The Dualized Standard Model and its Applications—an Interim Report

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Abstract

Based on a nonabelian generalization of electric-magnetic duality, the Dualized Standard Model (DSM) suggests a natural explanation for exactly 3 generations of fermions as the 'dual colour' $\widetilde{SU}(3)$ symmetry broken in a particular manner. The resulting scheme then offers on the one hand a fermion mass hierarchy and a perturbative method for calculating the mass and mixing parameters of the Standard Model fermions, and on the other testable predictions for new phenomena ranging from rare meson decays to ultra-high energy cosmic rays. Calculations to 1-loop order gives, at the cost of adjusting only 3 real parameters, values for the following quantities all (except one) in very good agreement with experiment: the quark CKM matrix elements $|V_{rs}|$, the lepton CKM matrix elements $|U_{rs}|$, and the second generation masses m_c, m_s, m_μ. This means, in particular, that it gives near maximal mixing $U_{\mu 3}$ between ν_μ and ν_τ as observed by SuperKamiokande, Kamiokande and Soudan, while keeping small the corresponding quark angles V_{cb}, V_{ts}. In addition, the scheme gives (i) rough order-of-magnitude estimates for the masses of the lowest generation, (ii) predictions for low energy FCNC effects such as $K_L \to e\mu$, (iii) a possible explanation for the long-standing puzzle of air showers beyond the GZK cut-off. All these together, however, still represent but a portion of the possible physical consequences derivable from the DSM scheme the majority of which are yet to be explored.

The Dualized Standard Model (DSM) is an attempt to answer some of the questions left open by the Standard Model, such as the reason for the existence of 3 fermion generations and of Higgs fields, and to explain the observed values of some of the Standard Model's many parameters. In contrast to most attempts with a similar purpose which extend or modify the SM's theoretical framework, the DSM scheme remains within it, only going beyond the SM in recognizing and exploiting a nonabelian generalization of electric-magnetic duality discovered in Yang-Mills theory a couple of years ago. Thus, it is just by assigning physical significance to the newly discovered dual symmetries which are claimed to be inherent already in the SM gauge theory that the DSM obtains its new physical results.

In this paper we shall briefly review its basic assumptions, outline its main features and summarize the results so far obtained from its phenomenological applications.[1] This is merely an interim report in that only a portion of the new potentials opened up by the scheme has been explored, and even in that portion there are indications that some of the assumptions and approximations made could (or perhaps even need to) be both tightened and refined. Besides, there are yet unanswered questions of consistency both within the scheme itself and of the scheme with nature. However, even with these limitations, the results so far obtained are, we believe, already encouraging enough to be interesting.

As mentioned above, the DSM scheme is based on a generalization [2] of the familiar electric-magnetic duality of electromagnetism to nonabelian Yang-Mills theory, the full development of which requires a fair amount of theoretical apparatus formulated in loop space, and is therefore beyond the scope of the present review aimed mainly at phenomenological applications. Fortunately, for our present purpose, very little detail of the theory is required, which we shall have no difficulty later briefly to supply. For the reader, however, who is interested in the theoretical bases, we have written a companion paper to the present one giving a short review of the steps involved in deriving them [3].

1 Nonabelian Duality and Basic Ingredients of DSM

To illustrate how nonabelian duality enters in the Standard Model, we begin with a short resumé of (abelian) duality in electromagnetism. The Maxwell theory has long been known to possess a symmetry under the interchange of electricity and magnetism:

$$F_{\mu\nu}(x) \longleftrightarrow {}^*\!F_{\mu\nu}(x), \tag{1.1}$$

where the *-operation (Hodge star):

$${}^*\!F_{\mu\nu}(x) = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}(x), \tag{1.2}$$

we may call the abelian dual transform. Electric charges are sources of the field F but monopoles of the field ${}^*\!F$, while magnetic charges are monopoles of F but sources of ${}^*\!F$, where the strength of the quantized electric charges e is related to that of the also quantized magnetic charges $\tilde{e}$ by the famous Dirac condition:

$$e\tilde{e} = 2\pi. \tag{1.3}$$

At any point x in space-time free (locally) of electric and magnetic charges, both $F_{\mu\nu}(x)$ and its dual ${}^*\!F_{\mu\nu}(x)$ are, by virtue of the Maxwell equations, 'gauge fields' derivable from potentials,

[1]Readers preferring an even shorter summary is referred to our report with J. Bordes [1] given at ICHEP'98 Vancouver.

thus:

$$F_{\mu\nu}(x) = \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x), \tag{1.4}$$

$$^*F_{\mu\nu}(x) = \partial_\nu \tilde{A}_\mu(x) - \partial_\mu \tilde{A}_\nu(x). \tag{1.5}$$

It follows then that the theory is invariant under the 2 independent gauge transformations:

$$A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu \alpha(x), \tag{1.6}$$

$$\tilde{A}_\mu(x) \longrightarrow \tilde{A}_\mu(x) + \partial_\mu \tilde{\alpha}(x). \tag{1.7}$$

In other words, the theory is invariant under the doubled gauge symmetry $U(1)\times\tilde{U}(1)$, although obviously, given (1.2), the dual fields F and $^*F$ represent just the same physical degrees of freedom. Notice that this 'doubled' gauge symmetry is inherent in the Maxwell theory itself and not an additional assertion imposed from outside. The only reason we are less familiar with, and have not made much used of, the dual gauge symmetry $\tilde{U}(1)$, which theoretically is on an equal footing with the other gauge symmetry $U(1)$, is just that in the physical world we have observed electric charges but not so far (apparently) their magnetic counterparts.

Nonabelian Yang-Mills theory is not symmetric under the *-operation of (1.2) [4] so that it was not known for some time whether the dual symmetry of electromagnetism generalizes to Yang-Mills theory. But it has now been shown in [2] that there is a generalized nonabelian dual transform $\tilde{}$, reducing to * in the abelian case, under which Yang-Mills theory is symmetric. Unfortunately, the explicit formula for the generalized transform $\tilde{}$ is known at present only in the language of loop space [5, 6] and is for that reason a little cumbersome. However, for our present discussion, we need only to note the consequences of the symmetry it implies, as follows.

Dual to a Yang-Mills field $F_{\mu\nu}$ derivable from a potential A_μ:

$$F_{\mu\nu}(x) = \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x) + ig\,[A_\mu(x), A_\nu(x)], \tag{1.8}$$

there is a field $\tilde{F}_{\mu\nu}$ (the relation of which to $F_{\mu\nu}$ is known though complicated) which is also derivable from a potential, $\tilde{A}_\mu$:

$$\tilde{F}_{\mu\nu}(x) = \partial_\nu \tilde{A}_\mu(x) - \partial_\mu \tilde{A}_\nu(x) + i\tilde{g}\,[\tilde{A}_\mu(x), \tilde{A}_\nu(x)], \tag{1.9}$$

where the coupling strengths g and $\tilde{g}$ are related by a generalized Dirac condition [7]:

$$g\tilde{g} = 4\pi. \tag{1.10}$$

'Colour' (electric) charges appear as sources of the field F but monopoles of the dual field $\tilde{F}$, while 'dual colour' (magnetic) charges appear as monopoles of F but sources of $\tilde{F}$ [2, 8]. The symmetry claimed in [2] then means that Yang-Mills theory is invariant under the 2 independent gauge transformations:

$$A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu \Lambda(x) + ig\,[\Lambda(x), A_\mu(x)], \tag{1.11}$$

and

$$\tilde{A}_\mu(x) \longrightarrow \tilde{A}_\mu(x) + \partial_\mu \tilde{\Lambda}(x) + i\tilde{g}\,[\tilde{\Lambda}(x), \tilde{A}_\mu(x)], \tag{1.12}$$

giving thus to a theory with structure group G a 'doubled' gauge symmetry $G \times \tilde{G}$, in close analogy to the situation in electromagnetism. This doubling of the gauge symmetry, as in the abelian case, is claimed to be an inherent property of the gauge theory, not an additional input. Again, there is no doubling in the physical degrees of freedom.

Given this nonabelian duality let us now examine its implications in the Standard Model with the structure group $SU(3) \times SU(2) \times U(1)$.[2] From the discussion in the preceding paragraph, it follows that the theory will have, in addition to the familiar gauge symmetry $SU(3) \times SU(2) \times U(1)$, also the dual gauge symmetry $\widetilde{SU}(3) \times \widetilde{SU}(2) \times \tilde{U}(1)$. This dual gauge symmetry being, according to our observations above, an inherent property of the gauge theory, what we call the Dualized Standard Model [9] which utilizes this dual gauge symmetry is, as at present understood, no different in principle as a gauge theory from the Standard Model itself. As physical schemes, however, they differ, but only in the DSM scheme's recognition of the existence of and its assignment of physical meanings to the dual gauge symmetry which is ignored in the usual SM treatment. In other words, the new DSM results are deduced just by exploring how this 'pre-existing' dual gauge symmetry is likely to manifest itself in the physical world. The exploration is based on two main assumptions, namely the identification of 2 ingredients inherent in the theory to 2 physical objects, as we shall now explain.

The first assumption concerns the physical interpretation of dual colour. Indeed, the DSM results so far obtained are all from exploitations of only the $\widetilde{SU}(3)$ dual colour symmetry, and it is to this symmetry that our considerations in this paper will be restricted. Colour symmetry $SU(3)$ being known from experiment to be confined, 't Hooft's famous arguments [10] then suggest that the dual colour symmetry $\widetilde{SU}(3)$ should be broken.[3] Now the idea has long been toyed with that fermion generations may be considered as a broken 'horizontal' symmetry. In most schemes, this new symmetry will have to be introduced *ad hoc*. Here, however, given the fact that because of duality and the arguments of 't Hooft, there is inherent in the SM gauge theory already a broken symmetry $\widetilde{SU}(3)$, it seems natural to explore the possibility of making it into the 'horizontal' symmetry of generations. The idea is made particularly attractive by the fact that recent LEP experiments have determined the number of generations of light neutrinos to be 3 to a high accuracy [11]. Hence, the first basic assumption one makes in the DSM scheme is that dual colour is to be identified with generations. This means, first, that there will be just 3 generations, no more no less, and second, that any particle carrying a generation index will carry a dual colour charge, implying, as explained above, that it is a colour monopole of the Yang-Mills field F. In particular a quark will be a colour 'dyon' with both colour (electric) and dual colour (magnetic) charges.

Now, 't Hooft's arguments suggest that the dual colour symmetry is broken, but they do not tell us explicitly how this breaking will occur. The interesting thing is that within the DSM gauge theory, there are quantities which can figure as Higgs fields, and if so identified will imply a particular symmetry breaking pattern. The candidates as Higgs fields in the DSM are the frame vectors in internal symmetry space, which in the case of dual colour is the space of $\widetilde{SU}(3)$. The idea of using frame vectors as dynamical variables is made familiar already in the theory of relativity where in the Palatini treatment or the Einstein-Cartan-Kibble-Sciama formalism [12] the space-time frame vectors or vierbeins are used as dynamical variables. In gauge theory, frame vectors in internal symmetry space are not normally given a dynamical role, but it turns out that in the dualized framework they seem to acquire some dynamical

[2]Here, for ease of presentation, we ignore the subtle differences between gauge groups with the same algebra, although of course global properties of the gauge group are of primary importance in considerations of monopole charges. (See e.g. [6].)

[3]That the definition of duality given in [2] is consistent with 't Hooft's definition given in [10] is shown in [7].

properties, in being patched, for example, in the presence of monopoles [8]. Moreover, they are space-time scalars belonging to the fundamental representations of the internal symmetry group, i.e. doublets in electroweak $SU(2)$ and triplets in dual colour $\widetilde{SU}(3)$, and have finite lengths (as vev's). They thus seem to have just the right properties to be Higgs fields, at least as borne out by the familiar example of the Salam-Weinberg breaking of the electroweak theory. Hence, one makes the second basic assumption in the DSM scheme, namely that these frame vectors are indeed the physical Higgs fields required for the spontaneously broken symmetries.

Identifying dual colour with generations and frame vectors with Higgs fields are of course assumptions. It is worth noting, however, that even if one does not choose to do so, the niches in the form of dual colour and frame vectors will still exist and manifest themselves physically in some way which will need to be accounted for in another manner. Opting for the identification, on the other hand, not only offers a hope of determining some of the SM parameters but also gives a much desired geometric significance both to generations and to Higgs fields which has been sadly lacking to an otherwise highly geometric theory.

Having made these basic assumptions, let us now explore the consequences. First, making the frame vectors in internal symmetry space into dynamical variables and identifying them with Higgs fields mean that for dual colour $\widetilde{SU}(3)$, we introduce 3 triplets of Higgs fields $\phi_a^{(a)}$, where $(a) = 1, 2, 3$ labels the 3 triplets and $a = 1, 2, 3$ their 3 dual colour components. Further, the 3 triplets having equal status, it seems reasonable to require that the action be symmetric under their permutations, although the vacuum need not be so symmetric. An example of a Higgs potential which breaks both this permutation symmetry and also the $\widetilde{SU}(3)$ gauge symmetry completely is as follows [9]:

$$V[\phi] = -\mu \sum_{(a)} |\phi^{(a)}|^2 + \lambda \left\{ \sum_{(a)} |\phi^{(a)}|^2 \right\}^2 + \kappa \sum_{(a) \neq (b)} |\bar{\phi}^{(a)}.\phi^{(b)}|^2, \tag{1.13}$$

a vacuum of which can be expressed without loss of generality in terms of the Higgs vev's:

$$\phi^{(1)} = \zeta \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}, \ \phi^{(2)} = \zeta \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}, \ \phi^{(3)} = \zeta \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix}, \tag{1.14}$$

with

$$x^2 + y^2 + z^2 = 1, \tag{1.15}$$

and

$$\zeta = \sqrt{\mu/2\lambda}, \tag{1.16}$$

x, y, z, and ζ being all real and positive. Indeed, this vacuum breaks not just the symmetry $\widetilde{SU}(3)$ but even the larger symmetry $\widetilde{SU}(3) \times \tilde{U}(1)$ completely giving rise to 9 massive dual gauge bosons. And of the 18 real components in $\phi_a^{(a)}$, 9 are thus eaten up, leaving just 9 (dual colour) Higgs bosons.

Next, we turn to the fermion fields. In analogy to the electroweak theory, the left-handed fermion fields are assigned the triplet **3** and the right-handed fermions the singlet **1** representation in dual colour, so that one can construct their Yukawa couplings as:

$$\sum_{(a)[b]} Y_{[b]} \bar{\psi}_L^a \phi_a^{(a)} \psi_R^{[b]}, \tag{1.17}$$

which, by the same reasons as for the Higgs potential, we have made symmetric under permutations of the Higgs fields $\phi^{(a)}$. As a result of this permutation symmetry, the tree-level fermion mass matrix takes the following factorized form:

$$m = \zeta \begin{pmatrix} x \\ y \\ z \end{pmatrix} (a, b, c), \tag{1.18}$$

where a, b, c are just abbreviations for the Yukawa couplings $Y_{[b]}$. Written in the hermitized convention of e.g. [13], which is basically $\sqrt{mm^\dagger}$ in terms of the m above and is the matrix of relevance to the mass spectrum, it becomes:

$$m = m_T \begin{pmatrix} x \\ y \\ z \end{pmatrix} (x, y, z), \tag{1.19}$$

where m_T is a mass scale dependent on the fermion type $T = U, D, L, N$, i.e. whether U-type quarks (U), D-type quarks (D), charged leptons (L) or neutrinos (N), but the vector (x, y, z) given in terms of the Higgs vev's is independent of the fermion type.

One immediate consequence of such a factorized mass matrix is that at tree-level there is only one state with mass, represented by the eigenvector (x, y, z) with eigenvalue m_T, the other 2 states having zero eigenvalues. This we interpret as embryo fermion mass hierarchy, with one generation much heavier than the other two generations. Such an arrangement is not a bad first approximation to the experimental situation where the highest generation states of U, D, L are all more than one order of magnitude heavier than the lower states. Another immediate consequence is that at tree-level the CKM matrix is the identity, since the CKM matrix is the matrix giving the relative orientations between the physical states of the U- and D-type quarks, and these orientations at tree-level are both given in (1.19) by the vector (x, y, z). Again, this is not a bad first approximation to the experimental CKM matrix whose off-diagonal elements are at most of order 20 percent. Though reasonable as a first approximation, this tree-level description is obviously too degenerate to be realistic. We shall see, however, that the degeneracy will be lifted at higher orders where nonzero masses for the two lower generations and nonzero off-diagonal elements for the CKM matrix will both result from loop corrections.

2 Quark Mixing and Light Masses as Loop Corrections

Radiative corrections to the tree-level mass matrix (1.19) have already been calculated to 1-loop level [14]. There are in principle many diagrams to calculate. There are first the usual diagrams of the Standard Model with loops of gluons, γ, Z, W, and electroweak Higgses. Then there are new diagrams with loops of dual gluons and dual Higgses. All these diagrams, however, share the common feature that they preserve the factorizability of the fermion mass matrix, namely that after the corrections the mass matrix m' is of the form:

$$m' = m'_T \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} (x', y', z'). \tag{2.1}$$

The reason is that only those bosons carrying dual colour can affect the factorizability, and of these the dual gluon couples only to the left-handed fermion while the dual Higgses have themselves factorizable couplings. Indeed, it is believed, though not formally proved, that factorizability of m' will remain intact to all orders.

Only a small number of these diagrams, however, need be considered for the purpose of this paper. The reason is as follows. The value of the normalization factor m'_T in (2.1) is actually not calculable perturbatively since it receives contributions from diagrams with a dual gauge boson loop, and these are proportional to the square of the dual gauge coupling $\tilde{g}$ which is large according to the Dirac condition (1.10), given the known empirical value of the colour coupling g. The normalization m'_T has thus to be treated in any case as an empirical parameter. In other words, the calculation of any diagram which affects only this normalization will not add to our present level of understanding and might as well be ignored. What one can calculate perturbatively, on the other hand, is the orientation of the vector (x', y', z') and this is affected by only a small subset of the diagrams, namely those shown in Figure 1. For example, it is clear that diagrams with only loops of gluons or W-bosons will not affect the orientation of (x', y', z') since these bosons do not carry dual colour. Next, of the remaining 3 diagrams as in Figure 1, the first 2 can give only negligible effects. This last conclusion is arrived at as follows. The diagrams (a) and (b) both give rotations to (x', y', z') but these are of the order $\tilde{g}^2 m_T^2/\mu_N^2$, where μ_N represents the masses of the dual gauge bosons which are constrained by present experimental bounds on flavour-changing neutral current (FCNC) effects to be rather large. The identification of dual colours to generations means that particles carrying generation indices can interact by exchanging dual gluons, which would lead to generation-changing or FCNC effects, such as $K_L \to e\mu$ decay or an anomalous $K_L - K_S$ mass difference, and such effects are strongly bounded by experiment. An analysis using the latest data [15, 16] which will be outlined in Section 5 below, shows that a violation to present experimental bounds can be avoided only if $\mu_N^2/\tilde{g}^2$ is of order at least a few hundred TeV, which means contributions from diagrams (a) and (b), even for the top quark, are of at most 10^{-6}. Neglecting then contributions of this order, the calculation becomes rather simple depending on only the (dual) Higgs loop diagram of Figure 1 (c) [14].

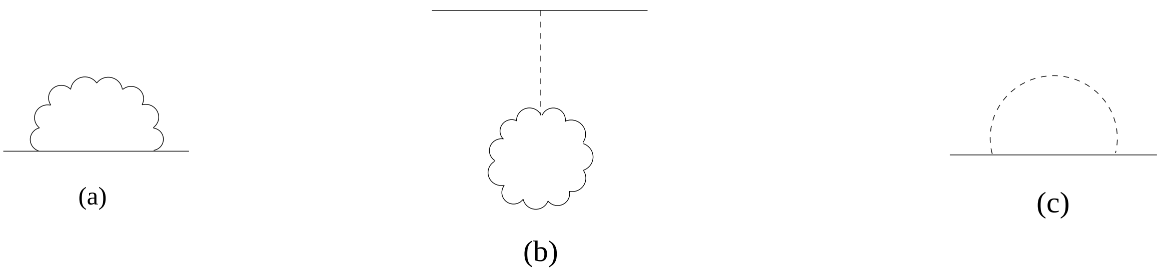

Figure 1: One loop diagrams rotating the fermion mass matrix.

The rotation given to the normalized vector (x', y', z') by the remaining Higgs loop diagram Figure 1 (c) has been calculated. It is found to depend on the energy scale μ as follows:

$$\frac{d}{d(\ln \mu^2)} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \frac{5}{64\pi^2} \rho^2 \begin{pmatrix} \tilde{x}'_1 \\ \tilde{y}'_1 \\ \tilde{z}'_1 \end{pmatrix}, \tag{2.2}$$

with

$$\tilde{x}'_1 = \frac{x'(x'^2 - y'^2)}{x'^2 + y'^2} + \frac{x'(x'^2 - z'^2)}{x'^2 + z'^2}, \quad \text{cyclic}, \tag{2.3}$$

and $\rho^2 = |a|^2 + |b|^2 + |c|^2$ being the Yukawa coupling strength. As μ varies the normalized vector traces out a curve on the unit sphere. It is easily seen from (2.2) and (2.3) that the points $(1,0,0)$ and $\frac{1}{\sqrt{3}}(1,1,1)$ are fixed points under scale changes, and for decreasing energy, the point (x',y',z') runs away from $(1,0,0)$ towards $\frac{1}{\sqrt{3}}(1,1,1)$. The trajectory along which (x',y',z') runs depends in principle on the Higgs boson masses logarithmically, but because of a 'happy accident' that we shall make clear later, even this weak dependence can be ignored. The only parameters remaining are then just the initial values of the components of the normalized vector (x',y',z') identifiable with the Higgs vev's which are the same for all fermion types, and the Yukawa coupling strength ρ_T, one for each fermion type T. The information being now encoded in the vector (x',y',z') running along a trajectory on the unit sphere, the remaining question is just how to extract the fermion mass and mixing parameters from this rotating vector.

Let us first consider quarks of the U-type and ask what is the physical state vector the of t quark in dual colour or generation space. Now, according to (2.1), (x',y',z') is the vector with the single nonzero eigenvalue of the loop-corrected fermion mass matrix at the energy scale where (x',y',z') is evaluated. It ought therefore to be identified with the state vector of the heaviest generation, except that (x',y',z') has to be evaluated at the scale μ equal to the top mass m_t. Inputting then the empirical value of $m_t \sim 176$ GeV we run (x',y',z') to the scale $\mu = m_t$. This gives us the physical state vector $|\mathbf{v}_t\rangle$ and also fixes the value of the parameter m'_U at the same scale. This calculation, however, still depends on the starting value (x,y,z) of the rotating vector (x',y',z') and also on the Yukawa coupling strength ρ_U.

Next, we ask what is the physical state vector of the c quark. Being an independent physical entity, the c quark ought to have a state vector orthogonal to $|\mathbf{v}_t\rangle$. This means it must have a zero eigenvalue of the mass matrix (2.1) evaluated at the scale $\mu = m_t$. But there are two linearly independent such vectors and we do not as yet know which linear combination of these should correspond to the c quark. Nor can we identify the mass m_c as the zero eigenvalue for this is evaluated at the wrong scale m_t. Let us first extract the mass submatrix in the 2-dimensional subspace orthogonal to $|\mathbf{v}_t\rangle$ and run it down to lower scale. Being a submatrix of a rank 1 matrix, it is of rank at most 1 at this lower scale but need no longer be a zero matrix since the nonzero eigenvector (x',y',z') of the full mass matrix (2.1) has already rotated from the direction $|\mathbf{v}_t\rangle$ and can have thus a component in the subspace orthogonal to $|\mathbf{v}_t\rangle$ which contains $|\mathbf{v}_c\rangle$. For consistency with the way we fixed the vector $|\mathbf{v}_t\rangle$ before, the procedure to determine $|\mathbf{v}_c\rangle$ is now clear. We ought to run the 2×2 mass submatrix down in energy until the scale μ equals the empirical value m_c of the c quark mass. Then its only nonzero eigenvector at that scale is to be identified with $|\mathbf{v}_c\rangle$. This vector is, of course, by definition orthogonal to $|\mathbf{v}_t\rangle$ as it should be. Its eigenvalue of the mass submatrix, however, will not in general be the same as the input value of m_c. But our calculation, we recall, still depends on (x,y,z) and ρ_U, and by adjusting ρ_U we can make the output eigenvalue of $|\mathbf{v}_c\rangle$ the same as the input value of m_c. This fixes ρ_U, leaving now only (x,y,z) as parameters.

Once the state vectors of the t and c quarks are determined, then obviously the state vector of the u quark is also fixed as the vector orthogonal to both. We have thus the whole triad of physical state vectors of the U-type quarks in terms of the 2 remaining parameters in the normalised vector (x,y,z) representing the Higgs vev's.

The above procedure can now be repeated for the D-type quarks and for the charged leptons to determine the triad of their physical state vectors. (The problem for neutrinos is somewhat different as we shall make clear later.) To do so, we shall have to input the empirical masses of the two highest generations in each case, namely b and s for the D-type quarks and τ and μ for the charged leptons, and then, as before for the U-type quarks, to adjust the Yukawa coupling strengths ρ_D and ρ_L to obtain consistency. The values of the ρ's determined in this way need not of course be the same for D and L nor as the value obtained before for the U-type quarks. Again the triads so obtained still depend on (x, y, z), which is the same for all fermion types.

Now the matrix relating the orientations in generation space of the two triads of physical state vectors of respectively the U-type and D-type quarks is what is known in the literature as the Cabibbo-Kobayashi-Moskawa (CKM) matrix. These orientations being now known, the CKM matrix can be calculated in terms of the two remaining parameters in (x, y, z). By adjusting these, we can then try to fit the empirical CKM matrix. There are actually 4 independent degrees of freedom in the CKM matrix, but in our treatment up to 1-loop order, there is no CP-violating phase, the vector (x', y', z') being real. We are thus left with 3 real quantities to fit with our two remaining parameters, which is still nontrivial but has been achieved rather well. For the matrix of absolute values $|V_{rs}|$, for $r = u, c, t$ and $s = d, s, b$, we obtained [14] [4]:

$$|V_{rs}| = \begin{pmatrix} 0.9755 & 0.2199 & 0.0044 \\ 0.2195 & 0.9746 & 0.0452 \\ 0.0143 & 0.0431 & 0.9990 \end{pmatrix}, \tag{2.4}$$

as compared with the experimental values [17]:

$$|V_{rs}| = \begin{pmatrix} 0.9745 - 0.9757 & 0.219 - 0.224 & 0.002 - 0.005 \\ 0.218 - 0.224 & 0.9736 - 0.9750 & 0.036 - 0.046 \\ 0.004 - 0.014 & 0.034 - 0.046 & 0.9989 - 0.9993 \end{pmatrix}. \tag{2.5}$$

The above fit to the CKM matrix fixes all the parameters in the problem, the values of which so obtained show two very intriguing features. First, the vev's of the Higgs fields (x, y, z) turn out to have values very close to the high energy fixed point $(1, 0, 0)$. Second, and even more intriguingly, the Yukawa coupling strengths ρ_T turn out to be T-independent to a surprising accuracy [14]. Indeed, for fermion masses taken at the (geometric) median of experimental values, the fitted values of ρ_T for $T = U, D, L$ differ by only 1.5 parts in a thousand, while varying the masses within the experimental error bars still give differences of at most a few percent. This last 'happy accident' seems to indicate some hidden symmetry in the scheme, the reason for which we have at present only some ideas not yet fully understood. In practical terms, on the other hand, it is a bonus since it simplifies greatly not only the calculation but also the presentation of the results. It means, first, that the 3 originally different parameters ρ_T can now be treated as just a single parameter ρ; second, that all fermion types run on the same trajectory at the same speed; and third, even the originally already weak dependence of the calculation on the Higgs boson masses can now be completely ignored.

Next, we turn to the masses of the lowest generation. Since we know already the physical state vectors of the lowest generation states for all the 3 fermion types U, D, L, namely u, d, e, we can simply evaluate their expectation values of the mass matrix m' at any scale. The actual masses of the 3 states are given by these expectation values evaluated at the scales

[4]This fit was obtained using data given in the databook of 1996 [17]. For this reason the comparison with experiment given in this paper refers for consistency also to data from the same source. Although much of the data have since been updated, the changes are not large. For a parallel fit to the latest data and comparison with them, see [18], which arrives at a similar conclusion to that presented in this paper.

	Calculation	*Experiment*
m_c	1.327GeV	$1.0-1.6$GeV
m_s	0.173GeV	$100-300$MeV
m_μ	0.106GeV	105.7MeV
m_u	209MeV	$2-8$MeV
m_d	15MeV	$5-15$MeV
m_e	6MeV	0.511MeV
m_{ν_1}	10^{-15}eV	< 10eV
B	400TeV	?

Table 1: Predicted fermion masses compared with experiment. Notice, however, that for the u- and d-quarks, the calulated masses are defined each at the scale equal to its value, and are not directly comparable to the quoted experimental values defined at the scale of 1 GeV.

equal to the values themselves, and can also be readily calculated. However, the calculation of these lowest generation masses is an extrapolation on a logarithmic scale and depends also on the normalization m'_T of the mass matrix m' in (2.1), whose variation with scale, as we have already explained, is not calculable perturbatively. It is thus expected to be far less reliable than the above calculation for the mixing parameters. Nevertheless, assuming simply that the normalization m'_T is scale-independent, one may hope to get rough order-of-magnitude estimates for the lowest generation masses. The result of such a calculation is shown in Table 1. We notice that the mass of the electron is within about an order of magnitude of the experimental value, which we regard as reasonable. As for the quarks, the d-quark comes out about right but the u quark is too large by nearly two orders of magnitude. However, we should note that light quark masses are notoriously difficult to define, being sensitive to nonperturbative QCD corrections below around 2 GeV. Moreover, the u and d masses given in the table are defined each at the scale equal to its value whereas the quoted experimental values are defined at the scale of 1 GeV. The two sets of values are therefore not directly comparable. Indeed, if the expectation value in the u-state of the running mass matrix m' is taken at 1 GeV, a value of order only 1 MeV is obtained, although it is also unclear whether this should be compared with the empirical value quoted. In all cases, at least, the masses are hierarchical as they should be.

3 Neutrino Oscillations

Next, we turn our attention to neutrinos. The case for neutrino oscillations has recently been much strengthened by the atmospheric neutrino data from SuperKamiokande [19] confirming earlier results of the last decade [20]–[22]. These have not only given convincing evidence for the phenomena, but have even provided quite restrictive bounds on the relevant parameters which would be a challenge for theoreticians to explain.

If the basic idea in the DSM scheme of identifying generations with dual colour is adhered to, then there will be 3 and only 3 generations of neutrinos as for any other fermion type. In principle then, there is nothing to stop us applying the same procedure as that applied above to quarks and charged leptons to determine also the masses and physical state vectors of neutrinos. Indeed, in strict adherence to the scheme, it would be incumbent upon us to do so. Since the Higgs vev's (x, y, z) are already known, all we need to do so would be to input the (Dirac) masses of the two heaviest neutrinos as we did for quarks and charged leptons. In fact, since the Yukawa couplings ρ turned out to be so close for all the other fermion types, it seems reasonable to assume the same value also for neutrinos. We shall need then to input only one (Dirac) mass. However, neutrinos differ from the other fermions in that they can also have Majorana masses, and it is from these together with their Dirac masses that one obtains their physical masses via the well-known seesaw mechanism [23] by diagonalizing the matrix:

$$\mathbf{M}_r = \begin{pmatrix} 0 & M_r \\ M_r & B \end{pmatrix}, \tag{3.1}$$

where it turns out that in the DSM scheme as at present understood, all 3 generations of neutrinos will have to have the same Majorana mass B for consistency [24]. With then B as an extra parameter, we need to input two masses to perform the proposed calculation.

Information on the (physical) mass of the heaviest neutrino, usually denoted by m_3, is obtained from the muon anomaly in atmospheric neutrinos. From [20, 19], we have an estimate of the difference between the physical masses of the two heaviest neutrinos. Since in the DSM scheme, masses are supposed to be hierarchical, meaning $m_3 \gg m_2 \gg m_1$, we can put the mass itself equal to the difference $m_3^2 \sim 10^{-3} - 10^{-2}$ eV^2, which we can take as one input, leaving thus just one more mass to be determined.

To do so, we draw on the information from solar neutrino data. There one has 2 estimates for the (physical) mass of the second generation neutrino, again taking the masses to be hierarchical. From the LWO solution [25], one has $m_2^2 \sim 10^{-10}$ eV^2, and from the MSW solution [26] $m_2^2 \sim 10^{-5}$ eV^2. With either of these as input, one has in principle enough information to determine the Dirac mass M_3 and hence complete the DSM calculation of the leptonic CKM matrix.

It turns out, however, that inputting the MSW estimate for m_2 and the estimate of [20, 19] for m_3, one obtains no sensible DSM solution for neutrinos. The reason is that in the DSM scheme, lower generation masses come only as a 'leakage' from the mass of the highest generation and this leakage mechanism does not easily admit a ratio m_2/m_3 as large as that wanted by the MSW solution. One concludes thus that the DSM scheme, as at present understood, disfavours the MSW solution to the solar neutrino problem.[5]

On the other hand, inputting the LWO estimate for $m_2^2 \sim 10^{-10}$ eV^2 and the estimate of $m_3^2 \sim 10^{-3} - 10^{-2}$ eV^2 from [20, 19], a solution is readily found. The state vectors of the neutrinos so determined then allow one immediately to calculate the leptonic CKM matrix. For $m_3^2 = 10^{-3}$eV^2, one obtains [24]:

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 0.97 & 0.24 & 0.07 \\ 0.22 & 0.71 & 0.66 \\ 0.11 & 0.66 & 0.74 \end{pmatrix}, \tag{3.2}$$

all elements being real at the 1-loop level that we are working. The result is insensitive to the actual values (in the above range) of m_2 and m_3 used. Notice that apart from inputting the rough values of m_2 and m_3 from experiment in the way explained, the calculation involves no adjustment of parameters which have all been fixed by our earlier calculation of the quark CKM matrix [14].

[5]It is interesting to note in this context that the latest SuperKamiokande data on day-night variations and flux reported at the Vancouver ICHEP'98 conference and at the APS meeting (DPF) at UCLA in fact also favour the LWO over the MSW solution for solar neutrinos [27, 19].

The result (3.2) should be compared with the leptonic CKM matrix extracted from experiment by, for example, [28][6] where the bounds on $U_{\mu 3}$ comes mainly from atmospheric neutrino data and the bounds on U_{e3} comes mainly from reactor data such as [29] and the estimate for U_{e2} comes from the solar neutrino data as interpreted by either the large angle MSW [26] or the LWO [25] scenario:

$$= \begin{pmatrix} * & 0.4 - 0.7 & 0.0 - 0.2 \\ * & * & 0.5 - 0.8 \\ * & * & * \end{pmatrix}. \tag{3.3}$$

Since we are ignoring for the present the CP-violating phase, there are only 3 independent elements of the matrix we need consider. We notice that the theoretical predictions for both the angles $U_{\mu 3}$ and U_{e3} fall neatly into the middle of the experimental range. A more detailed comparison of our prediction with experiment based again on the analysis in [28] is shown in Figures 2 and 3 for a range of m_3 values and for m_2 within the range allowed by [25]. One sees that the agreement is consistently good.

The prediction, however, for the other angle U_{e2} relevant to solar neutrinos does not score so well, being about a factor 2 too small and lying some way outside the experimental limits. We shall see later the reason why this element is particularly hard for the DSM scheme to get correct.

In addition to the mixing angles, the calculation gives predictions also for the masses of the lightest and the 'right-handed' neutrinos, namely m_{ν_1} and B, as listed in Table 1. The present experimental bound on m_{ν_1} is too weak to be a test. As for B, there is no direct information. However, given B, one can estimate within the scheme a value for the life-time of neutrinoless double beta decays. One notes that the value of B we obtained is several orders of magnitude lower than that usually given, say for example, from grand unified theory models. The reason is that one usually assumes for the heaviest neutrino a Dirac mass similar in value to the mass of the τ or t, i.e. of order GeV or higher, whereas the value we obtain above by fitting m_3 and m_2 in the DSM scheme gives a value only of order MeV (and B is proportional to the square of this estimate). Such a big difference between the (Dirac) masses of the charged leptons and neutrinos need not be a worry since the same is known already to occur between the U- and D-type quarks. But as a result of this lower value for B, the rate for neutrinoless double beta decays predicted here will be much more accessible to experiment. In particular, a rough estimate shows that the predicted 0ν half-life for ^{76}Ge is only about 2–3 orders longer than the present experimental limit.

To conclude, one sees that the DSM with no freedom left after fitting the quark CKM matrix, reproduces quite well the general features of neutrino oscillations as observed in experiment, and gives in addition some interesting and in principle testable predictions.

4 Features of Mixing from Differential Geometry

It is instructive to compare the quark (2.5) and leptonic (3.3) CKM matrices as now experimentally known. We note in particular the following outstanding features:

[6] This analysis [28] was done with the Kamiokande not with the SuperKamiokande data, for which no parallel analysis as far as we know has yet been done. The result, however, is expected to be similar, with a somewhat tighter bound but roughly the same central value for the $\mu 3$ element, but a looser bound on the $e3$ element. For a comparison with the latest data from SuperKamiokande, see [18].

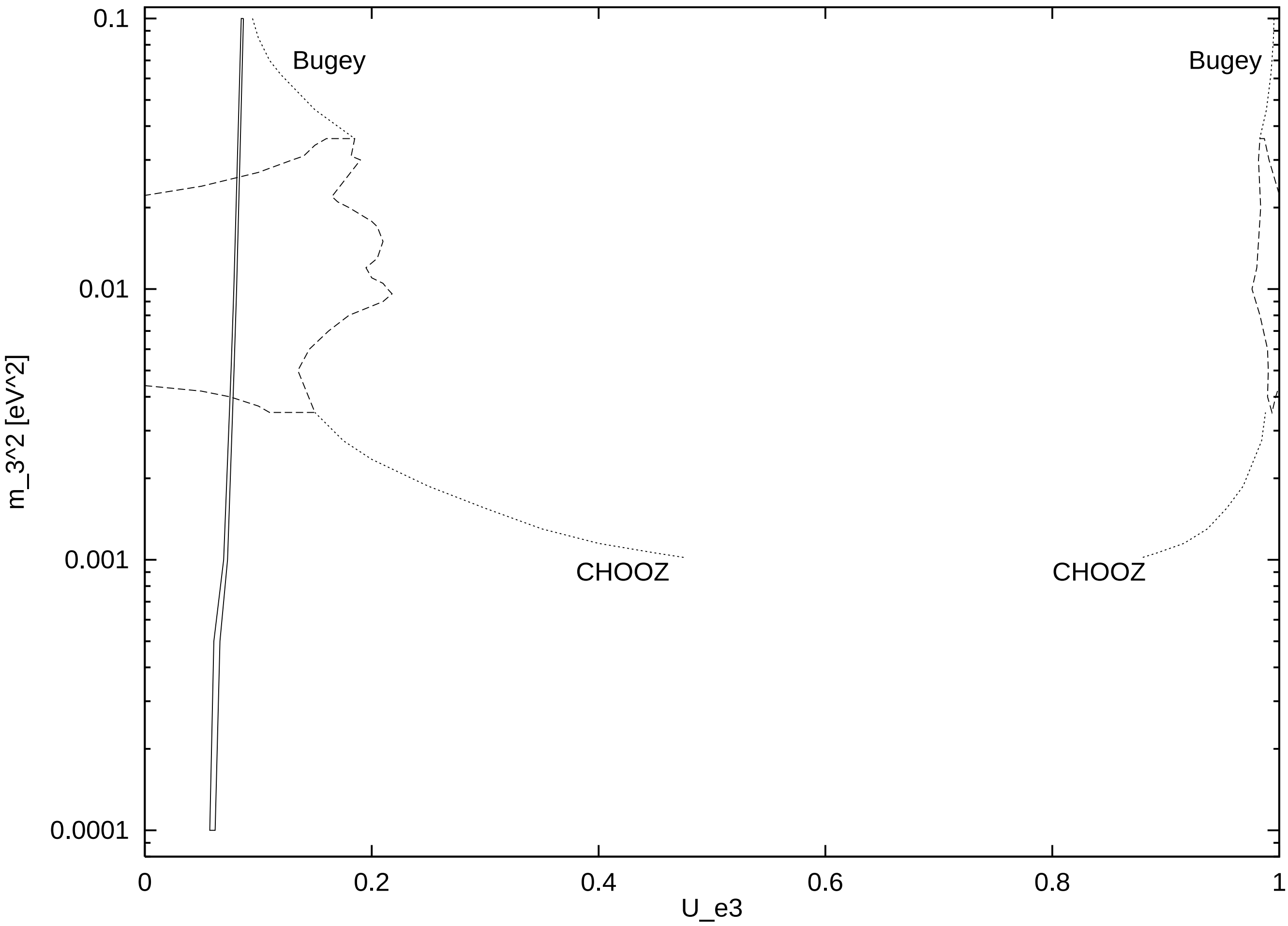

Figure 2: 90 % CL limits on the CKM element U_{e3} compared with the result of our calculation.

(a) The corner elements 13 and 31 are much smaller than the others for both quarks and leptons.

(b) All off-diagonal elements for quarks are much smaller than the diagonal elements.

(c) The 23 element is much smaller for quarks than for leptons.

These will need to be accounted for in any scheme aiming to explain the fermion mixing phenomenon. In the preceding sections we have already shown that the DSM scheme is able quantitatively to reproduce these features in terms of just a few parameters without explaining why it should be so. What we shall do in this section is to gain an intuitive understanding why the CKM matrices have the qualitative features they do and to show that they emerge from the basic structure of the scheme as simple consequences of classical differential geometry and can be deduced, almost quantitively in some cases, without a detailed calculation [30].

We recall that the fermion mass matrix in the DSM scheme is factorized even after loop corrections, and all the information needed for our consideration of the CKM matrix is encoded

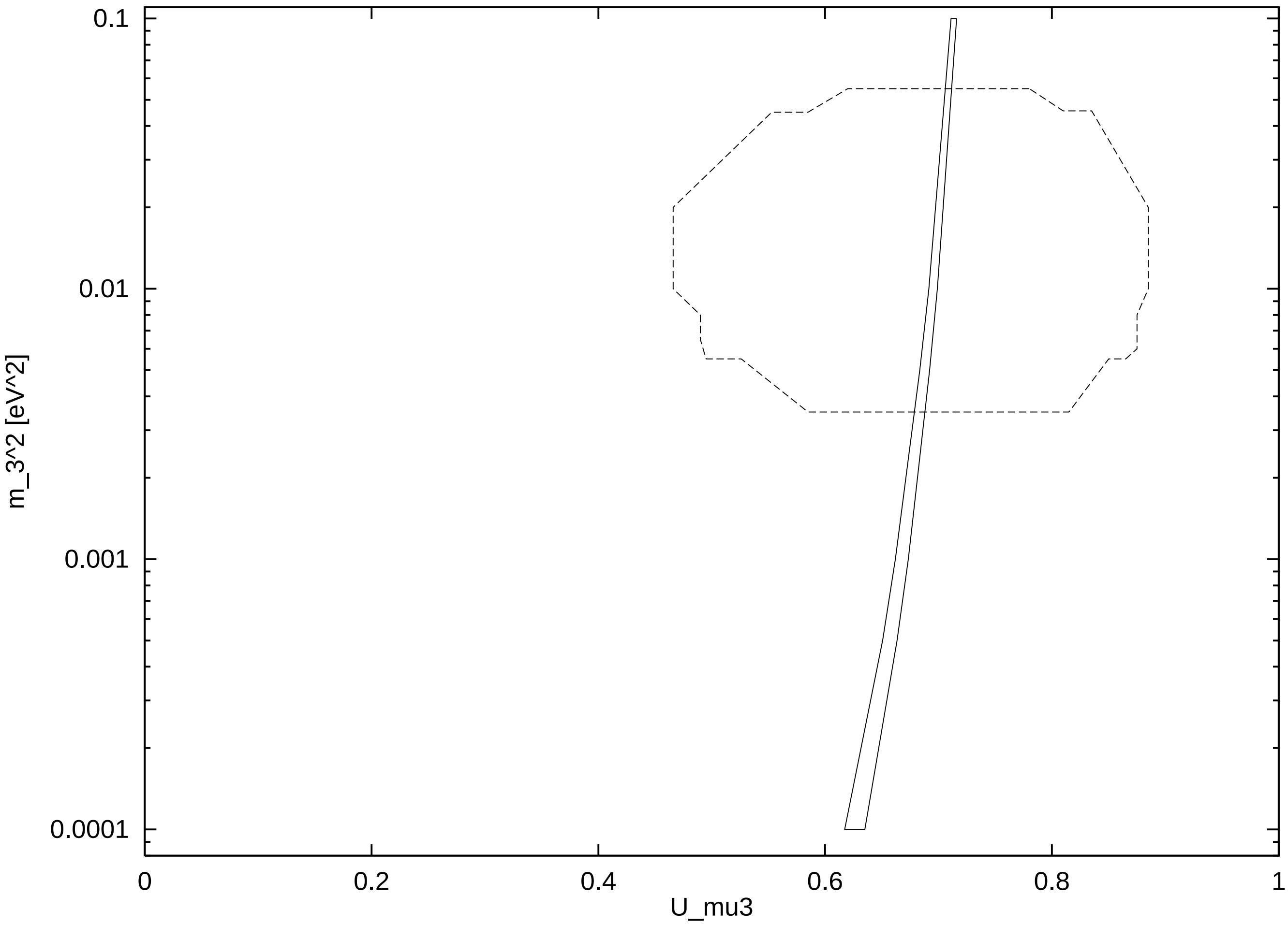

Figure 3: 90 % CL limits on the CKM element $U_{\mu 3}$ compared with the result of our calculation.

in the normalized vector (x', y', z'). This vector rotates with the energy scale, tracing out a trajectory on the unit sphere, which trajectory is the same for all fermion types T, i.e. whether U- or D-type quarks, charged leptons or neutrinos. The various physical states, however, differ in the locations they occupy on this trajectory. Figure 4 shows the actual trajectory and locations of the 12 fermion states obtained in the fit of [14, 24].

The state vectors of the various physical states are given in terms of the rotating vector (x', y', z') as follows. (i) Evaluated at the scale of the top mass, (x', y', z') is the state vector $|\mathbf{v}_1\rangle$ of t, as shown in Figure 5. (ii) At the scale of the charm mass, the vector (x', y', z') is rotated to another direction, say $|\tilde{\mathbf{v}}_1\rangle$ in Figure 5, with thus a zonzero component orthogonal to $|\mathbf{v}_1\rangle$, the direction of which gives the state vector $|\mathbf{v}'_2\rangle$ of c. (iii) The state vector of the u-quark is $\mathbf{v}'_3 = \mathbf{v}_1 \wedge \mathbf{v}'_2$. Similar constructions apply to the other 3 fermion types D, L, N.

If we make the approximation that the locations of the t and c quarks on the trajectory are close together (as is seen to be true in Figure 4), then the 3 state vectors of t, c, u of Figure 5 form in that limit an orthonormal triad at the t position. If we do the same for the D-type

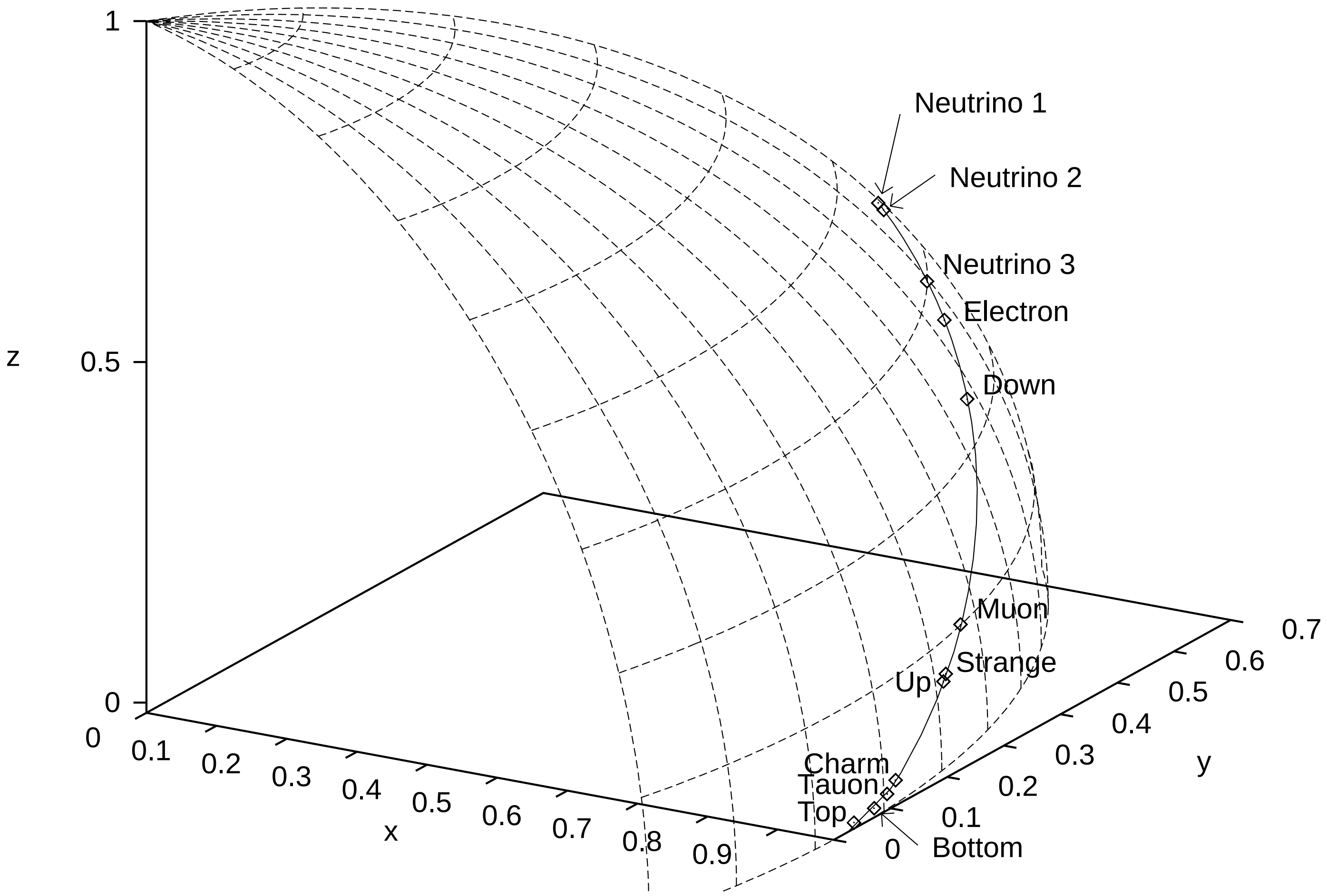

Figure 4: The trajectory traced out by (x', y', z') and the locations on it of the 12 fermion states.

quarks, we have another such triad at the b position, as illustrated in Figure 6. The entries of the (quark) CKM matrix are nothing but the direction cosines between the vectors of these 2 triads. The leptonic CKM matrix is similar.

Since the trajectory lies on the unit sphere, the c vector is the tangent $\mathbf{T}$ to the trajectory and the t vector the normal $\mathbf{N}$ to the surface, so that they form, together with the u vector $\mathbf{B} = \mathbf{N} \wedge \mathbf{T}$, what is known in elementary differential geometry as the 'Darboux triad'. Differentiating then with respect to the arc-length, we get the following formulae similar to the well-known Serret–Frenet formulae for space curves [31]:

$$\begin{aligned} \mathbf{N}' &= -\kappa_n \mathbf{T} - \tau_g \mathbf{B}, \\ \mathbf{T}' &= \kappa_g \mathbf{B} + \kappa_n \mathbf{N}, \\ \mathbf{B}' &= -\tau_g \mathbf{N} - \kappa_g \mathbf{T}. \end{aligned} \tag{4.1}$$

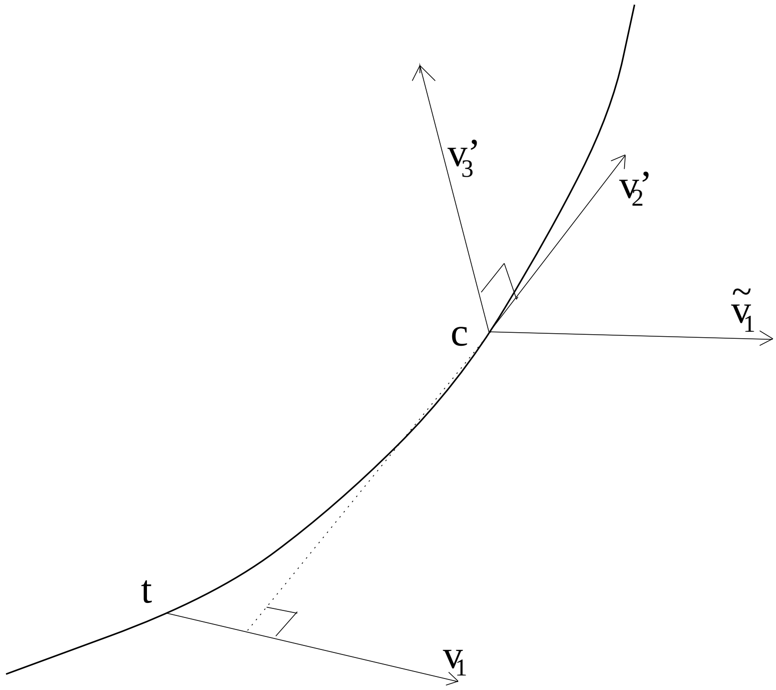

Figure 5: The state vectors of the 3 physical states belonging to the 3 generations of the U-type quark.

Here κ_g is the geodesic curvature, κ_n the normal curvature, and τ_g the geodesic torsion of the curve on the surface. Equivalently, to first order in arc-length Δs, (4.1) can be rewritten in the form of a CKM matrix with entries arranged in the conventional order:

$$\begin{pmatrix} 1 & -\kappa_g \Delta s & -\tau_g \Delta s \\ \kappa_g \Delta s & 1 & \kappa_n \Delta s \\ \tau_g \Delta s & -\kappa_n \Delta s & 1 \end{pmatrix}. \tag{4.2}$$

In our case of the unit sphere, $\tau_g = 0$ and $\kappa_n = 1$. It follows then from (4.2) that [30]:

(a) The corner elements of both the quark CKM matrix (V_{ub}, V_{td}) and the leptonic CKM matrix $(U_{e3}, U_{\tau 1})$ are small since they vanish to first order in the separation between the corresponding fermion types.

(b) The 4 other off-diagonal elements of the quark CKM matrix are small compared to the diagonal elements since they are of first order in the separation between the t and b quarks, which is small as seen in Figure 4.

(c) The elements V_{cb}, V_{ts} for quarks are much smaller than their counterparts $U_{\mu 3}, U_{\tau 2}$ for leptons, since they are to first order proportional to the separation, which is much smaller for quarks than for leptons as seen in Figure 4.

These 3 points are all borne out by experiment as already noted above. Indeed, it is amusing to note that even the approximate values for the 4 elements in (c), as quoted above from either experiment (2.5), (3.3) or the DSM calculation (2.4), (3.2), can simply be read off by measuring the separations between t and b and between τ and μ on the trajectory in Figure 4!

Further, we note that since the geodesic curvature κ_g, in contrast to the geodesic torsion τ_g and the normal curvature κ_n on a sphere, depends both on the location and on the trajectory,

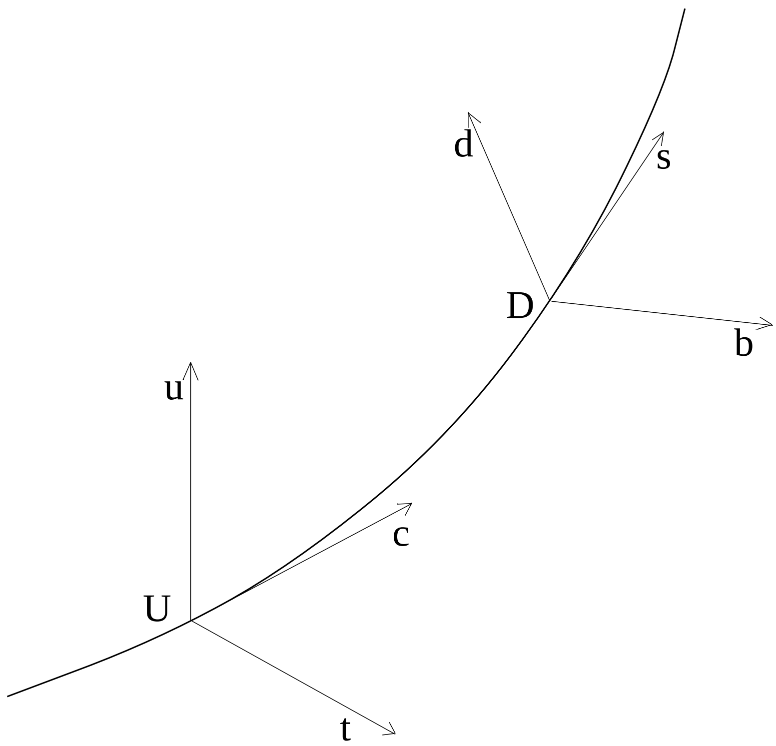

Figure 6: Two triads of state vectors for two fermion types transported along a common trajectory.

so do the values of the remaining pair of off-diagonal elements of the CKM matrix, namely the 'Cabibbo angles' (V_{us}, V_{cd} for quarks, and $U_{e2}, U_{\mu 1}$ for leptons). This explains why the Cabbibo angle is so large even though the separation between t and b is small. It also means that the 12 elements are much more sensitive to the details of the fit and explains why our calculation has been less successful in predicting U_{e2} than with the other leptonic mixing angles.

That all these features in the mixing matrices echoing experimental data can be derived *without* detailed calculations is very encouraging, for it means that the agreement with experiment reported in the 2 sections before are much less likely to be just numerical accidents of the calculation.

5 FCNC Effects from Dual Gluon Exchange

Besides explaining the features of the Standard Model, any scheme which attempts to go beyond has of course also to examine its own predictions for the possibility of their violating already some known experimental limits, and if not, for the feasibility of their being tested by future experiment. For the DSM scheme, one obvious direction to probe in this respect is the new interactions arising from exchanges of the dual colour gauge bosons. Dual colours in DSM having been identified with generations, it follows that any particle carrying a generation index can acquire a new interaction by exchanging these bosons, leading to generation-changing or flavour-changing neutral current (FCNC) effects. These gauge bosons are presumably quite heavy or otherwise they would already have been discovered. There are thus two areas where one can look for their influence. One can either look for effects at energies low compared with their mass where the effects of their exchange would be suppressed, or else for sizeable effects at ultra-high energies. We shall consider examples of both, at low energies in this section, and at high energies in the next.

At low energy, flavour-changing neutral currents can manifest themselves in rare decays and in mass differences between charge conjugate neutral meson pairs. For DSM, as for other 'horizontal gauge symmetry' models [32], these effects arise already at the level of one-(FCNC) gauge boson exchange and can thus be estimated once given the masses of the gauge bosons and their couplings to the fermions involved. What distinguishes DSM, however, is that the scheme has been made so restrictive by what has gone before that detailed estimates can now be given for all these FCNC effects at the one-boson exchange level in terms of only one additional parameter.

In view both of the intrinsic structure built into the scheme and of the calculations already performed which are summarized above, most of the 'fundamental' parameters of DSM as at present formulated are now known. First, by virtue of the Dirac quantization condition [7]:

$$g_3\tilde{g}_3 = 4\pi, \quad g_2\tilde{g}_2 = 4\pi, \quad g_1\tilde{g}_1 = 2\pi, \tag{5.1}$$

the coupling strengths $\tilde{g}_i$ of the dual gauge bosons are derivable from the coupling strengths g_i of the ordinary colour and electroweak gauge bosons measured in present experiments. Secondly, the branching of these couplings $\tilde{g}_i$ into the various physical fermion states are given by the rotation matrices relating these physical states to the 'gauge states' in generation or dual colour space, thus:

$$\psi^T_{gauge,L} = S^T \psi^T_{physical,L} \tag{5.2}$$

where the index T runs over the four types of fermions U, D, L and N. These matrices S^T were already determined in the calculation of fermion mixing matrices [14, 24], where for example the (quark) CKM matrix was obtained as $(S^U)^\dagger S^D$ in Section 2 and there found to be in excellent agreement with experiment. Finally, in tree-level approximation, the masses of the dual gauge bosons are given in terms of the vacuum expectation values of the dual colour Higgs fields, the ratios x, y, z between which are among the parameters determined in the calculation [14] by fitting the CKM matrix. Thus the only remaining unknown among the quantities required is the actual strength ζ of the vev's, which, though also entering in principle in the calculations of Standard Model parameters outlined in Section 2, turns out to be hardly restricted there. That being the case, one can now calculate in the DSM scheme all one-dual gauge boson exchange diagrams between any two fermions in terms of this single mass parameter ζ.

There are altogether 9 dual gauge bosons (including that corresponding to $\tilde{U}(1)$) which can be exchanged, whose masses at tree-level are given by diagonalizing a mass matrix dependent on the dual couplings $\tilde{g}_3, \tilde{g}_1$ and on ζ and (x, y, z). Given that, as mentioned in Section 2, the value obtained for (x, y, z) from fitting the quark CKM matrix [14] is very close to the fixed point $(1, 0, 0)$, the mass matrix for the dual gauge bosons can be readily diagonalized [16] yielding one particular state with mass:

$$M^2 = \zeta^2 z^2 \frac{3}{4} \frac{\tilde{g}_3^2}{1 + \frac{3}{16}\frac{\tilde{g}_3^2}{\tilde{g}_1^2}}, \tag{5.3}$$

which is much lower than the rest. As a result, the calculation for FCNC effects becomes quite simple, being dominated by the exchange of just this one boson.

At energies much lower than the mass of this dual gauge boson, the effects can then be summarized in terms of an effective Lagrangian thus [16]:

$$L_{eff} = \frac{1}{2\zeta^2 z^2} \sum_{T,T'} f^{T,T'}_{\alpha,\beta;\alpha',\beta'} (J_T^{\mu\dagger})^{\alpha,\beta} (J_{\mu,T'})^{\alpha',\beta'}, \tag{5.4}$$

with currents of the usual $V - A$ form:

$$(J_\mu^T)_{\alpha,\beta} = \bar{\psi}_{L,\alpha}^T \gamma_\mu \psi_{L,\beta}^T, \tag{5.5}$$

and a group factor which, for reactions involving changes of flavour, reduces to:

$$f_{\alpha,\beta;\alpha',\beta'}^{T,T'} = S_{3,\alpha}^{T*} S_{3,\beta}^T S_{3,\alpha'}^{T'*} S_{3,\beta'}^{T'}, \tag{5.6}$$

which is given entirely in terms of the matrices $S_{\alpha,\beta}^T$, so that the only remaining free parameter in (5.4) is the mass scale ζz.

However, the effective Lagrangian (5.4) describes only interactions between quarks and leptons. To make contact with actual experiment on hadrons, we follow the usual procedures adopted in these contexts. For example, the effective action gives a contribution to the $K_L - K_S$ mass difference of the form:

$$\Delta m_K = \frac{1}{\zeta^2 \ z^2} |f_{2,3;2,3}^{D,D}| \langle K^0 | \left[\bar{s}_L \gamma^\mu d_L \right]^2 | \bar{K}^0 \rangle. \tag{5.7}$$

Evaluating the matrix elements in the vacuum insertion approximation one obtains:

$$\Delta m_K = \frac{1}{\zeta^2 \ z^2} |f_{2,3;2,3}^{D,D}| \frac{f_K^2 \ m_K}{3}, \tag{5.8}$$

where f_K is the K decay constant and m_K is the K mass. Mass differences between other charge conjugate neutral mesons are treated similarly. On the other hand, for hadron decays, in order to minimize the uncertainties in the hadron structure we take quotients between the rare and Standard Model-allowed processes which contain the same or similar hadronic matrix elements. For instance, for K^+ decays we take:

$$\frac{Br \left(K^+ \to \pi^+ l_\alpha l_\beta \right)}{Br \left(K^+ \to \pi^0 \nu_\mu \mu^+ \right)} = |f_{2,3;\alpha,\beta}^{D,L}|^2 \left(\frac{v}{\zeta \ z} \right)^4 \frac{2}{\sin^2 \theta_c}, \tag{5.9}$$

where $v = \frac{0.246}{\sqrt{2}}$ TeV and θ_c is the Cabibbo angle, $\sin \theta_c = 0.23$. Similarly, for the leptonic decays of the neutral K-mesons, we take:

$$\frac{\Gamma \left(K_{L(S)}^0 \to l_\alpha l_\beta \right)}{\Gamma \left(K^+ \to \mu^+ \nu_\mu \right)} = |f_{2,3;\alpha,\beta}^{D,L}|^2 \left(\frac{v}{\zeta \ z} \right)^4 \frac{1}{\sin^2 \theta_c}. \tag{5.10}$$

Then from these formulae, given the total widths of the K's and their widths in the Standard Model-allowed modes as measured in experiment, one can easily calculate the branching ratios of the various rare modes of K-decay. These procedures for dealing with the complexities of hadronic effects are of course far from foolproof but are likely to give the rough order of magnitudes correctly.

All predictions we obtain in this way are still dependent on the single parameter ζz, so that without any further input we can give no numerical value for the predicted quantities. So long as ζz remains undetermined, our predictions will of course lead to no violation of present experiment. However, given the experimental bound on any one quantity, a bound on ζz is implied, which will then allow us to give the correlated bound on all the others. The most stringent lower bound on ζz obtained in this way turns out to be that from the experimentally measured $K_L - K_S$ mass difference, namely $\Delta m_K(K_L - K_S) = 3.5 \times 10^{-12}$ MeV which is of

	Theory	*Experiment*
$Br(K^+ \to \pi^+ e^+ e^-)$	4×10^{-15}	2.7×10^{-7}
$Br(K^+ \to \pi^+ \mu^+ \mu^-)$	2×10^{-15}	2.3×10^{-7}
$Br(K^+ \to \pi^+ e^+ \mu^-)$	2×10^{-15}	7×10^{-9}
$Br(K^+ \to \pi^+ e^- \mu^+)$	2×10^{-15}	2.1×10^{-10}
$Br(K^+ \to \pi^+ \nu\bar{\nu})$	2×10^{-14}	2.4×10^{-9}
$Br(K_L \to e^+ e^-)$	2×10^{-13}	4.1×10^{-11}
$Br(K_L \to \mu^+ \mu^-)$	7×10^{-14}	7.2×10^{-9}
$Br(K_L \to e^\pm \mu^\mp)$	1×10^{-13}	5.1×10^{-12}
$Br(K_S \to \mu^+ \mu^-)$	1×10^{-16}	3.2×10^{-7}
$Br(K_S \to e^+ e^-)$	3×10^{-16}	2.8×10^{-6}

Table 2: Branching ratios for rare leptonic and semileptonic K decays. The first column shows the DSM predictions from one-dual gauge boson exchange with the lowest v.e.v. ζz of the Higgs fields taken at 400 TeV. The second column gives either the present experimental limits on that process if not yet observed or the actual measured value for that process. In the latter case, it means that the process can go by other mechanisms such as second-order weak so that our predictions with dual gauge boson exchange will appear as corrections to these. Except for the entry for the decay $K_L \to e\mu$ from [33] mentioned in the text, the other entries are from the databook [17].

roughly the right order of magnitude expected from second order weak interactions. Requiring that the FCNC effect due to dual gauge boson exchange be smaller than this value gives the bound [16]:

$$\zeta\ z \geq 400 \text{ TeV}. \tag{5.11}$$

The correlated (upper) bounds on other FCNC effects due to dual gauge boson exchange can then be estimated.

As will be seen in the next section, there is a possible upper bound on the parameter ζz coming from a rather unexpected angle which turns out to be similar to the lower bound quoted in (5.11). If that is the case, then the above bounds for FCNC effects can be treated as actual order of magnitude estimates.

For $\zeta z = 400$ TeV, the predicted branching ratios [16] of some rare K-decay modes are given in Table 2 and compared with the experimental limits/measurements [17]. One notes that most of the predictions are way beyond the present experimental sensitivity, while some others, such as $K_L \to e^+e^-, \mu^+\mu^-$, can also go by second order weak interactions which are expected to give similar or even somewhat larger contributions and so will overshadow the present predicted effects. Only the mode $K_L \to e^\pm \mu^\mp$, which is inaccessible to second order weak interactions unless neutrinos mix, is interesting in having a predicted branching ratio less than two orders of magnitude down from the present experimental limits [33] and so may be accessible in the near future. Its observation at this level may be considered as a confirmation either of the DSM prediction or that neutrinos mix and hence of interest in either case.

Similar tables have been compiled for rare D and B meson decays but the predicted branching ratios are in all cases much below the present experimental sensitivity and therefore not of immediate interest.

Mass differences between the conjugate neutral D and B meson pairs are given in a similar way to that for the K's. The contribution of one-dual gauge boson exchange to the mass splitting in D is [16]:

$$\Delta m_D = \frac{m_D}{\zeta^2 z^2} \frac{f_D^2}{3} |f_{2,3;2,3}^{U,U}|. \tag{5.12}$$

Taking the values $f_D^2 = 10^{-8}$ TeV^2 for the D-meson coupling and $m_D = 1.865$ GeV for the mass we have:

$$\Delta m_D = 5 \times 10^{-12} \text{ MeV}. \tag{5.13}$$

This is one-and-a-half orders of magnitude off the present experimental limits $\Delta m_D \leq 1.4 \times 10^{-10}$ MeV and could be accessible to planned experiments in the near future. Applying the same procedure to the mass-splitting between the neutral B-mesons, one finds that the contribution from dual gauge boson exchange is 6 orders of magnitude smaller than that from the Standard Model and thus not likely to be accessible.

We conclude therefore, for this section, that the DSM predictions on low energy FCNC effects so far made do not seem in contradiction with any existing experiment, and that for a couple of cases, namely $K_L \to e^\pm \mu^\mp$ and $\Delta m(D - \bar{D})$, apart of course from $\Delta m(K_L - K_S)$ itself, they may be testable in the near future.

6 Air Showers beyond the GZK Cut-off

At energies higher than the mass scale of the dual gauge bosons, FCNC effects will no longer be suppressed and, given the strength of their couplings $\tilde{g}_i$ as governed by the Dirac quantization conditions (5.1), the interactions due to their exchange will become strong. Hence, DSM would predict a new strong interaction at ultra-high energies for all particles carrying a generation index. In particular, even the neutrinos corresponding to the usual 3 generations of charged leptons will acquire strong interactions at high energies. At first sight, this seems alarming until one recalls from the estimate given in the last section for the mass scale involved of order $\zeta z > 400$ TeV, which is way beyond anything that has been achieved in terrestial experiments or can be achieved in the foreseeable future. Nor are such energies accessible to astrophysical or cosmological considerations except in the very early universe. There is in fact only one instance known to us that energies of that order have been experimentally observed, namely in air showers produced by cosmic rays with energy beyond the Greisen-Zatsepin-Kuz'min (GZK) cutoff.

Air showers with energy $E > 10^{20}$ eV [34]–[40], though rare in occurrence, pose a long-standing and intriguing question of fundamental physical interest. High energy air showers are usually thought to be due to protons, but protons with such an energy would interact with the 2.7 K microwave background via, for example, the reaction:

$$p + \gamma_{2.7K} = \Delta + \pi, \tag{6.1}$$

and degrade quickly in energy. Indeed, according to Greisen, Zatsepin and Kuz'min [41, 42], the cosmic ray spectrum for protons should cut off sharply at around 5×10^{19} eV (the GZK cut-off) if they come from further than 50 Mpc away. And within such distances, there does not seem to be any likely source for producing particles of so high an energy.

One possible solution would be that these post-GZK air showers are produced not by protons but by some other (stable, electrically neutral) particles which would not be so degraded in energy by the microwave background. The possibility can thus be considered that they are produced by neutrinos, which is feasible, of course, only if for some reason neutrinos acquire at high energy a new strong interaction, for otherwise they would not interact sufficiently with air nuclei to produce air showers. But this is exactly what is predicted by DSM as proposed in the preceding paragraph. So, it would appear that this prediction not only may escape contradiction with existing experiment as one might at first have feared, but may even offer an explanation for the long-standing puzzle of air showers beyond the GZK cutoff [43, 44, 45].

However, a strong interaction for neutrinos, though necessary, is by itself insufficient to guarantee a large cross section with air nuclei, which is needed for them to produce air showers in the atmosphere, since whatever the strength of the interaction, the cross section will remain small if the interaction is short-ranged. Now, the dual gauge bosons in DSM being supposedly very heavy, it looks at first sight that the interactions they mediate will be short-ranged and therefore not lead to large high energy cross sections for neutrinos. But this need not be the case for the following reason. The relation of dual colour to colour is similar to that of magnetism to electricity in electrodynamics which has only the physical degrees of freedom corresponding to the one photon, despite having two separate, electric *and* magnetic, gauge symmetries, as explained in Section 1. Hence, dual colour, though a different gauge degree of freedom to colour, represents just the same physical degree of freedom as colour [2]. As a result, it is argued [44] that a dual gluon can 'metamorphose' into a gluon in hadronic matter, thus giving neutrinos at high energy an interaction of hadronic range inside the nucleus. They will then interact coherently with the air nucleus and acquire with it a hadronic size cross section.[7] This assertion is admittedly rather conjectural. If it fails, then the suggested explanation for post-GZK showers no longer stands, but the assertion that the prediction of strong neutrino interactions at high energy contradicts no existing experiment still remains valid.

Since this suggested explanation for post-GZK air showers depends crucially on the assumed identification of generations with dual colour, it is worth examining its feasibility in some detail. Suppose that neutrinos do acquire strong interactions and a large enough cross section with air nuclei to produce air showers at high energy. The energy at which this begins to happen, according to DSM, is given by the scale estimated before to be > 400 TeV in the centre of mass. For a neutrino impinging on a nucleon at rest in the atmosphere, this corresponds to a primary energy of around 8×10^{19} eV, namely just above the GZK cut-off, exactly the sort of energy one wants. That being the case, let us now examine in more detail whether the hypothesis can accommodate the few observed facts known about the post-GZK showers, most of which have difficulties in being explained by protons as the primary particle.

(A) First, one asks how neutrinos at such a high energy can be produced. One does not actually know at present a truly realistic mechanism even for protons, but according to Hillas [47] one can at least write down the condition that a source must satisfy in order to produce such energetic particles:

$$BR > E/Z, \tag{6.2}$$

where B is the magnetic field in μG, R the size of the source in kpc, E the energy in EeV =

[7]There has appeared a paper by Burdman, Halzen and Gandhi [46] subsequent to [44] claiming, among other things, that neutrinos cannot on general grounds acquire hadronic size cross sections. We think that their claim is ill-founded. Their arguments used only first order perturbation theory which is far from adequate for hadron reactions which are notoriously nonperturbative in character. Indeed, with their arguments, one would be unable to deduce that protons have hadronic cross sections. They also claimed their conclusions to be a consequence of s-wave unitarity but gave neither justifications nor references to substantiate this claim, and a repeated effort by one of us (CHM) in correspondence with Francis Halzen has not produced any clarification. We do not think s-wave unitarity can constrain the cross section the way they claim it does since high energy cross sections involve all partial waves. For more details of this discussion see [45, 16].

10^{18} eV and Z the charge of the particle. There are only a few types of objects known which satisfy this condition, namely neutron stars, radio galaxies and active galactic nuclei. Of these, both the neutron stars and active galactic nuclei are surrounded by strong electromagnetic fields. The difficulty with protons is that even if the source can accelerate them to the required high energy, they would not be able to escape from the intense fields surrounding the source. However, there does not seem to be the same difficulty with neutrinos. By hypothesis based on the DSM, neutrinos interact strongly at high energy so that any source capable of accelerating protons to these energies will be able also copiously to produce neutrinos by say proton-proton collisions. Once produced, however, neutrinos will not be deterred by the intense e.m. fields and will be able to escape where protons fail.

(B) Secondly, one asks whether neutrinos will be able to survive a long journey through the 2.7 K microwave background. There is no problem, for in colliding with a (massless) photon at this temperature, a neutrino even at 10^{20} eV will produce only about 200 MeV C.M. energy, and at this energy a neutrino has still only weak interactions. The same applies also to its collision with the neutrino background in the intervening space.

(C) Thirdly, one asks whether a neutrino when it arrives on earth will be able to produce air showers with the observed angular and depth distributions. Neutrinos with only weak interactions will have immense penetrating power, and even if an enormous neutrino flux is assumed sufficient to produce air showers at these rare occasions, the showers will be mostly horizontal and have a near constant distribution in depth. This is in contradiction to what is observed for post-GZK showers which are mostly near vertical and occur in the upper atmosphere. However, once neutrinos are ascribed a hadronic cross section with air nuclei, they will interact like hadrons and both the angular and depth distributions will automatically fall into place.

(D) It was noted [48] that out of the few post-GZK events seen, three pairs coincide in incident angles to within the experimental error of about 2°. The probability of this occurring at random is very small and the obvious conclusion would be that the two members of each pair originate from the same source. They have however different energies and if they are protons should be deflected differently by the intervening magnetic fields and hence arrive at different angles, contrary to what is observed. If they are neutrinos, on the other hand, they will not be deflected by magnetic fields and will arrive on earth in the same direction they started out.

(E) The highest energy event at 3×10^{20} eV observed by the Fly's Eye [37] was noted to point in the direction of a very powerful Seyfert galaxy 900 Mpc away [49]. If that is taken to be its source and if it is due to a proton, one would wonder why many more showers with lower energies are not observed from the same direction, for a powerful source capable of accelerating protons to such a high energy would surely also produce protons at lower energies as well. For neutrinos interacting strongly only at high energy, this is not a problem. At low energies, neutrinos are weakly interacting and would first of all not be produced at source, and even if produced would not give rise to air showers when they arrive on earth.

It thus seems that the neutrino hypothesis has survived all the above tests (A)–(E) on post-GZK showers which pose difficulties for their having been produced by protons. In spite of this, however, the hypothesis must clearly be subjected to many more quantitative tests before it can be taken seriuosly. Fortunately, some such tests [44, 16] are available, as follows.

(I) If we accept our previous argument that a neutrino at high energy would interact not only strongly but coherently with an air nucleus, it is easy to deduce from a geometric picture that the cross section of a neutrino with the nucleus would be about half that of a proton. To both the neutrino and the proton, the nucleus would appear as a black disc, say of radius r_A. The neutrino, with as yet no known internal structure, would appear to the nucleus still as a point, but the proton will appear again as a black disc of radius, say, r_p. One concludes thus that the (geometric) cross section of the neutrino with the nucleus is roughly:

$$\sigma_T(\nu A) = \pi r_A^2, \tag{6.3}$$

while that of the proton with the nucleus is roughly:

$$\sigma_T(pA) = \pi(r_p + r_A)^2. \tag{6.4}$$

Assuming that:

$$r_A \sim A^{1/3} r_p, \tag{6.5}$$

with A around 15 for an air nucleus, one easily deduces the above estimate:

$$\frac{\sigma_T(\nu A)}{\sigma_T(pA)} \sim \frac{1}{2}. \tag{6.6}$$

This means that neutrinos at post-GZK energies are expected to be about twice as penetrating as protons, and hence that neutrino-produced air showers will occur at a lower depth on the average than proton-produced air showers. Folding in the air density as a function of height, it is easy to evaluate the penetration probability as a function of depth. This was done in [44] which finds that:

$$\begin{aligned} \text{Most probable height of } p\text{−produced showers} &\sim 21 \text{ km}, \\ \text{Most probable height of } \nu\text{−produced showers} &\sim 15 \text{ km}. \end{aligned} \tag{6.7}$$

Hence, one predicts that post-GZK air showers which are supposedly produced by neutrinos would occur most probably at a height of only around 15 km, in contrast to lower energy showers produced by protons which would occur most probably at a height of around 21 km in our atmosphere. Present detectors do not locate the primary vertices of air showers readily. For this reason, we have found up to the present only one tentative piece of information for testing this prediction. The development profile of the highest energy event obtained by the Fly's Eye shows that light began to be observed at a (vertical equivalent) height of around 12 km. If we interpret this as the primary vertex for the event, then it is much more likely, according to the preceding arguments, to be a neutrino-produced shower than a proton-produced one, for which the probability is estimated to be less than 5 %. This conclusion should not as yet be taken too seriously, but with new projects such as Auger [40], capable of collecting sizeable statistics, this could be a very useful test for the hypothesis that post-GZK showers are neutrino-produced.

(II) As far as particle physics proper is concerned, the post-GZK air shower events, if interpreted as due to neutrinos, are useful in providing a rough upper bound to the dual gauge boson mass. Translated to the language of Section 5, this means an upper bound on the parameter ζz of around 500 TeV [16], remarkably close to the lower bound of around 400 TeV obtained there from the $K_L - K_S$ mass difference. Acceptance of this upper bound then converts the bounds estimated in Section 5 on rare meson decays and mass differences into actual order of magnitude predictions, and hence affords a second test for the neutrino hypothesis here for post-GZK air showers.

7 Concluding Remarks

The basic tenets and applications to-date of the DSM scheme are summarized in the flow chart of Figure 7, which shows that starting from a previously derived result of nonabelian duality, one is led on the one hand to a calculation of some of the Standard Model's fundamental parameters, and on the other to new testable predictions ranging from FCNC effects at low energy to air showers from cosmic rays at the extreme end of the detected energy scale.

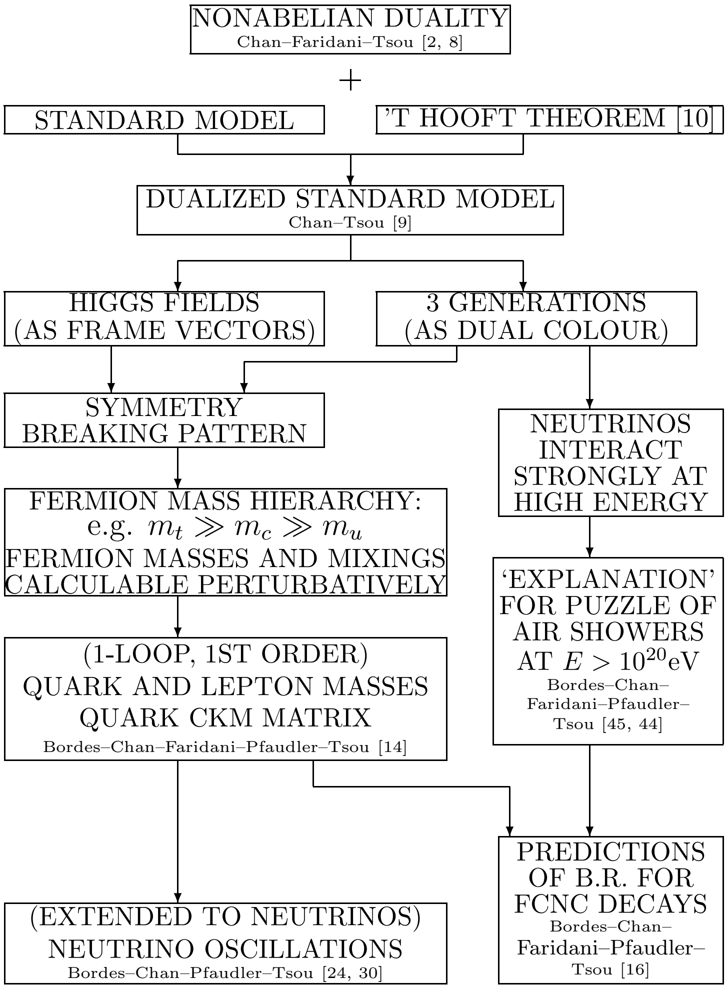

Figure 7: Summary flow-chart

One of the most attractive features of DSM is undoubtedly its offer of a possible explanation for the existence both of exactly 3 fermion generations and of scalar Higgs fields. In the conventional formulation of the Standard Model, the necessity to introduce by hand both of these, neither having any known geometrical significance, must be regarded as rather a blotch on a gauge theory otherwise so beautifully founded on geometry. The identification thus of generations as dual colour and of Higgs fields as frame vectors in internal symmetry space, giving each a geometrical significance, seems very attractive. Besides, according to [2], nonabelian duality is an intrinsic property of the Standard Model (as of any gauge theory) which then brings with it automatically a 3-fold broken dual colour symmetry and frame vectors in internal symmetry space playing a dynamical role. In other words, the niches for 3 fermion generations and Higgs fields already 'pre-exist' in the Standard Model. Hence, it seems appropriate to assign them to just these features we see in nature, because even if we do not, we shall still have to account for them physically in some other way.

Implementing these identifications with some seemingly natural assumptions as detailed in Section 1, one is then led to a scheme with a hierarchical fermion mass spectrum where the mixings between fermion types and lower generation masses are calculable as loop corrections in terms of a few parameters. The present score from the 1-loop calculation carried out to-date is as follows. By adjusting 3 parameters, namely the (common) Yukawa coupling strength ρ and the 2 ratios between the 3 vev's (x, y, z) of the dual colour Higgs fields, one has calculated the following 14 among the 26 or so of the Standard Model's fundamental parameters: the 3 independent parameters in the quark CKM matrix $|V_{rs}|$, the 3 corresponding parameters in the leptonic CKM matrix $|U_{rs}|$, and the 8 masses $m_c, m_s, m_\mu, m_u, m_d, m_e, m_{\nu_1}, B$. Of these 14 calculated quantities, 2 (i.e. m_u, U_{e2}) compare unsatisfactorily at present with experiment, and another 2 (i.e. the mass of the lightest neutrino m_{ν_1} and that of the right-handed neutrino B) are untested being experimentally yet unknown. The other 10, however, (namely, $|V_{rs}|, |U_{e3}|, |U_{\mu 3}|, m_c, m_s, m_\mu, m_d, m_e$), all agree as well as can be expected with their known empirical values. This, we think, is not a bad score for a first attempt based on some rather crude approximations, such as taking ρ and m_T as scale-independent constants. With more experience and sophistication, the score can possibly be improved.

However, even if considered successful, this score by itself does not constitute a stringent test for the basic assumption of DSM that dual colour is generations. As emphasized in a recent paper [18], the same result can be obtained just by assuming generations to be a broken $U(3)$ symmetry independent of whether it is identified with dual colour. The only physical consequence considered in this paper which relies crucially on that identification is the explanation suggested in Section 6 for air showers beyond the GZK cut-off, which is still far from established. An urgent task for this scheme is thus to device some further tests for the dual colour hypothesis.

Besides this, there are many further questions needing answers for checking the consistency of DSM, both within itself and with nature. Of these we list in particular the following. First, there is the question of CP-violation which, though known experimentally, has not yet made an appearance in DSM at the 1-loop level and might indicate a deficiency. Secondly, there is the question of the rotating mass matrix, which one has made use of in the calculation of mass and mixing parameters, and this might have other physical consequences yet waiting to be explored. Thirdly, there is the intriguing question of the 'accidental' near equality between Yukawa coupling strengths ρ for all fermion types, and the proximity of the Higgs vev's (x, y, z) to the fixed point $(1, 0, 0)$, which presumably reflect a deeper intricacy in the problem than we have yet understood. Fourthly, there is the question of linkage between the breaking of the dual colour symmetry, studied so far in isolation, with the breaking of the electroweak symmetry, which may be related to the point raised immediately above. Fifthly, there is the subtle question of whether the duality assertion that gauge and dual gauge bosons represent the same physical degrees of freedom might give rise to a new class of phenomena, called 'metamorphosis' by us in [9], of which post-GZK air showers are but one example of many possible manifestations. Sixthly, going even further afield, there is the question of the symmetry $\widetilde{SU}(2)$ dual to electroweak $SU(2)$, which by the same logic adopted for $SU(3)$ colour here, ought to give rise to another level of confinement deeper than colour, and this should be amenable to experimental investigation, but only by deep inelastic scattering at ultra-high energies. And there will be other questions too which we have not yet learned even to formulate. One has thus the feeling that what has been attempted so far is but scratching the surface of a possibly very rich vein.

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