

PROGRESS IN MAGNET DESIGN BY COMPUTER

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Abstract

The Partial Differential and Integral Equation formulations for the solution of the non-linear magnetostatic problems are reviewed. The advantages of the latter for numerical work are stressed. A brief survey of the literature of computer programs developed in various countries is made. The need for on-line programs for use by magnet designers is discussed. The Rutherford Laboratory interactive magnet design program GFUN, which features optimisation, is described. Some results for both two and three dimensional problems are presented with comparisons of the TRIM program with measurement.

I. Introduction

Some ten years ago magnet designers were predicting that in the not too distant future a magnet design would be adequately carried out entirely by purely computational techniques.¹ Has this prediction been realised? The answer to this question is only a qualified yes. A designer today can certainly expect to obtain from a computer a reliable approximation to the field for those magnets in which the current and iron regions can be idealised in two dimensions. Indeed there are many excellent computer programs available which are capable of achieving good accuracy, and can be used for assessing the performance of high homogeneity magnets.² Furthermore, since the power of computers has increased a greater degree of sophistication is now possible. Already programs which handle three dimensional geometries are being developed and tested.

However there is more in design than computing the field for a given geometry. The more pertinent question is the inverse of this: compute the geometry to achieve a specified field shape. This is a problem of optimisation and optimisation of one type or another is the essence of design. Several workers, notably J Colonias³, and K Halbach⁴ at LRL Berkeley have been very active in applying the computer to optimisation, and later in this paper an account of some recent work carried out at the Rutherford Laboratory will be given.

At the heart of any computer program for field optimisation is the 'analysis program', the algorithm for computing the field for a specified configuration. There is an extensive literature on the subject of field analysis programs, as the excellent bibliographies prepared by A Halacsy⁵ at the University of Nevada demonstrate.

The first part of this paper surveys the various techniques that have been used in some of these analysis programs, and tries to highlight their range of applicability and limitations.

II. Alternative formulations of the Magnetostatic Problem

The magnetostatic problem can be conveniently formulated in two ways. Consider first the classical approach in which the partial differential equations describing the field are solved. This method has proved most useful in the analytic solution of field problems.

These equations are:

$$\text{curl } \underline{H} = \underline{J} \quad (1)$$

$$\text{div } \underline{B} = 0 \quad (2)$$

where \underline{H} and \underline{B} are the magnetic field intensity and magnetic flux density at a point respectively and \underline{J} is the current density.

The field vectors \underline{H} and \underline{B} are related by the constitutive equation:

$$\underline{B} = \mu(\underline{H})\underline{H} \quad (3)$$

where μ is the permeability a known property of the material depending upon \underline{B} or \underline{H} . The material is assumed to be isotropic. It is usual to solve the field equations by introducing associated potential functions of various kinds. The effect of introducing these potential functions is to transform the coupled first order differential equations to a single second order equation of the elliptic type. The numerical procedure is to replace the continuous partial differential equation by a system of linear algebraic equations which connect the values of the potential function at neighbouring points of a mesh spanning the whole domain of the magnet. The known values of the potential or its derivatives at the boundaries ensure that the number of unknowns is the same as the number of interior mesh points.

The second approach, the direct method, in which there has been a great deal of interest shown recently following the pioneering work of A Halacsy⁶ and others, is to solve an integral equation for the magnetic field inside the iron. The field at a point inside the iron is the sum of the field due to external current sources \underline{H}_s and that due to the internal magnetisation sources \underline{H}_m . If the iron is considered as a conglomeration of dipoles then the field at a

point \underline{r} due to the magnetisation sources is given by:

$$\underline{H}_m(\underline{r}) = - \operatorname{grad} \int_{V'} M(\underline{r}') \cdot \operatorname{grad} \left[\frac{1}{|\underline{r}' - \underline{r}|} \right] dV' \quad (4)$$

where M is the magnetisation at the source point \underline{r}' and the integration extends over the volume V' of the iron. H is further related to M by the classical expression for the magnetisation:

$$M = \chi H \quad (5)$$

where χ is the susceptibility.

Hence the field at a point inside the iron can be expressed as an integral equation:

$$\underline{H}(\underline{r}) = \underline{H}_s(\underline{r}) - \operatorname{grad} \int_{V'} \chi(\underline{r}') \underline{H}(\underline{r}') \cdot \operatorname{grad} \left[\frac{1}{|\underline{r}' - \underline{r}|} \right] dV' \quad (6)$$

Of course there are, as with the partial differential equations method, alternative forms. The numerical procedure is to replace the continuous integral equation by a set of algebraic equations which connect the values of the field function at every point of a mesh spanning the domain of the iron only. The values for the field due to the current sources are readily available by direct integration of the known current density over the volume of the conductor. This ensures that the set of algebraic equations can be solved provided the value of χ is known.

The partial differential equation method has several limitations which do not arise in the integral equation method. In the first place consider the boundary values, which in the PDE method usually have to be prescribed at a distance far away from the region of interest to achieve a satisfactory solution, whereas in the IE method the boundary values are taken care of in the formulation. This corresponds to the physical situation when the boundary of the field is at infinity and so the IE method can compute fringe fields and far fields naturally. In the second place the potential function used in the PDE method has to be differentiated to obtain the field components and numerical differentiation is a difficult process. In the third place the mesh used in the PDE method has to span the entire geometry of the magnet and the space around which introduces several difficulties, for example it is difficult to represent interface boundaries and regions with small gaps and sharp corners, although the elegant mesh generator introduced by A Winslow⁷ in his TRIM program to some extent solves this problem.

In the IE method there is a far greater degree of freedom in the choice of mesh since only the magnetic material itself has to be discretised. A wide variety of element shapes can be easily used in the same magnet. This is of great importance in the extension to three dimensions where it is relatively easy to per-

form integrations over volumes but seemingly a daunting task to generate an irregular mesh embracing solid objects in three dimensions. The resulting set of equations in the two cases are different in character, the set in the PDE method are of a banded nature with a sparse matrix of coefficients and it is customary to solve these by iterative means, eg. successive over relaxation. The corresponding set in the IE method whilst fewer in number are not banded and generally not even symmetric so solution by direct methods is preferable to iterative methods.

In both approaches the form of the field equations and the way in which the field equations are discretised provide several alternative methods for computation, and some of the computer programs based on these methods are discussed briefly in the next section.

III. Selected Computer Programs

1. Potential Functions

Before listing some of the computer programs available to magnet designers, it is convenient to state the three main potential functions upon which they are based.

- a. Scalar Potential V. This is the ordinary magnetostatic potential which is widely used in analytic techniques for iron-air regions. If the field \underline{H}_s due to the current sources is considered as known then by equation (1) \underline{H}_m the field due to the magnetic material can be expressed as the gradient of a scalar potential V and so by equations (2) and (3) it can be seen that V satisfies:

$$\nabla \cdot (\mu \nabla V) = \operatorname{div} (\mu \underline{H}_s) \quad (7)$$

with the boundary condition that

$$V \rightarrow 0 \text{ at infinity}$$

The interface condition between air and iron required is obtained by applying the condition that the normal component of \underline{B} is continuous

$$\mu_1 \left(\frac{\partial V}{\partial n} \right)_1 - \mu_2 \left(\frac{\partial V}{\partial n} \right)_2 = (\mu_1 - \mu_2) H_{sn} \quad (8)$$

Hence the solution of the problem is obtained by solving for V in equation (7) subject to the interface condition in equation (8), and the boundary condition that V is zero at infinity. This approach appears to be the most successful to date for three dimensions but apart from infinite permeability problems has rarely been used in two dimensions.

- b. Vector Potential A. The vector potential which has been widely used in the analytic solutions of problems involving current and iron regions, has been successfully used as the basis of many computer programs.

The vector potential \underline{A} is introduced through the relation:

$$\underline{B} = \text{Curl } \underline{A} \quad (9)$$

This does not determine \underline{A} uniquely since the gradient of some scalar function may be added to \underline{A} to obtain the same result. This difficulty is avoided by imposing an extra constraint on \underline{A} , namely:

$$\text{Div } \underline{A} = 0 \quad (10)$$

From equations (1) and (9) the PDE for A is obtained:

$$\nabla \times (\frac{1}{\mu} \nabla \times \underline{A}) = \underline{J} \quad (11)$$

Since \underline{A} has only one component for problems in two dimensions equation (11) reduces to:

$$\nabla \cdot (\frac{1}{\mu} \nabla \cdot \underline{A}) = J_z \quad (12)$$

The constraint imposed by equation (10) can be further used to simplify equation (12).

The interface conditions of A following from the continuity of the normal component of \underline{B} at an air-iron boundary:

$$\frac{1}{\mu_1} \left(\frac{\partial A}{\partial n} \right)_1 = \frac{1}{\mu_2} \left(\frac{\partial A}{\partial n} \right)_2 \quad (13)$$

Hence the two dimensional problem is solved by solving equation (12) subject to interface conditions equation (13) and the boundary condition of zero A at infinity.

- c. Modified Scalar Potential ψ . The modified scalar potential introduced by M H Blewett¹ for the computation of AGS magnets has been used as the basis of many two dimensional programs.

Consider an associated vector \underline{C} which like \underline{H} satisfies the field equation (1). It then follows that:

$$\text{Curl } (\underline{H} - \underline{C}) = 0 \quad (14)$$

and a modified scalar potential ψ will exist.

$$-\text{grad } \psi = (\underline{H} - \underline{C}) \quad (15)$$

The vector \underline{C} , unlike \underline{H} , has not been uniquely defined and so in general the divergence of \underline{C} may be permitted to have non zero values. Hence by taking the divergence of both sides of equation (15) will result in:

$$\nabla^2 \psi = -\text{div } \underline{C} \quad (16)$$

In order to define \underline{C} it is usual to stipulate that \underline{C} has only an x component from which it follows that:

$$\underline{C} = - \int J_x dy + \text{const} \quad (17)$$

The value of the constant can be chosen to further simplify the problem. From equation (17) it can be seen that the modified scalar potential is identical to the ordinary scalar potential for all points outside the current region. The boundary values for ψ can be found from equation (15) by integrating along the boundary contour,

$$\psi = \psi_o + \int_{S_o}^S (\underline{H} - \underline{C}) \cdot d\underline{s} \quad (18)$$

and so for problems where the iron boundaries are essentially equipotentials, the field is everywhere normal to the boundary, the value of ψ at the boundaries depends only on the choice of the arbitrary constant in equation (17). It is usual to use this method for the air-coil region and it can be used directly for low field problems. For problems with saturation a two potential method is used - modified potential in the air-coil region and vector potential inside the iron.

2. Programs which are based on the PDE formulation.

The entries listed in Table 1 give some information on those programs known to the author. No attempt has been made to discuss the very important topic of the type of relaxation that some of these codes use. Neither has any attention been given to such matters as accuracy nor to the economics of using them in magnet design. These matters are covered in the references cited.

The advantage of the two potential method over the vector potential, for programs using a rectangular mesh, is that it avoids the complicated boundary conditions arising with the vector potential in the case of irregular contours. This complication is avoided in the TRIM program by the use of the irregular triangular mesh.

One of the earliest programs to be successfully used to solve the non-linear problem was SYBYL written by R Christian¹¹ in 1963. This program has been the model for a series of codes using the two potential approach. One of the most sophisticated of these is the CERN code MARE, written by R Perin and S van de Meer²⁰ in 1965 which has been successfully used in the design of the ISR. The two potential approach is inherently less satisfactory for magnets with highly saturated iron regions since the boundaries of the iron regions will have tangential components of field.

The vector potential method in principle can be applied quite generally and several important codes have been developed. These include the SLAC code NUTCRACKER (1965) by E Burfine, L Anderson and H Brechner¹⁸, the LRL code TRIM (1964) by A Winslow⁷ already mentioned, and the BNL code GRACY (1970) by G Parzen and

TABLE I. Summary of Computer Programs which solve the non-linear PDE for Magnetostatics in 2 dimensions

Date	Name/Centre	Authors	Method	Discretion	Remarks
1963	SIBYL MURA-LRL (Other variants include LYNDA)	R Christian ¹¹ P Dahl G Parzen J Dorst ¹² J Colonias ¹³	Two potential finite difference. SOR	Fixed topology and geometry. Rectangular meshes in the two regions	Design tool for HEP apparatus at LRL Good agreement with measurements
1964		F C Trutt ¹⁴	Scalar potential	Fixed topology and fixed geometry. Rectangular mesh Interfaces have to coincide with mesh lines.	
1964	TRIM/LRL Modifi- cation	A Winslow ⁷ J Colonias ³ N Diserens ¹⁵ R Lari ¹⁶ T Khoa	Vector potential. Variational and finite difference SOR	Fixed topology and variable geometry. Triangular mesh.	Design tool for all kinds of magnets. In regular international use at LRL, RHEL, NAL, etc. Good agreement with measurements over wide range of fields into saturation.
1965	NUTCRACKER /SLAC	E Burfine ¹⁸ L Anderson H Brechne L R Anderson ¹⁹	Vector potential finite difference SOR	Fixed topology and geometry. Rectangular mesh. Later version appears to have variable geometry for currents	At SLAC used for HEP; apparatus. Good agreement with measurement
1966	MARE/ CERN	R Perin ²⁰ S van der Meer	Two potential. Extensive use made of J Hornsby ²¹ program for solving elliptic PDE. Finite difference and SOR	Fixed topology and geometry	Powerful design tool at CERN. Used for design of ISR and numerous HEP magnets. Good agreement with measurement.
1966	Colorado Univ.	E A Erdelyi ²² S V Ahmed R E Hopkins	Vector potential Finite difference. Novel method of SOR	Rectangular and polar meshes. Variable geometry.	Design tool for electromagnetic machinery
1968		K Reichert ²³	Vector potential SOR	Rectangular and polar lattices	Applied to MDH magnets and to various rotationally symmetric magnets. Good agreement with experiment.
1970	McGill Univ.	P Silvester ²⁴ M Chari	Vector potential. Finite element (Variational method.) Gaussian for the linear part, Newton-Raphson for the non-linear variable part.	Variable topology and geometry. Triangular mesh.	One of the first attempts in applying the finite element method. Good agreement with measurements.

Date	Name/Centre	Authors	Method	Discretisation	Remarks
1970	FATIMA CERN	C Iselin ²⁵	Vector potential Finite element (with first & second order versions). SOR	Fixed geometry and variable topology Triangular mesh	Treats anisotropy effects. Good agree- ment with measure- ment
1970	GRACY	G Parzen ²⁶ K Jellett	Vector potential. Finite difference SOR	Fixed geometry and topology Rectangular mesh polar mesh	Used in study of saturated yokes in superconducting dipoles

TABLE II. Summary of Computer Programs which solve the PDE for Magnetostatics in 3 Dimensions

Date	Name/Centre	Authors	Method	Discretisation	Remarks
1967	Carnegie	M Foss ²⁷	Scalar potential Finite difference	Three dimensional lattice	Written for CDC 6600 in local software Little information available.
1970	MIMI A CERN	S Caeymax ²⁸	Modified scalar potential. Air-iron only at present Infinite μ boundaries. Finite difference SOR	Three dimensional lattice with variable spacing in Z direction	Used for assessing the performance of beam handling mag- nets with end effects. Good agreement with measurements.
1971	AEG- Telefunken Germany	W Muller ²⁹	Scalar potential Finite difference SOR	Cylindrical polar lattice Variable geometry	To be used in the design of turbine generators.
1971	LRL	S Sackett ³⁰ J Colonias	Vector potential Finite element SOR	Triangular prism elements irregular in xy plane	Under development
1971	RHEL	N Diserens C Trowbridge O Zienkiewicz	Scalar potential Finite element Frontal solution	Isoparametric elements	Under development
1971	LAMPP	R Christian ³¹ H Vogel	Scalar potential Infinite boundaries Use made of analytic functions for corner regions	Three dimensional lattice	Used for design of C magnets. Reason- ably good agreement with test models

K Jellett.²⁶ TRIM is a very versatile program and has travelled to many parts of the world. Many improvements to the code have been made to suit local requirements.^{15,16} One of the most exciting developments has been by J Colonias³ at LRL where TRIM has been used in conjunction with an on-line graphics display system.

In the formulation of programs such as SYBYL, MARE, NUTCRACKER and GRACY finite difference methods were employed to discretise the PDE. In this technique the mesh topology is fixed and since boundaries and interfaces between regions do not in general coincide with mesh lines

special irregular star' equations have to be introduced. The TRIM program mesh has variable geometry which ensures that the mesh lines coincide with boundaries while still keeping the mesh topology fixed. Several mathematical techniques can be used for obtaining the difference equations including the variational method which is the basis of the Finite Element Method.³⁷

The finite element method has been used for many years in the solution of structural problems and has recently been applied to the magneto-static problem. This approach enables both the topology and geometry of the mesh to be varied and a two dimensional program based on this method has been developed by P Silvester and M Chari.²⁴ The program FATIMA by C Iselin²⁵ also uses a finite element formulation and includes the treatment of anisotropy effects in the iron.

Finally a mention of some of the attempts that have been made to correct the errors introduced by the use of false external boundaries. N J Diserens¹⁵ has modified the TRIM program for the axisymmetric problem so that the field is forced to follow a dipole law at the false boundary. A Riche³⁸ describes an iterative

method which combines the relaxation of the mesh equations with a polynomial expression representing the solution up to infinity.

The programs listed in Table 2 give some information on three dimensional programs. Some of these programs are still under development and all are limited in one way or another. It is hard to believe that programs based on the PDE method, with iteration relaxation of the enormous set of equations over meshes spanning the entire magnet, will be as successful in three dimensions as they have been in two dimensions.

3. Programs which are based on the IE formulation.

Nearly all the programs listed in Table 3 were either conceived as three dimensional or can readily be extended to three dimensions.

The entries in Table III indicate that a great deal of interest is being shown in the integral equation formulations and the author regrets that time has not permitted a closer study of this work. The experience gained by A Halacsy³³ with the RENO program demonstrated that there are problems with convergence and symmetry. He also stresses the severe limit-

TABLE III. Summary of Computer Programs which use the IE Method

Date	Name/Centre	Authors References	Method	Discretisation	Remarks
1967	TAMI	Ref. S Sackett ³²	Vector potential Dipole magnetisa- tion. SOR	Rectangular Blocks	Experimental only Reported instabilities in the relaxation process.
1967- 1970	RENO	A Halacsy ³³ J Schneider ³⁴	Scalar potential Dipole magnetisa- tion Non relaxation method. Iterative loop for variable permeability.	Regular cubes	Well established program but owing to limitations in the size of memory result only available for simple cases.
1970	University of Toronto	S Zaky ³⁵ S Robertson ³⁶ H Karmarker	Three methods used. Scalar potential magnetisation currents, and vector potential	Variety of meshes used in 2D. Regular blocks in 3D	Under development. Three D version tested on constant μ problem. Good agreement with measurements
1971	GFUN RHEL	M Newman ¹⁰ C W Trowbridge L Turner	Dipole magnetisa- tions. Equation solved for magnet- isation field vector by Gaussian elimination. Non linear loop for variable μ by simple iterative scheme	Current and iron elements for a wide variety of shapes for 2D.	2D version fully oper- ational with estab- lished reliability. 3D version under advanced development with good agreement with measurement.

ation of computer memory. The experience gained with the GFUN program at RHEL¹⁰ has shown that agreement to one per cent can be achieved for a 3D problem with relatively few elements. It is a feature of this program that relatively large elements are used - the basic assumption made is that the magnetisation over the volume of each element is constant. In the papers by S Zaky et al 35, 36 several alternative formulations are given with a mathematical analysis of the effects of discontinuities in permeability.

IV. GFUN Program

There is always a requirement at Laboratories specialising in High Energy Physics apparatus to assess rapidly the performance of proposed magnet designs. Two years ago new graphics facilities became available at RHEL and this fact coupled with the development of on-line⁸ and graphics software⁹ created the right conditions for the development of a new magnet design program.

The new program should be user interactive. In one sense all design work performed by the computer is interactive. Magnet design is a problem of minimising some function of many variables subject to certain constraints. Examples of typical functions are performance to cost ratio, and field homogeneity. Usually both the function and the constraints are non-linear. A great deal of progress has been made in providing automatic optimisation routines for such problems, but explicit formulation is often very complicated. Designers usually break the problem down into smaller problems and ignore the coupling. Even then, unless the initial values are not too far from the optimum, automatic routines will fail. Furthermore, there appears to be no automatic routine which will find a global minimum among several local minima. In practice, computer programs for magnet design such as TRIM⁷ or MARE²⁰ for example which normally run in the batch are complicated and slow so that all optimisation is done by the designer while the computer is used to compute the function. In this situation the limiting factor in the time to optimise a design is the computer turn round time. The designer using the batch system is at the mercy of the turn round time and may have to wait for periods varying from minutes to hours depending upon the work load. Even for the most trivial data changes he has to wait - there is no feedback enabling him to have second thoughts. Because of this the designer usually takes many days, sometimes weeks, to prepare data for a complex magnet shape.

For these reasons the use of on-line computing will speed up the design; with the right combination of magnet design and expertise and skill at using the program, the time for a design should speed up dramatically. If use can be made of automatic optimisation routines guided by the designer this progress will be

even more rapid, and frequently a better optimum found.

Another important advantage is that the designer has the opportunity to concentrate his mind on the problem in hand for relatively long periods thus reducing the possibility of losing his train of thought.

Structure of the Program

The GFUN program is described in detail in the paper by Newman, Trowbridge and Turner.¹⁰ The program is controlled by a user at a remote terminal which consists of a Computek 400/15 display on line to the Rutherford Laboratory IBM 360/195 Computer via a Honeywell DDP224 Satellite.

The program consists of a collection of routines which perform three principal tasks.

1. Data input.
2. Data processing.
3. Data retrieval.

Each task is initiated by a set of commands and parameters. To enable parameters to be checked against a dictionary of valid parameters a special message decoding system was developed. Data input is utilised mainly by the DRAW command which allows the user to enter a wide variety of element shapes to represent his magnet. Further commands allow these elements to be erased, modified, replaced and rotated. Also the material data in the form of a table of flux density values versus magnetisation field values can be entered. At any stage the data can be named and the complete file stored on a direct access disk. Earlier magnet data files can be read into the program from the disk as well, thus ensuring continuity between different on-line runs and users.

The data processing task for the calculation magnetisation is initiated by the GETM command. For reasons outlined in the second section of this paper the IE method is used. Firstly, the coefficients of the set of algebraic equations are computed by specially written routines which perform integrations over the volumes of each iron element. Secondly, these set of equations are solved for the magnetic field at the centre of each iron element assuming that the magnetic susceptibility is constant everywhere. This process is then repeated successively with the updated values of susceptibility for each element now available. At the end of each iteration the changes in field are detected and the largest change is displayed allowing the user to assess the convergence of the problem. Usually some 10 to 100 iterations are required to converge the solution to a few gauss in several kilogauss. After the convergence the magnetisation is computed and can then be stored on the named magnet file.

The final task of data retrieval is to interrogate the results. The command GETM is used to plot the fields long lines on the display or to store values on a mesh. The command MAP is used to draw contours over defined regions. Examples can be seen in Figure 1, 2 and 3.

NB: All the figures shown in this paper were software generated hard copies of the pictures appearing on the Computek screen.

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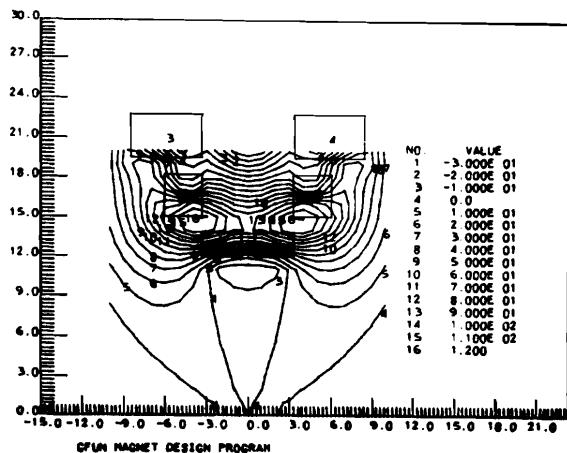


Fig. 1. Contour plots of field after optimisation for a superconducting polarised target magnet currently under design.

A further auxiliary task is to compute the Harmonic content of the field. The command HARM calls routines which Fourier analyse the field over a defined region, by least squares fitting. Another task is to optimise the shape of conductor region to achieve maximum homogeneity over a defined region. Here the command OPTI is used to vary in a continuous manner the geometry and position of current elements to minimise the sum of squares of the field homogeneity over an elliptical region. After each OPTI process the magnetisation has to be recalculated so the user controls a cycle of OPTI followed by GETM commands in order to achieve an optimum. A complete sequence of user and automatic optimisation for a dipole is given in the paper already cited.¹⁰

V. The PDE Method compared with IE Method

In this section the results from the TRIM program are compared with those for GFUN. TRIM has been in regular use at Rutherford Laboratory for many years and until recently was the only program available for computing magnets with saturation effects. The accuracy of a new program must be established, and it is usual to make analytic comparison to start with and then proceed to comparisons with other established programs and finally with measurement.

a. Magnetic cylinder

The elementary problem of the magnetic thick cylinder placed in a transverse uniform magnetic field was chosen as the first comparison. The low field limit can be compared with value calculated exactly. Table IV shows this result and also the results for a range of external fields into the saturation region. Figure 2 shows the GFUN error as a function of distance for the low field case. For this problem the accuracy is of the order of one part in 10^4 .

TABLE IV. Thick Cylinder TRIM/GFUN Comparison

Permeability	External Field (Gauss)	Central Field (Gauss)		
		GFUN	TRIM	Analytic
Fixed 100	1,000	50.50	48.20	51.61
Variable	10,000	6,498	5,824	-
Variable	20,000	18,060	17,394	-
Variable	30,000	28,820	27,988	-

The results presented in Table IV show a difference of about 4% at all field levels. This difference between the two programs can be accounted for by the fact that the external field for the TRIM program has to be specified along a boundary at some finite distance from the cylinder. Whereas in GFUN the boundary is at infinity by virtue of the formulation so this problem does not arise.

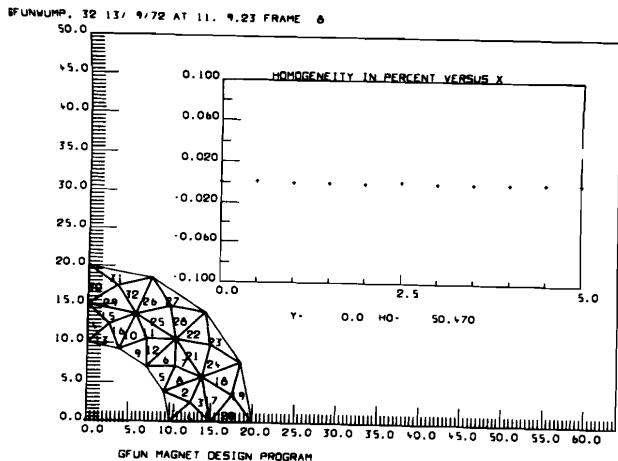


Fig. 2. Field homogeneity plot inside a magnetic cylinder immersed in a uniform magnetic field of 1000 G.

Cylinder dimensions: Outside radius = 20 cm
Inside radius = 10 cm

b. Dipole magnet

Consider the magnet shown in Figure 3; this magnet is a possible design for a superconducting dipole to operate with good field quality up to 5.5 Tesla. The current region consists of 6 layers of the layer² type and the iron region a thick cylindrical shell. Figure 3 shows the discretisation used in the GFUN program and Figure 4 the mesh generated for the TRIM program. Figure 3 also shows the GFUN field plot in the median plane with the TRIM computed values added for comparison. The data for the GFUN program was entered and the fields obtained in one single half-hour on-line session. The data for the TRIM run was prepared by an expert user and it took him approximately 6 hours. This included calculation of boundary points, a relatively easy task in this case since the boundaries are circular arcs, and the punching of 240 cards. This time could be reduced by a suitably written special pre-program for this type of magnet. Two test runs were needed to check and eliminate errors in the data. The computing time and storage requirements for these two programs are given in Table V.

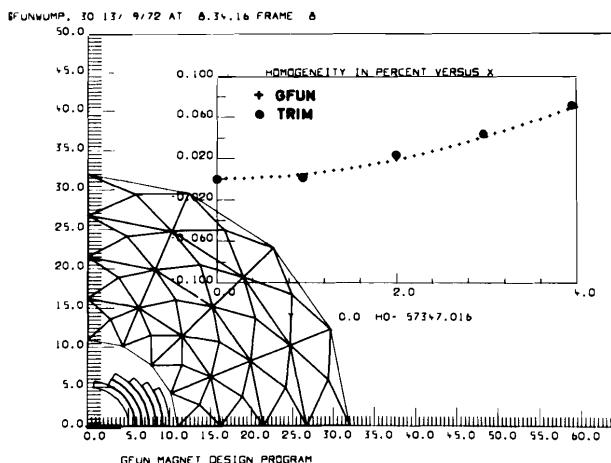


Fig. 3. Comparison of GFUN and TRIM for a six layer design for a superconducting dipole

TABLE V. Program Statistics for the TRIM-GFUN run on IBM 360/195

Program	Core K/Bytes	CPU sec	Mesh	Cycles for convergence
TRIM	330	132	4950 points	600
GFUN	220	60	64 elements	40

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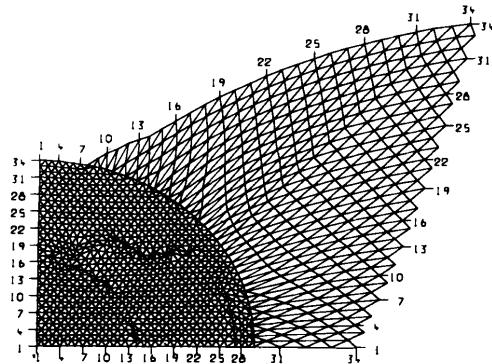


Fig. 4. The generated triangular mesh used in the six layer dipole computations by the TRIM program.

VI. Three Dimensional GFUN compared with measurement

For this comparison the measured results for the Rutherford Laboratory Helium Bubble Chamber Magnet were used. The details of the magnet and discretisation used in GFUN are given in the paper by Newman, Trowbridge and Turner.¹⁰ It is a feature of this 3D version to display the geometry stereoscopically and thus enable errors to be detected quickly on-line. Figure 5 shows stereo views of one octant of the iron and Figure 6 a plot of the field as a function of x in the xy plane at a height of 20 cm above the median plane. The measured values are also shown for comparison. In fact the predicted field agreed with the measurements to better than 1% everywhere in the bubble chamber volume.

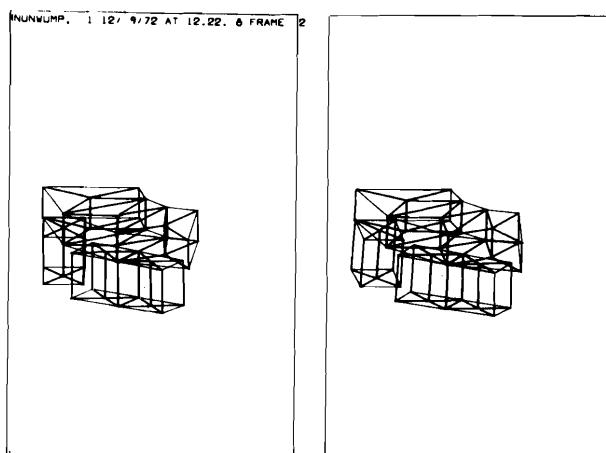


Fig. 5. Stereo views of one octant of the Helium Bubble Chamber Magnet, divided into twenty-four iron elements.

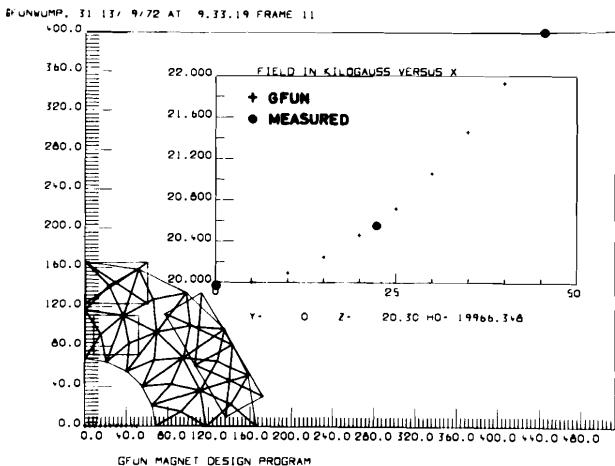


Fig. 6. Comparison of GFUN with the measured values of the Helium Bubble Chamber magnet. The plot shows the variation of the total field in the xy plane at a height of 20 cm. The yoke is divided into 52 elements.

VI. Conclusions

1. The application of the Integral Equation Method for the computer solution of the magneto-static problem has been used successfully at many centres and overcomes some of the limitations of the partial differential equation method. The progress made towards three dimensional solutions using the integral equation formulation is very encouraging but techniques which minimise computer memory requirements must be sought if complex systems are to be represented.
2. The advantages of user-interaction with programs for magnet design are numerous, and efforts must be made to discover the best way of implementing the necessary hardware and software in order to achieve an acceptable balance between 'batch' and 'on-line' processing.

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Comment by Charles G. Dols, LBL:

It seems appropriate here to underscore some of Bill Trowbridge's comments relating to the flexible, widely-used program "TRIM". 1) C.F. Iselin of CERN has included a second-order correction in the basic algorithm in his triangular-mesh program, "FATIMA". 2) N.J. Diserens, RHEL, expects to add such a correction to TRIM.

Incidentally, one of Dick Christian's many pioneering contributions is the use of girls' names for computer programs. When I asked him about the origin of the name "SIBYL", he was suspiciously vague. However, he did say that it is a girl's name. LINDA is the name of a friend of one of Dick's student programmers at Purdue.