Science & Technology Facilities Council

Technical Report
RAL-TR-2016-013

The VISAR. A Simple Analysis

JRJ Bennett

September 2016

Enquiries concerning this report should be addressed to:

RAL Library
STFC Rutherford Appleton Laboratory
Harwell Oxford
Didcot
OX11 0QX

Tel: +44(0)1235 445384
Fax: +44(0)1235 446403
email: libraryral@stfc.ac.uk

Science and Technology Facilities Council reports are available online at: http://epubs.stfc.ac.uk

ISSN 1358-6254

UKNF/Target/N3/2010

15 August 2010

The VISAR
A Simple Analysis

J. R. J. Bennett

Rutherford Appleton Laboratory, Chilton, Didcot, Oxon. OX11 0QX

The VISAR - A Simple Analysis

1. Introduction

The **V**elocity **I**nterferometer **S**ystem for **A**ny **R**eflector (VISAR) [1,2], also known as the Delay Leg Interferometer, Laser Velocity Interferometer or Doppler Velocity Interferometer, is analysed. A number of analyses of the operation of the VISAR are given in the literature [1-7], culminating in that of Goosman [8]. More recently a detailed account with extensive references has been given by Dolan [9]. The VISAR basically measures acceleration.

In this note the analysis considers the times for the light waves at a particular phase to traverse different parts of the VISAR, arriving at the difference in time for the interfering waves and avoiding the pitfalls of working in wavelengths and frequencies,. In this respect the analysis is similar to that of Goosman [8], but a simpler derivation of the solution for the VISAR with an accelerating reflector is given in Section 5. The resultant expressions are identical to those of Goosman.

2. The VISAR

Figure 1 shows the VISAR schematically. A laser, near 'A', sends out a beam of light of wavelength λ, (frequency f, angular frequency ω and velocity of light c). It hits a surface at 'B' and is reflected to 'C', where it is split into two legs: the delay leg, 'CDE' of length δ and finally passing along 'EF' to the detector at 'F', and the un-delayed leg 'CF'. The delayed and un-delayed beams produce interference patterns where they combine at F.

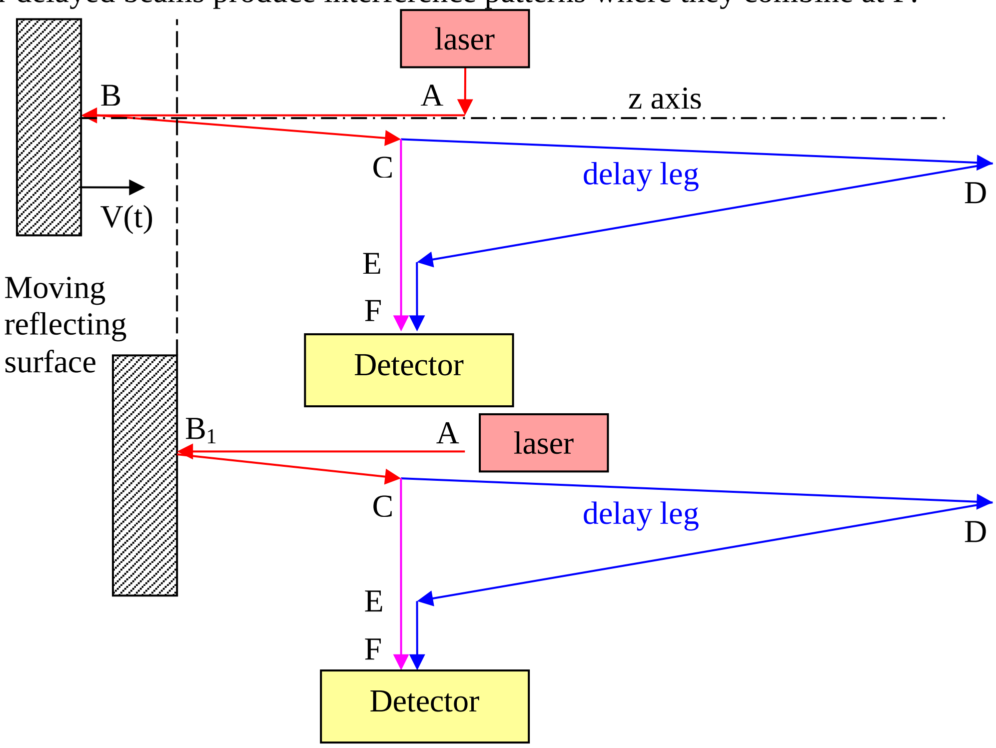

Figure 1. Schematic diagram of the VISAR. For practical views of the optical arrangement, see Dolan [8].

It is important to note that in this report the two beams that interfere at F are termed the *delayed* and *un-delayed* beam, by which is meant the beam which travels down the delay leg and the beam which does not travel down the delay leg, respectively. Thus the un-delayed beam sets off from A *after* the delayed beam.

3.0 Stationary Reflector

Assume that the reflector is stationary – see upper diagram of Figure 1. A wave from A taking the un-delayed route takes time, t_{ud} to travel to the detector at F via B and C. The delayed wave travelling down the delay leg from A to B, C, D, E to F, takes time,

$$t_d = t_{ud} + \tau \tag{3.1}$$

where τ is the delay time, $\tau = \delta/c$.

Thus the phase difference between the delayed beam (i.e. the beam travelling down the delay leg) and the un-delayed beam interfering at F is constant in time. Note that the delayed beam starts out from A at a time $\Delta t = \tau$ ***before*** the un-delayed beam. The electric fields from the two beams at A are,

$$\begin{aligned} E_d(t) &= \frac{E_0}{2}\sin\omega(t + \delta t - \Delta t) \\ E_{ud}(t) &= \frac{E_0}{2}\sin\omega(t + \delta t) \end{aligned} \tag{3.2}$$

where the amplitude of the electric field from the laser is E_0 before it is split into equal parts at the point C, t is the time at A and δt is a phase constant to account for the phase of the beam at A. For convenience let $\delta t = 0$. The resultant fields at F are,

$$\begin{aligned} E_d(t_F) &= \frac{E_0}{2}\sin\omega(t + t_d - \Delta t) \\ E_{ud}(t_F) &= \frac{E_0}{2}\sin\omega(t + t_d) \end{aligned} \tag{3.3}$$

where t_F is the time at F. The result of the two fields adding at F is,

$$E(t_F) = E_d(t_F) + E_{ud}(t_F)$$

and the intensity is,

$$\begin{aligned} I(t_F) &= E(t_F)^2 \\ I(t_F) &= \frac{E_o^{\,2}}{4}\left[\sin\omega(t + t_d) + \sin\omega(t + t_d - \Delta t)\right]^2 \end{aligned} \tag{3.4}$$

This equation may be expressed as,

$$I(t_E) = E_0^{\,2}\cos^2\left(\frac{1}{2}\omega\Delta t\right)\sin^2\frac{1}{2}\omega(t + t_d - \Delta t) \tag{3.5}$$

or,

$$I(t_F) = \frac{E_0^{\,2}}{2}(1 + \cos\omega\Delta t)\sin^2\left(\omega t + t_d - \frac{\omega\Delta t}{2}\right) \tag{3.6}$$

or,

$$I(t_F) = \frac{E_0^{\,2}}{4}\left[1 + \cos\omega\Delta t - \cos 2\omega\left(t + t_d - \frac{\Delta t}{2}\right) - \frac{1}{2}\left(\cos 2\omega(t + t_d) + \cos 2\omega(t + t_d - \Delta t)\right)\right] \tag{3.7}$$

The terms in ωt of equations 3.4, 3.5 and 3.6 are too high in frequency to be observable, but the amplitude of the intensity is modulated by the term $\cos^2(½\omega\Delta t)$ (or $(1 + \cos\omega\Delta t)$). For (3.4) and (3.5) the time averaged value of the high frequency terms over one or more periods of the light are ½, and Goosman [8] shows that the time integrals of the last three terms of (3.6) are very small and can be ignored, so the intensity becomes,

$$I(t_F) = \frac{E_0^{\,2}}{4}(1 + \cos\omega\Delta t) \tag{3.8}$$

Thus, for a reflector at rest the observable intensity of the interference pattern at F varies with the length of the delay leg, δ, given by (see Figure 2),

$$I(\delta) = \frac{E_0{}^2}{4}\left(1 + \cos\frac{\omega\delta}{c}\right) \tag{3.9}$$

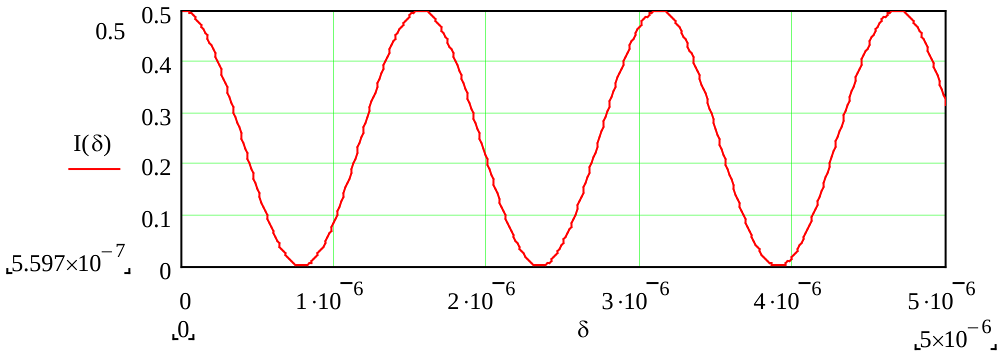

Figure 2. Variation of intensity with delay leg length, δ, in metres, $E_0 = 1$, $\omega = 1.209\text{x}10^{15}$ s^{-1}, $\lambda = 1.559\text{x}10^{-6}$ m; reflector stationary. In practice the delay leg is of the order of a meter long.

Note that the maximum intensity that can be achieved on recombination at F is *one half* of that before the laser beam hit the beam splitter, unless $\omega t_{ud} = \omega t_d$, - i.e. the delay leg is zero. (Furthermore, the long-time average intensity before the beam splitter is $\frac{1}{2}E_0^2$.

4.0 The Reflector at Constant Velocity

For the beam passing down the delay leg and the beam bypassing the delay leg to arrive at F simultaneously, the un-delayed beam must have started from A at time Δt *later* than the delayed beam, given by,

$$\Delta t = t_d - t_{ud} = \frac{2\text{BB}_1}{c} + \tau \tag{4.1}$$

Note that Δt is independent of AB_1, B_1C, CE and EF, so that we can choose to make these distances zero. Any change in any of these distances affects the delayed and un-delayed beams equally. Let us put B_1C CE and EF equal to zero, then as the reflector moves at constant velocity V from B to B_1 in the time t_1, the delayed beam from B passes along the delay leg to F. Thus,

$$t_1 = \frac{\text{B}_1\text{B}}{V}$$

$$t_1 = \frac{\text{B}_1\text{B}}{c} + \tau \tag{4.2}$$

and hence,

$$\frac{B_1B}{V} = \frac{B_1B}{c} + \tau$$

Rearranging this equation gives,

$$B_1B = \frac{V\tau}{\left(1 - \frac{V}{c}\right)}$$

Then the time difference for the delayed and un-delayed beams becomes,

$$\Delta t = \frac{2\tau}{\left(1 - \frac{V}{c}\right)}\frac{V}{c} + \tau = \frac{1 + \frac{V}{c}}{1 - \frac{V}{c}}\tau$$

For convenience, put,

$$\beta = \frac{V}{c}$$

Then,

$$\Delta t = \frac{1+\beta}{1-\beta}\tau \tag{4.3}$$

Ignoring terms of higher order in *β*,

$$\Delta t \cong (1-2\beta)\tau \tag{4.4}$$

Once again the interference pattern does not vary with time – but will vary with *changing* velocity (i.e. acceleration).

Note that Δ*t* is the difference in time that particular phases of the delayed and un-delayed beam set off from A and that these phases are retained as the beams pass through the optical system to the detector. That is, if the delayed beam is at a peak of the wave and the un-delayed beam at a trough, for example, they will still be at a peak and a trough throughout the system. (Even though the frequency may vary due to the Doppler Effect after the beams hit the moving reflector – see section 7.)

5.0 The Reflector in Motion; Varying Velocity –i.e. Acceleration.

Let us consider the interference of the delayed and un-delayed beams at F. It is assumed that the reflector is stationary at *t* = 0 and then moves at a varying velocity *V*(*t*). There are 2 time regimes to be considered:

1) The delayed beam is reflected from the stationary reflector but the un-delayed beam hits the reflector at *t* > 0 when the reflector is moving.
2) The reflector is moving when both the delayed and un-delayed beams hit the reflector. This is the section (5.2.1) that departs from the analysis by Goosman [8].

5.1 Time Regime 1; time, 0 < *t* <*τ*

At *t* = 0, the reflector starts to move and the un-delayed beam from this time forward is reflected from a moving reflector. At time *dt* the reflector moves a distance *dz* towards A and B and the time taken for the beam to reach F, but not travelling down the delay leg is,

$$t_{ud} = t_{BC} - 2dt$$

The time that the beam took to travel from the stationary reflector down the delay leg to F is,

$$t_d = t_{BC} + \tau$$

The un-delayed beam leaving the reflector at *dt* will arrive at F at time,

$$t_F = t_{ud} + 2dt$$

The delayed beam arrives at F at the same instant in time to interfere with the un-delayed beam, and the phase difference is,

$$\Delta t = \omega(\tau + 2dt) = \omega\left(\tau + \frac{2dz}{c}\right) = \omega\left(\tau + \frac{2V(t)dt}{c}\right) \tag{5.1}$$

where the velocity is given by $\frac{dz}{dt} = V(t)$. From (3.7) the intensity at the detector becomes,

$$I(t) = \frac{E_0^{\ 2}}{4}\left[1 + \cos\omega\left(\tau + \frac{2z(t)}{c}\right)\right] \tag{5.2}$$

In terms of the velocity of the reflector, the intensity becomes,

$$I(t) = \frac{E_0^{\ 2}}{4}\left[1 + \cos\omega\left(\tau + 2\int_0^t \frac{V(t)}{c}dt\right)\right] \tag{5.3}$$

Note that the intensity will only change with time if the velocity varies with time – i.e. there is acceleration. At constant velocity the intensity is constant at some value between 0 and $\tfrac{1}{2}E_0^2$ depending on the value of the cosine term.

5.2.1 Time Regime 2; time, $t > \tau$

The time taken for the delayed laser beam to go from A to F via the delay leg (see Figure 1) is given by (3.1) and the time for the un-delayed beam is given by (3.2), but the time difference, given by (4.2) and (4.3), is incorrect since the velocity varies. However, for a very short delay leg *dt*, the equations (4.2) and (4.3) are exact, so that the time difference becomes,

$$d\Delta t(t) = \frac{1+\beta(t)}{1-\beta(t)}dt \tag{5.4}$$

and, on integration with respect to time from t to $t\text{-}\tau$, $\Delta t(t)$ becomes,

$$\Delta t(t) = \int_{t-\tau}^{t} \frac{1+\beta(t)}{1-\beta(t)}dt \tag{5.5}$$

The intensity of the interference signal at F is,

$$I(t) = \frac{E_0^{\,2}}{4}\left[1+\cos\omega\int_{t-\tau}^{t}\frac{1+\beta(t)}{1-\beta(t)}dt\right] \tag{5.6}$$

5.2.2 Time Regime 2; time, $t > \tau$. Approximate Solution – see [1-7].

The distance BB_1 can be expressed as,

$$BB_1 = z(t) - z(t-\tau) \tag{5.7}$$

and from (4.3),

$$\Delta t(t) = 2\frac{\left(z(t) - z(t-\tau)\right)}{c} + \tau \tag{5.8}$$

The intensity at the detector, at F, is,

$$I(t) = \frac{E_0^{\,2}}{4}\left[1+\cos\omega\left(1+\Delta t(t)\right)\right] \tag{5.9}$$

$$I(t) = \frac{E_0^{\,2}}{4}\left\{1+\cos\omega\left[\tau + 2\frac{\left(z(t)-z(t-\tau)\right)}{c}\right]\right\} \tag{5.10}$$

In terms of the velocity at the mid-point in time, this becomes,

$$I(t) = \frac{E_0^{\,2}}{4}\left[1+\cos\omega\tau\left(1+\frac{2V\left(t-\tfrac{1}{2}\tau\right)}{c}\right)\right] \tag{5.11}$$

6.0 The Fringe Constant

From (5.9), the fringe pattern will change by one wavelength when,

$$2\pi = \frac{2\omega\tau}{c}\Delta V$$

where ΔV is the change in velocity. For the case where the velocity is initially zero, $\Delta V = V$, this becomes,

$$V = \frac{\lambda}{2\tau}$$

This value of V is called the Fringe Constant K_f ; it corresponds to a *change* of velocity producing one fringe shift of the interference pattern at the detector.

$$K_f = \frac{\lambda}{2\tau} = \frac{c\lambda}{2\delta} \tag{6.1}$$

A slightly better approximation to take account of the varying velocity is,

$$K_f = \Delta V(t - ½\tau) = \frac{\lambda}{2\tau} \tag{6.2}$$

6.1 The Fringe Count

Generally, for *n* fringe shifts (the fringe count)

$$V = n\frac{\lambda}{2\tau} = nK_f \tag{6.3}$$

If the initial velocity of the reflector at $t = 0$ is $V(0)$ then the velocity is,

$$V = V(0) + nK_f \tag{6.4}$$

where *n* is the number of fringe shifts counted from $t = 0$.

Using (4.3) for $\Delta t(t)$, the number of fringe shifts (fringe count) as a function of time becomes

$$\int_{t=0}^{t} \frac{d}{dt} n(t)dt = \int_0^t \frac{\tau c}{\lambda} \frac{d}{dt}\left(\frac{1+\beta(t)}{1-\beta(t)}\right)dt \tag{6.5}$$

The number of wavelength changes is,

$$\Delta n(t) = \frac{\tau c}{\lambda}\left[\frac{1+\beta(t)}{1-\beta(t)}\right]_{t=0}^{t} = \frac{\tau c}{\lambda}\left[\frac{1+\beta(t)}{1-\beta(t)} - \frac{1+\beta(0)}{1-\beta(0)}\right] \tag{6.6}$$

When the initial velocity is zero, $\beta(t) = 0$, and the fringe count becomes,

$$\Delta n(t) = \frac{\tau c}{\lambda}\left[\frac{1+\beta(t)}{1-\beta(t)} - 1\right] = \frac{\tau c}{\lambda}\left(\frac{2\beta(t)}{1-\beta(t)}\right) = \frac{2\tau V(t)}{\lambda(1-\beta(t))} \tag{6.7}$$

Goosman [8] gives the fringe count as,

$$\Delta n(t) = \frac{\tau c}{\lambda}\left(1 - 2\beta(t - ½\tau)\right) \tag{6.8}$$

This can be obtained from (6.7) by ignoring the terms of higher order than β and using the approximate correction for the time (t-½ τ) in the velocity term.

The VISAR can only measure whilst accelerating. If the reflector is originally at rest and then moves to reach a constant velocity *V* from $t = 0$, the fringes only change during the acceleration of the mirror. After this the delayed and un-delayed beams are both reflected from the mirror travelling at constant velocity.

In practice the VISAR is generally only useful at high accelerations such as in explosions where velocities can exceed the speed of sound producing shock waves. Thermal shock waves in materials travel at the speed of sound; measuring these motions presents real challenges for the VISAR.

6.2 Constant Acceleration

If the acceleration is constant

$$\frac{dV(t)}{dt} = a$$

$V(t) = at + d$ where *d* is a constant.

Putting this into equation (6.7) with initial velocity zero, $d = 0$, gives

$$\Delta n(t) = \frac{2\tau at}{\lambda\left(1 - \frac{at}{c}\right)} \approx \frac{2\tau at}{\lambda} \text{ when } at/c \ll 1$$

This gives a linear increase in Δn with time while $\beta(t)$ is << 1; and of course $\beta(t)$ can not exceed 1.

6.3 Sinusoidal Variation in Velocity

An example of the fringe count for a mirror moving vibrating at a sinusoidal velocity with time is given in this section. This is the type of signal to be expected by observing the radial or longitudinal vibration of a wire subject to thermal shock as a result of passing a short, high current pulse down the wire - see [10]. In practice the accelerations produced in these experiments [10] are rather low for the VISAR (the resulting signal to noise was too low to allow measurements of the motions), although measurements of the frequency were just obtainable by Fourier analysis of the signal. The laser Doppler vibrometer proved to be a much more suitable instrument for these measurements.

If $V(t)$ is sinusoidal,

$$V(t) = V_0 \sin \omega_s t$$

$$\Delta n(t) = \frac{2\tau V_0 \sin(\omega_s t)}{\lambda\left(1 - \frac{V_0 \sin \omega_s t}{c}\right)}$$

This is shown in Figure 3 for two different values of the delay time τ. Also shown is the acceleration,

$$a(t) = \frac{dV(t)}{dt} = V_0 \omega_s \cos(\omega_s t)$$

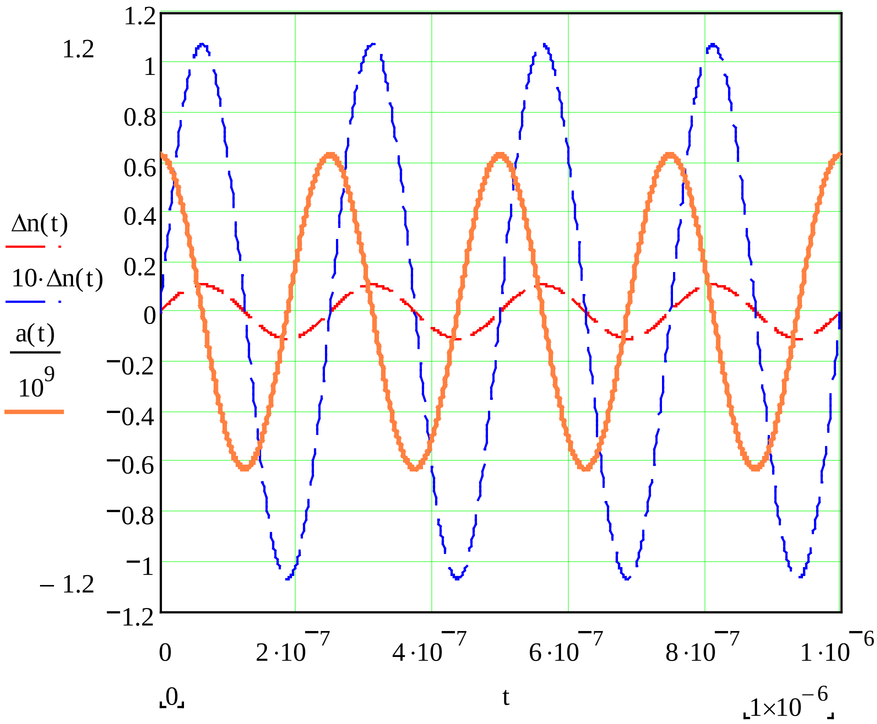

Figure 3. The number of fringes in the interference pattern as a function of time for delay leg lengths δ = 1 m (red dashed curve) and 10 m (blue dashed curve); ω_s = 2.5x10^7 radian s^{-1}, V_0 = 25 m s^{-1}, λ = 1.6x10^{-6} m. The bold orange curve shows the acceleration divided by 10^9 - to get the curve on the figure; the peak value is 6.28x10^8 m s^{-1}.

7.0 The Doppler Effect

So far the Doppler Effect has not entered the analysis, mainly because it is unnecessary, yet is often cited [1-6] as the cause of the fringe shifts. However, it is useful to consider the effect, since there is undoubtedly a Doppler shift in frequency of the light reflected from the moving reflector.

The Doppler Effect arises when a source of light moves towards or away from an observer. Consider a particular phase of the light wave of frequency f and wavelength λ, emitted at time t from the source moving towards an observer at velocity V. At time $t + T$, where T is the periodic time of the oscillation, the source will have emitted one wave. But the source has moved towards the observer a distance of VT, so the wavelength appears to the observer as,

$$\lambda^{|} = \lambda - VT = \lambda\left(1 - \frac{V}{c}\right)$$

Special relativity has been ignored and this relation is only approximate for small values of V/c. The relativistic result is,

$$\lambda^{|} = \lambda \frac{1-\beta}{\sqrt{1-\beta^2}} = \lambda\sqrt{\frac{1-\beta}{1+\beta}}$$

or, in terms of the angular frequency,

$$\omega^{|} = \omega\sqrt{\frac{1+\beta}{1-\beta}}$$

and is true for the relative motion of observer and source towards each other.

The result for the moving reflector is different [9]. Consider a reflector M, moving towards the source S and observer O, at velocity V as in Figure 4. Assume S and O are coincident and distance X from M at time $t = 0$; also the waves from the source reach M at $t = 0$. The reflector travels from M to S in time t, moving a distance $X = Vt$. The observer counts the number of wavelengths received at O from $t = 0$ until the reflector arrives at O.

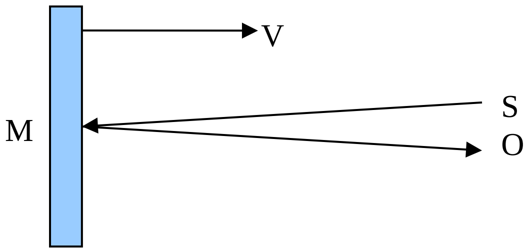

Figure 4. Doppler Effect with a moving mirror.

The total number of waves is the number that were filling the path SM at $t = 0$ plus the number emitted by the source in the time that the reflector took to travel from M to S (and O),

$$N = \frac{Vt}{\lambda} + \frac{ct}{\lambda}$$

where f is the frequency and λ the wavelength of the waves emitted from the source. These waves are received at O over the time $t + X/c$, so the Doppler shifted wave angular frequency observed at O is,

$$\omega^{|} = \frac{N}{t + \frac{X}{c}} = \omega\frac{1+\beta}{1-\beta} \qquad \text{or} \qquad \lambda^{|} = \lambda\frac{1-\beta}{1+\beta} \tag{7.1}$$

where $\beta = V/c$. Note that special relativity does not enter into the analysis since all the observations are in one frame of reference – the source and observer are in the same frame of reference and there is no discussion of the frequency observed by the moving reflector. Goosman [8] notes that the relation is correct for general relativity except at very high reflector velocities.

7.1 The Doppler Effect and the VISAR

An analysis of the VISAR was given in preceding sections without reference to the Doppler Effect. Yet the frequency of the light coming from the moving reflector is Doppler shifted in frequency. The previous analysis has deliberately avoided the use of frequency but used the times for the light to take to pass down the two legs of the VISAR to calculate the time differences that the light from the source took to interfere at the detector. Since the analysis followed a particular phase of the vibration at the source to the detector it is correct to use the frequency of the source and not the Doppler shifted frequency; the difference in the times for the two interfering beams takes into account the effective change in frequency. The difference in time for the interfering beams is given by (from section 4.2.1),

$$\Delta t(t) = \left(\frac{1+\beta(t)}{1-\beta(t)} \right) \tau$$

From section 6.0, this becomes,

$$\Delta t(t) = \frac{\omega^{|}(t)}{\omega} \tau \qquad (7.2)$$

where $\omega^{|}(t)$ is the Doppler shifted frequency as a function of time. However, it is necessary to take into account frequency modulation theory, which is not discussed here.

References

[1] L.M. Barker, Fine Structure of Compressive and Release Wave Shapes in Aluminium Measured by the Velocity Interferometer Technique, in Behaviour of Dense Media under High Dynamic Pressures, Gordon and Breach, New York, p.483,1968.

[2] W.F. Hemsing, Rev. Sci. Instrum. **50**, 73 (1979).

[3] L.M. Barker and R.E. Hollenbach, J. Appl. Phys. **41**, 4208 (1970).

[4] R.J. Clifton, J Appl. Phys. 41, 5335 (1970).

[5] L.M. Barker and R.E. Hollenbach, J. Appl. Phys. **43**, 4669 (1972).

[6] L.M. Barker and K.W. Schuler, J. Appl. Phys. 45, 3692 (1974)

[7] J.R. Asay and L.M. Barker, J. Appl. Phys. 45, 2540 (1974)

[8] D.R. Goosman, J. Appl. Phys. **46**, 3516 (1975).

[9] Daniel H. Dolan, Sandia Report, SAND2006-1950, April 2006.

[10] JRJ Bennett, A thermal impulse method to measure the strength of conducting materials at high temperatures and strain rates. RAL Technical Reports RAL-TR-2016-012.