HARMONIC LASING IN AN FEL AMPLIFIER

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Abstract

Recent proof-of-principle simulations have demonstrated a scheme that allows a planar undulator FEL to lase so that the interaction with an odd harmonic of the radiation field dominates that of the fundamental. This harmonic lasing of the FEL is achieved by disrupting the electron interaction with the fundamental while allowing that of one of the harmonic interactions to evolve unhindered. The disruption is achieved by a series of relative phase changes between electrons and the ponderomotive potentials of both the fundamental and harmonic fields. Such phase changes are relatively easy to implement and some current FEL designs would require no structural modification to test the scheme.

INTRODUCTION

Planar undulators allow a resonant on-axis FEL interaction with odd harmonics of the fundamental resonant radiation field. The main limiting factor in accessing the exponential harmonic interaction in a high gain FEL is the dominance of the interaction at the fundamental. In this paper a scheme of suppressing the fundamental FEL interaction is proposed which allows an harmonic to evolve to saturation.

The scheme, first outlined in [1], uses a series of relative phase changes between the electrons and ponderomotive potentials of the resonant fields at a series of points along the FEL interaction. The phase of the electrons with respect to the ponderomotive potential of the fundamental resonant wavelength is defined as ϑ_j where $j = 1..N$ and N is the number of electrons. The phase of the electrons with respect the ponderomotive potentials of the nth harmonic field is then $\vartheta_{nj} = n\vartheta_j + \phi_n$ where ϕ_n is the relative phase between the ponderomotive potential of the fundamental and nth harmonic. If the phase of the electrons with respect to the fundamental ponderomotive potential is changed at a point-like region of the interaction by a relative phase $\Delta\vartheta_j = 2\pi/k$ then the corresponding phase change for the harmonics will be $\Delta\vartheta_{nj} = 2\pi n/k$. Hence, if $k = n$ the electrons will be re-phased within the ponderomotive potential of the nth harmonic by 2π whereas that for the fundamental will be $2\pi/n$. While the 2π electron re-phasing of the nth harmonic should not adversely effect its subsequent FEL interaction, the $2\pi/n$ electron re-phasing of the fundamental can be expected to disrupt its exponential growth. A schematic of electron phase space of the fundamental and third harmonic interaction well before saturation and immediately following a relative phase shift of $2\pi/3$ is shown in Fig. 1. (Note that here the relative phase shift has been applied to the ponderomotive potentials rather than the electron phases.) It can be seen that while such a phase shift does not effect the relative phases of the electrons with respect to the separatrix of the third harmonic, the mean electron bunching phase has shifted with respect to the ponderomotive potential of the fundamental. It is this shift that disrupts the exponential FEL instability of the fundamental.

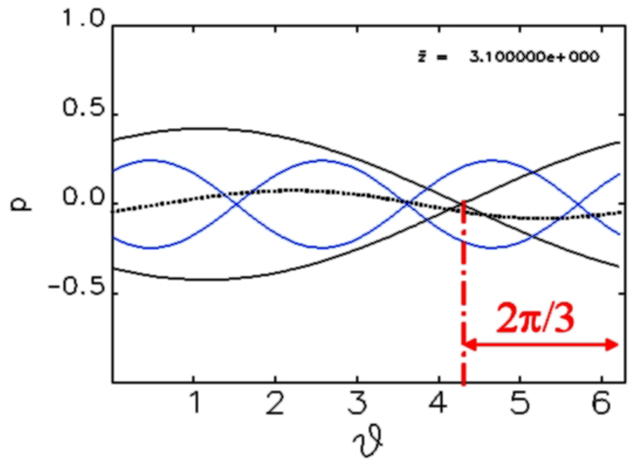

Figure 1: Schematic of electron phase space showing the fundamental seperatrix (black) and 3rd harmonic seperatrix (blue) following a relative phase shift of $2\pi/3$ by both fundamental and 3rd harmonic ponderomotive potentials.

THE 1D MODEL

The harmonic bunching of electrons in a high gain FEL was investigated in the 1-D limit in [2]. A similar notation for the FEL equations is used here:

$$\frac{d\vartheta_j}{d\bar{z}} = p_j \tag{1}$$

$$\frac{dp_j}{d\bar{z}} = -\sum_{h,odd} F_h(\xi_w)\left(A_h e^{ih\vartheta_j} + c.c.\right) \tag{2}$$

$$\frac{dA_h}{d\bar{z}} = F_h(\xi_w)\left\langle e^{-ih\vartheta}\right\rangle, \tag{3}$$

where $j = 1..N$ are the total number of electrons, $h = 1, 3, 5...$ are the odd harmonic components of the field, $\xi_w = a_w^2/2(1 + a_w^2)$ and $F_h(\xi_w)$ are the usual difference of Bessel function factors associated with planar wiggler FELs. Other symbols having their usual meaning [2].

In the 1D scaling and analysis that follows both the undulator period, λ_w, and the initial electron average beam

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energy, γ, are assumed constant. Tuning of the fundamental resonant radiation wavelength is therefore achieved by variation in the undulator magnetic field alone.

The FEL interaction is investigated for *single* radiation wavelength operation using both the normal mode of operation (where the wavelength is the fundamental) and in the harmonic lasing mode as described above where the wavelength is at an odd harmonic. The undulator must therefore have two different settings. In the first mode the RMS undulator parameter $a_w = a_1$ and is set so that the fundamental resonant wavelength is $\lambda_f = \lambda_1$ giving harmonic resonant wavelengths $\lambda_h = \lambda_1/h, \ \ h = 3, 5, 7, \ldots$. In the second mode the undulator parameter is reset to $a_w = a_n$ so that the new resonant fundamental wavelength is the nth harmonic, one of the odd harmonic numbers, of the first mode setting, i.e. $\lambda_f = \lambda_n$. For the assumed fixed beam energy and undulator period, it is simple to show from the FEL resonance relation that a_1 and a_n must satisfy the relation:

$$\frac{1 + a_1^2}{1 + a_n^2} = n. \tag{4}$$

Hence, there are no real solutions a_n for $a_1 < a_c = \sqrt{n - 1}$, i.e. for $a_1 < a_c$ it is not possible to re-tune the undulator to a fundamental wavelength $\lambda_f = \lambda_n$.

The universal scaling parameter, $\rho = (a_w \omega_p / c k_w)^{2/3} / \gamma$ [3, 2], of (1..3) is that for the undulator parameter $a_w = a_1$ i.e. when the undulator is tuned so that the fundamental wavelength $\lambda_f = \lambda_1$. When the undulator is tuned so that the fundamental wavelength $\lambda_f = \lambda_n$ then using identical scaling as (1..3) and neglecting all harmonics $h > n$:

$$\frac{d\vartheta_j}{d\bar{z}} = \frac{p_j}{n} \tag{5}$$

$$\frac{dp_j}{d\bar{z}} = -\frac{a_n}{a_1} F_1(\xi_n) \left(A_n e^{in\vartheta_j} + c.c. \right) \tag{6}$$

$$\frac{dA_n}{d\bar{z}} = \frac{a_n}{a_1} F_1(\xi_n) \left\langle e^{-in\vartheta} \right\rangle. \tag{7}$$

The equations (1..3), truncated at $h = n$, and (5..7) form the working set of equations for the remainder of this paper.

LINEAR THEORY

Linear analysis using standard methods [3] allows the 1-D gain lengths of the FEL interaction to be calculated for the two single wavelength operation undulator modes as described in the previous section. As was shown in [2], the harmonic evolution of (1..3) have two separate regimes of evolution before the fundamental saturates. For small values of the bunching at the fundamental, both fundamental and harmonics are uncoupled and evolve exponentially with gain lengths determined only by the independent parameters. However, as the exponential growth of the bunching at the fundamental wavelength λ_1 progresses, the harmonics become strongly driven by the interaction at the fundamental. In the cold-beam limit this results in a dramatic reduction in the gain length of the harmonic to $1/h$th of that of the fundamental.

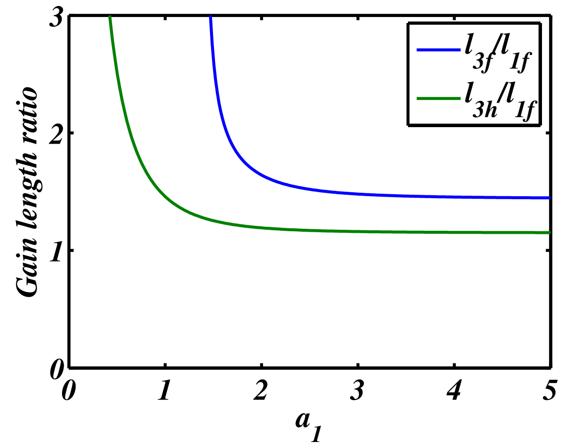

Figure 2: Comparison of gain lengths as a function of a_1 for a fixed wavelength using the undulator tuned for the harmonic lasing scheme of case 1): l_{3h} (green)) and tuned for lasing at the fundamental case 2): l_{3f} (blue).

Here, we assume, as will be shown in subsequent sections, that the scheme of disrupting the exponential growth of the fundamental works as described above and therefore that there is no growth of the fundamental. The gain lengths of a single wavelength $\lambda = \lambda_3$ are compared for the two cases: 1) 3rd harmonic interaction at wavelength λ_3 with gain length l_{3h} (fundamental is disrupted) and 2) undulator re-tuned to fundamental $\lambda_f = \lambda_3$ with gain length l_{3f}. To aid comparison these gain lengths are scaled with respect to the un-disrupted fundamental gain length l_{1f} of case 1). Using the results of [2, 4] it is simple to show that this results in:

$$\frac{l_{3h}}{l_{1f}} = \left(\frac{F_1^2(\xi_1)}{3F_3^2(\xi_1)} \right)^{1/3} \tag{8}$$

$$\frac{l_{3f}}{l_{1f}} = \left(\frac{a_1 F_1(\xi_1)}{a_3 F_1(\xi_3)} \right)^{2/3} \tag{9}$$

These expressions are plotted in Fig. (2) as a function of the undulator parameter a_1. The value of the re-tuned undulator parameter (case 2) is obtained from (4) with $n = 3$. It is seen from the plot for l_{3f}/l_{1f} that the gain length $l_{3f} \to \infty$ to as $a_1 \to \sqrt{2}$. This is the limit $a_3 \to 0$ in equation (4) corresponding to the minimum fundamental wavelength achievable and where FEL coupling no longer exists. Furthermore, note that the gain length for the disrupted fundamental scheme of case 1), $l_{3h} < l_{3f}$. This is generally true for all odd harmonics n. Thus, for a fixed period undulator and in the 1-D, cold beam limit, when tuning an FEL interaction to a shorter wavelength by a factor $1/n$ the FEL gain length is *always* shorter by using the disrupted fundamental scheme of lasing than by a simple re-tuning of the undulator magnetic field (where that is possible under the restrictions of equation (4)).

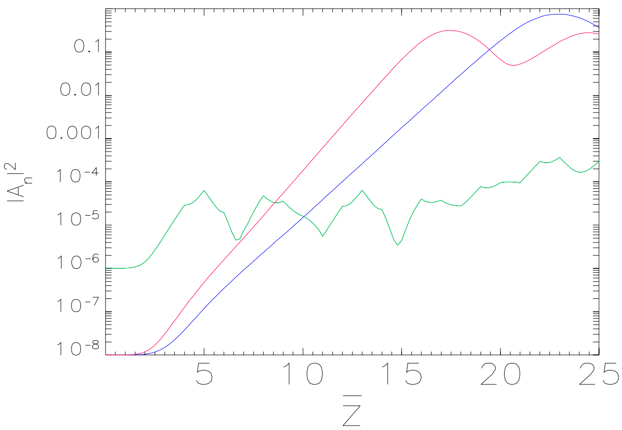

Figure 3: Scaled intensities of fundamental $|A_1|^2$ (green) and third harmonic $|A_3|^2$ (red) for undulator parameter $a_1 = 4$ demonstrating the effects of relative phase changes of $\Delta\vartheta = 2\pi/3$ at $\bar{z} = 4, 5, 6, \ldots, 24$. For the undulator parameter re-tuned to $a_3 = 2.16$, A_3 is the fundamental and a separate simulation shows how $|A_3|^2$ (blue) evolves.

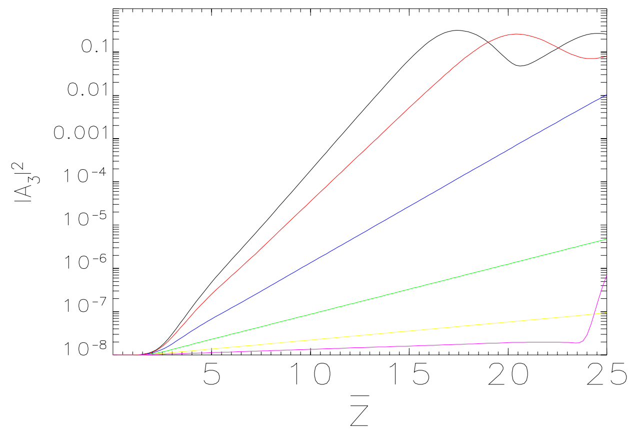

Figure 4: The effect of energy spread on harmonic lasing. The third harmonic $|A_3|^2$ of Fig. (3) (red) is plotted for scaled Gaussian energy spreads in p of $\sigma_p = 0.0$, 0.1, 0.2, 0.3, 0.4 and 0.5. The growth rate decreases monotonically with σ_p. The fundamental (green of Fig. (3)) is not shown.

NUMERICAL SIMULATION

The system of FEL equations (1..3) were solved numerically, with undulator parameter $a_1 = 4$, to demonstrate the harmonic lasing scheme as described above. The results are shown in Fig. (3). A numerical solution of equations (5..7) is also shown on the same scale to demonstrate the solution when the undulator parameter is re-tuned to $a_3 = 2.16$ so that λ_3 is the fundamental. The resonant, cold beam limit is assumed with an electron distribution $p_j = 0 \ \forall \ j$, so that the spread $\sigma_p = 0$. It is seen that for $\bar{z} > 4$ the exponential instability of the fundamental scaled intensity (green) is disrupted by the series of $\Delta\vartheta = 2\pi/3$ relative phase changes so that $|A_1|^2 \ll 1$ throughout the interaction. However, the simultaneous evolution of the third harmonic (red) is unaffected, remaining resonant and evolving exponentially to saturation. Note that the intensity $|A_1|^2 \gg |A_3|^2$ at the beginning of the interaction and acts as a seed field. If A_1 has good longitudinal coherence properties these may be transfered to the shorter wavelength interaction for A_3.

When the undulator is re-tuned to $a_3 = 2.16$, so that λ_3 is the fundamental (blue), it seen that the gain length is longer. The relative gain lengths agree with the linear theory of the previous section. Despite the lower growth rate, however, the saturation intensity is larger. It is seen by comparing equations (1) and (5) that, for the same energy spread, the phase velocity spread is a factor n times larger for the case of harmonic lasing. This reduces the electron energy spread and so the radiation intensity at saturation increases for the case of harmonic lasing.

The effect of an n-fold increase in the phase velocity spread for harmonic lasing also increases the homogeneous energy spread requirements of the electron beam at the beginning of the interaction. For normal FEL interaction at the fundamental this may be expressed as $\sigma_\gamma/\gamma < \rho$ or equivalently in the scaling used here $\sigma_p < 1$. For harmonic lasing this requirement is increased to $\sigma_p < 1/n$. The effect of this on harmonic lasing is clearly seen by comparing the results of Figs. (4) and (5) where the results of Fig. (3) are extended to include the effects of initial Gaussian energy spreads of $\sigma_p = 0, 0.1, 0.2, \ldots, 0.5$.

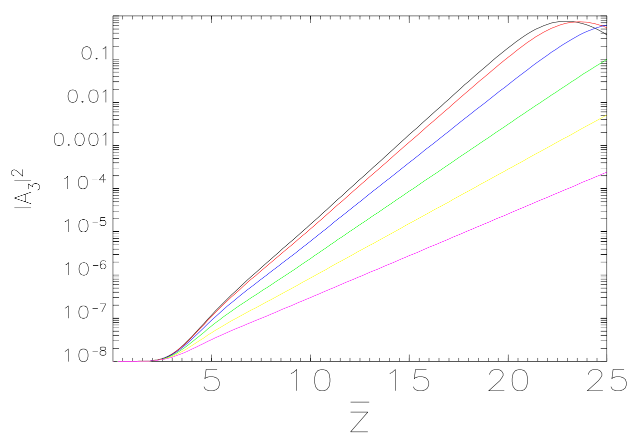

Figure 5: The effect of energy spread for a re-tuned undulator parameter of $a_3 = 2.16$ making A_3 is the fundamental. The fundamental $|A_3|^2$ of Fig. (3) (blue) is plotted for scaled Gaussian spread in p of $\sigma_p = 0.0$, 0.1, 0.2, 0.3, 0.4 and 0.5. The growth rate decreases monotonically with σ_p.

One can conclude that harmonic lasing at λ_3 is much more sensitive to the effects of electron beam energy spread than fundamental lasing at λ_3 (if $a_1 > a_c$ makes this possible). This may be explained intuitively by reference to Fig. (1) which shows the phase space and seperatrix of the associated ponderomotive wells of the fundamental and harmonic. For a given spread in p, because there are n times the number of harmonic ponderomotive wells than the fundamental, the spread appears n times as great to the harmonic interaction. The benefit of tuning the undulator so that the harmonic becomes the fundamental is therefore that the spread in p is reduced by $1/n$ (see equation 5) which may well improve the growth rate above that of the harmonic lasing scheme. Nevertheless, from equation (4), when $a_1 < a_c$ it is not possible to re-tune the undulator so that the harmonic becomes the fundamental. (Of course, this will also be true where both undulator period and gap are fixed and any tuning is achieved via variation of the electron beam energy.) In the examples above $a_1 = 4$, well above the critical value $a_c = \sqrt{2}$.

In addition to describing the uncoupled linear evolution of the fundamental and harmonic interactions, the work of [2] showed that evolution of the bunching at the fundamental also drives the nth harmonic field at a growth rate of n times that of the fundamental. This nonlinear coupling is seen in Fig. (6), where the fundamental and harmonic fields are plotted for a harmonic lasing scheme, here with $a_1 = 1 < a_c$. A Gaussian energy spread parameter of $\sigma_p = 0.1$ is used. The plot shows the evolution of the fundamental (black) and third harmonic (red) with phase changes of $4\pi/3$ at $\bar{z}$ =8, 9, 10,.... The third harmonic is also shown (green) for phase changes of $2\pi/3$ at $\bar{z}$ =8, 9, 10,.... (The fundamental is not shown for this case.) According to linear theory, there should be no difference between the harmonic lasing for the two cases of $4\pi/3$ and $2\pi/3$ phase changes and this is the case until $\bar{z} \approx 8.5$. Thereafter, for $4\pi/3$ phase changes (red), the harmonic is seen to attain a saturation intensity of approximately 2 orders of magnitude greater than that for $2\pi/3$ phase changes (green) and also, not shown, approximately one order of magnitude greater than that if no phase changes are applied and the fundamental evolves to saturation in the usual way. The difference in behaviour between the two cases is due to the fact that the electrons remain coupled to the fundamental which continues to drive the bunching at the harmonics. For the case of $4\pi/3$ phase changes the fundamental continues to bunch the electrons in a way similar to that of [1] when phase changes of π are used with the FEL interaction to enhance the bunching at the fundamental. For $2\pi/3$ phase changes, the fundamental interaction does not bunch the electrons as well, greatly reducing the non-linear driving of the harmonic.

CONCLUSIONS

A scheme for 'taming' the fundamental high-gain FEL instability in a planar undulator has been proposed in a way that allows an odd harmonic to remain resonant and evolve exponentially to saturation. The series of relative phase changes between electrons and the ponderomotive wells should not be difficult to implement. In particular, for FELs requiring long interaction lengths and employing many undulator sections, typical of VUV to x-ray FELs, methods of altering the relative phase already exist in the 'phase-matching' devices between undulator sections - an experiment to test the validity of the harmonic lasing scheme would therefore incur little or no cost. As with many schemes that attempt to exploit harmonics electron beam quality has been shown to be problematic. Nevertheless, the scheme does offer access to improved harmonic emission and may enable FELs to out-perform their original design specifications at shorter wavelengths e.g. it would be interesting to apply the scheme to the modulator section of a multiple undulator set-up such as that used in High Gain Harmonic Generation. No attempt has been made here to optimise the size of phase changes to attain higher saturation intensities. Trial simulations have demonstrated that this is possible. Other phase-changing schemes may also offer alternative opportunities to control the electron-radiation interaction. For example, phase changes post saturation of fundamental lasing may act as a form of tapering to enable further energy extraction from the electrons. In short, this method is a potentially useful tool and offers greater control of the FEL interaction.

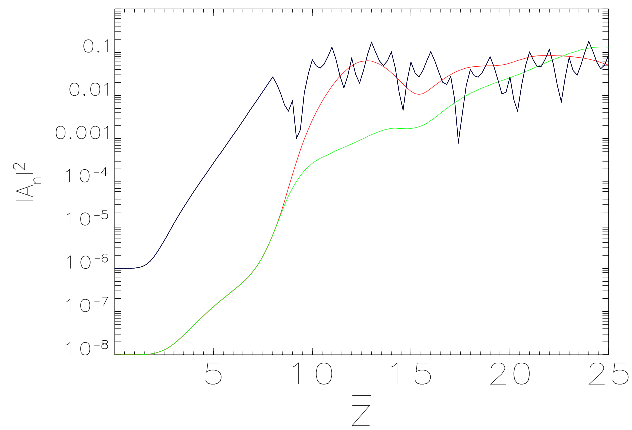

Figure 6: The scaled intensities of $|A_1|^2$ (black) and $|A_3|^2$ (red) for phase changes of $4\pi/3$ at $\bar{z} = 8, 9, \ldots, 24$ after non-linear harmonic coupling around $\bar{z} \approx 7$. For phase changes of $2\pi/3$, the growth of the harmonic $|A_3|^2$ (green) demonstrates less beneficial non-linear coupling to the fundamental.

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