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THE EFFECT OF BEAM TIME STRUCTURE ON COUNTING DETECTORS IN *SRS* EXPERIMENTS

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Abstract

Counting detector systems are increasingly used in x-ray experiments because of their attractive properties (linearity, large dynamic range and simple noise properties). In Sychrotron Radiation Source (SRS) applications of x-ray detectors counting rates are generally high enough to require deadtime correction. The time structure of an SRS beam interacts with the deadtime characteristics of the detector in a way that the simple stochastic deadtime models cannot always handle. This report generates mathematical models (analytical and montecarlo) which describe the rate performance of any given detector system when used with the typical beam structures encountered in an SRS.

1. Introduction

Counting detector systems are attractive for x-ray detection applications because of their inherent linearity, large potential dynamic range and well-defined (statistical) noise characteristics. Their main limitation is the inevitable deadtime losses which appear at data rates comparable with the detector deadtime. These deadtime losses are well understood when (as in the case of the detection of radioactive decay particles) the arrival of events is random in time [1]. Experiments on Sychrotron Radiation Sources, on the other hand, combine very high data rates with a highly time-structured beam which complicates the simple stochastic model. The bunch structure of the electron beam in a Sychrotron Radiation Source (SRS) produces an x-ray beam with the same time modulation. This time structure will (in general) interact with the intrinsic timing properties of any counting system used to detect the x-rays and can often limit the rate performance of the system.

Using mathematical models, this report investigates methods for assessing the performance of any given type of detector system when used with some typical SRS beam structures. The modelling approach required to describe the interaction between the beam structure and the detector system varies according to the relation between the counter deadtime and the beam structure. It is convenient to consider the three distinct types of beam structure separately.

In the following, the symbols N_{in} and N_{out} refer (respectively) to the time-averaged rate of events (i.e. over a period long compared to any beam structure) which a detector system would deliver if it had no deadtime (N_{in}) and the rate that it actually does deliver (N_{out}) in the context of a given beam structure. The two distinct types of deadtime model are denoted by the terms *counting above a simple discriminator* (i.e. a single deadtime period is operative for each event and is not re-triggerable during that period) and *pulse height analysis* in which a clean spectrum demands rejection of pile-up events.

2. SRS Beam Structures

For the purposes of developing the models we use the time structures used on the Daresbury Laboratory SRS. These are:

(i) Flat Fill
The ring is full of evenly spaced bunches of electrons which give continuously an x-ray pulse of width 0.2ns every 2ns.

(ii) Single Bunch
There is only one bunch in the ring which produces an x-ray pulse of width 0.2ns every circuit of the ring; i.e. every 320ns.

(iii) Gapped Beam
In this case there is a gap in the bunch structure so that x-rays are delivered as in the *flat fill* case for 200ns of each beam circuit and are absent for the remaining 120ns of the 320ns period.

Different SRS machines will have a cycle time determined by the machine diameter (3μs for the ESRF and about 1μs for Diamond) with a similar fine structure and various gap patterns.

3. Deadtime Modelling with a Flat Fill Beam

No counting detector in routine use can respond to the fine structure of the beam. (The fastest counter is probably a scintillator/PMT device which can operate with a deadtime of a few tens of nanoseconds.) Thus the flat fill case reduces (as far as the counter system is concerned) to uniform delivery of x-rays in time. For this we can use the standard deadtime models. (At any data capture rate accessible to the detector system there will be very much less than one detectable event per bunch so that the arrival of events will be effectively random in time.)

(a) Counting above a simple discriminator
In this case the response of the counting system (as a function of the input rate) is:

$$N_{out} = N_{in} / (1 + N_{in}\tau) \qquad (1)$$

where N_{out} is the data capture rate, N_{in} is the random x-ray rate driving the system and τ is the counting system deadtime (usually a combination of intrinsic detector deadtime and electronic deadtime).

In this case the data capture rate is 50% of N_{in} at $N_{in} = 1/\tau$ and the data capture rate, N_{out} asymptotes to $1/\tau$ as N_{in} goes to infinity.

(b) Pulse height analysis
In the case of pulse height analysis we require a clean signal free of pile-up distortion. This requires that a deadtime period must always be free of another event. Poisson statistics show that in this case:

$$N_{out} = N_{in} \exp(-N_{in}\tau)$$

This function peaks at a value of N_{in} / e at $N_{in} = 1/\tau$ and declines to zero as N_{in} increases further. It is important to understand the significance of τ in this equation: it represents the sum of the *occupancy times* (the period in which the analogue pulse of the detector is *significantly* above threshold) of the pulse under consideration and any preceding pulse. If we wish to be consistent with the definition of τ in equation (1) it is necessary to write this function as:

$$N_{out} = N_{in} \exp(-2N_{in}\tau) \qquad (2)$$

This is the actual counting rate that will be observed if a pile-up rejector is fitted to the system. If one is not fitted then the detected rate will be given by equation (1) above and equation (2) will give the rate of “good” events in the PHA spectrum with $N_{out}(1) - N_{out}(2)$ events in the “pile-up” spectrum. The word *significantly* in the above definition of the occupancy time sets the limits to the acceptable distortion in the pulse height spectrum - usually the occupancy time is defined as the period that the pulse is >1% of its amplitude above the baseline.

In conclusion we note that the rate performance of the detector systems is entirely governed by their own deadtime and is not affected by the beam structure.

4. Deadtime modelling with a Single Bunch Beam

In this case the situation is dominated by the fact that all the x-rays emitted during a bunch crossing are inseparable (in time) by any normal detector system. The important statistic is therefore the number of x-rays (M) detected per beam pulse of 0.2ns width. Two distinct case must be considered:

(a) The detector deadtime is shorter than the circulation period (τ_B).

In this situation the detector registers one count only whatever the value of M but is ready to trigger on the arrival of the next beam pulse. In order to evaluate the number (m) of counts recorded we must consider the Poisson distribution of the number of x-ray counts (x) actually produced for a mean number of M:

$$P(x,m) = e^{-M} M^x / x! \qquad \text{where } \Sigma_x P(x,M)=1 \qquad (3)$$

The number of events at each occurrence number (x) is:

$$xP(x,M) \text{ i.e. } e^{-M}M^x/ (x-1)! \text{ Where } \Sigma_x xP(x,M)=M$$

Summing the valid events we see that we can count the occurrences of single events but double, treble etc. events only count as one:

$$m = 1P(1,M) + 1P(2,M) + 1P(3,M) + \text{-------}$$

Using equation (3) we see that:

$$m = P(1,M) + \Sigma_{x=2} P(2,M)$$

$$\text{i.e. } m = P(1,M) + \{1 - P(0,M) - P(1,M)\}$$

$$\text{i.e. } m = Me^{-M} + 1 - e^{-M} - Me^{-M}$$

$$\text{i.e. } m = 1 - e^{-M}$$

Since we get M x-ray counts, of which we detect m at every beam pulse, our observed counting rate is:

$$N_{out} = (1 - e^{-M}) / \tau_B \qquad (4)$$

If N_{in} is the rate of events hitting the detector then:

$$N_{in} = M / \tau_B$$

and we can reformulate (4) as:

$$N_{out} = (1 - \exp(-N_{in}\tau_B)) / \tau_B \qquad (5)$$

As shown in figure 2 the data capture rate asymptotes to $1/\tau_B$ as the beam intensity (N_{in}) is increased. This is the case in which we are simply counting above a simple discriminator.

When uncorrupted data (i.e. no pile-ups) is required then we are allowed to count only the term P(1,M) in the Poisson distribution. This gives a detected rate of:

$$N_{out} = Me^{-M} / \tau_B$$

$$\text{i.e.} \quad N_{out} = N_{in}\tau_B \exp(-N_{in}\tau_B) / \tau_B$$

$$\text{i.e.} \quad N_{out} = N_{in} \exp(-N_{in}\tau_B) \qquad (5a)$$

This is the same expression as (2) above with the difference that the deadtime is the beam circulation period and (because the beam is synchronous in its temporal pattern, the factor of two is not present. This curve is also shown in figure 2.

In conclusion we note that the system throughput curves are very similar to the *flat fill* case **except** that the governing time constant is now the beam circulation time. Thus counting above a simple discriminator, the ultimate rate of any counter is just the beam circulation frequency. The "no pile-up" case is academic in this situation since it is impossible to make a pile-up rejector which can function within the 0.2ns of the fine structure. However, the "pile-up" curve (figure 2) is of interest in that it shows the rate of uncorrupted events with the difference between the two curves showing the rate of "piled up" events.

(b) The detector deadtime is longer than the beam circulation period

In this case any bunches which follow one with a detected event will not be seen by the detector until the deadtime of the detector has elapsed. We can apply the models derived above as follows:

Counting above a simple discriminator the detector the counting system sees a data capture rate governed by relation (1), where the input rate is controlled by the statistics of the countable events (m) i.e. relation (4). Thus:

$$N_{out} = \frac{(1-e^{-M}) / \tau_B}{\{1 + (1-e^{-M})\, \tau / \tau_B\}}$$

Substituting $N_{in} = M / \tau_B$ this becomes:

$$N_{out} = \frac{(1-\exp(-N_{in}\tau_B)) / \tau_B}{\{1 + (1-\exp(-N_{in}\tau_B))\tau/\tau_B\}}$$

To make this formula represent the true behaviour of the counting system we must observe that the single bunch mode the beam loss in the counter deadtime is digitised to the number of bunch crossings covered by τ. Thus the final formula for this case becomes:

$$N_{out} = \frac{(1-\exp(-N_{in}\tau_B)) / \tau_B}{\{1 + (1-\exp(-N_{in}\tau_B))\, n\}} \qquad (6)$$

where n is an integer defined by $n = INT(\tau / \tau_B)$.

Checking the limiting cases of (6) we see that if $\tau/\tau_B < 1$ (n=0) then (6) becomes (5) as required. If $\tau >> \tau_B$ and rates are high (6) limits to $1/\tau$ and if rates are low to equation (1).
Figure 3 shows the input/output curves given by equation (6) for a few values of n. A rough rule of thumb emerges from figure 3 for very high rates ($>1/\tau_B$), namely that $N_{out} \approx 1/\tau_B \; 1/(n+1)$. As noted above, at low rates N_{out} tends to the standard formula (1) for n>>1.

Similar reasoning can be applied to the pulse height analysis case (i.e. no pile-ups allowed). Pile-up in this case arises in two distinct settings: pile-up within a bunch, the fraction of non-piled up events being:

$$N_{out}/N_{in} = \exp(-N_{in}\tau_B)$$

and pile-up in the detector system:

$$N_{out}/N_{in} = \exp(-2N_{in}\tau)$$

Combining these two fractions the overall rate of "good" events is:

$$N_{out} = N_{in} \exp(-N_{in}\tau_B) \exp(-2N_{in}\tau_B n)$$

i.e.

$$N_{out} = N_{in} \exp\{-N_{in}\tau_B(2n + 1)\} \qquad (7)$$

where as usual $n = INT(\tau/\tau_B)$. In the limiting case of n=0 (no counter deadtime) we see the formula for the bunch pile-up as expected, and at large detector deadtimes (2n>>1) equation (7) reduces to the pile-up formula for the detector system alone as expected.

Figure 4 shows the behaviour of equation (7) for a range of values of n. In this figure the fraction N_{out}/N_{in} denotes the fraction of "clean" pulses in a pulse height spectrum. Pile-up protection on the counter circuit can only remove the fraction generated by the counter time constant and the unresolvable pile-up within the bunch remains. This contribution to the pile-up will only become small at rates such that $N_{in}\tau_B << 1$.

5. Deadtime modelling with a Gapped Beam

In the case of a *gapped beam* there are three interacting parameters describing the situation: the detector deadtime (τ), the basic beam cycle time (τ_B) and the length of the "on" time of the beam (τ_{on}). This leads to a situation which is analytically untractable, and to which the simplest modelling approach is a montecarlo computer program. The montecarlo model also provides the opportunity to check the analytical formulae developed for the simpler cases.

In order to compute the N_{out} versus N_{in} curves for the case of counting above a simple discriminator, we focus attention on a large number of beam cycles (about 1000) and generate events randomly in this interval building in the "ON" period. We express τ and τ_{on} as fractions of τ_B and work in units of τ_B. In order to apply the deadtime in the analogous fashion to an electronic system the chosen number of "input" pulses (a few thousand) are sorted into ascending order and only counted into the "output" total if they are more than one deadtime period from the preceding event. The input and output rates are expressed naturally as $N_{in}\tau_B$ and $N_{out}\tau_B$.

Figure 5 shows the plots obtained from the program when τ_{on} is set to the value appropriate for the SRS (τ_{on} = 0.625) and the detector deadtime is varied from τ = $0.1\tau_B$ to $10\tau_B$ (i.e. 32nS to 3.2μs corresponding to (say) a scintillation detector at one end to a germanium detector at the other). The input range of the counting rate is 0 to 6.25MHz and the output range 0 to 4.7MHz.

The program can be modified in a simple manner to generate the corresponding curves for the case in which we require uncorrupted events (i.e. a clean pulse height spectrum). The same logic is applied as above except that the deadtime period is extended forwards *and backwards* in time from the event under consideration.

Figure 6 shows the N_{out} versus N_{in} curves generated in this way with the same parameter ranges as in figure 5. As expected these curves do not asymptote to a maximum rate but peak at values of N_{in} and N_{out} inversely related to τ (but clearly not in any simple way).

Computation of the system throughput curves in terms of these generalised parameters means that the curves in figures 5 and 6 can be used for any detector used with the given pattern of gapped beam. If a different τ_{on} is required then the curves must be recalculated. Any pattern of beam and gap can be modelled if it is required.

6. Comparison of the Formulae with the Montecarlo Model

(a) Counting above a simple discriminator.
The simplest case of a *flat fill* beam can be represented in the model by setting τ_{on} = 1.0. If τ = 1 (using units of τ_B) then the plot of $N_{out}\tau_B$ (y) versus $N_{in}\tau_B$ (x) is just y =

1/(1+x) (formula (1). As figure 7 shows the agreement between the model and the function is perfect within the statistical noise of the montecarlo.

The model can represent a single bunch beam by setting τ_{on} = 0.001 (0.32ns). Figure 8 compares the output of the model with formulae (5) and (6). Again the agreement is perfect within the statistical error. Because the bunch is very narrow the deadtime curves switch abruptly with the value of **n**. In the real world with detector time jitter the transitions will be sloppier.

(b) Pulse height analysis
Figure 9 compares the rate of "clean" events in the case of a *flat fill* beam with τ in the range of 0.5 to 2, as predicted by formula (2) and the montecarlo model.

Figure 10 makes the comparison of the predictions of formula (7) and the model in the case of a *single bunch* beam when the detector occupancy time increases from less than τ_B to greater than τ_B.

In all cases the agreement is perfect within the statistical noise of the montecarlo process. Since the montecarlo model simply mimics the corresponding electronic function these results give confidence in the mathematical analysis (and vice versa in the case of the simplest formulae).

7. Conclusions

The models developed in this report permit detailed exploration of the performance of counting systems in any of the beam time structures generally met. As expected, counting above a simple discriminator leads to an asymptotic data capture rate and the rate of un-piled-up events peaks and thereafter declines. The main point to emerge is that in the *flat fill* case the detector deadtime (τ) is the controlling parameter and in the *single bunch* case the beam circulation time (τ_B) is the controlling parameter. The result of the latter effect is that a detector with a deadtime significantly less than τ_B will be limited to a rate of around $1/\tau_B$. For such a detector the *flat fill* is more advantageous. For very slow detectors ($\tau >> \tau_B$) the difference is small.

There is one fast detector system which can significantly reduce the losses associated with *single bunch* operation. Using a Gas Microstrip Detector (GMD)with the electron drift direction normal to the beam direction [2] one can use the drift time of the x-ray-generated electron clouds to randomise the times of arrival at the detector anodes. The 320ns period of the SRS corresponds to a drift distance of 15mm which is quite comfortable. For longer periods (up to several μs) it is possible to choose slower drift conditions without compromising the basic speed of the anode pulse. Operating in this way the GMD can randomise the events in the beam spike and detect them with a limit set by its own deadtime (about 100ns). On the SRS the gain in limiting rate is of the order of x3 but on the larger machines factors of >10 could be achieved in *single bunch* mode.

The performance of any detector system in *gapped beam* mode is obviously intermediate between the *single bunch* and *flat fill* cases. The interaction between the parameters is complex and the practical method of solving it is to use the montecarlo model with the parameters of the situation appropriately inserted. The simple model used is approximate but adequate to elucidate the key features of the situation.

The models show clearly the great difficulty of getting clean pulse height spectra (and clean discrimination) at high rates. As figures 6 and 10 show, if the occupancy time is larger than τ_B in *single bunch* or *gapped* beams, the occupancy time of the detector must be less than τ_B to achieve a useful fraction of "clean" data. Similarly in a *flat fill* beam (figure 9) the value of τ is critical to achieving "clean" data at high rates.

REFERENCES

[1] G F Knoll, Radiation Detection and Measurement, Chapter 17, Wiley, New York, 1989
[2] J E Bateman et al, A prototype Gas Microstrip wide angle x-ray detector, RAL-TR-1998-073

FIGURE CAPTIONS

1. The characteristic throughput curve predicted by formulae (1) for counting above a simple discriminator and by formula (2) for the uncorrupted rate in a pulse height spectrum. The natural parameters $N_{in}\tau$ and $N_{out}\tau$ are used where N_{in} is the event rate hitting the detector and N_{out} is the data capture rate. τ represents the detector/electronic deadtime in the first case and the event occupancy time in the second case. This is the case of random incidence with no detectable beam structure.

2. This shows the corresponding curves generated by formula (5) and formula (5a) for the case of a *single bunch beam* combined with a detector deadtime which is less than the beam circulation period τ_B. It will be noted that the detector deadtime is irrelevant if it is less than τ_B. In this figure (and in most of the subsequent figures) the natural parameters $N_{in}\tau_B$ and $N_{out}\tau_B$ are used.

3. The predictions of equation (6) are plotted for various values of n (= int(τ/τ_B)) for counting above a simple discriminator with deadtime longer than the beam circulation time in the case of a *single bunch* beam.

4. The equivalent predictions from formula (7) (pulse height analysis) when the event occupancy is greater than the beam circulation time.

5. The throughput characteristics predicted by the montecarlo model for a *gapped beam* with the "ON" time (τ_{on}) set at the SRS value (0.625τ_B) for counting above a simple discriminator with a range of detector deadtimes (expressed as fractions of τ_B).

6. The corresponding model predictions for the “clean” pulse height analysis rates with a range of occupancy times.

7. A comparison of the predictions of the montecarlo model in a *flat fill* beam (counting above a threshold rate) with formula (1). A range of deadtimes is shown.

8. A comparison of the predictions of the montecarlo model in a *single bunch* beam for the output rate (counting above a simple discriminator) with formula (6). A range of deadtimes is shown.

9. A comparison of the of the montecarlo model in a *flat fill* beam (“clean” pulse height analysis rate) with formula (2). A range of occupancy times is shown.

10. A comparison of the montecarlo model in a *single bunch* beam (“clean” pulse height analysis rate) with formula (7). A range of occupancy times is shown.

FIGURE 1

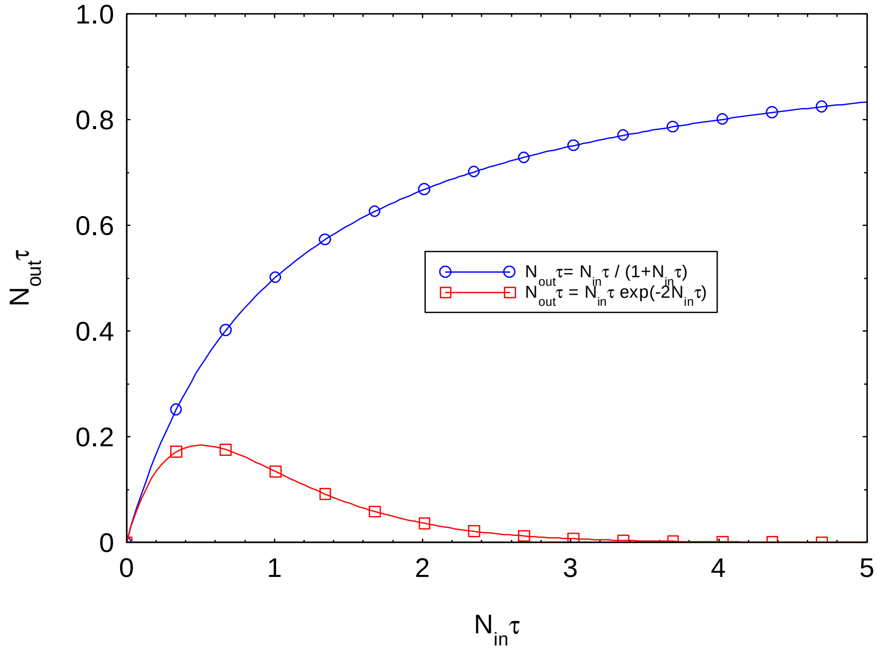

FIGURE 2

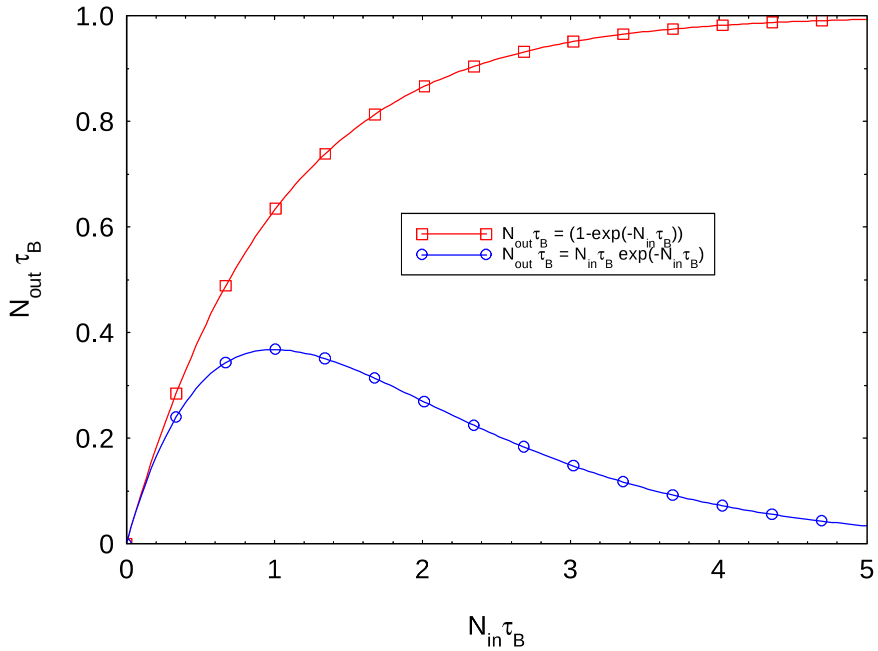

FIGURE 3

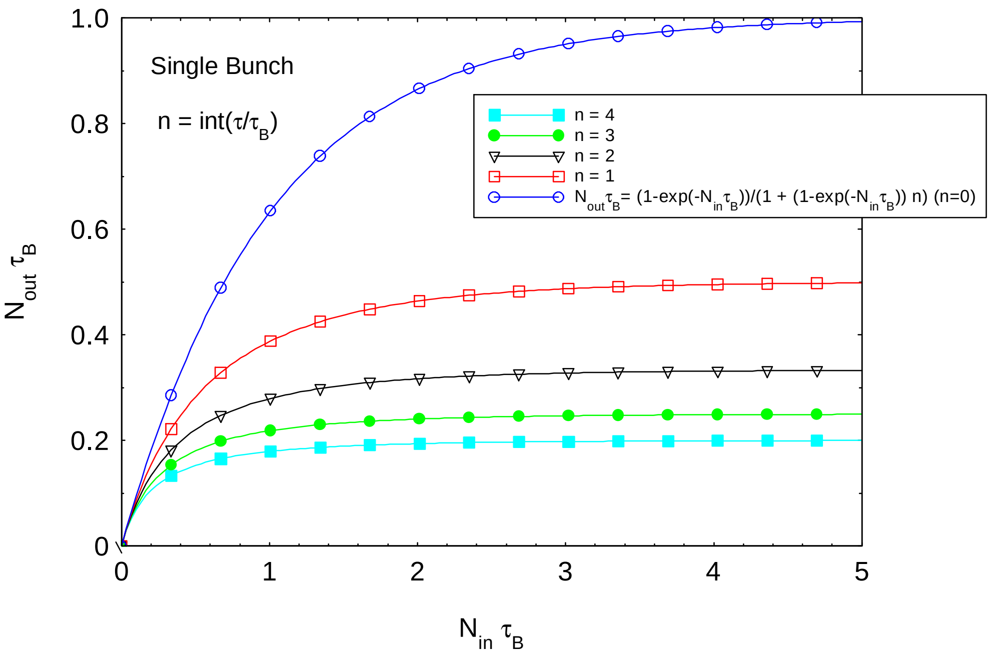

FIGURE 4

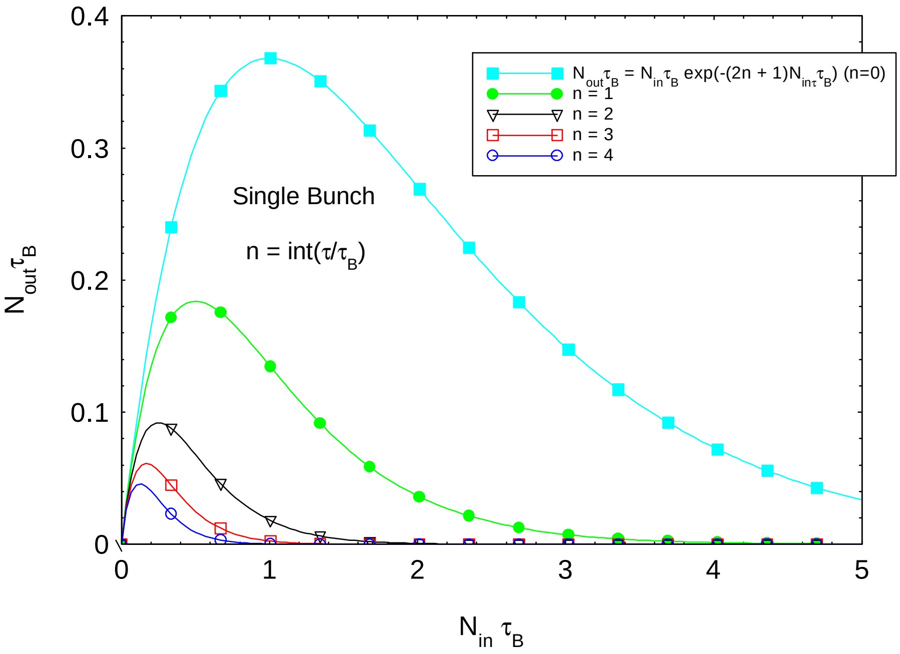

FIGURE 5

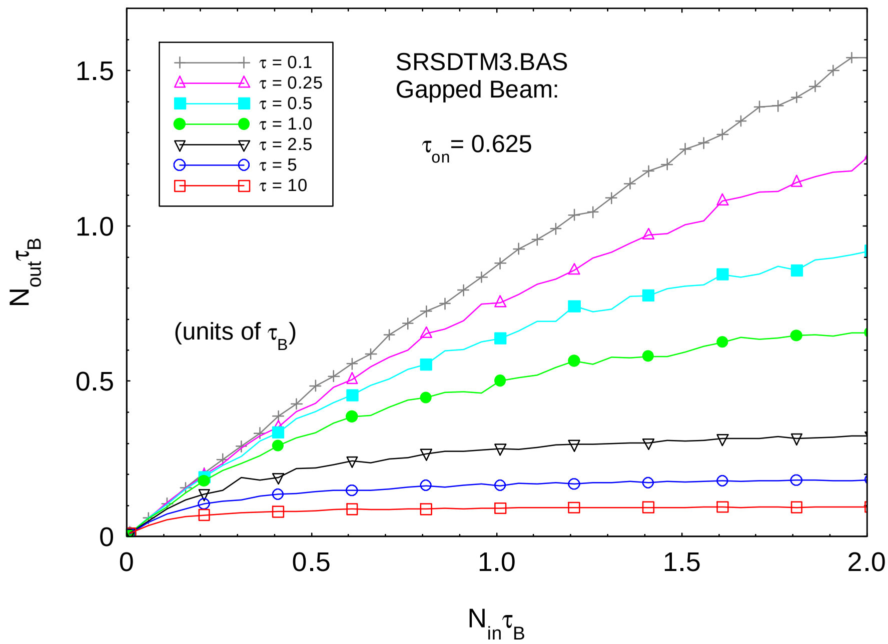

FIGURE 6

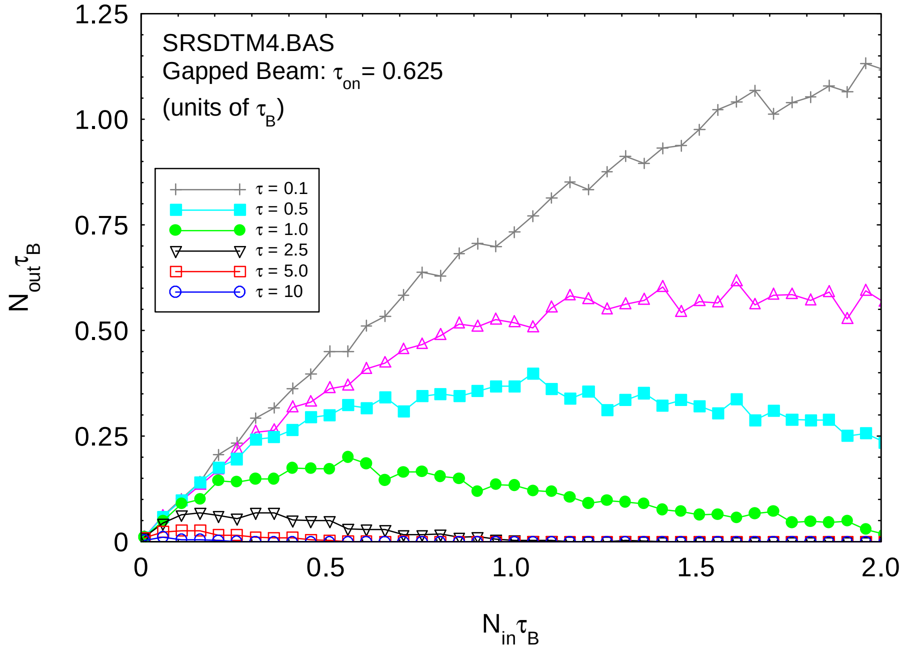

FIGURE 7

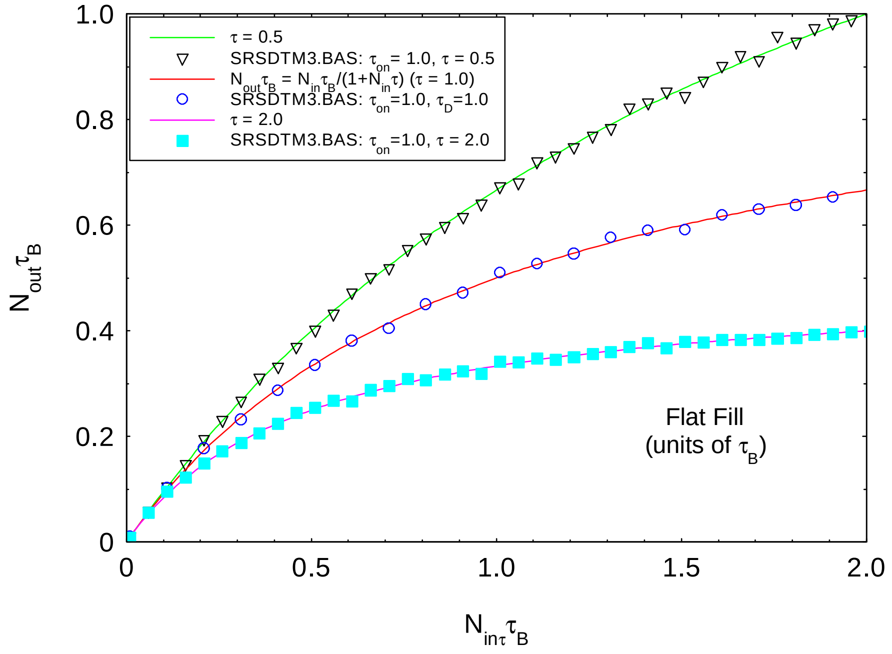

FIGURE 8

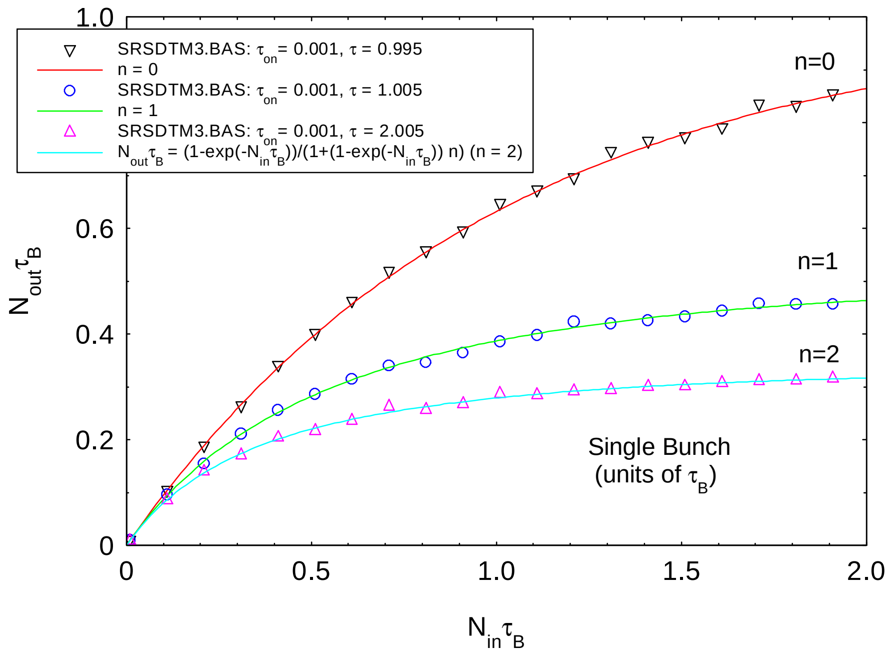

FIGURE 9

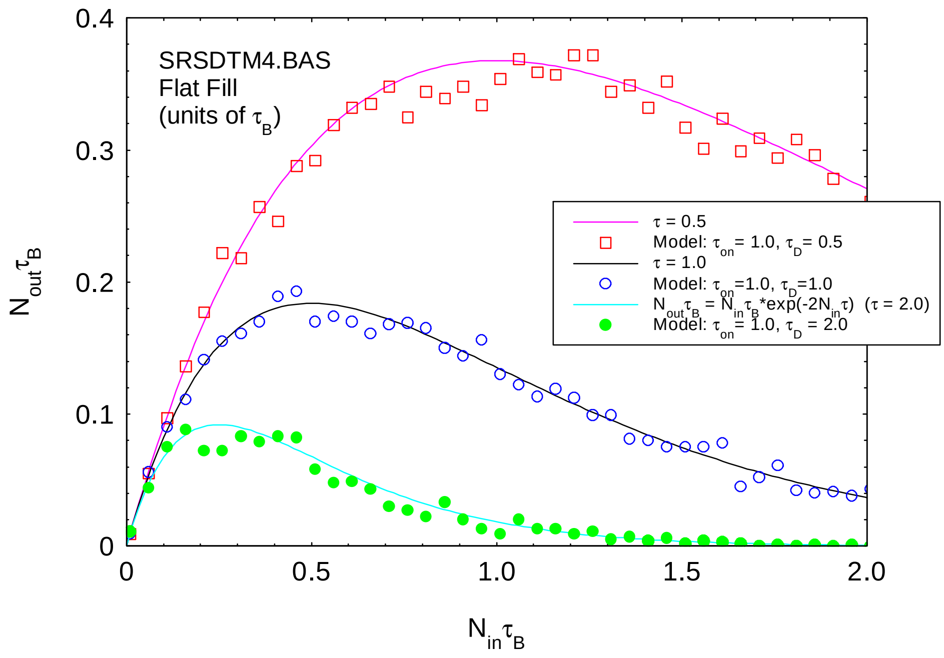

FIGURE 10