Analysis of Damping Requirements for Dipole Wake-Fields in RF Crab Cavities

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***Abstract*-Several current particle-antiparticle collider designs require the beams to collide at a small angle. In order to make the luminosity comparable to that of beams colliding head-on, transverse deflecting cavities known as crab cavities will be used. Additional modes, other than the required "crabbing" mode are excited by the beam and if these are excited resonantly then an appreciable dilution of the beam emittance will occur. We provide a single-mode analysis of the damping requirements on the dipole modes which will minimise the resulting angular kicks to the beams, by deriving an explicit solution for the frequency offset of the mode from a bunch harmonic for maximum deflection of bunches, using a novel analytical technique.**

I. Introduction

This paper uses a novel and efficient analytical approach to determine damping requirements for dipole wake-fields in RF crab cavities. Crab cavities are an important component of particle colliders where the beams collide at a small angle [1]. Calculations in this paper are based on requirements for the crab cavities of the International Linear Collider (ILC) [2].

The study of collision products from electrons interacting with positrons at energies well above the 209 GeV reach of LEP [3] and up to energies of 1 TeV is a key objective for complete understanding of the weak interaction. The ILC machine proposal to achieve this will aim to accelerate bunches of electrons and positrons in 11km long, opposing linacs for 250 GeV and collide them at a small crossing angle of 14 mrad. The favoured machine parameter set has a bunch size at the interaction point (IP) measuring 5.7 nm by 639 nm by 300,000 nm in height, width and length respectively. Also for the favoured parameter set, bunches contain 2 x 10^{10} charged particles spaced from their neighbours by 369.2 ns. Five groups of 2625 bunches will be accelerated per second. Colliding long thin bunches at an angle reduces the number of collisions between electrons and positrons. This reduction in luminosity will be restored using dipole RF cavities to impart a rotation to the bunches prior to the colliding at the interaction point. The dipole cavities are operated such that the magnetic field at their centres passes through zero at the instant when the centre of a bunch is at the centre of the cavity. Operated in this way, such RF cavities are known as crab cavities [4].

The dipole cavities being proposed for the ILC will have multiple cells and hence a large number of modes will be excited in addition to the π mode to be used for crab rotation. Each dipole mode has two polarisations, and hence there is a mode with the opposite polarisation to the operating π mode, known as the Same Order Mode (SOM), which will be excited by the beam.

As the bunches of charged particles are travelling at very nearly the speed of light the electric space-charge force is almost completely cancelled by the magnetic space-charge force. This means that the overall space charge forces acting to disrupt the bunch can be omitted in the analysis. However, the scattering of the electromagnetic field (e.m.) from the cavity itself and from various accelerator components can cause serious disruption in the beam. Energy in unwanted modes can adversely affect the shape and trajectory of later bunches in a bunch train through beam-cavity interactions, which will be referred to as wake-fields effects [5] in this paper. The transfer of energy from bunches into cavity modes can be explained with reference to the image charge that moves with the charged bunches along the beam pipes and accelerator cavity walls. Energy is left behind when the image charge takes a longer route around the cavity, and new image charge is created in the walls around the particle. The field excited by a charged bunch and its image charge is known as a wake-field. When electromagnetic energy left behind one bunch acts on a following bunch, the effect is referred to as an inter-bunch or long range wake-field, which we will concentrate on in this paper. It is also possible for the field associated with the front of a bunch to act on particles within the exiting bunch itself. This effect is known as an intra-bunch or short range wake-field, which we will neglect in this paper.

The crab cavities are located adjacent to the final focus quadrupole magnets, and hence when the wake-fields deflect the bunch the final focus magnets are not able to bring the bunch back on axis at the IP. As a consequence there are constraints on the maximum acceptable wake-fields and hence this sets damping requirements for unwanted modes. This paper is organised as follows. Calculation of unwanted transverse dipole modes excited by the ultra-relativistic beam transiting the ILC crab cavities are reported in section II. In section III the long-range sum wake-fields of these modes are derived analytically and are then used to derive the damping requirements as a function of the frequency and beam coupling of the mode. In section IV we use a new method to explicitly derive frequency offsets of modes from bunch repetition frequency harmonics giving rise to maximum deflection. The damping requirements are then obtained using these frequency offsets from a single-mode analysis. In section V we apply these results to a realistic deflecting cavity using both single-mode and multiple-mode calculations. Finally, typical damping requirements are specified.

II. Wake-field and Modal Kick Factor Analysis

In RF cavities the wake-field is conveniently decomposed as a sum over eigen-modes excited by the beam [6, 7]. Unwanted dipole modes, whose main effect is to give a transverse kick, are of most concern with respect to luminosity degradation at the IP. Provided the offset of the bunch from the cavity axis is small compared to the iris radius of the cavity, shown in Fig. 1, then the dominant modes in the transverse wake-fields will be dipole in character and we limit this analysis to these modes. For a cavity consisting of perfectly aligned cells, these modes are only excited by bunches that travel off axis.

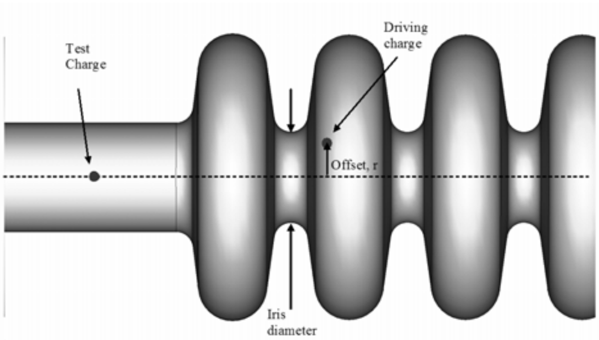

Fig. 1 Schematic of the crab cavity geometry and illustrating the essential parameters (offsets not to scale)

Successive bunches in the train will be kicked transversely to the longitudinal axis by these transverse dipole modes. The transverse multi-mode, single-bunch, wake-field, $W_{\perp,multi}$ is the transverse voltage, per unit driving charge with transverse offset *r,* acting upon trailing charged particles travelling at essentially the velocity of light c, transiting the cavity and trailing the driving charge by a distance, *s*:

$$W_{\perp,multi} = 2r\sum_{m=1}^{M} K_m \sin\left(\omega_m s/c\right)\exp\left(-\frac{\omega_m s}{2Q_n c}\right) \quad (1)$$

Here the kick factor K_m for the m^{th} mode is given by

$$K_m = \frac{c}{4}\left(\frac{R}{Q}\right)_m = \frac{c\left|V_m(a)\right|^2}{4\omega_m U_m a^2} \quad (2)$$

and the longitudinal voltage $V_m(a)$ is evaluated at radial location *a*, U_m is the total energy for the mode, $(R/Q)_m$ is the geometric shunt impedance, and $\omega_m/2\pi$ is the eigen-frequency of the mode.

We now consider the wake-field at the location of each bunch in a train of N bunches. In particular, we focus on the sum of the wake at the location of each bunch. For a series of bunches passing through the cavity the transverse multi-mode wake-field given in (1), summed over each bunch, we refer to as the sum wake-field $S_{\perp,multi}$, in Volts per Coulomb, as

$$S_{\perp,multi} = 2r\sum_{n=1}^{N-1}\sum_{m=1}^{M} K_m \sin\left(\omega_m n T_b\right)\exp\left(-\frac{\omega_m n T_b}{2Q_n}\right) \quad (3)$$

where T_b is the temporal separation between neighbouring bunches. The subscript multi refers to the multiple modes that are included in the summation. The sum wake-field is an important quantity to consider when alignment tolerances and luminosity considerations are analysed [8]

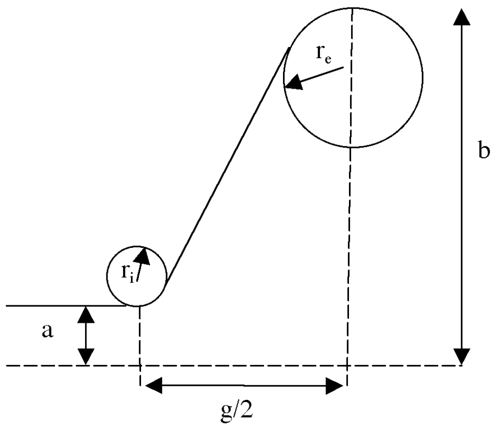

Fig. 2: Cell Shape Parameters

		mid-cell	trans-cup	End-cup
half cell length	g/2	19.2mm	19.2mm	18.6mm
iris radius	a	15.0mm	15.0mm	18.0mm
iris curvature	r_i	5.5mm	5.5mm	5.5mm
equator radius	b	47.18mm	47.37mm	47.37mm
equator curvature	r_e	11.41mm	11.41mm	11.41mm

Table 1: Cell Shape Parameters.

The frequencies and the $(R/Q)_m$ ratios have been calculated for the 9-cell deflecting-mode cavity being proposed for the ILC crab cavity using the 2D eigen-solver of the computer code MAFIA [9]. A 2D code is sufficient here as the crab cavity is almost cylindrically symmetric. The crab cavity design is a 9-cell version of the FNAL-CKM cavity The wake-fields of the CKM cavity have been previously calculated in [10] for a 13 cell cavity. However a 9-cell version of this cavity will have slightly different modes and we make such calculations and have performed additional analysis in this paper. These cavities have an iris radii of 15 mm for the mid-cells, and 18mm for the end cells, shown in Fig. 2 and Table 1. For these irises the TE_{11} cut-off frequencies are 5.86 GHz and 4.88 GHz, respectively for these cells. In order to adequately simulate the e.m. field in the cavity a study of modal convergence with increasing mesh was performed. As a result of this study 350,000 mesh elements were used corresponding to an average mesh spacing around the iris of 0.3 mm. This corresponds to between 56 and 250 lines per wavelength for the frequencies of interest. Furthermore, beam pipes, relatively long compared to individual cell lengths (150 mm versus 38.4 mm) were used to ensure non-propagating or trapped modes were properly attenuated before the e.m. field reached the planes used within the calculation to bound the cavity at either end. The calculations were subsequently verified using additional simulations with 500 mm beam pipes for a selected number of higher order dipole modes. Validation that the fields were independent of beam pipe boundary conditions was achieved by performing eigen-mode simulations with perfect electric and then perfect magnetic boundaries. By calculating the difference in the resonant frequency of each mode for both boundary conditions, we are able to discern the perturbation to that mode by the fields at the end of the beam-pipe [11]. It was found that for the 13 modes with the highest $(R/Q)_m$, the field is essentially independent of the boundary conditions and does not vary by more than 0.5% for most of the modes. One of the modes used, at 7.1 GHz, was found to vary by 1% and further analysis of this mode is required. The presence of modes localised, or trapped within a limited number of cells, was ascertained using detailed e.m. field plots. Trapped modes give rise to large wake-field kicks as they grow in the middle of the cavity with little damping.

Our calculations verify that the lowest frequency dipole, or deflecting, mode operates with a π phase advance per cell at a frequency of 3.9 GHz and has an $(R/Q)_m$ of 315 Ω/cm^2. The operating mode is excited resonantly from an RF power source and is not considered in this paper. However, the SOM has an equally large $(R/Q)_m$ and needs to be strongly damped and is included in our calculations. This latter mode is referred to as the same order mode. The other modes in the first pass-band have little coupling to the beam as they are not synchronous with the beam phase velocity. Some of the modes in higher bands will maintain synchronism with the beam and consequently have significant kick factors. Modes with particularly large kick factors are located in higher bands at 7.1 GHz, 8.0 GHz, 10.0 GHz, 13.0 GHz and 17.5 GHz. The frequencies of the most significant modes are plotted in Fig. 3 along with their $(R/Q)_m$ factors. In particular, the mode at 8 GHz is will require significant damping as it has very low fields in the end cells and the beam pipes and hence is liable to be trapped within the middle of the cavity.

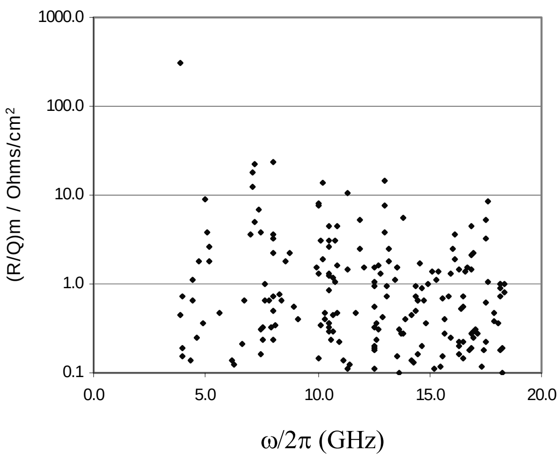

Fig. 3 R/Q of dipole modes in the 9-cell cavity.

III. DAMPING REQUIREMENT TO CONTAIN TRANSVERSE OFFSETS AT THE IP

The multi-mode method of calculating damping requirements covers a large range in parameter space. In order to study the damping requirements, that ensure the beam does not exceed a specified deflection, in a more efficient manner we undertake a single-mode analysis of the sum wake-field [12] and hence we will drop the subscript m. Following from (3), the single-mode transverse sum wake-field, $S_{\perp,\sin gle}$, for a beam offset, *r*, from the axis, summed at each trailing bunch for a beam consisting of a train of N_b bunches, is given by:

$$\frac{S_{\perp,single}}{\left(\frac{rc}{2}\left(\frac{R}{Q}\right)_m\right)} = \sum_{n=1}^{N_b-1} \exp\left(-n\frac{T_b}{T_d}\right)\sin\left(\omega n T_b\right) \tag{4}$$

$$= \mathrm{Im}\left\{-\frac{e^{i\delta-d}}{e^{i\delta-d}-1}+\frac{e^{(i\delta-d)N_b}}{e^{i\delta-d}-1}\right\} = F_I = F_I^{\infty} + F_I^{N_b}$$

where the two components in parentheses are given by:

$$F_I^{\infty} = \frac{\sin\delta}{2\left(\cosh d - \cos\delta\right)}$$
$$F_I^{N_b} = \frac{-e^{-N_b d}\left[e^{d}\sin N_b\delta + \sin(\delta - N_b\delta)\right]}{2\left(\cosh d - \cos\delta\right)} \tag{5}$$

with, $\omega = \frac{2\pi h}{T_b} + \Delta\omega$, $\delta = \Delta\omega T_b$, $d = T_b/T_d$, and h is a integer giving the bunch repetition frequency harmonic. Here we have decomposed the dipole angular frequency into harmonics of the bunch frequency plus an offset Δω. Also, T_b is the temporal separation between neighbouring bunches and T_d is the damping constant:

$$T_d = \frac{2Q}{\omega_n}. \tag{6}$$

In practice, for a sufficiently long bunch train, the second term in parenthesis in (4) is negligible and $F_{I\max}^{\infty}$ serves as an adequate description of the sum wake. We shall see that this is indeed the case for the ILC bunch train and under this approximation we arrive at a condition on the single-mode wake-field for a specified angular deflection, θ_{max}:

$$\theta_{max} = \frac{qS_{\perp,single}}{E} \tag{7a}$$

hence,

$$F_{I\max}^{\infty} = \frac{2\theta_{\max}E}{c\,q\,r\left(\frac{R}{Q}\right)_m} \tag{7b}$$

where q is the bunch charge and E the beam energy in eV. For a transfer matrix element R_{12} [13], which relates the angular direction at the cavity to the offset at the IP of the bunch, the maximum allowable deflection is

$$\theta_{\max} = \frac{\Delta y_{ip}}{R_{12}} \quad (8)$$

Here we have assumed the beam moves en masse and we refer to this as the stiff-beam approximation. The maximum offset allowed at the interaction point which gives rise to a tolerable luminosity dilution is: $\Delta y_{IP} = \sigma_{y,IP}/4$ [14]. In order to obtain the damping requirements we rearrange (5) to give

$$F^{\infty}_{I\max} x^2 - \left[2F^{\infty}_{I\max}\cos\delta + \sin\delta\right] x + F^{\infty}_{I\max} = 0 \quad (9)$$

where the bunch train has been approximated by an infinitely long train of bunches and

$$x = \exp(-d) \equiv \exp(-T_b / T_d) \quad (10)$$

The solution to this quadratic equation, for $0 \le x \le 1$, is readily obtained as

$$x = \frac{\left[2F^{\infty}_{I\max}\cos\delta + \sin\delta\right] - \sqrt{\left[2F^{\infty}_{I\max}\cos\delta + \sin\delta\right]^2 - 4(F^{\infty}_{I\max})^2}}{2F^{\infty}_{I\max}} \quad (11)$$

This equation determines the damping factor T_d as is required to prevent offsets larger than $\sigma_{y,IP}/4$. We can determine the associated loaded Q, using (6) and (10), i.e.

$$Q = \frac{\omega T_b}{2\ln(1/x)} \quad (12)$$

It now remains to investigate the influence on the sum wake-field, and hence the kick, of small deviations in the frequency offset of the dipole mode.

IV. Analysis of Resonant Excitation of Dipole Modes

The transverse single-mode wake-field is a component of the full single-mode wake-field [6].

$$\underline{S}_{single} = \mathrm{Re}\left(\omega_m \left(\frac{R}{Q}\right)_m (r\cos)^2 \theta\hat{z} - i K_m r\left(\hat{r}\cos\theta - \hat{\theta}\sin\theta\right)\right) F\} \quad (13)$$

where $F = F_R + iF_I$ and the angular variation is described by θ and F_R is given by

$$F_R = -\frac{1}{2} - \sum_{n=1}^{N_b-1} \cos(n\omega T_b)\exp\left(-n\frac{T_b}{T_d}\right) = \frac{\sinh d}{2(\cosh d - \cos\delta)} - \frac{1}{2} \quad (14)$$

We focus here on the real part of (13) as the imaginary part gives the kick on a test particle made to traverse the cavity with a 90 degree phase shift to the bunches in the actual train. The component F_R determines the interaction with longitudinal wake-fields, which accelerate or decelerate the particles in the bunch, and is in phase with the beam current. The dependence of F_I, and hence the transverse wake-field, on frequency is shown in Fig. 4a and F_R, which gives the longitudinal wake-field and transverse velocity spread along the bunch train, is indicated by the dashed curve.

The maximum longitudinal wake-field occurs when $\Delta\omega=0$, when the mode frequency is equal to a harmonic of the bunch repetition frequency. For transverse wake-fields, the part of the wake-field giving the kick, i.e. the transverse E and B fields, is 90 degrees out of phase with the part responsible for the build up of energy in the cavity, the longitudinal electric field, which is in phase with the bunch. As a consequence, the maximum of the component F_I^{∞}, giving the transverse kick, occurs at frequency slightly shifted from the bunch frequency, $\Delta\omega_{\max}$. The frequency offset of the maximum kick is strongly dependent on the damping Q.

Due to manufacturing errors it could be possible for any mode to have a frequency that causes the maximum offset, hence it is important to calculate the worst case. The frequency offset of the mode from a bunch harmonic for maximum deflection of bunches is determined from

$$\frac{dF_I^{\infty}}{d\omega} = 0 \quad (15)$$

The frequency offset where the maximum kick is obtained is therefore determined as

$$\Delta\omega_{\max} = \frac{1}{T_b}\cos^{-1}\left[\mathrm{sech}(T_b / T_d)\right] \quad (16)$$

The solution to (11), for the worst case wake-fields, can then be found by inserting (16) to give

$$x = e^{-d} = \frac{2F^{\infty}_{I\max}\mathrm{sech}(d) + \tanh(d)}{2F^{\infty}_{I\max}} - \frac{\sqrt{\left[2F^{\infty}_{I\max}\mathrm{sech}(d) + \tanh(d)\right]^2 - 4(F^{\infty}_{I\max})^2}}{2F^{\infty}_{I\max}} \quad (17)$$

solving this equation for x , for $0 \le x \le 1$, gives

$$x = \frac{-1 + \sqrt{1 + (2F^{\infty}_{I\max})^2}}{2\left|F^{\infty}_{I\max}\right|} \quad (18)$$

The corresponding damping requirement is then obtained by inserting (18) into (12) to give:

$$Q = \frac{\omega T_b}{2\ln\left[\frac{1}{2|F^{\infty}_{I\max}|} + \sqrt{1 + \frac{1}{(2F^{\infty}_{I\max})^2}}\right]} \quad (19)$$

where $F^{\infty}_{I\max}$ is defined in (7b) and the ratio of shunt impedance to Q (i.e. $(R/Q)_m$) in (7b) which is a geometric quantity.

The analysis has therefore provided a design for the damping required to prevent offsets larger than $\sigma_{y,IP}/4$ for any given wake-field mode.

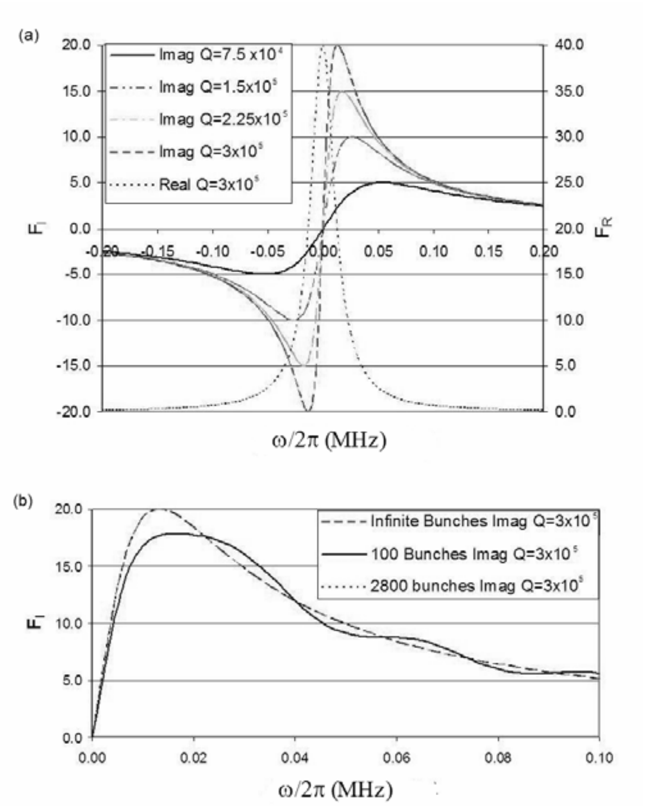

Fig. 4 a) The dependence of the imaginary part of F as a function of Δf for several values of Q and the real part of F for a Q of $3\text{x}10^5$ b) The imaginary part of F for a varying number of bunches in a train. Both figures show the fundamental dipole mode at 3.9 GHz.

Additionally if the damping of a cavity is specified we can insert (16) into (5) in order to find the maximum kick which will ensue:

$$F^{\infty}_{\mathrm{I\,max}} = \frac{1}{2}\mathrm{cosech}(d) \qquad (20)$$

Finally, we note that this approach is valid for infinitely long bunch trains. A bunch train can be considered effectively infinite when $T_d << T_{train}$ and for the ILC, in which there are 2650 bunches this is certainly a valid approximation, as can be seen in Figure 4b. Here we have considered the beam dynamics for interaction with a single dipole mode. In practice, many dipole modes are excited by the beam. However, in general a small number of modes tend to be dominant as the largest loss factors are concentrated in a limited number of modes. It now remains to apply this analysis to the parameters of crab cavities for the ILC to obtain typical deflections at the IP due to these higher order modes excited in the crab cavities. This is undertaken in the following section.

V. Application to the ILC Crab Cavities

Here we focus on an application of the analytic theory to the crab cavity which the beam encounters on exiting the main linac. The derivative of the transverse short-range wake-field with respect to the longitudinal distance is approximately 16 times larger in the crab cavities than that in the main linacs because the iris radius is smaller by a factor of two and the derivative of the transverse wake-field is proportional to the fourth power of the iris radius size [15]. The modes that constitute the long-range wake-field also have enhanced kick factors [12] compared to their linac counterparts and unless they are adequately damped an appreciable dilution of beam luminosity will occur.

An important parameter in characterising the interaction between the beam and the e.m. modes of the cavity is the bunch repetition frequency. The ILC crab cavities operate at the 1314th harmonic of the bunch repetition frequency [2]. Higher order dipole modes excited at a frequency close to a harmonic of the bunch repetition frequency, will be resonantly excited and are liable to cause significant transverse kicks to the bunch and a concomitant degradation in the beam luminosity will occur. This luminosity dilution can be reduced to within the tolerances by damping the e.m. modes as prescribed by (19). We have calculated the kick factors for more than 240 modes which were shown to have narrowband impedances in previous time-domain studies [10] and limited our subsequent analysis to 13 modes all of which have $(R/Q)_m$ values larger than 5 Ω/cm^2, shown in Table 2. In theory all 240 modes should be calculated however in practice most modes will be damped to Q values of much less than 10^7 hence have a negligible contribution to the overall wake-field and only a few modes are required.

frequency (GHz)	$(R/Q)_m$ (Ohms/cm^2)	External Q (x10^6)
3.907	315.4	0.03
7.082	10.3	1.46
7.136	29.3	0.51
7.178	20.8	0.73
7.39	9.7	1.60
8.039	24.6	0.69
10.029	10.0	2.12
10.054	7.4	2.86
12.98	7.9	3.49
12.996	16.0	1.71
13.014	9.8	2.81
17.533	4.5	8.19
17.541	8.1	4.55

Table 2. Frequencies, R/Q and external Q for the 13 modes used in the calculations.

The frequency which maximizes the luminosity dilution is shown in eq. (16) and the resultant tolerable damping is presented in Fig. 5. for these dominant modes. Here we have taken R_{12} ~ 2.4 m/rad and the beam was initially offset by $\sigma_{y,Crab}$(~34 μm), which is the maximum expected offset in the crab cavity in the ILC, in the first cell of the crab cavity. The energy of the beam was set to 250 GeV.

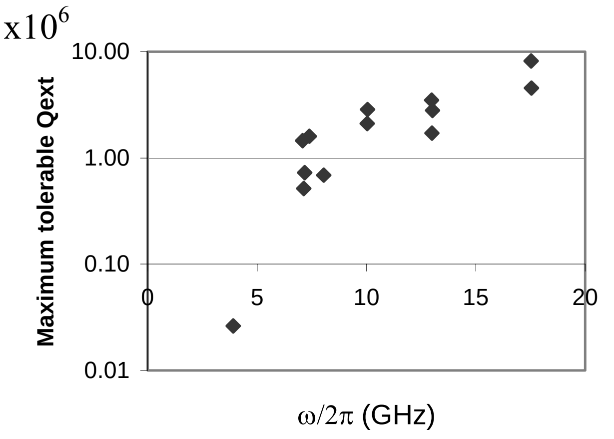

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Fig. 5 Tolerances of the external Q factor to limit Δy_{IP} to be below $\sigma_{y,IP}$ /4 for dipole modes due to the centre of mass kick. Calculation based on the vertical kick inflicted on a single 9 cell cavity with $\sigma_{y,IP}$=5.7 nm and R_{12}=2.4m/rad

The analytical results described in this paper all rely on the dominant effect being described by the interaction with a single mode. In reality there may be a significant excitation of multiple modes. To ascertain the degree to which the single-mode approximation is valid the multi-mode wake-field, summed over successive bunches, described by (3), is also summed over 13 modes and the mode frequencies are systematically varied by an amount Δf_{error} by up to 3MHz which is the inverse of the bunch repetition frequency. In the real cavity all the modes will not be detuned by the same amount, however this method does show the effect of a number of other modes on the wake-field and shows the damping calculated by (19) does keep the deflection below the required tolerances. The deflection of the beam at the IP that results from this calculation is illustrated in Fig. 6, and all parameters used in the calculation are defined in Table 3.

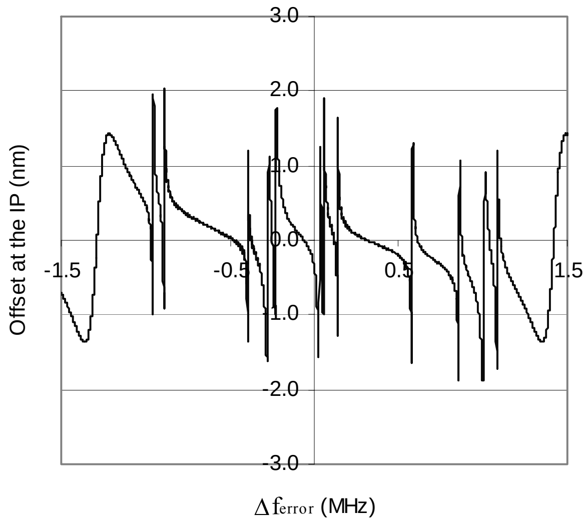

Fig. 6 Vertical offset, Δy, of the bunches leaving a single crab cavity due to interaction with the transverse wake-field versus a systematic shift in the mode frequencies from their nominal calculated values

There are distinct resonances at certain frequency offsets and these reach peak values at a deviation of 0.888 MHz which give rise to an vertical deflection angle, θ, of 0.84 nrad, which is equivalent to a vertical offset of 2.03 nm. This is a larger deflection than that predicted from our analytical calculations, which contained the offset to no more than 1.4 nm.

The reason for the larger deflection is that for a given frequency offset, two modes are resonant close to a harmonic of the bunch repetition frequency and these act together to produce an increased deflection. The effect of a transverse long-range wake can be significant at frequency offsets much wider than the modes bandwidth (as can be seen in Figure 4a) and hence the mode at 3.9GHz, which has a very wide bandwidth, increases the kick for most of the other modes.

$\sigma_{y,IP}$	5.7 nm
$\sigma_{y,crab}$	34 μm
E_{beam} (CM)	500 GeV
R_{12}	2.4 m/rad
τ_b	369.2 ns

Table 3. Parameters used in the calculations

In this case the deflection given to each bunch is constant, however in realistic case the bunch offset and arrival time, and hence the deflection, will vary bunch-to-bunch. As long as the modes are damped to Q factors less than the values prescribed here these effects should not cause deflections above the stated tolerances.

Thus, in practice the single-mode analysis provides an analytical tool to provide an estimate of the damping requirement. However, detailed numerical calculations are necessary in order to fully understand the emittance dilution that will occur in the interaction with a realistic set of cavity modes. Nonetheless, the analytical model presented enables the fundamental modal interaction to be ascertained.

VI. Conclusions

An analysis has been provided to calculate the deflection that will occur due to higher order modes excited in deflecting mode cavities. The single mode analytical result provides a basis for the estimating the damping requirements. Careful numerical studies indicate that under specific systematic frequency shifts, multiple higher order deflecting modes can be excited which can seriously damage the beam emittance if the damping is not increased to ameliorate this situation.

Finally we note that in fabricating the crab cavities there are several processes involved in finalizing the crab cavities prior to installation. Each of these processing steps introduces small geometrical errors. These errors change the effective dipole frequencies of the cavities and can push the beam into a regime in which significant luminosity reduction occurs. To avoid this, a careful study based on the techniques detailed herein will be undertaken to ascertain the damping requirement under the worst-case resonant condition. The analysis presented here has been confined to dipole modes excited with the opposite polarization from the deflecting mode. Clearly, the modes excited in the horizontal and longitudinal planes will also need damping. A similar analysis can be employed and will be reported a in a future publication.

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