# FEL SCHEME WITH OPTICAL CAVITY ROUND-TRIP FREQUENCY AT MULTIPLE OF ELECTRON BUNCH REPETITION RATE 

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#### Abstract

A study is carried out to develop an optimised scheme for a free electron laser operating in the far-infrared, based upon an electron beam delivered from a prototype superconducting linac module [1]. This collaboratively developed module is capable of delivering 123 pC of bunch charge at a repetition rate of 13 MHz , or alternatively, higher bunch charges are achievable at lower repetition rates ( 200 pC at 4.33 MHz for example). A conventional oscillator-type FEL design is compared with a proposed alternative design; wherein the radiation makes three passes of the optical cavity per electron bunch arrival. For both schemes, the output photon properties are predicted through simulations using Genesis 1.3 in combination with the optics code OPC, and tolerances to mirror alignment are investigated and compared to predictions from theory. The further potential of schemes where the radiation makes multiple passes of the optical cavity per electron bunch arrival is discussed.


## INTRODUCTION

Previous studies [2] investigated whether a FEL using an electron beam delivered from a prototype superconducting linac module [2] could deliver output meeting the requirements of the IR-FEL for the now-discontinued 4GLS project [3]. The electron beam parameters assumed were 25 MeV beam energy and 123 pC bunch charge at 13 MHz repetition rate. It was found that pulse energies in the range $15-25 \mu \mathrm{~J}$ were attainable over a wavelength range of $35-200 \mu \mathrm{~m}$. The requirement of [3] was for $>30 \mu \mathrm{~J}$ over the full wavelength range.

It is known from previous studies that higher photon pulse energies may be obtained by increasing the bunch charge. By operating the linac module with an electron bunch repetition rate of 4.33 MHz , the electron bunch charge can be increased to 200 pC .

A conventional oscillator-type FEL with a single optical pulse propagating in the cavity, has a cavity length, D , given by:

$$
\begin{equation*}
D=\frac{c}{2 f} \tag{1}
\end{equation*}
$$

where f is the electron bunch repetition rate and c is the speed of light in vacuum. For a repetition rate of $\mathrm{f}=4.33$ MHz , this gives a cavity length of 34.62 m , which would lead to excessively large mode sizes at the mirrors, given the divergence of optical beams at the operating radiation wavelengths. Hence, a less conventional FEL design is proposed; a cavity length of 11.54 m is chosen to operate with electron bunches at 4.33 MHz repetition rate: the radiation
makes three passes of the cavity per electron bunch arrival. The higher bunch charge should allow more gain, however there will be more losses through absorption in the mirrors and hole-outcoupling. Such a design can be modelled using Genesis 1.3 [4] in combination with the optics code OPC [5], since arbitrary cavity layouts can be specified. The predicted output photon properties will be compared to the previous design [2], modelled using the same method. A method has been devised which uses the optics code to enable visualisation of the evolution of the transverse pulse shape in the cavity. This method propagates the radiation field output from Genesis 1.3 in a series of small steps and outputs the transverse cross-section and transverse statistics at each step. The graphical representation of the statistical output from this method is demonstrated in Figs. 1 and 2, together with schematics of the two FEL schemes compared in the paper. The graphical representation of the cross-section information is demonstrated in Figs. 6 and 8.


Figure 1: Schematic of the conventional FEL oscillator (above) and a diagram to show how the evolution of the radiation over one pass of the cavity is represented in subsequent figures (below), plotted is $\pm$ the rms width of the optical field in the $x$-direction against intracavity distance.

Also to be investigated are tolerances to mirror alignment, since it is thought that multiple passes of the radiation through the cavity may compound mirror alignment errors.


Figure 2: Schematic of the FEL design in which radiation makes 3 passes of the cavity per electron bunch arrival (above), and an example of how the evolution of the radiation over three passes of the cavity is represented graphically.

## MAXIMUM OUTPUT POWER

To find the maximum output power from the two FEL schemes (when operating at $35 \mu \mathrm{~m}$ resonant wavelength), the cavity geometry has been varied by changing the radius of curvature (ROC) of the mirrors simultaneously such that the waist position of the radiation is kept in the centre of the undulator. Also varied is the radius of the hole in the downstream mirror, used for outcoupling radiation from the FEL. Figs. 3 and 4 show the variation of peak FEL output with varying outcoupling hole radius and cavity mirrors ROC.


Figure 3: Peak FEL output power variation with varying mirror radii of curvature and outcoupling hole radius, for the conventional FEL oscillator scheme.

The results show that higher peak output powers are attainable from the conventional FEL oscillator scheme and that these are available over a wider range of cavity configurations. The highest output power from the conventional oscillator scheme is 12.3 MW while the highest out-


Figure 4: Peak FEL output power variation with varying mirror radii of curvature and outcoupling hole radius, for the multiple radiation pass scheme.
put power averaged over three passes for the multiple pass scheme is 4.5 MW , although powers from a single pass may be up to 10.8 MW . The nature of the variation of peak output power with cavity mirror ROC and hole radius may be explained by considering the radiation profile in the cavity. For smaller values of the radius of curvature, e.g. point (B) of Fig. 3, the mode size on the cavity mirrors is larger and the hence the optimum hole size is larger than for a case with larger cavity mirror radii of curvature, e.g. point (A). The transverse profiles of these two cases are plotted in Fig. 5.

It should be noted that both FEL schemes will emit radiation pulses at 13 MHz , i.e. the multiple radiation pass case will emit three radiation pulses per electron bunch. It might be assumed that the first pulse in the cycle (the pulse which co-propagates with the electron bunch) would have higher powers than the second which would in turn be higher than the third. However, by selection of the cavity mirror radii of curvature and the radius of the outcoupling hole, con-


Figure 5: The rms width of the optical field in the x direction for two cavity configurations labelled in Fig. 3
figurations can be chosen where this is not the case. This is evident in Fig. 4. For example we can choose a configuration in which the output power is approximately equal from each pass, or where one of the passes has significantly higher powers than the others. The behaviour can be explained by considering the transverse mode profile in the cavity, for example, for the configuration with mirror ROC $=7.6 \mathrm{~m}$ and hole radius $=2.2 \mathrm{~mm}$, the transverse mode shape is such that the on-axis power at the downstream mirror is low for the first two passes relative to the third pass. Hence the output power is significantly higher on the third pass; this is shown in Fig. 6. For the configuration with mirror $\mathrm{ROC}=7.4 \mathrm{~m}$ and hole radius $=0.8 \mathrm{~mm}$, the output power varies by $<5 \%$ from pass to pass.

The peak output powers from the single-radiation-pass scheme are consistent with the results predicted using simple design formulae in previous studies [2]. Since the peak output powers for the multiple radiation pass scheme are lower than for the single pass case, we can predict that the energy per photon pulse will also be lower, if we assume similar photon pulse length; time-dependent Genesis 1.3 simulations have been used to verify that this is the case. Due to the CPU-expensive nature of time-dependent Genesis 1.3 simulations, a complete investigation of output power and energy per photon pulse variation with cavity detuning value has not been performed. It is, however safe to predict that neither FEL design will yield the required pulse energies.

There are several reasons why the wider range of FELoperable cavity set-ups available in the single-radiationpass scheme is advantageous: the FEL may be operated at a different cavity set-up than the optimum in order to scan over a range of wavelengths with a single mirror set
or because of errors in mirror manufacture or mirror deformation during operation.

## MIRROR ALIGNMENT ERRORS

Also investigated are tolerances to mirror alignment, since it is thought that multiple passes of the radiation through the cavity may compound mirror alignment errors. The OPC code allows us to model mirror tilts and transverse offsets. For these simulations, cavity parameters are chosen for near-optimum FEL output power: a value of 6.4 m is used for the mirror radii of curvature and the outcoupling hole radius is set to 2 mm . A simple approach is adopted, whereby only a single alignment error (i.e. either a tilt or an offset) on the downstream mirror is considered. A more comprehensive approach would consider both tilts and offsets applied simultaneously to both mirrors. It can be shown that mirror tilt and offset are equivalent for spherical mirrors.

## Downstream Mirror Tilt

For the two FEL schemes, simulations were carried out for a range of downstream mirror tilts. The variation of peak FEL output power is plotted against mirror tilt in Fig. 7 for both schemes. There is shown to be no significant difference between the two FEL schemes in terms of the tolerance to downstream mirror tilt.

For the conventional oscillator scheme, these results can be compared to theory [6]. To keep the optical axis within the divergence angle of the optical beam, the alignment tolerance, $\theta$, is restricted by the relation

$$
\begin{equation*}
\theta \ll\left(\frac{2 \lambda_{R}}{\pi d}\right)^{\frac{1}{2}}(1-g)^{\frac{1}{4}}(1+g)^{\frac{3}{4}} \tag{2}
\end{equation*}
$$

where $\lambda_{R}$ is the radiation wavelength, $d$ is the cavity length and

$$
\begin{equation*}
g=1-d / R \tag{3}
\end{equation*}
$$

where $R$ is the mirror radius of curvature (i.e. assuming both cavity mirrors to have the same radius of curvature). Substituting into equation (2) gives us $\theta \ll 0.473 \mathrm{mrad}$ and the simulations are in good agreement with this. The theoretical limit is shown in Fig. 7 (a) and the transverse mode profile against intracavity distance is shown for a case with 0.4 mrad tilt in Fig. 8 .

## Downstream Mirror Offset

For the two FEL schemes, simulations were carried out for a range of downstream mirror offsets. The equivalence of tilts and offsets for spherical mirrors was confirmed.

## CONCLUSION

Simulations have shown that the proposed FEL scheme, in which the radiation pulse makes three passes of the cavity per electron bunch arrival, is workable. For this particular set of parameters however, it is better to operate with a


Figure 6: The variation of the transverse mode profile in the $x$-direction with intracavity distance is shown for the cavity configuration with mirror $\mathrm{ROC}=7.6 \mathrm{~m}$ and hole radius $=2.2 \mathrm{~mm}$. The downstream mirror position is shown.


Figure 7: Variation of peak FEL output power with varying downstream mirror tilt for the two FEL schemes. Shown for the conventional FEL scheme is the theoretical limit of FEL operation
conventional FEL design as higher peak powers are attainable. Also high powers are attainable over a wider range of cavity configurations hence the FEL may be operated at a different cavity set-up than the optimum in order to scan over a range of wavelengths with a single mirror set or because of errors in mirror manufacture or mirror deformation during operation. The optics code has been used to allow us to observe the evolution of the transverse pulse shape in the cavity and this has allowed us to better understand the behaviour of the system.

The output from the multiple radiation pass scheme has been shown to have a strong dependence on the mode shape in the cavity. The results suggest that configurations could be chosen to supply a train of pulses with approximately equal powers at a multiple of the frequency of the electron bunch repetition rate. It is likely that the hole outcoupling


Figure 8: The variation of the transverse mode profile in the x -direction with intracavity distance is shown for the cavity configuration with mirror $\mathrm{ROC}=6.4 \mathrm{~m}$ and hole radius $=2$ mm , and a tilt in the x -direction on the downstream mirror of 0.4 mrad
method would be integral to such a scheme.
Also, a study of mirror alignment tolerances has shown that, for the conventional FEL scheme, the results of simulations match with theory and that there should be little significant difference between the two FEL schemes in this respect.

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