. Thermal noise through a charge amp and CR - RC filter in the time domain.

$$\sigma_{Vout}^2(t_{01}) = \int_{t=0}^{t=t_{01}} \pi Q_t h_r^{\,2}(t)dt$$

$$h_r(t_{m1}-t_p) = \frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\left(\frac{1}{(\tau)} - \frac{t_{m1}-t_p}{(\tau)^2}\right)e^{-\frac{(t_{m1}-t_p)}{\tau}}$$

$$\sigma_{Vout}^2(t_{01}) = \pi Q_t\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)^2 \int_0^{t_{01}}\left(\frac{1}{\tau}-\frac{t}{\tau^2}\right)^2 e^{-\frac{2t}{\tau}}dt$$

$$\sigma_{Vout}^2(t_{01}) = \pi Q_t\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)^2 \int_0^{t_{01}}\left(\frac{1}{\tau^2}-\frac{2t}{\tau^3}+\frac{t^2}{\tau^4}\right)e^{-\frac{2t}{\tau}}dt$$

$$\sigma_{Vout}^2(t_{01}) = \pi Q_t\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)^2 \left[\left(\frac{-1}{2\tau}-2\left(-\frac{t\tau}{2}-\frac{\tau^2}{4}\right)\frac{1}{\tau^3}+\left(-\frac{t^2\tau}{2}-\frac{t\tau^2}{2}-\frac{\tau^3}{4}\right)\frac{1}{\tau^4}\right)e^{-\frac{2t}{\tau}}\right]_0^{t_{01}}$$

$$\sigma_{Vout}^2(t_{01}) = \pi Q_t\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)^2 \left[\left(\frac{-1}{2\tau}+\frac{t}{\tau^2}+\frac{1}{2\tau}-\frac{t^2}{2\tau^3}-\frac{t}{2\tau^2}-\frac{1}{4\tau}\right)e^{-\frac{2t}{\tau}}\right]_0^{t_{01}}$$

$$\sigma_{Vout}^2(t_{01}) = \pi Q_t\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)^2 \left[\left(\frac{t}{2\tau^2}-\frac{t^2}{2\tau^3}-\frac{1}{4\tau}\right)e^{-\frac{2t}{\tau}}\right]_0^{t_{01}}$$

$$\sigma_{Vout}^2(t_{01}) = \pi Q_t\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)^2 \left(\left(\frac{-1}{4\tau}+\frac{t_{01}}{2\tau^2}-\frac{t_{01}^{\ 2}}{2\tau^3}\right)e^{-\frac{2t_{01}}{\tau}}+\frac{1}{4\tau}\right)$$

$$\sigma_{Vout}^2(t_{\infty}) = 2kTr\left(\frac{A\ c_{tot}}{(c_{tot}+A\ c_f)}\right)^2 \frac{1}{4\tau}$$

B. Thermal noise through a charge amp and CR - RC filter in the frequency domain.

$$\sigma^2_{Vout}(t_\infty) = \int_0^\infty Q_t \left|H_r(jw)\right|^2 dw$$

$$H_r(s) = \frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)} \frac{s\tau}{(1+s\tau)^2}$$

$$\sigma^2_{Vout}(t_\infty) = \int_0^\infty Q_t \left|H_r(jw)\right|^2 dw$$

$$\sigma^2_{Vout}(t_\infty) = \int_0^\infty Q_t \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)}\right)^2 \frac{(w\tau)^2}{\left(1+(w\tau)^2\right)^2} dw$$

$$\sigma^2_{Vout}(t_\infty) = Q_t \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)}\right)^2 \tau^2 \left(\left[\frac{-w}{2\tau^2\left(1+(w\tau)^2\right)}\right]_0^\infty + \frac{1}{2\tau^2}\int_0^\infty \frac{dw}{1+(w\tau)^2}\right)$$

$$\sigma^2_{Vout}(t_\infty) = Q_t \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)}\right)^2 \tau^2 \left[\frac{1}{2(\tau)^3}\tan^{-1}(w\tau)\right]_0^\infty$$

$$\sigma^2_{Vout}(t_\infty) = Q_t \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)}\right)^2 \frac{\pi}{4\tau}$$

$$\sigma^2_{Vout}(t_\infty) = kTr \left(\frac{A\ c_{tot}}{\left(c_{tot} + A\ c_f\right)}\right)^2 \frac{1}{2\tau}$$

B. Shot noise through a charge amplifier and a CR-RC filter in the time domain.

$$\sigma^2_{Vout}(t_{01}) = \int_{t=0}^{t=t_{01}} \pi Q_s h_i^2(t)dt$$

$$h_i(t_{m1} - t_p) = \frac{A}{c_{tot} + A\,c_f}\frac{(t_{m1} - t_p)}{\tau}e^{-\frac{(t_{m1}-t_p)}{\tau}}$$

$$\sigma^2_{Vout}(t_{01}) = \pi Q_s\left(\frac{A}{c_{tot} + A\,c_f}\right)^2\frac{1}{\tau^2}\int_0^{t_{01}} t^2 e^{-\frac{2t}{\tau}}dt$$

$$\sigma^2_{Vout}(t_{01}) = \pi Q_s\left(\frac{A}{c_{tot} + A\,c_f}\right)^2\frac{1}{\tau^2}\left[e^{-\frac{2t}{\tau}}\left(\frac{-t^2\tau}{2} - \frac{\tau^2 t}{2} - \frac{2\tau^3}{8}\right)\right]_0^{t_{01}}$$

$$\sigma^2_{Vout}(t_{01}) = \pi Q_s\left(\frac{A}{c_{tot} + A\,c_f}\right)^2\frac{1}{\tau^2}\left(e^{-\frac{2t_{01}}{\tau}}\left(\frac{-t_{01}{}^2\tau}{2} - \frac{\tau^2 t_{01}}{2} - \frac{2\tau^3}{8}\right) + \frac{2\tau^3}{8}\right)$$

$$\sigma^2_{Vout}(t_\infty) = iq\left(\frac{A}{c_{tot} + A\,c_f}\right)^2\frac{\tau}{4}$$

B. Shot noise through a charge amplifier and a CR-RC filter in the frequency domain.

$$\sigma_{Vout}^2(t_\infty) = \int_0^\infty Q_s \left|H_i(jw)\right|^2 dw$$

$$H_i(s) = \frac{A}{c_{tot} + A\,c_f}\frac{\tau}{(1+s\tau)^2}$$

$$\sigma_{Vout}^2(t_\infty) = \int_0^\infty Q_s\left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{\tau^2}{\left(1+(w\tau)^2\right)^2} dw$$

$$\sigma_{Vout}^2(t_\infty) = Q_s\left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \tau^2\left(\left[\frac{w}{2\left(1+(w\tau)^2\right)}\right]_0^\infty + \frac{1}{2}\int_0^\infty \frac{dw}{1+(w\tau)^2}\right)$$

$$\sigma_{Vout}^2(t_\infty) = Q_s\left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \tau^2\left[\frac{w}{2\left(1+(w\tau)^2\right)^2} + \frac{1}{2\tau}\tan^{-1} w\tau\right]_0^\infty$$

$$\sigma_{Vout}^2(t_\infty) = Q_s\left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \tau^2 \frac{1}{2\tau}\frac{\pi}{2}$$

$$\sigma_{Vout}^2(t_\infty) = iq\left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{\tau}{4}$$

B. Flicker noise through a charge amp and a CR-RC filter in the frequency domain.

$$\sigma_{Vout}^2(t_\infty) = \int_0^\infty \frac{Q_f}{w} \left| H_r(jw) \right|^2 dw$$

$$H_r(s) = \frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)} \frac{s\tau}{(1+s\tau)^2}$$

$$\sigma_{Vout}^2(t_\infty) = \int_0^\infty \frac{Q_f}{w} \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)} \right)^2 \frac{(w\tau)^2}{\left(1+(w\tau)^2\right)^2} dw$$

$$\sigma_{Vout}^2(t_\infty) = Q_f \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)} \right)^2 \int_0^\infty \frac{w\tau^2}{\left(1+(w\tau)^2\right)^2} dw$$

$$\sigma_{Vout}^2(t_\infty) = Q_f \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)} \right)^2 \left[\frac{-1}{2\left(1+(w\tau)^2\right)} \right]_0^\infty$$

$$\sigma_{Vout}^2(t_\infty) = Q_f \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)} \right)^2 \frac{1}{2}$$

for a FET

$$\sigma_{Vout}^2(t_\infty) = \frac{Kf_2}{WLC_{ox}} \left(\frac{A\ c_{tot}}{\left(c_{tot} + A\ c_f\right)} \right)^2 \frac{1}{2}$$

B. Thermal noise through a charge amp and CR-RC^{2}filter in the time domain.

$$\sigma^2_{Vout}(t_\infty) = \int_{t=0}^{t=\infty} Q_t h_r^2(t)dt$$

$$h_r(t) = \left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)}\right)\left[\frac{t}{\tau^2} - \frac{t^2}{2\tau^3}\right]e^{-\frac{t}{\tau}}$$

$$\sigma^2_{Vout}(t_\infty) = Q_t\left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)}\right)^2 \int_0^\infty \left[\frac{t}{\tau^2} - \frac{t^2}{2\tau^3}\right]^2 e^{-\frac{2t}{\tau}}dt$$

$$\sigma^2_{Vout}(t_\infty) = Q_t\left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)}\right)^2 \int_0^\infty \left(\frac{t^2}{\tau^4} - \frac{t^3}{2\tau^5} + \frac{t^4}{4\tau^6}\right)e^{-\frac{2t}{\tau}}dt$$

ignoring the terms integrated to give $t^n e^{-\frac{2t}{\tau}}$ as these will give zero in these limits

$$\sigma^2_{Vout}(t_\infty) = Q_t\left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)}\right)^2 \left[\left(\left(-\frac{\tau^3}{4}\right)\frac{1}{\tau^4} - \left(-\frac{3\tau^4}{4}\right)\frac{1}{2\tau^5} + \left(-\frac{3\tau^5}{4}\right)\frac{1}{4\tau^6}\right)e^{-\frac{2t}{\tau}} + ..\text{ignored}\right]_0^\infty$$

$$\sigma^2_{Vout}(t_\infty) = Q_t\left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)}\right)^2 \left(\frac{1}{4\tau} - \frac{3}{8\tau} + \frac{3}{16\tau}\right)$$

$$\sigma^2_{Vout}(t_\infty) = kTr\left(\frac{A\ c_{tot}}{(c_{tot} + A\ c_f)}\right)^2 \frac{1}{8\tau}$$

B. Thermal noise through a charge amp and CR-RC^2 filter in the frequency domain.

$$\sigma_{Vout}^2(t_\infty) = \int_0^\infty Q_t \left|H_r(jw)\right|^2 dw$$

$$H_r(s) = \frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)} \frac{s\tau}{(1+s\tau)^3}$$

$$\sigma_{Vout}^2(t_\infty) = \int_0^\infty Q_t \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)}\right)^2 \frac{(w\tau)^2}{\left(1+(w\tau)^2\right)^3} dw$$

$$\sigma_{Vout}^2(t_\infty) = Q_t \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)}\right)^2 \tau^2 \left(\left[\frac{-w}{4\tau^2\left(1+(w\tau)^2\right)^2}\right]_0^\infty + \frac{1}{4\tau^2}\int_0^\infty \frac{dw}{\left(1+(w\tau)^2\right)^2}\right)$$

$$\sigma_{Vout}^2(t_\infty) = Q_t \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)}\right)^2 \left(\left[\frac{w}{2\left(1+(w\tau)^2\right)}\right]_0^\infty + \frac{1}{2}\int_0^\infty \frac{dw}{1+(w\tau)^2}\right)$$

$$\sigma_{Vout}^2(t_\infty) = Q_t \left(\frac{A\ c_{tot}}{gm\left(c_{tot} + A\ c_f\right)}\right)^2 \left[\frac{1}{\tau}\tan^{-1}(w\tau)\right]_0^\infty$$

$$\sigma_{Vout}^2(t_\infty) = kTr \left(\frac{A\ c_{tot}}{\left(c_{tot} + A\ c_f\right)}\right)^2 \frac{1}{8\tau}$$

B. Shot noise through a charge amplifier and a CR-RC^2 filter in the time domain.

$$\sigma^2_{Vout}\left(t_{01}\right) = \int_{t=0}^{t=t_{01}} \pi Q_s h_i^2(t)dt$$

$$h_i\left(t_{m1} - t_p\right) = \frac{A}{c_{tot} + A\,c_f}\frac{\left(t_{m1} - t_p\right)^2}{2\tau^2} e^{-\frac{\left(t_{m1} - t_p\right)}{\tau}}$$

$$\sigma^2_{Vout}\left(t_{01}\right) = \pi Q_s\left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{1}{4\tau^4}\int_0^{t_{01}} t^4 e^{-\frac{2t}{\tau}}dt$$

$$\sigma^2_{Vout}\left(t_{01}\right) = \pi Q_s\left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{1}{4\tau^4}\left[24e^{-\frac{2t}{\tau}}\left(\frac{-\tau^5}{32} - \frac{\tau^4 t}{16} - \frac{\tau^3 t^2}{16} - \frac{\tau^2 t^3}{24} - \frac{\tau t^4}{48}\right)\right]_0^{t_{01}}$$

$$\sigma^2_{Vout}\left(t_{01}\right) = \pi Q_s\left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{24}{4\tau^4}\left(e^{-\frac{2t_{01}}{\tau}}\left(\frac{-\tau^5}{32} - \frac{\tau^4 t_{01}}{16} - \frac{\tau^3 t_{01}^{\ 2}}{16} - \frac{\tau^2 t_{01}^{\ 3}}{24} - \frac{\tau t_{01}^{\ 4}}{48}\right) + \frac{\tau^5}{32}\right)$$

$$\sigma^2_{Vout}\left(t_{\infty}\right) = iq\left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{3\tau}{16}$$

B. Shot noise through a charge amplifier and a CR-RC^2 filter in the frequency domain.

$$\sigma^2_{Vout}(t_\infty) = \int_0^\infty Q_s \left|H_i(jw)\right|^2 dw$$

$$H_i(s) = \frac{A}{c_{tot} + A\,c_f}\frac{\tau}{(1+s\tau)^3}$$

$$\sigma^2_{Vout}(t_\infty) = \int_0^\infty Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{\tau^2}{\left(1+(w\tau)^2\right)^3} dw$$

$$\sigma^2_{Vout}(t_\infty) = Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \tau^2 \left(\left[\frac{w}{4\left(1+(w\tau)^2\right)^2}\right]_0^\infty + \frac{3}{4}\int_0^\infty \frac{dw}{\left(1+(w\tau)^2\right)^2}\right)$$

$$\sigma^2_{Vout}(t_\infty) = Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{3\tau^2}{4} \left(\left[\frac{w}{2\left(1+(w\tau)^2\right)}\right]_0^\infty + \frac{1}{2}\int_0^\infty \frac{dw}{\left(1+(w\tau)^2\right)}\right)$$

$$\sigma^2_{Vout}(t_\infty) = Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{3\tau^2}{4} \left[\frac{w}{2\left(1+(w\tau)^2\right)^2} + \frac{1}{2\tau}\tan^{-1} w\tau\right]_0^\infty$$

$$\sigma^2_{Vout}(t_\infty) = Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{3\tau^2}{4}\frac{1}{2\tau}\frac{\pi}{2}$$

$$\sigma^2_{Vout}(t_\infty) = iq \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{3\tau}{16}$$

B. Flicker noise through a charge amp and CR-RC² filter.

$$\sigma^2_{Vout}(t_\infty) = \int_0^\infty \frac{Q_f}{w}\left|H_r(jw)\right|^2 dw$$

$$H_r(s) = \frac{A\,c_{tot}}{gm\left(c_{tot} + A\,c_f\right)}\frac{s\tau}{(1+s\tau)^3}$$

$$\sigma^2_{Vout}(t_\infty) = \int_0^\infty \frac{Q_f}{w}\left(\frac{A\,c_{tot}}{gm\left(c_{tot} + A\,c_f\right)}\right)^2 \frac{(w\tau)^2}{\left(1+(w\tau)^2\right)^3} dw$$

$$\sigma^2_{Vout}(t_\infty) = Q_f\left(\frac{A\,c_{tot}}{gm\left(c_{tot} + A\,c_f\right)}\right)^2 \int_0^\infty \frac{w\tau^2}{\left(1+(w\tau)^2\right)^3} dw$$

$$\sigma^2_{Vout}(t_\infty) = Q_f\left(\frac{A\,c_{tot}}{gm\left(c_{tot} + A\,c_f\right)}\right)^2 \left[\frac{-1}{4\left(1+(w\tau)^2\right)^2}\right]_0^\infty$$

$$\sigma^2_{Vout}(t_\infty) = Q_f\left(\frac{A\,c_{tot}}{gm\left(c_{tot} + A\,c_f\right)}\right)^2 \frac{1}{4}$$

for a FET input

$$\sigma^2_{Vout}(t_\infty) = \frac{Kf_2}{WLC_{ox}}\left(\frac{A\,c_{tot}}{\left(c_{tot} + A\,c_f\right)}\right)^2 \frac{1}{4}$$

B. Thermal noise through a charge amp and CR-RC^n filter in the frequency domain.

$$\sigma^2_{Vout}(t_\infty) = \int_0^\infty Q_t \left|H_r(jw)\right|^2 dw$$

$$H_r(s) = \frac{A\, c_{tot}}{gm\left(c_{tot} + \mathrm{A}\, c_f\right)} \frac{s\tau}{(1+s\tau)^{n+1}}$$

$$\sigma^2_{Vout}(t_\infty) = \int_0^\infty Q_t \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + \mathrm{A}\, c_f\right)}\right)^2 \frac{(w\tau)^2}{\left(1+(w\tau)^2\right)^{n+1}} dw$$

$$\sigma^2_{Vout}(t_\infty) = Q_t \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + \mathrm{A}\, c_f\right)}\right)^2 \int_0^\infty \frac{(w\tau)^2}{\left(1+(w\tau)^2\right)^{n+1}} d(w\tau)^2$$

$$\sigma^2_{Vout}(t_\infty) = Q_t \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + \mathrm{A}\, c_f\right)}\right)^2 \frac{1}{2\tau} \int_0^\infty \frac{w\tau}{\left(1+(w\tau)^2\right)^{n+1}} d(w\tau)^2$$

$$\sigma^2_{Vout}(t_\infty) = Q_t \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + \mathrm{A}\, c_f\right)}\right)^2 B\left(\frac{3}{2}, n - \frac{1}{2}\right)$$

$$\sigma^2_{Vout}(t_\infty) = \frac{kTr}{\pi\tau} \left(\frac{A\, c_{tot}}{\left(c_{tot} + \mathrm{A}\, c_f\right)}\right)^2 B\left(\frac{3}{2}, n - \frac{1}{2}\right)$$

B. Shot noise through a charge amplifier and a CR-RC^n filter in the frequency domain.

$$\sigma^2_{Vout}(t_\infty) = \int_0^\infty Q_s \left|H_i(jw)\right|^2 dw$$

$$H_i(s) = \frac{A}{c_{tot} + A\,c_f} \frac{\tau}{(1+s\tau)^{n+1}}$$

$$\sigma^2_{Vout}(t_\infty) = \int_0^\infty Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{\tau^2}{\left(1+(w\tau)^2\right)^{n+1}} dw$$

$$\sigma^2_{Vout}(t_\infty) = Q_s \int_0^\infty \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{1}{\left(1+(w\tau)^2\right)^{n+1}} \frac{1}{2w} d(w\tau)^2$$

$$\sigma^2_{Vout}(t_\infty) = \frac{Q_s}{2} \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \tau \int_0^\infty \frac{1}{\left(1+(w\tau)^2\right)^{n+1}} \frac{1}{w\tau} d(w\tau)^2$$

$$\sigma^2_{Vout}(t_\infty) = \frac{iq\tau}{2\pi} \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 B\left(\frac{1}{2}, n+\frac{1}{2}\right)$$

B. Flicker noise through a charge amp and a CR-RC^n filter in the frequency domain.

$$\sigma^2_{Vout}(t_\infty)=\int_0^\infty \frac{Q_f}{w}\left|H_r(jw)\right|^2 dw$$

$$H_r(s)=\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\frac{s\tau}{(1+s\tau)^{n+1}}$$

$$\sigma^2_{Vout}(t_\infty)=\int_0^\infty \frac{Q_f}{w}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2\frac{(s\tau)^2}{\left(1+(w\tau)^2\right)^{n+1}}dw$$

$$\sigma^2_{Vout}(t_\infty)=Q_f\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2\int_0^\infty \frac{(s\tau)^2}{\left(1+(w\tau)^2\right)^{n+1}}dw$$

$$\sigma^2_{Vout}(t_\infty)=\frac{Q_f}{2}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2\int_0^\infty \frac{1}{\left(1+(w\tau)^2\right)^{n+1}}d(w\tau)^2$$

$$\sigma^2_{Vout}(t_\infty)=\frac{Q_f}{2}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2\frac{1}{n}$$

For FET input

$$\sigma^2_{Vout}(t_\infty)=\frac{Kf_2}{2WLC_{ox}}\left(\frac{A\,c_{tot}}{\left(c_{tot}+A\,c_f\right)}\right)^2\frac{1}{n}$$

B. Thermal noise through a charge amp and a CR^2-RC bipolar filter in the frequency domain.

$$\sigma^2_{Vout}(t_\infty) = \int_0^\infty Q_t \left| H_r(jw)^2 \right| dw$$

$$H_r(s) = \left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)} \right) \frac{s^2 \tau^2}{(1+s\tau)^3}$$

$$\sigma^2_{Vout}(t_\infty) = \int_0^\infty Q_t \left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)} \right)^2 \frac{w^4 \tau^4}{\left(1+(w\tau)^2\right)^3} dw$$

$$\sigma^2_{Vout}(t_\infty) = Q_t \left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)} \right)^2 \tau^4 \int_0^\infty \frac{w^4}{\left(1+(w\tau)^2\right)^3} dw$$

$$\sigma^2_{Vout}(t_\infty) = Q_t \left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)} \right)^2 \tau^4 \left(\frac{1}{\tau^2} \int_0^\infty \frac{w^2}{\left(1+(w\tau)^2\right)^2} dw - \frac{1}{\tau^2} \int_0^\infty \frac{w^2}{\left(1+(w\tau)^2\right)^3} dw \right)$$

$$\sigma^2_{Vout}(t_\infty) = Q_t \left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)} \right)^2 \tau^2 \left(0 + \left[\frac{1}{2\tau^3} \tan^{-1} w\tau \right]_0^\infty - \left[\frac{1}{8\tau^3} \tan^{-1} w\tau \right]_0^\infty \right)$$

$$\sigma^2_{Vout}(t_\infty) = Q_t \left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)} \right)^2 \frac{3}{8\tau} \frac{\pi}{2}$$

$$\sigma^2_{Vout}(t_\infty) = \left(\frac{A\ c_{tot}}{(c_{tot} + A\ c_f)} \right)^2 \frac{3kTr}{8\tau}$$

B. Shot noise through a charge amplifier and a CR^2-RC bipolar filter in the frequency domain.

$$\sigma^2_{Vout}(t_\infty)=\int_0^\infty Q_s\left|H_i(jw)\right|^2 dw$$

$$H_i(s)=\frac{A}{c_{tot}+A\,c_f}\frac{s\tau^2}{(1+s\tau)^3}$$

$$\sigma^2_{Vout}(t_\infty)=\int_0^\infty Q_s\left(\frac{A}{c_{tot}+A\,c_f}\right)^2\frac{-w^2\tau^4}{\left(1+(w\tau)^2\right)^3}dw$$

$$\sigma^2_{Vout}(t_\infty)=Q_s\left(\frac{A}{c_{tot}+A\,c_f}\right)^2\tau^4\left(0+\left[-\frac{1}{8\tau^3}\tan^{-1}w\tau\right]_0^\infty\right)$$

$$\sigma^2_{Vout}(t_\infty)=Q_s\left(\frac{A}{c_{tot}+A\,c_f}\right)^2\frac{\tau\pi}{16}$$

$$\sigma^2_{Vout}(t_\infty)=iq\left(\frac{A}{c_{tot}+A\,c_f}\right)^2\frac{\tau}{16}$$

B. Flicker noise through a charge amp and RC^2-CR bipolar filter in the frequency domain.

$$\sigma^2_{Vout}(t_\infty) = \int_0^\infty \frac{Q_f}{w} \left| H_r(jw) \right|^2 dw$$

$$H_r(s) = \frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)} \frac{s^2 \tau^2}{(1 + s\tau)^3}$$

$$\sigma^2_{Vout}(t_\infty) = \int_0^\infty \frac{Q_f}{w} \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)} \right)^2 \frac{w^4 \tau^4}{\left(1 + (w\tau)^2\right)^3} dw$$

$$\sigma^2_{Vout}(t_\infty) = \frac{Q_f}{w} \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)} \right)^2 \int_0^\infty \frac{w^3 \tau^4}{\left(1 + (w\tau)^2\right)^3} dw$$

$$\sigma^2_{Vout}(t_\infty) = \frac{Q_f}{w} \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)} \right)^2 \left(\int_0^\infty \frac{w\tau^2}{\left(1 + (w\tau)^2\right)^2} dw - \int_0^\infty \frac{w\tau^2}{\left(1 + (w\tau)^2\right)^3} dw \right)$$

$$\sigma^2_{Vout}(t_\infty) = \frac{Q_f}{w} \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)} \right)^2 \left(\left[\frac{-1}{2\left(1 + (w\tau)^2\right)} \right]_0^\infty - \left[\frac{-1}{4\left(1 + (w\tau)^2\right)^2} \right]_0^\infty \right)$$

$$\sigma^2_{Vout}(t_\infty) = \frac{Q_f}{w} \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)} \right)^2 \frac{1}{4}$$

For FET input

$$\sigma^2_{Vout}(t_\infty) = \frac{Kf_2}{WLC_{ox}} \left(\frac{A\, c_{tot}}{\left(c_{tot} + A\, c_f\right)} \right)^2 \frac{1}{4}$$

B. Thermal noise through a charge amp and cusp filter in the time domain.

$$\sigma_{Vout}^2(t_{01}) = \int_{t_p=t_0}^{t_p=t_{m1}} \pi Q_t h_r^2(t_{m1}-t_p)\,dt_p$$

$$h_r(t_{m1}-t_p) = \frac{1}{\tau}\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)\left(1-e^{-\frac{t_s}{\tau}}\right)e^{-\frac{(t_{m1}-(t_p+t_s))}{\tau}} \qquad t_0 < t_p < t_{m1}-t_s$$

$$h_r(t_{m1}-t_p) = -\frac{1}{\tau}\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)\left(e^{-\frac{(t_p+t_s-t_{m1})}{\tau}} - e^{-\frac{t_s}{\tau}}\right) \qquad t_{m1}-t_s < t_p < t_{m1}$$

$$\sigma_{Vout}^2(t_{01}) = \int_{t_p=t_0}^{t_p=t_{m1}-t_s} \frac{\pi Q_t}{\tau^2}\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)^2\left(1-e^{-\frac{t_s}{\tau}}\right)^2 e^{-\frac{2(t_{m1}-(t_p+t_s))}{\tau}}\,dt_p$$

$$+\int_{t_p=t_{m1}-t_s}^{t_p=t_{m1}} \frac{\pi Q_t}{\tau^2}\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)^2\left(e^{-\frac{2(t_p+t_s-t_{m1})}{\tau}} - 2e^{-\frac{(t_p+2t_p-t_{m1})}{\tau}} + e^{-\frac{2t_s}{\tau}}\right)dt_p$$

$$\sigma_{Vout}^2(t_{01}) = \frac{\pi Q_t}{\tau^2}\tau\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)^2 \frac{1}{2}\left[\left(1-e^{-\frac{t_s}{\tau}}\right)^2 e^{-\frac{2(t_{m1}-(t_p+t_s))}{\tau}}\right]_{t_0}^{t_{m1}-t_s}$$

$$+\frac{\pi Q_t}{\tau^2}\tau\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)^2\left[\left(-\frac{1}{2}e^{-\frac{2(t_p+t_s-t_{m1})}{\tau}} + 2e^{-\frac{(t_p+2t_s-t_{m1})}{\tau}} + t_p e^{-\frac{2t_s}{\tau}}\right)\right]_{t_{m1}-t_s}^{t_{m1}}$$

$$\sigma_{Vout}^2(t_{01}) = \frac{\pi Q_t}{\tau}\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)^2 \frac{1}{2}\left(1-e^{-\frac{t_s}{\tau}}\right)^2\left(e^{-\frac{2(t_{m1}-(t_{m1}-t_s+t_s))}{\tau}} - e^{-\frac{2(t_{m1}-(t_0+t_s))}{\tau}}\right)$$

$$+\frac{\pi Q_t}{\tau}\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)^2$$

$$\left(-\frac{1}{2}e^{-\frac{2(t_{m1}+t_s-t_{m1})}{\tau}} + 2e^{-\frac{(t_{m1}+2t_s-t_{m1})}{\tau}} + t_{m1}e^{-\frac{2t_s}{\tau}} + \frac{1}{2}e^{-\frac{2(t_{m1}-t_s+t_s-t_{m1})}{\tau}} - 2e^{-\frac{(t_{m1}-t_s+2t_s-t_{m1})}{\tau}} - (t_{m1}-t_s)e^{-\frac{2t_s}{\tau}}\right)$$

$$\sigma_{Vout}^2(t_{01}) = \frac{\pi Q_t}{\tau}\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)^2 \frac{1}{2}\left(1-2e^{-\frac{t_s}{\tau}} + e^{-\frac{2t_s}{\tau}}\right)\left(1-e^{-\frac{(t_{m1}-(t_0+t_s))}{\tau}}\right)$$

$$+\frac{\pi Q_t}{\tau}\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)^2\left(-\frac{1}{2}e^{-\frac{2t_s}{\tau}} + 2e^{-\frac{2t_s}{\tau}} + t_{m1}e^{-\frac{2t_s}{\tau}} + \frac{1}{2} - 2e^{-\frac{t_s}{\tau}} - (t_{m1}-t_s)e^{-\frac{2t_s}{\tau}}\right)$$

$$\sigma_{Vout}^2(t_{01}) = \frac{\pi Q_t}{\tau}\left(\frac{A\ c_{tot}}{gm(c_{tot}+A\ c_f)}\right)^2 \frac{1}{2}\left(1-e^{-\frac{2(t_{m1}-(t_0+t_s))}{\tau}} - 2e^{-\frac{t_s}{\tau}} + e^{-\frac{2(t_{m1}-(t_0+t_s))-2t_s}{\tau}} + e^{-\frac{2t_s}{\tau}} - e^{-\frac{2(t_{m1}-t_0-t_s)+2t_s}{\tau}}\right)$$

B. Thermal noise through a charge amp and cusp filter in the time domain (cont).

$$+\frac{\pi Q_t}{\tau}\left(\frac{A\ c_{tot}}{\mathrm{gm}\left(c_{tot}+A\ c_f\right)}\right)^2\left[\left(\frac{1}{2}-2e^{-\left(\frac{t_s}{\tau}\right)}+\left(\frac{3}{2}+t_s\right)e^{-\frac{2t_s}{\tau}}\right)\right]$$

$$\sigma^2_{Vout}\left(t_{01}\right)=\frac{\pi Q_t}{\tau}\left(\frac{A\ c_{tot}}{\mathrm{gm}\left(c_{tot}+A\ c_f\right)}\right)^2\left(\frac{1}{2}-\frac{1}{2}e^{-\frac{2(t_{m1}-t_0-t_s)}{\tau}}-e^{-\frac{t_s}{\tau}}+\frac{1}{2}e^{-\frac{2(t_{m1}-t_0)}{\tau}}+\frac{1}{2}e^{-\frac{2t_s}{\tau}}-\frac{1}{2}e^{-\frac{2(t_{m1}-t_0)}{\tau}}\right)$$

$$+\frac{\pi Q_t}{\tau}\left(\frac{A\ c_{tot}}{\mathrm{gm}\left(c_{tot}+A\ c_f\right)}\right)^2\left[\left(\frac{1}{2}-2e^{-\frac{t_s}{\tau}}+\left(\frac{3}{2}+t_s\right)e^{-\frac{2t_s}{\tau}}\right)\right]$$

$$\sigma^2_{Vout}\left(t_{01}\right)=\frac{\pi Q_t}{\tau}\left(\frac{A\ c_{tot}}{\mathrm{gm}\left(c_{tot}+A\ c_f\right)}\right)^2\left(1-\frac{1}{2}e^{-\frac{2(t_{m1}-t_0-t_s)}{\tau}}-\frac{5}{2}e^{-\frac{t_s}{\tau}}+\left(2+t_s\right)e^{-\frac{2t_s}{\tau}}\right)$$

Let $t_{m1}-t_0$ tend to infinity gives

$$\sigma^2_{Vout}\left(t_\infty\right)=\frac{\pi Q_t}{\tau}\left(\frac{A\ c_{tot}}{\mathrm{gm}\left(c_{tot}+A\ c_f\right)}\right)^2\left(1-\frac{5}{2}e^{-\frac{t_s}{\tau}}+\left(2+t_s\right)e^{-\frac{2t_s}{\tau}}\right)$$

Let t_s tend to infinity gives

$$\sigma^2_{Vout}\left(t_\infty\right)=\frac{2kTr}{\tau}\left(\frac{A\ c_{tot}}{\left(c_{tot}+A\ c_f\right)}\right)^2$$

B. Thermal noise through a charge amp and cusp filter in the frequency domain

$$\sigma^2_{Vout}(t_\infty)=\int_{w=0}^{w=\infty} Q(w)\left|H_r(jw)\right|^2 dw$$

$$\sigma^2_{Vout}(t_\infty)=Q_t\int_{w=0}^{w=\infty}\left|H_r(jw)\right|^2 dw$$

$$h_r(t)=\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\frac{1}{\tau}e^{-\frac{|t|}{\tau}}$$

$$H_r(jw)=\Im(h_r(t))$$

$$H_r(jw)=\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\frac{1}{\tau}\int_{-\infty}^{\infty}e^{-\frac{|t|}{\tau}}e^{-iwt}dt$$

$$H_r(jw)=\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\frac{1}{\tau}\int_{0}^{\infty}e^{-\frac{t}{\tau}}e^{-iwt}dt+\frac{1}{\tau}\int_{-\infty}^{0}e^{-\frac{t}{\tau}}e^{-iwt}dt$$

$$H_r(jw)=\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\frac{1}{\tau}\int_{0}^{\infty}e^{-t\left(\frac{1}{\tau}+iw\right)}dt+\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\frac{1}{\tau}\int_{-\infty}^{0}e^{-t\left(\frac{-1}{\tau}+iw\right)}dt$$

$$H_r(jw)=-\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\frac{1}{\tau}\left[\frac{-e^{-t\left(\frac{1}{\tau}+iw\right)}}{\left(\frac{1}{\tau}+iw\right)}\right]_0^{\infty}-\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\frac{1}{\tau}\left[\frac{e^{-t\left(\frac{-1}{\tau}+iw\right)}}{\left(-\frac{1}{\tau}+iw\right)}\right]_{-\infty}^{0}$$

$$H_r(jw)=\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\left(\frac{1}{\tau\left(\frac{1}{\tau}+iw\right)}-\frac{1}{\tau\left(\frac{1}{\tau}-iw\right)}\right)$$

$$H_r(jw)=\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\frac{-2iw}{\tau\left(\frac{1}{\tau^2}+w^2\right)}=\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\frac{-2iw\tau}{1+(w\tau)^2}$$

$$\sigma^2_{Vout}(t_\infty)=\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2\frac{4Q_t}{\tau}\int_{w=0}^{w=\infty}\frac{w^2\tau^2}{\left(1+(w\tau)^2\right)^2}d(w\tau)$$

$$\sigma^2_{Vout}(t_\infty)=\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2\frac{4Q_t}{\tau}\left[\frac{-w\tau}{2\left(1+(w\tau)^2\right)}-\frac{1}{2}\tan^{-1}wt\right]_0^{\infty}$$

$$\sigma^2_{Vout}(t_\infty)=\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2\frac{4Q_t}{\tau}\left[\frac{\pi}{4}\right]=\left(\frac{A\,c_{tot}}{\left(c_{tot}+A\,c_f\right)}\right)^2\frac{2kTr}{\tau}$$

B. Shot noise through a charge amp and cusp filter in the time domain.

$$\sigma_{Vout}^2(t_{01}) = \int_{t_p=t_0}^{t_p=t_{m1}} \pi Q_s h_i^2(t_{m1} - t_p)\,dt_p$$

$$h_i(t_{m1} - t_p) = \frac{A}{C_{tot} + A\,c_f}\left(1 - e^{-\frac{t_s}{\tau}}\right) e^{-\frac{(t_{m1}-(t_p+t_s))}{\tau}} \qquad t_0 \langle t_p \langle t_{m1} - t_s$$

$$h_i(t_{m1} - t_p) = \frac{A}{C_{tot} + A\,c_f}\left(e^{-\frac{(t_p+t_s-t_{m1})}{\tau}} - e^{-\frac{t_s}{\tau}}\right) \qquad t_{m1} - t_s \langle t_p \langle t_{m1}$$

$$\sigma_{Vout}^2(t_{01}) = \int_{t_p=t_0}^{t_p=t_{m1}-t_s} \pi Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \left(1 - e^{-\frac{t_s}{\tau}}\right)^2 e^{-\frac{2(t_{m1}-(t_p+t_s))}{\tau}}\,dt_p$$

$$+ \int_{t_p=t_{m1}-t_s}^{t_p=t_{m1}} \pi Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \left(e^{-\frac{2(t_p+t_s-t_{m1})}{\tau}} - 2e^{-\frac{(t_p+2t_s-t_{m1})}{\tau}} + e^{-\frac{2t_s}{\tau}}\right) dt_p$$

$$\sigma_{Vout}^2(t_{01}) = \pi Q_s \tau \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \left[\frac{1}{2}\left(1 - e^{-\frac{t_s}{\tau}}\right)^2 e^{-\frac{2(t_{m1}-(t_p+t_s))}{\tau}}\right]_{t_0}^{t_{m1}-t_s}$$

$$+ \pi Q_s \tau \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \left[\left(-\frac{1}{2} e^{-\frac{2(t_p+t_s-t_{m1})}{\tau}} + 2e^{-\frac{(t_p+2t_p-t_{m1})}{t}} + t_p e^{-\frac{2t_s}{\tau}}\right)\right]_{t_{m1}-t_s}^{t_{m1}}$$

$$\sigma_{Vout}^2(t_{01}) = \pi Q_s \tau \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{1}{2}\left(1 - e^{-\frac{t_s}{\tau}}\right)^2 \left(e^{-\frac{2(t_{m1}-(t_{m1}-t_s+t_s))}{\tau}} - e^{-\frac{2(t_{m1}-(t_0+t_s))}{\tau}}\right)$$

$$+ \pi Q_s \tau \left(\frac{A}{c_{tot} + A\,c_f}\right)^2$$

$$\left(-\frac{1}{2} e^{-\frac{2(t_{m1}+t_s-t_{m1})}{\tau}} + 2e^{-\frac{(t_{m1}+2t_s-t_{m1})}{\tau}} + t_{m1} e^{-\frac{2t_s}{\tau}} + \frac{1}{2} e^{-\frac{2(t_{m1}-t_s+t_s-t_{m1})}{\tau}} - 2e^{-\frac{(t_{m1}-t_s+2t_s-t_{m1})}{\tau}} - (t_{m1} - t_s) e^{-\frac{2t_s}{\tau}}\right)$$

$$\sigma_{Vout}^2(t_{01}) = \pi Q_s \tau \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{1}{2}\left(1 - 2e^{-\frac{t_s}{\tau}} + e^{-\frac{2t_s}{\tau}}\right)\left(1 - e^{-\frac{2(t_{m1}-(t_0+t_s))}{\tau}}\right)$$

$$+ \pi Q_s \tau \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \left(-\frac{1}{2} e^{-\frac{2t_s}{\tau}} + 2e^{-\frac{2t_s}{\tau}} + t_{m1} e^{-\frac{2t_s}{\tau}} + \frac{1}{2} - 2e^{-\frac{t_s}{\tau}} - (t_{m1} - t_s) e^{-\frac{2t_s}{\tau}}\right)$$

B. Shot noise through a charge amp and cusp filter in the time domain (cont)

$$\sigma_{Vout}^2(t_{01}) = \pi Q_s \tau \left(\frac{A}{C_{tot} + A\,c_f} \right)^2 \frac{1}{2} \left(1 - e^{-\frac{2(t_{m1}-(t_0+t_s))}{\tau}} - 2e^{-\frac{t_s}{\tau}} + e^{-\frac{2(t_{m1}-(t_0+t_s))+t_s}{\tau}} + e^{-\frac{2t_s}{\tau}} - e^{-\frac{2(t_{m1}-t_0-t_s)+2t_s}{\tau}} \right)$$

$$+ \pi Q_s \tau \left(\frac{A}{C_{tot} + A\,c_f} \right)^2 \left[\left(\frac{1}{2} - 2e^{-\left(\frac{t_s}{\tau}\right)} + \left(\frac{3}{2} + t_s \right) e^{-\frac{2t_s}{\tau}} \right) \right]$$

$$\sigma_{Vout}^2(t_{01}) = \pi Q_s \tau \left(\frac{A}{C_{tot} + A\,c_f} \right)^2 \left(\frac{1}{2} - \frac{1}{2} e^{-\frac{2(t_{m1}-t_0-t_s)}{\tau}} - e^{-\frac{t_s}{\tau}} + \frac{1}{2} e^{-\frac{2(t_{m1}-t_0)}{\tau}} + \frac{1}{2} e^{-\frac{2t_s}{\tau}} - \frac{1}{2} e^{-\frac{2(t_{m1}-t_0+t_s)}{\tau}} \right)$$

$$+ \pi Q_s \tau \left(\frac{A}{C_{tot} + A\,c_f} \right)^2 \left[\left(\frac{1}{2} - 2e^{-\frac{t_s}{\tau}} + \left(\frac{3}{2} + t_s \right) e^{-\frac{2t_s}{\tau}} \right) \right]$$

$$\sigma_{Vout}^2(t_{01}) = \pi Q_s \tau \left(\frac{A}{C_{tot} + A\,c_f} \right)^2 \left(1 - \frac{1}{2} e^{-\frac{2(t_{m1}-t_0-t_s)}{\tau}} - \frac{5}{2} e^{-\frac{t_s}{\tau}} + \frac{1}{2} e^{-\frac{(t_{m1}-t_0)}{\tau}} + (2 + t_s) e^{-\frac{2t_s}{\tau}} - \frac{1}{2} e^{-\frac{(t_{m1}-t_0)}{\tau}} \right)$$

Letting $t_{m1} - t_0$ tend to infinity gives

$$\sigma_{Vout}^2(t_\infty) = iq\tau \left(\frac{A}{C_{tot} + A\,c_f} \right)^2 \left(1 - \frac{5}{2} e^{-\frac{t_s}{\tau}} + (2 + t_s) e^{-\frac{2t_s}{\tau}} \right)$$

Letting t_s tend to infinity gives

$$\sigma_{Vout}^2(t_\infty) = iq\tau \left(\frac{A}{C_{tot} + A\,c_f} \right)^2$$

B. Shot noise through a charge amp and cusp filter in the frequency domain.

$$\sigma^2_{Vout}(t_\infty) = \int_{w=0}^{w=\infty} Q(w) |H_i(jw)|^2 \, dw$$

$$\sigma^2_{Vout}(t_\infty) = Q_s \int_{w=0}^{w=\infty} |H_i(jw)|^2 \, dw$$

$$h_i(t) = \frac{A}{c_{tot} + A\, c_f} e^{-\frac{|t|}{\tau}}$$

$$H_i(jw) = \Im(h_i(t))$$

$$H_i(jw) = \frac{A}{c_{tot} + A\, c_f} \int_{-\infty}^{\infty} e^{-\frac{|t|}{\tau}} e^{-iwt} \, dt$$

$$H_i(jw) = \frac{A}{c_{tot} + A\, c_f} \int_{0}^{\infty} e^{-\frac{|t|}{\tau}} e^{-iwt} \, dt + \frac{A}{c_{tot} + A\, c_f} \int_{-\infty}^{0} e^{-\frac{|t|}{\tau}} e^{-iwt} \, dt$$

$$H_i(jw) = \frac{A}{c_{tot} + A\, c_f} \int_{0}^{\infty} e^{-t\left(\frac{1}{\tau} + iw\right)} dt + \frac{A}{c_{tot} + A\, c_f} \int_{-\infty}^{0} e^{-t\left(\frac{-1}{\tau} + iw\right)} dt$$

$$H_i(jw) = \frac{A}{c_{tot} + A\, c_f} \left[\frac{e^{-t\left(\frac{1}{\tau} + iw\right)}}{-\left(\frac{1}{\tau} + iw\right)} \right]_0^{\infty} + \frac{A}{c_{tot} + A\, c_f} \left[\frac{e^{-t\left(\frac{-1}{\tau} + iw\right)}}{-\left(\frac{-1}{\tau} + iw\right)} \right]_{-\infty}^{0}$$

$$H_i(jw) = \frac{A}{c_{tot} + A\, c_f} \left(\frac{1}{\left(\frac{1}{\tau} + iw\right)} + \frac{1}{\left(\frac{1}{\tau} - iw\right)} \right)$$

$$H_i(jw) = \frac{A}{c_{tot} + A\, c_f} \frac{2}{\tau\left(\frac{1}{\tau^2} + w^2\right)}$$

$$\sigma^2_{Vout}(t_\infty) = \left(\frac{A}{c_{tot} + A\, c_f} \right)^2 Q_s \frac{4}{\tau^2} \int_{w=0}^{w=\infty} \frac{1}{\left(\frac{1}{\tau^2} + w^2\right)^2} dw$$

$$\sigma^2_{Vout}(t_\infty) = \left(\frac{A}{c_{tot} + A\, c_f} \right)^2 \frac{Q_s 4\tau^2}{2\tau^2} \left[\frac{w}{\left(\frac{1}{\tau^2} + w^2\right)} + \tau \tan^{-1} \tau w \right]_0^{\infty}$$

$$\sigma^2_{Vout}(t_\infty) = \left(\frac{A}{c_{tot} + A\, c_f} \right)^2 \frac{Q_s 4\tau^2}{2\tau^2} \frac{\tau\pi}{2}$$

$$\sigma^2_{Vout}(t_\infty) = \left(\frac{A}{c_{tot} + A\, c_f} \right)^2 iq\tau$$

B. Flicker noise through a charge amp and cusp filter in the frequency domain.

$$\sigma^2_{Vout}(t_\infty)=\int_{w=0}^{w=\infty}\frac{Q_f}{w}\left|H_r(jw)\right|^2 dw$$

$$h_r(t)=\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)\frac{-1}{\tau}e^{-\frac{|t|}{\tau}}$$

$$H_r(jw)=\Im(h(t))$$

$$H_r(jw)=\frac{-1}{\tau}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)\int_{-\infty}^{\infty}e^{-\frac{|t|}{\tau}}e^{-iwt}dt$$

$$H_r(jw)=\frac{-1}{\tau}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)\int_{0}^{\infty}e^{-\frac{t}{\tau}}e^{-iwt}dt-\frac{1}{\tau}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)\int_{-\infty}^{0}e^{\frac{t}{\tau}}e^{-iwt}dt$$

$$H_r(jw)=\frac{-1}{\tau}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)\int_{0}^{\infty}e^{-t\left(\frac{1}{\tau}+iw\right)}dt-\frac{1}{\tau}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)\int_{-\infty}^{0}e^{-t\left(-\frac{1}{\tau}+iw\right)}dt$$

$$H_r(jw)=\frac{-1}{\tau}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)\left[\frac{e^{-t\left(\frac{1}{\tau}+iw\right)}}{\left(\frac{1}{\tau}+iw\right)}\right]_0^{\infty}-\frac{1}{\tau}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)\left[\frac{e^{-t\left(-\frac{1}{\tau}+iw\right)}}{\left(\frac{1}{\tau}-iw\right)}\right]_{-\infty}^{0}$$

$$H_r(jw)=\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\left(\frac{1}{\tau\left(\frac{1}{\tau}+iw\right)}-\frac{1}{\left(\frac{1}{\tau}-iw\right)}\right)$$

$$H_r(jw)=\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\frac{2iw}{\tau\left(\frac{1}{\tau^2}+w^2\right)}$$

$$\sigma^2_{Vout}(t_\infty)=\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2\frac{-4Q_f}{\tau^2}\int_{w=0}^{w=\infty}\frac{w}{\left(\frac{1}{\tau^2}+w^2\right)^2}dw$$

$$\sigma^2_{Vout}(t_\infty)=\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2\frac{-4Q_f}{\tau^2}\left[\frac{1}{2\left(\frac{1}{\tau^2}+w^2\right)}\right]_0^{\infty}$$

$$\sigma^2_{Vout}(t_\infty)=\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2\frac{4Q_f}{\tau^2}\frac{\tau^2}{2}=\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2 2Q_f$$

|For FET input

$$\sigma^2_{Vout}(t_\infty)=\left(\frac{A\,c_{tot}}{\left(c_{tot}+A\,c_f\right)}\right)^2\frac{2Kf_2}{WLC_{ox}}$$

B. Thermal noise through a charge amp and a trapezoidal filter in the time domain.

$$\sigma^2_{Vout}(t_{01}) = \int_{t_p=t_0}^{t_p=t_{m1}} \pi Q_t h_r^2 (t_m - t_p) dt_p$$

$$h_r(t_{m1} - t_p) = 0 \qquad t_0 \langle t_p \langle t_{m1} - (t_{rise} + t_{flat} + t_{fall})$$

$$h_r(t_{m1} - t_p) = \frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)} \frac{1}{t_{fall}} \qquad t_{m1} - (t_{rise} + t_{flat} + t_{fall}) \langle t_p \langle t_{m1} - (t_{rise} + t_{flat})$$

$$h_r(t_{m1} - t_p) = 0 \qquad t_{m1} - (t_{rise} + t_{flat}) \langle t_p \langle t_{m1} - t_{rise}$$

$$h_r(t_{m1} - t_p) = \frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)} \frac{-1}{t_{rise}} \qquad \mathrm{t_{m1}} - t_{rise} \langle t_p \langle t_{m1}$$

$$\frac{\sigma^2_{Vout}(t_{01})}{\pi Q_t \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)} \right)^2} = \int_{t_p=t_{m1}-(t_{rise}+t_{flat}+t_{fall})}^{t_p=t_{m1}-(t_{rise}+t_{flat})} \left(\frac{1}{t_{fall}} \right)^2 dt_p + \int_{t_p=t_{m1}-T_{rise}}^{t_p=t_{m1}} \left(\frac{-1}{t_{rise}} \right)^2 dt_p$$

$$= \frac{1}{t^2_{fall}} \left[t_p \right]_{t_{m1}-(t_{rise}+t_{flat}+t_{fall})}^{t_{m1}-(t_{rise}+t_{flat})} + \frac{1}{t^2_{rise}} \left[t_p \right]_{t_{m1}-T_{rise}}^{t_{m1}}$$

$$= \frac{1}{t^2_{fall}} (t_{m1} - t_{rise} - t_{flat}) - \frac{1}{t^2_{fall}} (t_{m1} - t_{rise} - t_{flat} - t_{fall}) + \frac{1}{t^2_{rise}} t_{m1} - \frac{1}{t^2_{rise}} (t_{m1} - t_{rise})$$

$$\sigma^2_{Vout}(t_{01}) = 2kTr \left(\frac{A\, c_{tot}}{(c_{tot} + A\, c_f)} \right)^2 \left(\frac{1}{t_{fall}} + \frac{1}{t_{rise}} \right)$$

B. Thermal noise through charge amp and a trapezoidal filter in the frequency domain.

Transfer function of trapezoidal step response with $t_{rise} = t_{fall}$

$$H_v(s) = \frac{1}{t_{rise}}\left(\frac{1}{s^2} - \frac{e^{st_{rise}}}{s^2} - \frac{e^{s(t_{rise}+t_{flat})}}{s^2} + \frac{e^{s(2t_{rise}+t_{flat})}}{s^2}\right)$$

$$|H_v(jw)|^2 =$$

$$\frac{1}{w^4 t_{rise}^2}\left(1 - e^{jwt_{rise}} - e^{jw(t_{rise}+t_{flat})} + e^{jw(2t_{rise}+t_{flat})}\right)\left(1 - e^{-jwt_{rise}} - e^{-jw(t_{rise}+t_{flat})} + e^{-jw(2t_{rise}+t_{flat})}\right)$$

$$|H_v(jw)|^2 w^4 t_{rise}^2 =$$

$$1 - e^{-jwt_{rise}} - e^{-jw(t_{rise}+t_{flat})} + e^{-jw(2t_{rise}+t_{flat})} - e^{jwt_{rise}} + 1 + e^{-jw(t_{flat})} - e^{-jw(t_{rise}+t_{flat})}$$

$$- e^{jw(t_{rise}+t_{flat})} + e^{jw(t_{flat})} + 1 - e^{-jwt_{rise}} + e^{jw(2t_{rise}+t_{flat})} - e^{jw(t_{rise}+t_{flat})} - e^{jwt_{rise}} + 1$$

$$|H_v(jw)|^2 w^4 t_{rise}^2 = 4 - 2e^{-jwt_{rise}} - 2e^{jwt_{rise}} - 2e^{-jw(t_{rise}+t_{flat})} - 2e^{jw(t_{rise}+t_{flat})} + e^{-jw(t_{flat})} + e^{jw(t_{flat})}$$

$$+ e^{-jw(2t_{rise}+t_{flat})} + e^{jw(2t_{rise}+t_{flat})}$$

$$|H_v(jw)|^2 w^4 t_{rise}^2 = 4 - 4\cos wt_{rise} - 4\cos w(t_{rise} + t_{flat}) + 2\cos wt_{flat} + 2\cos w(2t_{rise} + t_{flat})$$

$$|H_v(jw)|^2 w^4 t_{rise}^2 = 4 - 4\cos wt_{rise} - 4\cos w(t_{rise} + t_{flat}) + 4\cos w(t_{rise} + t_{flat})\cos w(t_{rise})$$

$$|H_v(jw)|^2 w^4 t_{rise}^2 = 4 - 4\cos wt_{rise} - 4(1 - \cos wt_{rise})\cos w(t_{rise} + t_{flat})$$

$$|H_v(jw)|^2 w^4 t_{rise}^2 = 4(1 - \cos wt_{rise}) - 4(1 - \cos wt_{rise})\cos w(t_{rise} + t_{flat})$$

$$|H_v(jw)|^2 w^4 t_{rise}^2 = 4(1 - \cos wt_{rise})(1 - \cos w(t_{rise} + t_{flat}))$$

$$|H_v(jw)|^2 w^4 t_{rise}^2 = 4(2\sin^2 wt_{rise})(2\sin^2(t_{rise} + t_{flat}))$$

$$|H_v(jw)|^2 = \frac{16\sin^2 \frac{wt_{rise}}{2} \sin^2\left(\frac{t_{rise} + t_{flat}}{2}\right)}{w^4 t_{rise}^2}$$

Transfer function of trapezoidal impulse response

$$|H_v(jw)|^2 = \frac{16\sin^2 \frac{wt_{rise}}{2} \sin^2\left(\frac{t_{rise} + t_{flat}}{2}\right)}{w^2 t_{rise}^2}$$

B. Thermal noise in charge amp and trapezoidal filter in the frequency domain (cont).

$$\sigma_{Vout}^2(t_\infty) = \int_{w=0}^{w=\infty} Q_t \left|H_r(jw)\right|^2 dw$$

$$\left|H_r(jw)\right|^2 = \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)^2 \frac{16}{w^2 t_{rise}^2} \sin^2(w t_{rise}/2) \sin^2\left(w\left(t_{rise} + t_{flat}\right)/2\right)$$

$$\sigma_{Vout}^2(t_\infty) = Q_t \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)^2 \int_{w=0}^{w=\infty} \frac{16}{w^2 t_{rise}^2} \sin^2(w t_{rise}/2) \sin^2\left(w\left(t_{rise} + t_{flat}\right)/2\right) dw$$

if $t_{flat} = 0$

$$\sigma_{Vout}^2(t_\infty) = Q_t \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)^2 \int_{w=0}^{w=\infty} \frac{16}{w^2 t_{rise}^2} \sin^4(w t_{rise}/2) dw$$

$$\sigma_{Vout}^2(t_\infty) = Q_t \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)^2 \frac{16}{2 t_{rise}} \int_{w=0}^{w=\infty} \frac{4}{w^2 t_{rise}^2} \sin^4(w t_{rise}/2) d(w t_{rise}/2)$$

$$\sigma_{Vout}^2(t_\infty) = Q_t \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)^2 \frac{16}{2 t_{rise}} \frac{\pi}{4} = \frac{4kTr}{t_{rise}} \left(\frac{A\, c_{tot}}{\left(c_{tot} + A\, c_f\right)}\right)^2$$

if $t_{flat} = t_{rise}$

$$\sigma_{Vout}^2(t_\infty) = Q_t \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)^2 \int_{w=0}^{w=\infty} \frac{16}{w^2 t_{rise}^2} \sin^2(w t_{rise}/2) \sin^2(w t_{rise}) dw$$

$$\sigma_{Vout}^2(t_\infty) = Q_t \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)^2 \frac{16}{2 t_{rise}} \int_{w=0}^{w=\infty} \frac{4}{w^2 t_{rise}^2} \sin^2(w t_{rise}/2) \sin^2(w t_{rise}) d(w t_{rise}/2)$$

$$\sigma_{Vout}^2(t_\infty) = Q_t \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)^2 \frac{8}{t_{rise}} \frac{\pi}{4} = \frac{4kTr}{t_{rise}} \left(\frac{A\, c_{tot}}{\left(c_{tot} + A\, c_f\right)}\right)^2$$

B. Shot noise through charge amp and a trapezoidal filter in the time domain.

$$\sigma^2_{Vout}(t_{01}) = \int_{t_p=t_0}^{t_p=t_{m1}} \pi Q_s h_i^2(t_m - t_p)\,dt_p$$

$$h_i(t_m - t_p) = 0 \qquad t_0 \langle t_p \langle t_{m1} - (t_{rise} + t_{flat} + t_{fall})$$

$$h_i(t_m - t_p) = \frac{A}{c_{tot} + A\,c_f} \frac{(t_p + t_{rise} + t_{flat} + t_{fall}) - t_{m1}}{t_{fall}} \qquad t_{m1} - (t_{rise} + t_{flat} + t_{fall}) \langle t_p \langle t_{m1} - (t_{rise} + t_{flat})$$

$$h_i(t_m - t_p) = 1 \qquad t_{m1} - (t_{rise} + t_{flat}) \langle t_p \langle t_{m1} - t_{rise}$$

$$h_i(t_m - t_p) = \frac{A}{c_{tot} + A\,c_f} \frac{t_{m1} - t_p}{t_{rise}} \qquad t_{m1} - t_{rise} \langle t_p \langle t_{m1}$$

$$\frac{\sigma^2_{Vout}(t_{01})}{\pi Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2} =$$

$$= \int_{t_p=t_{m1}-(t_{rise}+t_{flat}+t_{fall})}^{t_p=t_{m1}-(t_{rise}+t_{flat})} \left(\frac{(t_p + t_{rise} + t_{flat} + t_{fall}) - t_{m1}}{t_{fall}}\right)^2 dt_p + \int_{t_p=t_{m1}-(t_{rise}+t_{flat})}^{t_p=t_{m1}-t_{rise}} dt_p + \int_{t_p=t_{m1}-t_{rise}}^{t_p=t_{m1}} \left(\frac{t_{m1} - t_p}{t_{rise}}\right)^2 dt_p$$

substituting in first integral $t = t_{m1} - t_{rise} - t_{flat}$

$$= \frac{1}{t^2_{fall}} \int_{t_p=t-t_{fall}}^{t_p=t} (t_p - t + t_{fall})^2\,dt_p + \int_{t_p=t_{m1}-(t_{rise}+t_{flat})}^{t_p=t_{m1}-t_{rise}} dt_p + \frac{1}{t^2_{rise}} \int_{t_p=t_{m1}-T_{rise}}^{t_p=t_{m1}} (t_{m1} - t_p)^2\,dt_p$$

substituting in first integral $t = t_p - t$ and in third integral $t = t_p - t_{m1}$

$$= \frac{1}{t^2_{fall}} \int_{t=-T_{fall}}^{t=0} (t + t_{fall})^2\,dt + \int_{t_p=t_{m1}-(t_{rise}+t_{flat})}^{t_p=t_{m1}-t_{rise}} dt_p + \frac{1}{t^2_{rise}} \int_{t=-t_{rise}}^{t=0} t^2\,dt$$

$$= \frac{1}{t^2_{fall}} \int_{t=-t_{fall}}^{t=0} (t^2 + 2t_{fall}t + t^2_{fall})\,dt + \int_{t_p=t_{m1}-(t_{rise}+t_{flat})}^{t_p=t_{m1}-t_{rise}} dt_p + \frac{1}{t^2_{rise}} \int_{t=-t_{rise}}^{t=0} t^2\,dt_p$$

$$= \frac{1}{t^2_{fall}} \left[\frac{t^3}{3} + t_{fall}t^2 + tt^2_{fall}\right]_{-T_{fall}}^{0} + \left[t_p\right]_{t_{m1}-(t_{rise}+t_{flat})}^{t_{m1}-t_{rise}} + \frac{1}{t^2_{rise}} \left[\frac{t^3}{3}\right]_{-T_{rise}}^{0}$$

$$= \frac{1}{t^2_{fall}} \left(\frac{t^3_{fall}}{3} + t_{fall}t^2_{fall} - t_{fall}t^2_{fall}\right) + t_{m1} - t_{rise} - t_{m1} + (t_{rise} + t_{flat}) - \frac{(-t_{rise})^3}{3t^2_{rise}}$$

$$= \frac{t_{fall}}{3} + t_{flat} + \frac{t_{rise}}{3}$$

$$\sigma^2_{Vout}(t_{01}) = iq\left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \left(\frac{t_{fall}}{3} + t_{flat} + \frac{t_{rise}}{3}\right)$$

B. Shot noise in charge amp and trapezoidal filter in the frequency domain.

$$\sigma_{Vout}^2(t_\infty) = \int_{w=0}^{w=\infty} Q_s |H_i(jw)|^2 \, dw$$

$$|H_i(jw)|^2 = \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{16}{w^4 t_{rise}^2} \sin^2(wt_{rise}/2)\sin^2\left(w(t_{rise} + t_{flat})/2\right)$$

$$\sigma_{Vout}^2(t_\infty) = Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \int_{w=0}^{w=\infty} \frac{16}{w^4 t_{rise}^2} \sin^2(wt_{rise}/2)\sin^2\left(w(t_{rise} + t_{flat})/2\right) dw$$

if $t_{flat} = 0$

$$\sigma_{Vout}^2(t_\infty) = Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \int_{w=0}^{w=\infty} \frac{16}{w^4 t_{rise}^2} \sin^4(wt_{rise}/2) dw$$

$$\sigma_{Vout}^2(t_\infty) = Q_s 16 \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \int_{w=0}^{w=\infty} \frac{2}{w^4 t_{rise}^3} \sin^4(wt_{rise}/2) d(wt_{rise}/2)$$

$$\sigma_{Vout}^2(t_\infty) = Q_s 16 \frac{t_{rise}}{8} \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \int_{w=0}^{w=\infty} \frac{16}{w^4 t_{rise}^4} \sin^4(wt_{rise}/2) d(wt_{rise}/2)$$

$$\sigma_{Vout}^2(t_\infty) = Q_s 2\frac{\pi}{3} \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 t_{rise} = \frac{2qi}{3} \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 t_{rise}$$

if $t_{flat} = t_{rise}$

$$\sigma_{Vout}^2(t_\infty) = Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \int_{w=0}^{w=\infty} \frac{16}{w^4 t_{rise}^2} \sin^2(wt_{rise}/2)\sin^2(wt_{rise}) dw$$

$$\sigma_{Vout}^2(t_\infty) = Q_s 2t_{rise} \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \int_{w=0}^{w=\infty} \frac{16}{w^4 t_{rise}^4} \sin^2(wt_{rise}/2)\sin^2(wt_{rise}) d(wt_{rise}/2)$$

$$\sigma_{Vout}^2(t_\infty) = Q_s 2t_{rise} \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \int_{x=0}^{x=\infty} \frac{1}{x^4} \sin^2 x \sin^2 2x \, dx$$

$$\sigma_{Vout}^2(t_\infty) = Q_s 2\frac{5\pi}{6} t_{rise} \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 = \frac{5qi}{3} \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 t_{rise}$$

B. Flicker noise in charge amp and trapezoidal filter in the frequency domain.

$$\sigma^2_{Vout}(t_\infty) = \int_{w=0}^{w=\infty} \frac{Q_f}{w} |H_r(jw)|^2 dw$$

$$|H_r(jw)|^2 = \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)} \right)^2 \frac{16}{w^3 t_{rise}^2} \sin^2(wt_{rise}/2) \sin^2(w(t_{rise} + t_{flat})/2)$$

$$\sigma^2_{Vout}(t_\infty) = Q_f \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)} \right)^2 \int_{w=0}^{w=\infty} \frac{16}{w^3 t_{rise}^2} \sin^2(wt_{rise}/2) \sin^2(w(t_{rise} + t_{flat})/2) dw$$

if $t_{flat} = 0$

$$\sigma^2_{Vout}(t_\infty) = Q_f \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)} \right)^2 \int_{w=0}^{w=\infty} \frac{16}{w^3 t_{rise}^2} \sin^4(wt_{rise}/2) dw$$

$$\sigma^2_{Vout}(t_\infty) = Q_f \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)} \right)^2 \frac{16}{4} \int_{w=0}^{w=\infty} \frac{8}{w^3 t_{rise}^3} \sin^4(wt_{rise}/2) d(wt_{rise}/2)$$

$$\sigma^2_{Vout}(t_\infty) = Q_f \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)} \right)^2 \frac{16}{4} lin2 = Q_f \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)} \right)^2 4lin2$$

$$\sigma^2_{Vout}(t_\infty) = Q_f \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)} \right)^2 2.77$$

$$\sigma^2_{Vout}(t_\infty) = \frac{Kf_2}{WLC_{ox}} \left(\frac{A\, c_{tot}}{(c_{tot} + A\, c_f)} \right)^2 2.77$$

if $t_{flat} = t_{rise}$

$$\sigma^2_{Vout}(t_\infty) = Q_f \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)} \right)^2 \int_{w=0}^{w=\infty} \frac{16}{w^3 t_{rise}^2} \sin^2(wt_{rise}/2) \sin^2(wt_{rise}) dw$$

$$\sigma^2_{Vout}(t_\infty) = Q_f \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)} \right)^2 \frac{16}{4} \int_{w=0}^{w=\infty} \frac{8}{w^3 t_{rise}^3} \sin^2(wt_{rise}/2) \sin^2(wt_{rise}) d(wt_{rise}/2)$$

$$\sigma^2_{Vout}(t_\infty) = Q_f \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)} \right)^2 \frac{16}{4} lin \frac{9}{4} 3^{\frac{1}{4}} = Q_f \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)} \right)^2 4.34$$

$$\sigma^2_{Vout}(t_\infty) = \frac{Kf_2}{WLC_{ox}} \left(\frac{A\, c_{tot}}{(c_{tot} + A\, c_f)} \right)^2 4.34$$

B. Thermal noise through charge amp, RC filter and a

Correlated Double Sampler in the time domain.

$$\sigma_{sum}^2 = \int_{t_p=t_0}^{t_p=t_{m1}} \pi Q_t \left(-h_r\left(t_{m1}-t_p\right)+h_r\left(t_{m2}-t_p\right)\right)^2 dt_p + \int_{t_p=t_{m1}}^{t_p=t_{m2}} \pi Q_t \left(h_r\left(t_{m2}-t_p\right)\right)^2 dt_p$$

$$h_r\left(t_m - t_p\right) = \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)\frac{1}{\tau} e^{-\frac{t_m - t_p}{\tau}}$$

$$\sigma_{sum}^2 = \int_{t_p=t_0}^{t_p=t_{m1}} \pi Q_t \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)^2 \frac{1}{\tau^2}\left(e^{-\frac{t_{m2}-t_p}{\tau}} - e^{-\frac{t_{m1}-t_p}{\tau}}\right)^2 dt_p$$

$$+ \int_{t_p=t_{m1}}^{t_p=t_{m2}} \pi Q_t \left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)^2 \frac{1}{\tau^2}\left(e^{-\frac{t_{m2}-t_p}{\tau}}\right)^2 dt_p$$

$$\sigma_{sum}^2 = \frac{\pi Q_t}{\tau^2}\left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)^2 \left(\int_{t_p=t_0}^{t_p=t_{m1}}\left(e^{-2\frac{t_{m1}-t_p}{\tau}} - 2e^{-\frac{t_{m1}-t_p+t_{m2}-t_p}{\tau}} + e^{-2\frac{t_{m2}-t_p}{\tau}}\right)dt_p + \int_{t_p=t_{m1}}^{t_p=t_{m2}} e^{-2\frac{t_{m2}-t_p}{\tau}} dt_p\right)$$

$$\sigma_{sum}^2 = \frac{\pi Q_t}{\tau^2}\left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)^2 \left(\left[\frac{\tau}{2}e^{-2\frac{t_{m1}-t_p}{\tau}} - \tau e^{-\frac{t_{m1}-t_p+t_{m2}-t_p}{\tau}} + \frac{\tau}{2}e^{-2\frac{t_{m2}-t_p}{\tau}}\right]_{t_o}^{t_{m1}} + \left[\frac{\tau}{2}e^{-2\frac{t_{m2}-t_p}{\tau}}\right]_{t_{m1}}^{t_{m2}}\right)$$

$$\sigma_{sum}^2 = \frac{\pi Q_t}{\tau^2}\left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)^2 \left(\frac{\tau}{2} - \tau e^{-\frac{t_{12}}{\tau}} + \frac{\tau}{2}e^{-2\frac{t_{12}}{\tau}} - \frac{\tau}{2}e^{-2\frac{t_{01}}{\tau}} + \tau e^{-\frac{t_{01}+t_{02}}{\tau}} - \frac{\tau}{2}e^{-2\frac{t_{02}}{\tau}}\right)$$

$$+ \frac{\pi Q_t}{\tau^2}\left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)^2 \left(\frac{\tau}{2} - \frac{\tau}{2}e^{-2\frac{t_{12}}{\tau}}\right)$$

$$\sigma_{sum}^2\left(t_\infty\right) = \frac{\pi Q_t}{\tau}\left(\frac{A\, c_{tot}}{gm\left(c_{tot} + A\, c_f\right)}\right)^2 \left(1 - e^{-\frac{t_{12}}{\tau}}\right)$$

$$\sigma_{sum}^2\left(t_\infty\right) = \frac{2kTr}{\tau}\left(\frac{A\, c_{tot}}{\left(c_{tot} + A\, c_f\right)}\right)^2 \left(1 - e^{-\frac{t_{12}}{\tau}}\right)$$

B. Thermal noise of a charge amp and RC filter followed by a Correlated Double Sampler in the frequency domain.

$$\sigma_{sum}^2(t_\infty) = \int_0^\infty Q_t \left|H_r(jw)\right|^2 dw$$

$$H_r(s) = \frac{A\,c_{tot}}{gm\left(c_{tot} + A\,c_f\right)} \frac{1}{1+s\tau}\left(1 - e^{-st_{12}}\right)$$

$$\left|H_r(jw)\right|^2 = \left(\frac{A\,c_{tot}}{gm\left(c_{tot} + A\,c_f\right)}\right)^2 \frac{2\left(1-\cos wt_{12}\right)}{\left(1+w^2\tau^2\right)}$$

$$\sigma_{sum}^2(t_\infty) = Q_t\left(\frac{A\,c_{tot}}{gm\left(c_{tot} + A\,c_f\right)}\right)^2 \int_0^\infty \frac{2\left(1-\cos wt_{12}\right)}{1+(w\tau)^2} dw$$

$$\sigma_{sum}^2(t_\infty) = Q_t\left(\frac{A\,c_{tot}}{gm\left(c_{tot} + A\,c_f\right)}\right)^2 \left(\left[\frac{2}{\tau}\tan^{-1} w\tau\right]_0^\infty - \frac{\pi}{\tau} e^{-\frac{t_{12}}{\tau}}\right)$$

$$\sigma_{sum}^2(t_\infty) = Q_t\left(\frac{A\,c_{tot}}{gm\left(c_{tot} + A\,c_f\right)}\right)^2 \frac{\pi}{\tau}\left(1 - e^{-\frac{t_{12}}{\tau}}\right)$$

$$\sigma_{sum}^2(t_\infty) = Q_t\left(\frac{A\,c_{tot}}{gm\left(c_{tot} + A\,c_f\right)}\right)^2 \left(1 - e^{-\frac{t_{12}}{\tau}}\right)$$

$$\sigma_{sum}^2(t_\infty) = \frac{2kTr}{\tau}\left(\frac{A\,c_{tot}}{\left(c_{tot} + A\,c_f\right)}\right)^2 \left(1 - e^{-\frac{t_{12}}{\tau}}\right)$$

B. Shot noise through charge amp, RC filter and a

Correlated Double Sampler in the time domain.

$$\sigma_{sum}^2 = \int_{t_p=t_0}^{t_p=t_{m1}} \pi Q_s\left(-h\left(t_{m1}-t_p\right)+h\left(t_{m2}-t_p\right)\right)^2 dt_p + \int_{t_p=t_{m1}}^{t_p=t_{m2}} \pi Q_s\left(h\left(t_{m2}-t_p\right)\right)^2 dt_p$$

$$h_i\left(t_m-t_p\right)=\frac{A}{c_{tot}+A\ c_f}\left(1-e^{-\frac{t_m-t_p}{\tau}}\right)$$

$$\sigma_{sum}^2 = \int_{t_p=t_0}^{t_p=t_{m1}} \pi Q_s\left(\left(\frac{A}{c_{tot}+A\ c_f}\right)^2\left(e^{-\frac{t_{m1}-t_p}{\tau}}-e^{-\frac{t_{m2}-t_p}{\tau}}\right)\right)^2 dt_p$$

$$+\int_{t_p=t_{m1}}^{t_p=t_{m2}} \pi Q_s\left(\left(\frac{A}{c_{tot}+A\ c_f}\right)^2\left(1-e^{-\frac{t_{m2}-t_p}{\tau}}\right)\right)^2 dt_p$$

$$\sigma_{sum}^2 = \pi Q_s\left(\frac{A}{c_{tot}+A\ c_f}\right)^2\int_{t_p=t_0}^{t_p=t_{m1}}\left(e^{-2\frac{t_{m1}-t_p}{\tau}}-2e^{-\frac{t_{m1}-t_p+t_{m2}-t_p}{\tau}}+e^{-2\frac{t_{m2}-t_p}{\tau}}\right)dt_p$$

$$+Q_s\left(\frac{A}{c_{tot}+A\ c_f}\right)^2\int_{t_p=t_{m1}}^{t_p=t_{m2}}\left(1-2e^{-\frac{t_{m2}-t_p}{\tau}}+e^{-2\frac{t_{m2}-t_p}{\tau}}\right)dt_p$$

$$\sigma_{sum}^2 = \pi Q_s\left(\frac{A}{c_{tot}+A\ c_f}\right)^2\left[\frac{\tau}{2}e^{-2\frac{t_{m1}-t_p}{\tau}}-\tau e^{-\frac{t_{m1}-t_p+t_{m2}-t_p}{\tau}}+\frac{\tau}{2}e^{-2\frac{t_{m2}-t_p}{\tau}}\right]_{t_0}^{t_{m1}}$$

$$+Q_s\left(\frac{A}{c_{tot}+A\ c_f}\right)^2\left[\mathrm{t_p}-2\tau e^{-\frac{t_{m2}-t_p}{\tau}}+\frac{\tau}{2}e^{-2\frac{t_{m2}-t_p}{\tau}}\right]_{\mathrm{t_{m1}}}^{t_{m2}}$$

$$\sigma_{sum}^2 = \pi Q_s\left(\frac{A}{c_{tot}+A\ c_f}\right)^2\left(\frac{\tau}{2}-\tau e^{-\frac{t_{12}}{\tau}}+\frac{\tau}{2}e^{-2\frac{t_{12}}{\tau}}-\frac{\tau}{2}e^{-2\frac{t_{01}}{\tau}}+\tau e^{-\frac{t_{01}+t_{02}}{\tau}}-\frac{\tau}{2}e^{-2\frac{t_{02}}{\tau}}\right)$$

$$+Q_s\left(\frac{A}{c_{tot}+A\ c_f}\right)^2\left(t_{12}-2\tau+\frac{\tau}{2}+2\tau e^{-\frac{t_{12}}{\tau}}-\frac{\tau}{2}e^{-2\frac{t_{12}}{\tau}}\right)$$

$$\sigma_{sum}^2\left(t_\infty\right)=\pi Q_s\left(\frac{A}{c_{tot}+A\ c_f}\right)^2\left(t_{12}-\tau+\tau e^{-\frac{t_{12}}{\tau}}\right)$$

$$\sigma_{sum}^2\left(t_\infty\right)=iq\left(\frac{A}{c_{tot}+A\ c_f}\right)^2\left(t_{12}-\tau+\tau e^{-\frac{t_{12}}{\tau}}\right)$$

B. Shot noise of a charge amplifier and RC filter and a Correlated Double Sampler in the frequency domain.

$$\sigma_{sum}^2(t_\infty) = \int_0^\infty Q_s \left|H_i(jw)\right|^2 dw$$

$$H_i(s) = \left(\frac{A}{c_{tot} + A\,c_f}\right) \frac{1}{s(1+s\tau)}\left(1 - e^{-st_{12}}\right)$$

$$\sigma_{sum}^2(t_\infty) = \int_0^\infty Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{2\left(1-\cos wt_{12}\right)}{w^2\left(1+(w\tau)^2\right)} dw$$

$$\sigma_{sum}^2(t_\infty) = 2\tau^2 Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \int_0^\infty \left(\frac{1-\cos wt_{12}}{(w\tau)^2} - \frac{1}{1+(w\tau)^2} + \frac{\cos wt_{12}}{1+(w\tau)^2}\right) dw$$

$$\sigma_{sum}^2(t_\infty) = 2\tau^2 Q_s \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \left(\frac{\pi t_{12}}{2\tau^2} - \frac{\pi}{2\tau} + \frac{\pi}{2\tau} e^{-\frac{t_{12}}{\tau}}\right)$$

$$\sigma_{sum}^2(t_\infty) = iq \left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \left(t_{12} - \tau + \tau e^{-\frac{t_{12}}{\tau}}\right)$$

B. Flicker noise of charge amplifier with an CR filter followed by a Correlated Double Sampler in the frequency domain.

$$\sigma^2_{sum}(t_\infty) = \int_0^\infty \frac{Q_f}{w} |H_r(jw)|^2 \, dw$$

$$H_r(s) = \frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)} \frac{1}{1+s\tau}(1 - e^{-st_{12}})$$

$$|H_r(jw)|^2 = \left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)}\right)^2 \frac{2(1-\cos wt_{12})}{(1+w^2\tau^2)}$$

$$\sigma^2_{sum}(t_\infty) = \int_0^\infty \frac{Q_f}{w}\left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)}\right)^2 \frac{2(1-\cos wt_{12})}{(1+w\tau^2)} dw$$

$$= Q_f\left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)}\right)^2 \int_0^\infty \frac{2(1-\cos wt_{12})}{w(1+(w\tau)^2)} dw$$

Substituting $x = w\tau$

$$\sigma^2_{sum}(t_\infty) = Q_f\left(\frac{A\ c_{tot}}{gm(c_{tot} + A\ c_f)}\right)^2 \int_0^\infty \frac{2\left(1-\cos x\frac{t_{12}}{\tau}\right)}{x(1+x^2)} dx$$

$$\sigma^2_{sum}(t_\infty) = \frac{Kf_2}{WLCox}\left(\frac{A\ c_{tot}}{(c_{tot} + A\ c_f)}\right)^2 (\text{Factor1})$$

and

$$ENC^2 = Q_f c_{tot}^2 \frac{1}{\left(1-e^{-\frac{t_{12}}{\tau}}\right)^2} \int_0^\infty \frac{2(1-\cos wt_{12})}{w(1+(w\tau)^2)} dw$$

$$ENC^2 = \frac{Kf_2}{WLC_{ox}} c_{tot}^2 (\text{Factor2})$$

B. Flicker noise of charge amplifier with an RC filter followed by a Correlated Double Sampler in the frequency domain (cont).

Factor 1 and Factor 2 can be tabulated for different $\left(\frac{t_{12}}{\tau}\right)$ values:-

$\left(\frac{t_{12}}{\tau}\right)$	0.5	1	2	3	4	6	8	10
Factor 1	0.42	1.05	2.23	3.12	3.77	4.67	5.28	5.74
Factor 2	2.68	2.64	2.98	3.45	3.92	4.69	5.28	5.74

$\left(\frac{t_{12}}{\tau}\right)$	12	16	32	50	100	1000	10000
Factor 1	6.11	6.69	8.08	8.98	10.4	15.0	20.1
Factor 2	6.11	6.69	8.08	8.98	10.4	15.0	20.1

Best ENC at $\frac{t_{12}}{\tau}$ = 0.8 when Factor 2 = 2.62

B. Thermal noise of charge amp and CR-RC filter with triple sampled deconvolution.

$$\sigma_{sum}^2 = \int_{t_p=t_0}^{t_p=t_{m1}} \pi Q_t \left(b_1 h_r\left(t_{m1}-t_p\right)+b_2 h_r\left(t_{m2}-t_p\right)+b_3 h_r\left(t_{m3}-t_p\right)\right)^2 dt$$

$$+\int_{t_p=t_{m1}}^{t_p=t_{m2}} \pi Q_t \left(b_2 h_r\left(t_{m2}-t_p\right)+b_3 h_r\left(t_{m3}-t_p\right)\right)^2 dt_p + \int_{t_p=t_{m2}}^{t_p=t_{m3}} \pi Q_t \left(b_3 h_r\left(t_{m3}-t_p\right)\right)^2 dt_p$$

$$h_r\left(t_{m1}-t_p\right)=\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\left(\frac{1}{\tau}-\frac{\left(t_{m1}-t_p\right)}{(\tau)^2}\right)e^{-\frac{\left(t_{m1}-t_p\right)}{\tau}}$$

$$h_r\left(t_{m1}-t_p\right)=\frac{-1}{(\tau)^2}\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\left(\left(t_{m1}-t_p\right)-\tau\right)e^{-\frac{\left(t_{m1}-t_p\right)}{\tau}}$$

and $t_{m3}-t_{m2}=t_{m2}-t_{m1}=\Delta$ the multiple sample formula gives

$$\sigma_{\mathrm{sum}}^2 = \int_{t_p=t_0}^{t_p=t_{m1}} \pi Q_t \left(b_1 h_r\left(t_{m1}-t_p\right)+b_2 h_r\left(t_{m1}+\Delta-t_p\right)+b_3 h_r\left(t_{m1}+2\Delta-t_p\right)\right)^2 dt_p$$

$$+\int_{t_p=t_{m1}}^{t_p=t_{m1}+\Delta} \pi Q_t \left(b_2 h_r\left(t_{m1}+\Delta-t_p\right)+b_3 h_r\left(t_{m1}+2\Delta-t_p\right)\right)^2 dt_p$$

$$+\int_{t_p=t_{m1}+\Delta}^{t_p=t_{m1}+2\Delta} \pi Q_t \left(b_3 h_r\left(t_{m1}+2\Delta-t_p\right)\right)^2 dt_p$$

substituting $-t_p' = t_{m1}-t_p \text{or} t_p' = -t_{m1}+t_p \text{or} -t_p = -t_{m1}-t_p'$

$$\sigma_{\mathrm{sum}}^2 = \int_{t_p'=t_{01}}^{t_p'=0} \pi Q_t \left(b_1 h_r\left(-t_p'\right)+b_2 h_r\left(\Delta-t_p'\right)+b_3 h_r\left(2\Delta-t_p'\right)\right)^2 dt_p'$$

$$+\int_{t_p'=0}^{t_p'=\Delta} \pi Q_t \left(b_2 h_r\left(\Delta-t_p'\right)+b_3 h_r\left(2\Delta-t_p'\right)\right)^2 dt_p'$$

$$+\int_{t_p'=\Delta}^{t_p'=2\Delta} \pi Q_t \left(b_3 h_r\left(2\Delta-t_p'\right)\right)^2 dt_p'$$

$$\frac{\sigma_{sum}^2}{\frac{\pi Q_t}{\tau^4}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2} = \int_{t_p'=t_{01}}^{t_p'=0}\left(b_1\left(-t_p'-\tau\right)e^{-\frac{\left(-t_p'\right)}{\tau}}+b_2\left(\Delta-t_p'-\tau\right)e^{-\frac{\left(\Delta-t_p'\right)}{\tau}}+b_3\left(2\Delta-t_p'-\tau\right)e^{-\frac{\left(2\Delta-t_p'\right)}{\tau}}\right)^2 dt_p'$$

$$+\int_{t_p'=0}^{t_p'=\Delta}\left(b_2\left(\Delta-t_p'-\tau\right)e^{-\frac{\left(\Delta-t_p'\right)}{\tau}}+b_3\left(2\Delta-t_p'-\tau\right)e^{-\frac{\left(2\Delta-t_p'\right)}{\tau}}\right)^2 dt_p'$$

B. Thermal noise of charge amp and CR-RC filter with triple sampled deconvolution (cont).

$$+\int_{t_p'=\Delta}^{t_p'=2\Delta}\left(b_3(2\Delta-t_p'-\tau)e^{-\frac{(2\Delta-t_p')}{\tau}}\right)^2 dt_p'$$

$$\text{if} b_1=\frac{\tau}{\Delta}e^{-\left(\frac{\tau+\Delta}{\tau}\right)}, b_2=-\frac{2\tau}{\Delta}e^{-\left(\frac{\tau}{\tau}\right)}, b_3=\frac{\tau}{\Delta}e^{-\left(\frac{\tau-\Delta}{\tau}\right)}$$

$$\frac{\sigma_{sum}^2}{\frac{\pi Q_t}{\tau^4}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2}$$

$$=\int_{t_p'=t_{01}}^{t_p'=0}\left(\frac{\tau}{\Delta}e^{-\left(\frac{\tau+\Delta}{\tau}\right)}(-t_p'-\tau)e^{-\frac{(-t')}{\tau}}-\frac{2\tau}{\Delta}e^{-\left(\frac{\tau}{\tau}\right)}(\Delta-t_p'-\tau)e^{-\frac{(\Delta-t_p')}{\tau}}+\frac{\tau}{\Delta}e^{-\left(\frac{\tau-\Delta}{\tau}\right)}(2\Delta-t_p'-\tau)e^{-\frac{(2\Delta-t_p')}{\tau}}\right)^2 dt_p'$$

$$+\int_{t_p'=0}^{t_p'=\Delta}\left(-\frac{2\tau}{\Delta}e^{-\left(\frac{\tau}{\tau}\right)}(\Delta-t_p'-\tau)e^{-\frac{(\Delta-t_p')}{\tau}}+\frac{\tau}{\Delta}e^{-\left(\frac{\tau-\Delta}{\tau}\right)}(2\Delta-t_p'-\tau)e^{-\frac{(2\Delta-t_p')}{\tau}}\right)^2 dt_p'$$

$$+\int_{t_p'=\Delta}^{t_p'=2\Delta}\left(\frac{\tau}{\Delta}e^{-\left(\frac{\tau-\Delta}{\tau}\right)}(2\Delta-t_p'-\tau)e^{-\frac{(2\Delta-t_p')}{\tau}}\right)^2 dt_p'$$

$$\frac{\sigma_{sum}^2}{\frac{\pi Q_t}{\tau^2\Delta^2}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2}=\int_{t_p'=t_{01}}^{t_p'=0}\left((-t_p'-\tau)-2(\Delta-t'-\tau)+(2\Delta-t_p'-\tau)\right)^2 e^{-2\frac{(\tau+\Delta-t_p')}{\tau}}dt_p'$$

$$+\int_{t_p'=0}^{t_p'=\Delta}\left(-2(\Delta-t_p'-\tau)+(2\Delta-t_p'-\tau)\right)^2 e^{-2\frac{(\tau+\Delta-t_p')}{\tau}}dt_p'$$

$$+\int_{t_p'=\Delta}^{t_p'=2\Delta}(2\Delta-t_p'-\tau)^2 e^{-2\frac{(\tau+\Delta-t_p')}{\tau}}dt_p'$$

B. Thermal noise of charge amp and CR-RC filter with triple sampled deconvolution (cont).

$$\frac{\sigma_{sum}^2}{\frac{\pi Q_t}{\tau^2\Delta^2}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2} = 0 + \int_{t_p'=0}^{t_p'=\Delta} \left(t_p'+\tau\right)^2 e^{-2\frac{\left(\tau+\Delta-t_p'\right)}{\tau}}\,dt_p'$$

$$+\int_{t_p'=\Delta}^{t_p'=2\Delta} \left(4\Delta^2 - 2\left(t_p'+\tau\right)2\Delta + \left(t_p'+\tau\right)^2\right) e^{-2\frac{\left(\tau+\Delta-t_p'\right)}{\tau}}\,dt_p'$$

$$\frac{\sigma_{sum}^2}{\frac{\pi Q_t}{\tau^2\Delta^2}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2} = \int_{t_p'=0}^{t_p'=2\Delta} \left(t_p'+\tau\right)^2 e^{-2\frac{\left(\tau+\Delta-t_p'\right)}{\tau}}\,dt_p'$$

$$+\int_{t_p'=\Delta}^{t_p'=2\Delta} \left(4\Delta^2 - 2\left(t_p'+\tau\right)2\Delta\right) e^{-2\frac{\left(\tau+\Delta-t_p'\right)}{\tau}}\,dt_p'$$

$$\frac{\sigma_{sum}^2}{\frac{\pi Q_t}{\tau^2\Delta^2}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2} = \int_{t_p'=0}^{t_p'=2\Delta} \left(\left(t_p'\right)^2 + 2t_p'\tau + (\tau)^2\right) e^{-2\frac{\left(\tau+\Delta-t_p'\right)}{\tau}}\,dt_p'$$

$$+\int_{t_p'=\Delta}^{t_p'=2\Delta} 4\Delta\left(-t_p'-\tau+\Delta\right) e^{-2\frac{\left(\tau+\Delta-t_p'\right)}{\tau}}\,dt_p'$$

$$\frac{\sigma_{sum}^2}{\frac{\pi Q_t}{\tau^2\Delta^2}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2} = \left[\left(\frac{\left(t_p'\right)^2\tau}{2} - \frac{2t_p'\tau^2}{4} + \frac{2\tau^3}{8} + \frac{2t_p'\tau^2}{2} - \frac{2\tau^3}{4} + \frac{\tau^3}{2}\right) e^{-2\frac{\left(\tau+\Delta-t_p'\right)}{\tau}}\right]_0^{2\Delta}$$

$$-\left[4\Delta\left(\frac{t_p'\tau}{2} - \frac{\tau^2}{4} + \frac{(\tau-\Delta)\tau}{2}\right) e^{-2\frac{\left(\tau+\Delta-t_p'\right)}{\tau}}\right]_\Delta^{2\Delta}$$

$$\frac{\sigma_{sum}^2}{\frac{\pi Q_t}{\tau^2\Delta^2}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2} = \left[\left(\frac{\left(t_p'\right)^2\tau}{2} + \frac{2t_p'\tau^2}{4} + \frac{2\tau^3}{8}\right) e^{-2\frac{\left(\tau+\Delta-t_p'\right)}{\tau}}\right]_0^{2\Delta}$$

$$-\left[4\Delta\left(\frac{t_p'\tau}{2} - \frac{\tau^2}{4} + \frac{\left(\tau^2-\Delta\right)\tau}{2}\right) e^{-2\frac{\left(\tau+\Delta-t_p'\right)}{\tau}}\right]_\Delta^{2\Delta}$$

B. Thermal noise of charge amp and CR-RC filter with triple sampled deconvolution (cont).

$$\frac{\sigma_{sum}^2}{\frac{\pi Q_t}{r\tau^2\Delta^2}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2}=\left(\frac{(2\Delta)^2\tau}{2}+\frac{22\Delta\tau}{4}+\frac{2\tau^3}{8}\right)e^{-2\frac{(\tau+\Delta-2\Delta)}{\tau}}-\left(\frac{2\tau^3}{8}\right)e^{-2\frac{(\tau+\Delta)}{\tau}}$$

$$-4\Delta\left(\frac{2\Delta\tau}{2}+\frac{\tau^2}{4}-\frac{\Delta\tau}{2}\right)e^{-2\frac{(\tau+\Delta-2\Delta)}{\tau}}+4\Delta\left(\frac{\Delta\tau}{2}+\frac{\tau^2}{4}-\frac{\Delta\tau}{2}\right)e^{-2\frac{(\tau)}{\tau}}$$

$$\frac{\sigma_{sum}^2}{\frac{\pi Q_t}{r\tau^2\Delta^2}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2}=\left(\frac{(2\Delta)^2\tau}{2}+\frac{4\Delta\tau^2}{4}+\frac{2\tau^3}{8}\right)e^{-2\frac{(\tau-\Delta)}{\tau}}-\left(\frac{2\tau^3}{8}\right)e^{-2\frac{(\tau+\Delta)}{\tau}}$$

$$-\left(\frac{4\Delta^2\tau}{2}+\frac{4\Delta\tau^2}{4}\right)e^{-2\frac{(\tau-\Delta)}{\tau}}+4\Delta\left(\frac{\tau^2}{4}\right)e^{-2\frac{(\tau)}{\tau}}$$

$$\frac{\sigma_{sum}^2}{\frac{\pi Q_t}{r\tau^2\Delta^2}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2}=\left(\frac{2\tau^3}{8}\right)e^{-2\frac{(\tau-\Delta)}{\tau}}-\left(\frac{2\tau^3}{8}\right)e^{-2\frac{\tau+\Delta}{\tau}}+4\Delta\left(\frac{\tau}{4}\right)e^{-2\frac{(\tau)}{\tau}}$$

$$\sigma_{sum}^2=\pi Q_t\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2\frac{1}{4\Delta}\left(\left(\frac{\tau}{\Delta}\right)e^{-2\frac{(\tau-\Delta)}{\tau}}+4e^{-2}-\left(\frac{\tau}{\Delta}\right)e^{-2\frac{\tau+\Delta}{\tau}}\right)$$

$$\sigma_{sum}^2=\pi Q_t\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2\frac{1}{4\Delta}\left(-\left(\frac{\tau}{\Delta}\right)e^{-2\frac{(\tau+\Delta)}{\tau}}+4e^{-2}+\left(\frac{\tau}{\Delta}\right)e^{-2\frac{\tau-\Delta}{\tau}}\right)$$

if $\tau=2\Delta$

$$\sigma_{sum}^2=2kTr\left(\frac{A\,c_{tot}}{\left(c_f+A\,c_{tot}\right)}\right)^2\frac{1}{2\tau}\left(-2e^{-3}+4e^{-2}+2e^{-1}\right)$$

B. Thermal noise of charge amp and CR-RC filter with triple sampled in the frequency domain.

$$o_{\text{sum}}^2(t_\infty) = \int_0^\infty \mathrm{Q}_t |H_r(jw)|^2\, dw$$

if $\tau = 2\Delta$

$$|H_r(jw)|^2 = \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)}\right)^2 \frac{w^2\tau^2}{(1+w^2\tau^2)^2}\frac{4}{e^2}\left(e^{\frac{1}{2}} + e^{-\frac{1}{2}} - 2\cos\frac{(w\tau)}{2}\right)^2$$

$$o_{\text{sum}}^2(t_\infty) = \mathrm{Q}_t \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)}\right)^2 \int_0^\infty \frac{w^2\tau^2}{(1+w^2\tau^2)^2}\frac{4}{e^2}\left(e^{\frac{1}{2}} + e^{-\frac{1}{2}} - 2\cos\frac{(w\tau)}{2}\right)^2 dw$$

$$o_{\text{sum}}^2(t_\infty) = \frac{\mathrm{Q}_t}{\tau} \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)}\right)^2 \int_0^\infty \frac{w^2\tau^2}{(1+w^2\tau^2)^2}\frac{4}{e^2}\left(e^{\frac{1}{2}} + e^{-\frac{1}{2}} - 2\cos\frac{(w\tau)}{2}\right)^2 dw\tau$$

$$o_{\text{sum}}^2(t_\infty) = \frac{4\mathrm{Q}_t}{e^2\tau} \left(\frac{A\, c_{tot}}{gm(c_{tot} + A\, c_f)}\right)^2 \int_0^\infty \frac{x^2}{(1+x^2)^2}\left(e^{\frac{1}{2}} + e^{-\frac{1}{2}} - 2\cos\frac{x}{2}\right)^2 dx$$

$$o_{\text{sum}}^2(t_\infty) = \left(\frac{A\, c_{tot}}{(c_{tot} + A\, c_f)}\right)^2 \frac{8.7}{e^2}\frac{kTr}{\tau}$$

B. Shot noise of charge amp and CR-RC filter with triple sampled deconvolution.

$$\sigma_{sum}^2 = \int_{t_p=t_0}^{t_p=t_{m1}} \pi Q_s \left(b_1 h_i\left(t_{m1}-t_p\right)+b_2 h_i\left(t_{m2}-t_p\right)+b_3 h_i\left(t_{m3}-t_p\right)\right)^2 dt_p$$

$$+\int_{t_p=t_{m1}}^{t_p=t_{m2}} \pi Q_s \left(b_2 h_i\left(t_{m2}-t_p\right)+b_3 h_i\left(t_{m3}-t_p\right)\right)^2 dt_p + \int_{t_p=t_{m2}}^{t_p=t_{m3}} \pi Q_s \left(b_3 h_i\left(t_{m3}-t_p\right)\right)^2 dt_p$$

$$h_i\left(t_{m1}-t_p\right)=\frac{A}{c_{tot}+A\,c_f}\frac{\left(t_{m1}-t_p\right)}{\tau}e^{-\frac{\left(t_{m1}-t_p\right)}{\tau}}$$

with $t_{m3}-t_{m2}=t_{m2}-t_{m1}=t_{m1}-t_0=\Delta$ the multiple sample formula gives

$$\sigma_{sum}^2 = \int_{t_p=t_0}^{t_p=t_{m1}} \pi Q_s \left(b_1 h_i\left(t_{m1}-t_p\right)+b_2 h_i\left(t_{m1}+\Delta-t_p\right)+b_3 h_i\left(t_{m1}+2\Delta-t_p\right)\right)^2 dt_p$$

$$+\int_{t_p=t_{m1}}^{t_p=t_{m1}+\Delta} \pi Q_s \left(b_2 h_i\left(t_{m1}+\Delta-t_p\right)+b_3 h_i\left(t_{m1}+2\Delta-t_p\right)\right)^2 dt_p$$

$$+\int_{t_p=t_{m1}+\Delta}^{t_p=t_{m1}+2\Delta} \pi Q_s \left(b_3 h_i\left(t_{m1}+2\Delta-t_p\right)\right)^2 dt_p$$

substituting $-t_p' = t_{m1}-t_p \quad or \quad t_p' = t_{m1}+t_p \quad or \quad -t_p = -t_{m1}-t_p'$

$$\sigma_{sum}^2 = \int_{t_p'=t_{01}}^{t_p'=0} \pi Q_s \left(b_1 h_i\left(-t_p'\right)+b_2 h_i\left(\Delta-t_p'\right)+b_3 h_i\left(2\Delta-t_p'\right)\right)^2 dt_p'$$

$$+\int_{t_p'=0}^{t_p'=\Delta} \pi Q_s \left(b_2 h_i\left(\Delta-t_p'\right)+b_3 h_i\left(2\Delta-t_p'\right)\right)^2 dt_p'$$

$$+\int_{t_p'=\Delta}^{t_p'=2\Delta} \pi Q_s \left(b_3 h_i\left(2\Delta-t_p'\right)\right)^2 dt_p'$$

$$\frac{\sigma_{sum}^2}{\left(\frac{A}{c_{tot}+A\,c_f}\right)^2\frac{\pi Q_s}{\tau^2}} = \int_{t_p'=t_{01}}^{t_p'=0}\left(b_1\left(-t_p'\right)e^{-\frac{\left(-t_p'\right)}{\tau}}+b_2\left(\Delta-t_p'\right)e^{-\frac{\left(\Delta-t_p'\right)}{\tau}}+b_3\left(2\Delta-t_p'\right)e^{-\frac{\left(2\Delta-t_p'\right)}{\tau}}\right)^2 dt_p'$$

$$+\int_{t_p'=0}^{t_p'=\Delta}\left(b_2\left(\Delta-t_p'\right)e^{-\frac{\left(\Delta-t_p'\right)}{\tau}}+b_3\left(2\Delta-t_p'\right)e^{-\frac{\left(2\Delta-t_p'\right)}{\tau}}\right)^2 dt_p'$$

$$+\int_{t_p'=\Delta}^{t_p'=2\Delta}\left(b_3\left(2\Delta-t_p'\right)e^{-\frac{\left(2\Delta-t_p'\right)}{\tau}}\right)^2 dt_p'$$

B. Shot noise of charge amp and CR-RC filter with triple sampled deconvolution (cont).

if $b_1 = \frac{\tau}{\Delta} e^{-\left(\frac{\tau+\Delta}{\tau}\right)}, b_2 = -\frac{2\tau}{\Delta} e^{-\left(\frac{\tau}{\tau}\right)}, b_3 = \frac{\tau}{\Delta} e^{-\left(\frac{\tau-\Delta}{\tau}\right)}$

$$\frac{\sigma_{sum}^2}{\left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{\pi Q_s}{\tau^2}} =$$

$$\int_{t_p'=-t_{01}}^{t_p'=0} \left(\frac{\tau}{\Delta} e^{-\left(\frac{\tau+\Delta}{\tau}\right)} (-t_p') e^{-\frac{(-t_p')}{\tau}} - \frac{2\tau}{\Delta} e^{-\left(\frac{\tau}{\tau}\right)} (\Delta - t_p') e^{-\frac{(\Delta - t_p')}{\tau}} + \frac{\tau}{\Delta} e^{-\left(\frac{\tau-\Delta}{\tau}\right)} (2\Delta - t_p') e^{-\frac{(2\Delta - t_p')}{\tau}} \right)^2 dt_p'$$

$$+ \int_{t_p'=0}^{t_p'=\Delta} \left(-\frac{2\tau}{\Delta} e^{-\left(\frac{\tau}{\tau}\right)} (\Delta - t_p') e^{-\frac{(\Delta - t_p')}{\tau}} + \frac{\tau}{\Delta} e^{-\left(\frac{\tau-\Delta}{\tau}\right)} (2\Delta - t_p') e^{-\frac{(2\Delta - t_p')}{\tau}} \right)^2 dt_p'$$

$$+ \int_{t_p'=\Delta}^{t_p'=2\Delta} \left(\frac{\tau}{\Delta} e^{-\left(\frac{\tau-\Delta}{\tau}\right)} (2\Delta - t_p') e^{-\frac{(2\Delta - t_p')}{\tau}} \right)^2 dt_p'$$

$$\frac{\sigma_{\text{sum}}^2}{\left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{\pi Q_s}{\tau^2} \left(\frac{\tau}{\Delta}\right)^2} = \int_{t_p'=t_{01}}^{t_p'=0} \left((-t_p') - 2(\Delta - t_p') + (2\Delta - t_p') \right)^2 e^{-2\frac{(\tau+\Delta-t_p')}{\tau}} dt_p'$$

$$+ \int_{t_p'=0}^{t_p'=\Delta} \left(-2(\Delta - t_p') + (2\Delta - t_p') \right)^2 e^{-2\frac{(\tau+\Delta-t_p')}{\tau}} dt_p'$$

$$+ \int_{t_p'=\Delta}^{t_p'=2\Delta} (2\Delta - t_p')^2 e^{-2\frac{(\tau+\Delta-t_p')}{\tau}} dt_p'$$

$$\frac{\sigma_{\text{sum}}^2}{\left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{\pi Q_s}{\Delta^2}} = 0 + \int_{t_p'=0}^{t_p'=\Delta} (t_p')^2 e^{-2\frac{(\tau+\Delta-t_p')}{\tau}} dt_p'$$

$$+ \int_{t_p'=\Delta}^{t_p'=2\Delta} \left(4\Delta^2 - 2t_p'2\Delta + (t_p')2\right) e^{-2\frac{(\tau+\Delta-t_p')}{\tau}} dt_p'$$

$$\frac{\sigma_{\text{sum}}^2}{\left(\frac{A}{c_{tot} + A\,c_f}\right)^2 \frac{\pi Q_s}{\Delta^2}} = \left[\left(\frac{(t_p')^2 \tau}{2} - \frac{t_p'\tau^2}{2} + \frac{\tau^3}{4} \right) e^{-2\frac{(\tau+\Delta-t_p')}{\tau}} \right]_0^\Delta$$

B. Shot noise of charge amp and CR-RC filter with triple sampled deconvolution (cont).

$$+\left[\left(\frac{4\Delta^2\tau}{2}-\frac{2t_p'\tau 2\Delta}{2}+\frac{2\tau^2 2\Delta}{4}+\frac{(t_p')^2\tau}{2}-\frac{t_p'\tau^2}{2}+\frac{\tau^3}{4}\right)e^{-2\frac{(\tau+\Delta-t_p')}{\tau}}\right]_{\Delta}^{2\Delta}$$

$$\frac{\sigma_{sum}^2}{\left(\frac{A}{c_{tot}+A\,c_f}\right)^2\frac{\pi Q_s}{\Delta^2}}=\left(\frac{\Delta^2\tau}{2}-\frac{\Delta\tau^2}{2}+\frac{\tau^3}{4}\right)e^{-2}-\left(\frac{\tau^3}{4}\right)e^{-2\frac{(\tau+\Delta)}{\tau}}$$

$$+\left(\frac{4\Delta^2\tau}{2}-\frac{4\Delta\tau 2\Delta}{2}+\frac{2\tau^2\Delta}{4}+\frac{4\Delta^2\tau}{2}-\frac{2\Delta\tau^2}{2}+\frac{\tau^3}{4}\right)e^{-2\frac{(\tau-\Delta)}{\tau}}$$

$$-\left(\frac{4\Delta^2\tau}{2}-\frac{2\Delta\tau 2\Delta}{2}+\frac{2\tau^2 2\Delta}{4}+\frac{\Delta^2\tau}{2}-\frac{\Delta\tau^2}{2}+\frac{\tau^3}{4}\right)e^{-2}$$

$$\frac{\sigma_{sum}^2}{\left(\frac{A}{c_{tot}+A\,c_f}\right)^2\frac{\pi Q_s}{\Delta^2}}=-\left(\frac{\tau^3}{4}\right)e^{-2\frac{(\tau+\Delta)}{\tau}}+\left(\frac{\tau^3}{4}\right)e^{-2\frac{(\tau-\Delta)}{\tau}}-\left(\tau^2\Delta\right)e^{-2}$$

$$\sigma_{sum}^2=\left(\frac{A}{c_{tot}+A\,c_f}\right)^2\pi Q_s\frac{\tau^2}{4\Delta}\left(-\left(\frac{\tau}{\Delta}\right)e^{-2\frac{(\tau+\Delta)}{\tau}}-4e^{-2}+\left(\frac{\tau}{\Delta}\right)e^{-2\frac{(\tau-\Delta)}{\tau}}\right)$$

if $\tau=2\Delta$

$$\sigma_{sum}^2=\left(\frac{A}{c_{tot}+A\,c_f}\right)^2 iq\frac{2\tau}{4}\left(-2e^{-3}-4e^{-2}+2e^{-1}\right)$$

$$\sigma_{sum}^2=\left(\frac{A}{c_{tot}+A\,c_f}\right)^2\frac{iq}{e^2}\tau\left(-e^{-1}-2+e^{1}\right)$$

$$\sigma_{sum}^2=\left(\frac{A}{c_{tot}+A\,c_f}\right)^2\frac{iq}{e^2}\tau\ 0.35$$

B. Shot noise of charge amp and CR-RC filter with triple sampled deconvolution

$$o_{\text{sum}}^2(t_\infty) = \int_0^\infty Q_s |H_i(jw)|^2 \, dw$$

if $\tau = 2\Delta$

$$|H_i(jw)|^2 = \left(\frac{A}{(c_{tot} + A\,c_f)}\right)^2 \frac{\tau^2}{(1 + w^2\tau^2)^2} \frac{4}{e^2}\left(e^{\frac{1}{2}} + e^{-\frac{1}{2}} - 2\cos\frac{(w\tau)}{2}\right)^2$$

$$o_{\text{sum}}^2(t_\infty) = Q_s\left(\frac{A}{(c_{tot} + A\,c_f)}\right)^2 \int_0^\infty \frac{\tau^2}{(1 + w^2\tau^2)^2} \frac{4}{e^2}\left(e^{\frac{1}{2}} + e^{-\frac{1}{2}} - 2\cos\frac{(w\tau)}{2}\right)^2 dw$$

$$o_{\text{sum}}^2(t_\infty) = Q_s\left(\frac{A}{(c_{tot} + A\,c_f)}\right)^2 \int_0^\infty \frac{\tau}{(1 + w^2\tau^2)^2} \frac{4}{e^2}\left(e^{\frac{1}{2}} + e^{-\frac{1}{2}} - 2\cos\frac{(w\tau)}{2}\right)^2 dw\tau$$

$$o_{\text{sum}}^2(t_\infty) = \frac{4Q_s\tau}{e^2}\left(\frac{A}{(c_{tot} + A\,c_f)}\right)^2 \int_0^\infty \frac{1}{(1 + x^2)^2}\left(e^{\frac{1}{2}} + e^{-\frac{1}{2}} - 2\cos\frac{x}{2}\right)^2 dx$$

$$o_{\text{sum}}^2(t_\infty) = \left(\frac{A}{(c_{tot} + A\,c_f)}\right)^2 \frac{0.35}{e^2} iq\tau$$

B. Flicker noise of charge amp and CR-RC filter with triple sampled deconvolution.

$$\sigma^2_{sum}(t_\infty)=\int_0^\infty \frac{Q_f}{w}\left|H_r(jw)\right|^2 dw$$

$$b_1=\frac{\tau}{\Delta}e^{-\left(\frac{\tau+\Delta}{\tau}\right)}, b_2=-\frac{2\tau}{\Delta}e^{-\left(\frac{\tau}{\tau}\right)}, b_3=\frac{\tau}{\Delta}e^{-\left(\frac{\tau-\Delta}{\tau}\right)}$$

$$\frac{H_r(s)}{\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)}=\frac{s\tau}{(1+s\tau)^2}\left(\frac{\tau}{\Delta}e^{-\left(\frac{\tau+\Delta}{\tau}\right)}e^{s\Delta}-\frac{2\tau}{\Delta}e^{-\left(\frac{\tau}{\tau}\right)}+\frac{\tau}{\Delta}e^{-\left(\frac{\tau-\Delta}{\tau}\right)}e^{-s\Delta}\right)$$

$$\frac{\left|H_r(jw)\right|^2}{\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2}=\frac{w^2\tau^2}{(1+w^2\tau^2)^2}\left(\frac{\tau}{\Delta}\right)^2 e^{-2}\left(e^{-\left(\frac{\Delta}{\tau}\right)}e^{jw\Delta}-2+e^{-\left(\frac{-\Delta}{\tau}\right)}e^{-jw\Delta}\right)\left(e^{-\left(\frac{\Delta}{\tau}\right)}e^{-jw\Delta}-2+e^{-\left(\frac{-\Delta}{\tau}\right)}e^{jw\Delta}\right)$$

$$=\frac{w^2\tau^2}{(1+w^2\tau^2)^2}\left(\frac{\tau}{\Delta e}\right)^2$$

$$\left(e^{-\left(\frac{2\Delta}{\tau}\right)}-2e^{-\left(\frac{\Delta}{\tau}\right)}e^{jw\Delta}+e^{2jw\Delta}-2e^{-\left(\frac{\Delta}{\tau}\right)}e^{-jw\Delta}+4-2e^{-\left(\frac{-\Delta}{\tau}\right)}e^{jw\Delta}+e^{-2jw\Delta}-2e^{\left(\frac{\Delta}{\tau}\right)}e^{-jw\Delta}+e^{\left(\frac{2\Delta}{\tau}\right)}\right)$$

$$=\frac{w^2\tau^2}{(1+w^2\tau^2)^2}\left(\frac{\tau}{e\Delta}\right)^2\left(4+e^{\left(\frac{2\Delta}{\tau}\right)}+e^{-\left(\frac{2\Delta}{\tau}\right)}-2\left(e^{\left(\frac{\Delta}{\tau}\right)}+e^{-\left(\frac{\Delta}{\tau}\right)}\right)\left(e^{jw\Delta}+e^{-jw\Delta}\right)+e^{2jw\Delta}+e^{-2jw\Delta}\right)$$

$$=\frac{w^2\tau^2}{(1+w^2\tau^2)^2}\left(\frac{\tau}{e\Delta}\right)^2\left(\left(e^{\left(\frac{\Delta}{\tau}\right)}+e^{-\left(\frac{\Delta}{\tau}\right)}\right)^2-4\left(e^{\left(\frac{\Delta}{\tau}\right)}+e^{-\left(\frac{\Delta}{\tau}\right)}\right)(\cos w\Delta)+4(\cos w\Delta)^2\right)$$

$$=\frac{w^2\tau^2}{(1+w^2\tau^2)^2}\left(\frac{\tau}{e\Delta}\right)^2\left(e^{\left(\frac{\Delta}{\tau}\right)}+e^{-\left(\frac{\Delta}{\tau}\right)}-2\cos w\Delta\right)^2$$

$$\sigma^2_{sum}(t_\infty)=\int_0^\infty \frac{Q_f}{w}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2\frac{w^2\tau^2}{(1+w^2\tau^2)^2}\left(\frac{\tau}{e\Delta}\right)^2\left(e^{\left(\frac{\Delta}{\tau}\right)}+e^{-\left(\frac{\Delta}{\tau}\right)}-2\cos w\Delta\right)^2 dw$$

substituting $x=w\tau$

$$\sigma^2_{sum}(t_\infty)=Q_f\,\frac{1}{e^2}\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2\int_0^\infty\frac{x}{(1+x^2)^2}\left(\frac{\tau}{\Delta}\right)^2\left(e^{\left(\frac{\Delta}{\tau}\right)}+e^{-\left(\frac{\Delta}{\tau}\right)}-2\cos x\frac{\Delta}{\tau}\right)^2 dx$$

If $\tau=2\Delta$

$$\sigma^2_{sum}(t_\infty)=Q_f\left(\frac{A\,c_{tot}}{gm\left(c_{tot}+A\,c_f\right)}\right)^2\frac{2.95}{e^2}\quad\text{for FET input}\quad \sigma^2_{sum}(t_\infty)=\frac{Kf_2}{WLC_{ox}}\left(\frac{A\,c_{tot}}{\left(c_{tot}+A\,c_f\right)}\right)^2\frac{2.95}{e^2}$$