Technical Report
RAL-TR-95-071

CLRC

A Rederivation of the Spin-dependent Next-to-leading Order Splitting Functions

W Vogelsang

December 1995

COUNCIL FOR THE CENTRAL LABORATORY OF THE RESEARCH COUNCILS

Enquiries about copyright, reproduction and requests for
additional copies of this report should be addressed to:

The Central Laboratory for the Research Councils
Library and Information Services
Rutherford Appleton Laboratory
Chilton
Didcot
Oxfordshire
OX11 0QX
Tel: 01235 445384   Fax: 01235 446403
E-mail library@rl.ac.uk

ISSN 1358-6254

RAL-TR-95-071
November 1995

A Rederivation of the Spin-dependent Next-to-leading Order Splitting Functions

Werner Vogelsang

Rutherford Appleton Laboratory,
Chilton DIDCOT, Oxon OX11 0QX, England

Abstract

We perform a new calculation of the polarized next-to-leading order splitting functions, using the method developed by Curci, Furmanski and Petronzio. We confirm the results of the recent calculation by Mertig and van Neerven.

1 Introduction

The past few years have seen much progress in our knowledge about the nucleons' spin structure due to the experimental study of the spin asymmetries $A_1^N(x,Q^2) \approx g_1^N(x,Q^2)/F_1^N(x,Q^2)$ $(N = p,n,d)$ in deep-inelastic scattering (DIS) with longitudinally polarized lepton beams and nucleon targets. Previous data on A_1^p by the SLAC-Yale collaboration [1] have been succeeded by more accurate data from [2-4], which also cover a wider range in (x,Q^2), and results on A_1^n and A_1^d have been published in [5] and [6, 7], respectively.

On the theoretical side, it has become possible to perform a complete and consistent study of longitudinally polarized DIS in next-to-leading order (NLO) of QCD, since recently results for the spin-dependent two-loop anomalous dimensions, needed for the NLO evolution of polarized parton distributions, have been calculated for the first time [8] within the Operator Product Expansion (OPE). A first phenomenological NLO study has been presented in [9], later followed by the analyses [10].

The calculation of the NLO anomalous dimensions or splitting functions is in general very complicated. This is true in particular for the polarized case, where the Dirac matrix γ_5 and the antisymmetric tensor $\epsilon_{\mu\nu\rho\sigma}$ enter the calculation as projectors onto definite helicity states of the involved particles. These (genuinely *four*-dimensional) quantities lead to certain complications when dimensional regularization - which probably represents the only viable method of regularization in such a calculation - is used. In fact, [8] was recently revised since an error related to the treatment of γ_5 was found. Although the results of [8] now fulfil a relation motivated from Supersymmetry - which appears to be an important constraint - it seems necessary to perform an independent calculation of the polarized two-loop splitting functions to check the results of [8]. This is the purpose of this paper.

In the unpolarized case, two different methods have been used to obtain the next-to-leading order splitting functions. The first calculation [11] was performed within the OPE. Afterwards Curci, Furmanski and Petronzio developed a technique [12, 13] which is as close as possible to parton model intuition since it is based explicitly on the factorization properties of mass singularities in the light-like axial gauge and on the generalized ladder expansion [14]. Note that the results of [13] fulfilled the above mentioned supersymmetric relation [15], but were in disagreement with the first calculation [11] and led to the detection of an error in [11]. In this paper we exploit the method and results of [12, 13] to rederive the spin-dependent next-to-leading order splitting functions. To deal with γ_5 and the ϵ tensor we use the HVBM scheme [16], which still seems to be the most consistent prescription [17]. Section 2 sets the framework for the calculation, details of which are then given in section 3. In Section 4 we present our results.

2 Framework

In this section we outline the framework for our calculation. We mainly focus on the new features in the polarized case; more details on the method itself can be found in the original work [12].

The general strategy consists of a rearrangement of the perturbative expansion which makes explicit the factorization into a part which does not contain any mass singularity and another one which contains all (and only) mass singularities. Fig.1 represents the matrix element squared for polarized virtual (space-like) photon-quark scattering. The blob ΔM is expanded into 2PI kernels ΔC_0 and ΔK_0. In the axial gauge these 2PI kernels have been proven [18] to be finite as long as the external legs are kept unintegrated, such that all collinear singularities originate from the integrations over the momenta flowing in the lines connecting the various kernels. The generalized ladder in Fig.1 can be written as [12, 19]

$$\Delta M = \Delta C_0 \left(1 + \Delta K_0 + \Delta K_0^2 + ...\right) = \Delta C_0 \frac{1}{1-\Delta K_0} \equiv \Delta C_0 \Delta \Gamma_0 \quad . \tag{1}$$

ΔC_0 and $\Delta \Gamma_0$ are then decoupled by projectors which for the longitudinally polarized case read (ΔA, ΔB being two polarized kernels):

$$(\Delta A) P_F (\Delta B) = \left(\Delta A^{ij} (\not k \gamma_5)_{ij}\right) [\text{P.P.}] \left(\frac{(\gamma_5 \not n)^{kl}}{4kn} \Delta B_{kl}\right) \tag{2}$$

for polarized quarks and

$$(\Delta A)P_G(\Delta B) = \left(\Delta A_{\mu\nu}\epsilon^{\mu\nu\rho\sigma}\frac{k_\rho n_\sigma}{kn}\right)[\text{P.P.}]\left(\epsilon^{\alpha\beta\gamma\delta}\frac{k_\gamma n_\delta}{kn}\Delta B_{\alpha\beta}\right) \quad , \tag{3}$$

for polarized gluons, putting $k^2 = 0$ in the part containing the kernel ΔA and taking the pole-part (P.P.) of the projection on kernel ΔB. In eqs. (2),(3) i, j, k, l are spinor and the greek letters are Lorentz indices. n is the vector to be introduced in the axial gauge with $n^2 = 0$ for the light-like gauge. Performing the factorization of mass singularities [12], which in dimensional regularization ($d = 4 - 2\epsilon$) appear as poles in $1/\epsilon$, one ends up with the contribution to the (partonic) deep-inelastic structure function g_1:

$$g_1(\frac{Q^2}{\mu^2}, x, \alpha_s, \frac{1}{\epsilon}) = \Delta C(\frac{Q^2}{\mu^2}, x, \alpha_s) \otimes \Delta\Gamma(x, \alpha_s, \frac{1}{\epsilon}) \quad , \tag{4}$$

where the convolution $\otimes$ is defined as usual by

$$(f \otimes g)(x) \equiv \int_x^1 \frac{dz}{z} f(z) g(\frac{x}{z}) \quad . \tag{5}$$

In eq. (4) we have introduced the virtuality of the photon Q^2, the unit of mass μ in dimensional regularization, the Bjørken variable $x = Q^2/2pq$ and the strong coupling α_s. Eq. (4) has a clear partonic interpretation: $\Delta\Gamma(x, \alpha_s, \frac{1}{\epsilon})$ describes the density of partons in the parent quark, is independent of the hard process considered and contains all the collinear singularities (poles in ϵ), whereas $\Delta C(Q^2/\mu^2, x, \alpha_s)$ is the (process-dependent) short-distance cross section. As was shown in [12], $\Delta\Gamma$ does not depend on Q^2, which is a consequence of the finiteness of the kernel ΔK_0 in the axial gauge [18], and allows for the derivation of a 'renormalization group' equation for ΔC with $\Delta\Gamma$ related to the 'anomalous dimension'. Thus $\Delta\Gamma$, to be convoluted with bare ('unrenormalized') parton densities which must cancel its $1/\epsilon$ poles, is equivalent to the respective Altarelli-Parisi [20] kernels, e.g.,

$$\Delta\Gamma(x, \alpha_s, \frac{1}{\epsilon}) = 1 - \frac{1}{\epsilon}\left[\left(\frac{\alpha_s}{2\pi}\right)\Delta P^0_{qq}(x) + \frac{1}{2}\left(\frac{\alpha_s}{2\pi}\right)^2 \Delta P^1_{qq}(x) + ...\right] + \mathcal{O}(\frac{1}{\epsilon^2}) \tag{6}$$

for the non-singlet (NS) case. The final NLO expression for the (physical) spin-dependent structure function g_1^N then reads:

$$\begin{aligned} g_1(x, Q^2) &= \frac{1}{2}\sum_q^{N_f} e_q^2 \left\{\Delta q(x, Q^2) + \Delta\bar{q}(x, Q^2) + \right. \\ &+ \left. \frac{\alpha_s(Q^2)}{2\pi}\left[\Delta C_q \otimes (\Delta q + \Delta\bar{q}) + \frac{1}{N_f}\Delta C_g \otimes \Delta g\right](x, Q^2)\right\} \quad , \end{aligned} \tag{7}$$

where N_f is the number of active flavors, and where in the full singlet case two short-distance cross sections ΔC_q and ΔC_g for scattering off incoming polarized quarks or gluons, respectively, exist. Here, the polarized parton distributions $\Delta p \equiv p^{\uparrow} - p^{\downarrow}$ $(p = q, g)$ are to be evolved according to the polarized Altarelli-Parisi [20] evolution equations which to NLO read (see, e.g., [21])

$$\frac{d}{d\ln Q^2}(\Delta q + \Delta\bar{q} - \Delta q' - \Delta\bar{q}') = \frac{\alpha_s}{2\pi}(\Delta P_{qq} + \Delta P_{q\bar{q}}) \otimes (\Delta q + \Delta\bar{q} - \Delta q' - \Delta\bar{q}') \quad , (8)$$

$$\frac{d}{d\ln Q^2}(\Delta q - \Delta\bar{q}) = \frac{\alpha_s}{2\pi}(\Delta P_{qq} - \Delta P_{q\bar{q}}) \otimes (\Delta q - \Delta\bar{q}) \quad (9)$$

for the NS quark densities and

$$\frac{d}{d\ln Q^2}\begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} = \frac{\alpha_s}{2\pi}\begin{pmatrix} \Delta P_{qq} + \Delta P_{q\bar{q}} + \Delta P_{qq,PS} & \Delta P_{qg} \\ \Delta P_{gq} & \Delta P_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} \quad (10)$$

in the singlet sector, where $\Delta\Sigma \equiv \sum_q(\Delta q + \Delta\bar{q})$ and the argument (x, Q^2) has been omitted from all parton densities. To NLO, all splitting functions in (8-10) have the perturbative expansion

$$\Delta P_{ij} = \Delta P_{ij}^0 + \frac{\alpha_s}{2\pi}\Delta P_{ij}^1 \quad . \quad (11)$$

The entries $\Delta P_{q\bar{q}}$ and $\Delta P_{qq,PS}$ start to be non-zero only beyond the leading order. For future reference it is convenient to introduce the NLO combinations

$$\Delta P_{qq,NS}^{1,\pm} = \Delta P_{qq}^1 \pm \Delta P_{q\bar{q}}^1 \quad (12)$$

which according to (8,9) govern the NLO part of the evolution in the NS sector. $\Delta P_{qq,PS}$ is called the 'pure singlet' splitting function since it only appears in the singlet case.

3 The Calculation

Some representatives of the graphs to be evaluated in the calculation of the ΔP_{ij}^1 are shown in Fig.2. We do not need to calculate the contributions from genuine two-loop graphs to the diagonal splitting functions ΔP_{qq}^1 and ΔP_{gg}^1, which are $\sim \delta(1-x)$, since these are the same as for the unpolarized case.

Before giving details of the calculation, we note that the use of the light-like $(n^2 = 0)$ axial gauge in practical calculations has been a matter of debate for a long time now [22].

The great computational advantage and success it has brought for, e.g., perturbative NLO QCD calculations in DIS [12, 13, 23] or jet calculus [24] has not always been matched by the theoretical understanding of why it worked so well [12, 25]. The problems connected with the light-like axial gauge are due in the first place to the presence of spurious singularities in loop integrals coming from the vector propagator in this gauge, which are neither of ultraviolet nor of infrared origin. In [12, 13] 'phenomenological rules' were applied to the problem which consisted of the subtraction of spurious poles by hand and of using the Cauchy principle value (CPV) prescription to deal with $1/l \cdot n$ terms (where l is some momentum), entailing renormalization 'constants' depending on the infinite-momentum-frame (IMF) variable x. In view of the complications due to the spurious poles, but also in view of the fact that the calculations and the prescriptions in [12, 13] led to the *correct* answer, we find it very satisfactory that it turns out to be possible in our case to infer the effective contributions of the non-trivial [26] virtual, in particular the vertex correction, graphs to the polarized splitting functions from the known results [12, 13] for the *un*polarized P_{ij}^1, such that these contributions need not be calculated all over again. The strategy to do this is to calculate all real emission graphs to a given process for the unpolarized case and to subtract their sum from the corresponding final results listed in [12, 13]. This difference and knowledge of the pole parts (renormalization constants) of the virtual corrections [12] then straightforwardly allow for a determination of the result for each virtual correction in terms of a parametrization of the most general Dirac/Lorentz structure it can have [27], and make a transfer to the polarized case easily possible. In this way we avoid to have to deal with the spurious poles again but can simply rely on the success [28] of the approach used in the unpolarized calculation [12, 13].

The calculation of the real emission graphs is rather involved. This is true in particular for the polarized case when using the HVBM scheme since in this method the $d = 4 - 2\epsilon$ dimensional space-time is explicitly decomposed into the usual four dimensions in which γ_5 anticommutes with the other Dirac matrices and the -2ϵ dimensional part, where it commutes. Thus the squared matrix elements of the graphs will depend on the usual 'd-dimensional' scalar products like $l_1 \cdot l_2$ etc. (see Fig.2a for notation), but also on '$(d-4)$-dimensional' ones, denoted by $\hat{l}_1 \cdot \hat{l}_2$, $\hat{k}^2$ etc. [29]. It is most convenient to work in the

IMF parametrization of the momenta [12] which in our case takes the form:

$$
\begin{aligned}
p &= (P \ , \ \vec{0}_{xy} \ , \ P \ , \ \vec{0}_{d-4} \ ) \ , \\
n &= (\frac{pn}{2P} \ , \ \vec{0}_{xy} \ , \ -\frac{pn}{2P} \ , \ \vec{0}_{d-4} \ ) \ , \\
k &= \left(xP + \frac{k^2+\tilde{k}^2}{4xP} \ , \ \vec{k}_T \ , \ xP - \frac{k^2+\tilde{k}^2}{4xP}, \ \hat{\vec{k}} \right) \ , \\
l_1 &= (\ l_1^0 \ , \ \vec{l}_1^{xy} \ , \ l_1^z \ , \ \hat{\vec{l}}_1 \ ) \ ,
\end{aligned} \tag{13}
$$

where $x = kn/pn$ is interpreted as the IMF momentum fraction of the incoming momentum p carried by k, and $\tilde{k}^2 = k_x^2 + k_y^2 + \hat{k}^2 \equiv k_T^2 + \hat{k}^2$ is the total transverse momentum squared of k relative to the axis defined by p, n. We split the $(d-4)$-dimensional components of l_1 into a part $\hat{l}_1^{||}$ parallel to those of k and a transverse part $\hat{l}_1^{\perp}$. According to our definitions, only k, l_1, and $l_2 = p - k - l_1$ possess such components. When performing the phase space integrations one has to carefully take into account the $(d-4)$-dimensional terms. The contribution of each graph to $\Delta\Gamma(x, \alpha_s, \frac{1}{\epsilon})$ is given by the integration of the projected (see eqs.(2,3)) squared matrix elements over the phase space

$$
R \equiv \int \frac{d^dk}{(2\pi)^d} x\delta\left(x - \frac{kn}{pn}\right) \int \frac{d^dl_1}{(2\pi)^{d-1}} \int \frac{d^dl_2}{(2\pi)^{d-1}} (2\pi)^d \delta^{(d)}(p-k-l_1-l_2)\delta(l_1^2)\delta(l_2^2) \tag{14}
$$

which is conveniently written as

$$
\begin{aligned}
&x^{\epsilon}(1-x)^{1-2\epsilon} \int_{-Q^2}^{0} dk^2 (-k^2)^{1-2\epsilon} \int_0^1 d\tilde{\kappa} \, (\tilde{\kappa}(1-\tilde{\kappa}))^{-\epsilon} \int_0^1 dw \, (w(1-w))^{-\epsilon} \int_0^1 dv \, (v(1-v))^{-\frac{1}{2}-\epsilon} \\
&\times \left((-\epsilon) \int_0^1 d\hat{\kappa} \hat{\kappa}^{-1-\epsilon} \right) \left((-\frac{1}{2} - \epsilon) \int_0^1 d\lambda^{\perp} \left(\lambda^{\perp}\right)^{-3/2-\epsilon} \right) \left(\frac{1}{\pi} \int_0^1 d\lambda^{||} \left(\lambda^{||}(1-\lambda^{||})\right)^{-1/2} \right)
\end{aligned} \tag{15}
$$

where we have omitted trivial prefactors and defined

$$
\begin{aligned}
\hat{k}^2 &= -k^2(1-x)\hat{\kappa}\tilde{\kappa} \ , \\
\tilde{k}^2 &= -k^2(1-x)\tilde{\kappa} \ , \\
l_1^0 + l_1^z &= 2P(1-x)w = 2P\frac{l_1 n}{pn} \ , \\
(l_1^0)^2 - (l_1^z)^2 &= c_1^2 + v(c_2^2 - c_1^2) = \frac{1}{P}(l_1^0 + l_1^z)(l_1 p) \ , \\
\hat{l}_1^{||} &= \lambda_1 + \lambda^{||}(\lambda_2 - \lambda_1) \ , \\
(\hat{l}_1^{\perp})^2 &= v(1-v)\,(c_1 + c_2)^2\,\lambda^{\perp}
\end{aligned} \tag{16}
$$

with

$$c_{1,2} \equiv \sqrt{\frac{-k^2(1-x)w}{x}}\left[\sqrt{(1-w)(1-\tilde{\kappa})} \mp \sqrt{xw\tilde{\kappa}}\right] ,$$

$$\lambda_{1,2} = -\frac{1}{2}\frac{\hat{\kappa}}{\hat{k}w}\left((l_1^0)^2 - (l_1^z)^2 - c_1c_2\right) \mp (c_1+c_2)\sqrt{(1-\hat{\kappa})(1-\lambda^\perp)v(1-v)} .$$

Note that the last two integrals in (15) are all unity if no dependence on $(d-4)$-dimensional scalar products occurs, which of course is always the case in the unpolarized situation. If present, such $(d-4)$-dimensional terms only give contributions proportional to ϵ after the last three integrals in (15) have been performed.

Following [12, 13] we will regularize infrared divergencies appearing at $l_1n \to 0$ or $l_2n \to 0$ ($w \to 0$ or $w \to 1$), which are typical of the axial gauge, by the CPV prescription (see above):

$$\frac{1}{ln} \to \frac{ln}{(ln)^2 + \delta^2(pn)^2} . \tag{17}$$

All the resulting divergencies of this type can then be transformed into the basic integrals

$$I_i \equiv \int_0^1 dy \frac{y \ln^i y}{y^2+\delta^2} \qquad (i = 0, 1) . \tag{18}$$

I_0, I_1 have to cancel out in the final answer which, as well as the finiteness of each graph in the axial gauge [12, 18] before the final k^2 integration is performed, helps when extracting the contribution of the virtual corrections by the strategy outlined above. As mentioned above, the renormalization constants depend on the IMF fractions x or $1-x$ when the CPV prescription is used [12]. We finally note that whenever considering a genuine ladder graph with two parallel rungs, subtraction of the 'doubly collinear' graph (see Fig.2) is required within the method of [12]. The result for this is given by convoluting the n-dimensional leading order splitting function standing for the upper part with the four-dimensional one representing the lower part of the diagram, and including a factor $(1-x)^{-\epsilon}$ from phase space in the convolution. In $4-2\epsilon$ dimensions the polarized LO splitting functions read for $x \neq 1$ in the HVBM scheme [30]:

$$\Delta P_{qq}^0(x,\epsilon) = C_F\left(\frac{1+x^2}{1-x} + 3\epsilon(1-x)\right) ,$$

$$\Delta P_{qg}^0(x,\epsilon) = 2T_R N_f\left(2x-1-2\epsilon(1-x)\right) ,$$

$$\Delta P_{gq}^0(x,\epsilon) = C_F\left(2-x+2\epsilon(1-x)\right) ,$$

$$\Delta P^0_{gg}(x,\epsilon) \;=\; 2C_A\left(\frac{1}{1-x} - 2x + 1 + 2\epsilon(1-x)\right) \;, \tag{19}$$

where $C_F = 4/3$, $C_A = 3$, $T_R = 1/2$ and N_f is the number of active flavors.

4 Results

In the normalization of [12, 13] our $\overline{\text{MS}}$ results read:

$$\Delta P^{1,\pm}_{qq,NS}(x) \;=\; P^{1,\mp}_{qq,NS}(x) - 2\beta_0 C_F(1-x) \;, \tag{20}$$

$$\begin{aligned}
\Delta P^{1}_{qq,PS}(x) &= \Delta P^{([8])}_{qq,PS}(x) \;, \\
\Delta P^{1}_{qg}(x) &= \Delta P^{([8])}_{qg}(x) + 4C_F(1-x)\otimes \Delta P^0_{qg}(x) \;, \\
\Delta P^{1}_{gq}(x) &= \Delta P^{([8])}_{gq}(x) - 4C_F(1-x)\otimes \Delta P^0_{gq}(x) \;, \\
\Delta P^{1}_{gg}(x) &= \Delta P^{([8])}_{gg}(x) \;,
\end{aligned} \tag{21}$$

where $\beta_0 = 11C_A/3 - 4T_R N_f/3$ and $\Delta P^0_{ij}(x) \equiv \Delta P^0_{ij}(x,0)$ (see eq. (19)). The $\Delta P^{([8])}_{ij}$ [31] are the results of [8], and $P^{1,\pm}_{qq,NS}$ can be found in [12]. As was already discussed in [9, 32] and indicated in eq. (20), the '+' and '−' combinations of the NS splitting functions as defined in (12) interchange their role in the polarized case, such that, according to eqs.(8,12,20), the combination $\Delta P^{1,+}_{qq,NS} = P^1_{qq} - P^1_{q\bar{q}} - 2\beta_0 C_F(1-x)$ would govern the Q^2-evolution of, e.g., the polarized NS quark combination

$$\Delta A_3(x,Q^2) = \left(\Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d}\right)(x,Q^2) \;.$$

Since the first moment (integral) of the latter corresponds to the nucleon matrix element of the NS axial vector current $\bar{q}\gamma^\mu\gamma_5\lambda_3 q$ which is conserved, it has to be Q^2-independent [33]. Keeping in mind that the first moment of the unpolarized $P^1_{qq} - P^1_{q\bar{q}}$ vanishes already due to fermion number conservation [12], it becomes obvious that the additional term $-2\beta_0 C_F(1-x)$ in (20) spoils the Q^2 independence of the first moment of $\Delta A_3(x,Q^2)$. It is therefore necessary to perform a factorization scheme transformation to the results in (20,21) in order to remove this additional term which, as pointed out in [34, 30, 32], is typical of the HVBM scheme with its not fully anticommuting γ_5 and trivially would not be present in a scheme with a fully anticommuting γ_5 since then the two γ_5 matrices appearing in the relevant graphs could be removed by anticommuting them towards each other and using $\gamma_5^2 = 1$ (cf. Fig2a). We note that all these observations were already made in the original calculation of [8] in the OPE where, however, the removal of the additional term $\sim (1-x)$ corresponds to a finite renormalization rather than a factorization scheme transformation. It is nice to recover this analogy between our results and those of [8, 35]. The factorization scheme transformation for $\Delta P^{1,\pm}_{qq,NS}$ also affects the singlet sector since, according to eq. (10), $\Delta P^{1,+}_{qq,NS} + \Delta P^{1}_{qq,PS}$ occurs in the evolution of the NLO quark singlet $\Delta\Sigma$. The transformation reads in general (see, e.g., [8, 36]):

$$
\begin{aligned}
\Delta P^{1,\pm}_{qq,NS} &= \Delta \tilde{P}^{1,\pm}_{qq,NS} - 2\beta_0 z_{qq} \quad , \\
\Delta P^{1}_{qq,PS} &= \Delta \tilde{P}^{1}_{qq,PS} \quad , \\
\Delta P^{1}_{qg} &= \Delta \tilde{P}^{1}_{qg} + 4 z_{qq} \otimes \Delta P^{0}_{qg} \quad , \\
\Delta P^{1}_{gq} &= \Delta \tilde{P}^{1}_{gq} - 4 z_{qq} \otimes \Delta P^{0}_{gq} \quad , \\
\Delta P^{1}_{gg} &= \Delta \tilde{P}^{1}_{gg} \quad ,
\end{aligned}
\tag{22}
$$

where the $\Delta \tilde{P}^1_{ij}$ now are the NLO splitting functions on the left-hand-sides of eqs. (20,21) and the ΔP^1_{ij} are the *new* splitting functions after the scheme transformation. One immediately sees that the choice

$$
z_{qq} = -C_F(1-x) \tag{23}
$$

leads to $\Delta P^{1,\pm}_{qq,NS} = P^{1,\mp}_{qq,NS}$ and thus now yields the required vanishing of the first moment of $\Delta P^{1,+}_{qq,NS}$. Even more, the transformation (22,23) removes *all* additional terms on the right-hand-side of (20,21) simultaneously, bringing our final result into complete agreement with the revised one of [8]. We finally note that the above factorization scheme transformation also changes the quark short distance cross section (coefficient function) ΔC_q in (7), since the combination

$$
\Delta C_q - \frac{2\Delta P^{1,\pm}_{qq,NS}}{\beta_0}
$$

must be independent of the choice of the factorization scheme convention [36]. As was shown in [34, 37, 32], only *after* the transformation (22,23) takes ΔC_q in the $\overline{\mathrm{MS}}$ scheme the form

$$
\Delta C_q(x) \;=\; C_F \left[(1+x^2) \left(\frac{\ln(1-x)}{1-x} \right)_+ - \frac{3}{2} \frac{1}{(1-x)_+} - \frac{1+x^2}{1-x} \ln x + \right.
$$

$$+2+x-\left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\Bigg] \quad , \tag{24}$$

i.e., becomes the previously calculated [38] $\mathcal{O}(\alpha_s)$ quark-correction to g_1 giving rise to, e.g., the correct first order correction $1-\alpha_s/\pi$ to the Bjørken sum rule. In eq. (24),

$$\int_0^1 dz f(z)\,(g(z))_+ \equiv \int_0^1 dz\,(f(z)-f(1))\,g(z) \quad .$$

For completeness we note that the NLO $\overline{\rm MS}$ gluonic short distance cross section ΔC_g in eq. (7) remains unaffected by the transformation (22,23) and reads (see, e.g., [8])

$$\Delta C_g(x) = 2T_R N_f\left[(2x-1)\left(\ln\frac{1-x}{x}-1\right)+2(1-x)\right] \quad . \tag{25}$$

Our *final* results for the polarized $\overline{\rm MS}$ NLO splitting functions are given by:

$$\Delta P^{1,\pm}_{qq,NS} = P^{1,\mp}_{qq,NS} \quad , \tag{26}$$

$$\Delta P^{1}_{qq,PS}(x) = C_F T_R N_f\Big[2\,(1-x)-2\,(1-3x)\ln x-2\,(1+x)\ln^2 x\Big] \quad , \tag{27}$$

$$\begin{aligned}\Delta P^{1}_{qg}(x) = {} & C_F T_R N_f\Big[-22+27x-9\ln x+8\,(1-x)\ln(1-x)\\ & +\frac{1}{2}\delta p_{qg}(x)\left(4\ln^2(1-x)-8\ln(1-x)\ln x+2\ln^2 x-8\zeta(2)\right)\Big]\\ & +C_A T_R N_f\Big[2\,(12-11x)-8\,(1-x)\ln(1-x)+2\,(1+8x)\ln x\\ & -2\left(\ln^2(1-x)-\zeta(2)\right)\delta p_{qg}(x)-\left(2I_x-3\ln^2 x\right)\delta p_{qg}(-x)\Big] \quad ,\end{aligned} \tag{28}$$

$$\begin{aligned}\Delta P^{1}_{gq}(x) = {} & C_F T_R N_f\left[-\frac{4}{9}(x+4)-\frac{4}{3}\delta p_{gq}(x)\ln(1-x)\right]\\ & +C_F^2\Big[-\frac{1}{2}-\frac{1}{2}(4-x)\ln x-\delta p_{gq}(-x)\ln(1-x)\\ & +\left(-4-\ln^2(1-x)+\frac{1}{2}\ln^2 x\right)\delta p_{gq}(x)\Big]\\ & +C_A C_F\Big[(4-13x)\ln x+\frac{1}{3}(10+x)\ln(1-x)+\frac{1}{9}(41+35x)\\ & +\frac{1}{2}\left(-2I_x+3\ln^2 x\right)\delta p_{gq}(-x)+\left(\ln^2(1-x)-2\ln(1-x)\ln x-\zeta(2)\right)\delta p_{gq}(x)\Big]\end{aligned} \tag{29}$$

$$\begin{aligned}\Delta P^{1}_{gg}(x) = {} & -C_A T_R N_f\left[4\,(1-x)+\frac{4}{3}(1+x)\ln x+\frac{20}{9}\delta p_{gg}(x)+\frac{4}{3}\delta(1-x)\right]\\ & -C_F T_R N_f\Big[10\,(1-x)+2\,(5-x)\ln x+2\,(1+x)\ln^2 x+\delta(1-x)\Big]\end{aligned}$$

$$+C_A^2\left[\frac{1}{3}(29-67x)\ln x-\frac{19}{2}(1-x)+4(1+x)\ln^2 x-2I_x\delta p_{gg}(-x)\right.$$
$$\left.+\left(\frac{67}{9}-4\ln(1-x)\ln x+\ln^2 x-2\zeta(2)\right)\delta p_{gg}(x)+\left(3\zeta(3)+\frac{8}{3}\right)\delta(1-x)\right] \quad (30)$$

where, as mentioned above, the unpolarized NS pieces $P_{qq,NS}^{1,\pm}$ can be found in [12] and [39]

$$\begin{aligned}\delta p_{qg}(x) &\equiv 2x-1 \;,\\ \delta p_{gq}(x) &\equiv 2-x \;,\\ \delta p_{gg}(x) &\equiv \frac{1}{(1-x)_+}-2x+1 \;.\end{aligned} \quad (31)$$

Furthermore we have in eqs. (26-30) $\zeta(2)=\pi^2/6$, $\zeta(3)\approx 1.202057$ and

$$I_x \equiv \int_{x/(1+x)}^{1/(1+x)}\frac{dz}{z}\ln\left(\frac{1-z}{z}\right) \;.$$

For relating our results to those of [8] the relation

$$I_x=-2Li_2(-x)-2\ln x\ln(1+x)+\frac{1}{2}\ln^2 x-\zeta(2)$$

is needed, where $Li_2(x)$ is the Dilogarithm [40]. In (30), the contributions $\sim\delta(1-x)$ to ΔP_{gg}^1 are the same as those for the unpolarized P_{gg}^1 [41]; they lead to satisfaction of the constraint [42]

$$\int_0^1 dx\Delta P_{gg}^1(x)=\frac{\beta_1}{4}\equiv\frac{17}{6}C_A^2-C_FT_RN_f-\frac{5}{3}C_AT_RN_f \;,$$

valid in the $\overline{\mathrm{MS}}$ scheme.

In conclusion, our calculation, which was based on the approach of [12] and on using the HVBM [16] prescription for γ_5, has confirmed the recent results of [8] for the spin-dependent two-loop splitting functions $\Delta P_{ij}^1(x)$. Our results also once more demonstrate the usefulness and applicability of the method of [12] and the light-like axial gauge in perturbative QCD calculations.

References

[1] M.J. Alguard et al., SLAC-Yale Collab. (E80), Phys. Rev. Lett. **37**, 1261 (1976);
G. Baum et al., Phys. Rev. Lett. **45**, 2000 (1980);
G. Baum et al., SLAC-Yale Collab. (E130), Phys. Rev. Lett. **51**, 1135 (1983).

[2] J. Ashman et al., EMC, Phys. Lett. **B206**, 364 (1988); Nucl. Phys. **B328**, 1 (1989).

[3] D. Adams et al., SMC, Phys. Lett. **B329**, 399 (1994); Erratum **B339**, 332 (1994).

[4] K. Abe et al., SLAC-E143 Collab., Phys. Rev. Lett. **74**, 346 (1995) and SLAC-PUB-6508 (1994).

[5] D.L. Anthony et al., SLAC-E142 Collab., Phys. Rev. Lett. **71**, 959 (1993).

[6] B. Adeva et al., SMC, Phys. Lett. **B302**, 533 (1993);
D. Adams et al., SMC, CERN-PPE/95-97 (Phys. Lett. B).

[7] K. Abe et al., SLAC-E143 Collab., Phys. Rev. Lett. **75**, 25 (1995) and SLAC-PUB-95-6734.

[8] R. Mertig and W.L. van Neerven, Univ. Leiden INLO-PUB-6/95 and NIKHEF-H/95-031, June 1995, November 1995 (revised).

[9] M. Glück, E. Reya, M. Stratmann, and W. Vogelsang, Univ. Dortmund DO-TH 95/13, Rutherford RAL-TR-95-042.

[10] T. Weigl and W. Melnitchouk, TU München report TUM/T39-95-9;
R.D. Ball, S. Forte, and G. Ridolfi, CERN-TH/95-266.

[11] E.G. Floratos, D.A. Ross, and C.T. Sachrajda, Nucl. Phys. **B129**, 66 (1977); E: **139**, 545 (1978); Nucl. Phys. **B152**, 493 (1979);
see also: A. Gonzales-Arroyo, C. Lopez, and F.J. Yndurain, Nucl. Phys. **B153**, 161 (1979); A. Gonzales-Arroyo and C. Lopez, Nucl. Phys. **B166**, 429 (1980); E.G. Floratos, C. Kounnas, and R. Lacaze, Phys. Lett. **98B**, 89,285 (1981); Nucl. Phys. **B192**, 417 (1981).

[12] G. Curci, W. Furmanski, and R. Petronzio, Nucl. Phys. **B175**, 27 (1980).

[13] W. Furmanski and R. Petronzio, Phys. Lett. **97B**, 437 (1980).

[14] K. Symanzik, Comm. Math. Phys. **23**, 49 (1971); **34**, 7 (1973);
A.H. Mueller. Phys. Rev. **D9**, 963 (1974).

[15] I. Antoniadis and E.G. Floratos, Nucl. Phys. **B191**, 217 (1981).

[16] G. 't Hooft and M. Veltman, Nucl. Phys. **B44**, 189 (1972);
P. Breitenlohner and D. Maison, Comm. Math. Phys. **52**, 11 (1977).

[17] G. Bonneau, Phys. Lett. **96B**, 147 (1980); Paris-LPTHE report PAR LPTHE 88-52 (unpublished); Int. J. Mod. Phys. **A5**, 3831 (1990).

[18] R.K. Ellis, H. Georgi, M. Machacek, H.D. Politzer, and G.G. Ross, Phys. Lett. **78B**, 281 (1978); Nucl. Phys. **B152**, 285 (1979).

[19] The following equations schematically apply to the flavor non-singlet as well as the flavor singlet sector.

[20] G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).

[21] W. Furmanski and R. Petronzio, Z. Phys. **C11**, 293 (1982).

[22] G. Leibbrandt, Rev. Mod. Phys. **59**, 1067 (1987).

[23] D.J. Pritchard and W.J. Stirling, Nucl. Phys. **B165**, 237 (1980).

[24] J. Kalinowski, K. Konishi, and T.R. Taylor, Nucl. Phys. **B181**, 221 (1981);
J. Kalinowski, K. Konishi, P.N. Scharbach, and T.R. Taylor, Nucl. Phys. **B181**, 253 (1981);
J.F. Gunion and J. Kalinowski, Phys. Rev. **D29**, 1545 (1984);
J.F. Gunion, J. Kalinowski, and L. Szymanowski, Phys. Rev. **D32**, 2303 (1985).

[25] A. Bassetto, Phys. Rev. **D47**, 727 (1993).

[26] By these we mean virtual corrections which do not enter just via their (known) renormalization constants or as pure quark loops (the last graph in Fig.2a is an example for both cases).

[27] Here it is crucial to observe that for each graph the *same* coefficients of such a parametrization contribute to the unpolarized and polarized results, appearing in even the same *combination* (when considering a vertex correction, the coefficient multiplying the tree-level vertex of course plays a special role; it can, however, only give rise to contributions proportional to the respective d-dimensional (un)polarized LO splitting function).

[28] We note that it seems desirable for the future to put the results of [12, 13] (and ours) on a somewhat field-theoretically firmer basis. First steps in this direction have been taken in ref. [25].

[29] We use the program TRACER for calculating Dirac traces in the HVBM scheme and performing contractions: M. Jamin and M.E. Lautenbacher, TU München report TUM-T31-20/91.

[30] L.E. Gordon and W. Vogelsang, Phys. Rev. **D48**, 3136 (1993).

[31] In this normalization the results of [8] have to be divided by a factor 8.

[32] M. Stratmann, W. Vogelsang, and A. Weber, Univ. Dortmund DO-TH 95/15 and Rutherford RAL-TR-95-043, to appear in Phys. Rev. **D**.

[33] J. Kodaira, S. Matsuda, K. Sasaki, and T. Uematsu, Nucl. Phys. **B159**, 99 (1979).

[34] A. Weber, Nucl. Phys. **B382**, 63 (1992).

[35] We note that in an original version of [8] the subtraction of the terms $\sim (1-x)$ was performed incorrectly which was responsible for the error mentioned in the introduction.

[36] M. Glück and E. Reya, Phys. Rev. **D25**, 1211 (1982).

[37] D. de Florian and R. Sassot, Phys. Rev. **D51**, 6052 (1995).

[38] J. Kodaira, S. Matsuda, T. Muta, K. Sasaki, and T. Uematsu, Phys. Rev. **D20**, 627 (1979);

P. Ratcliffe, Nucl. Phys. **B223**, 45 (1983).

[39] Needless to say that for $\delta p_{gg}(-x)$ the '+'-prescription has to be omitted.

[40] A. Devoto and D.W. Duke, Nuov. Cim. (Riv.) **7**, 1 (1984).

[41] R.T. Herrod and S. Wada, Phys. Lett. **96B**, 195 (1980).

[42] G. Altarelli and B. Lampe, Z. Phys. **C47**, 315 (1990).

Figure Captions

Fig.1 The matrix element squared for polarized photon-quark interaction, its expansion in terms of 2PI kernels ΔC_0 and ΔK_0, and its final factorized form.

Fig2. Some representative Feynman graphs to be evaluated in the calculation of **a:** $(\Delta)P^1_{qq}$, **b:** $(\Delta)P^1_{q\bar{q}}$, $(\Delta)P^1_{qq,PS}$, **c:** $(\Delta)P^1_{qg}$, **d:** $(\Delta)P^1_{gq}$, **e,f:** $(\Delta)P^1_{gg}$. Subtraction of 'doubly collinear' graphs is indicated.

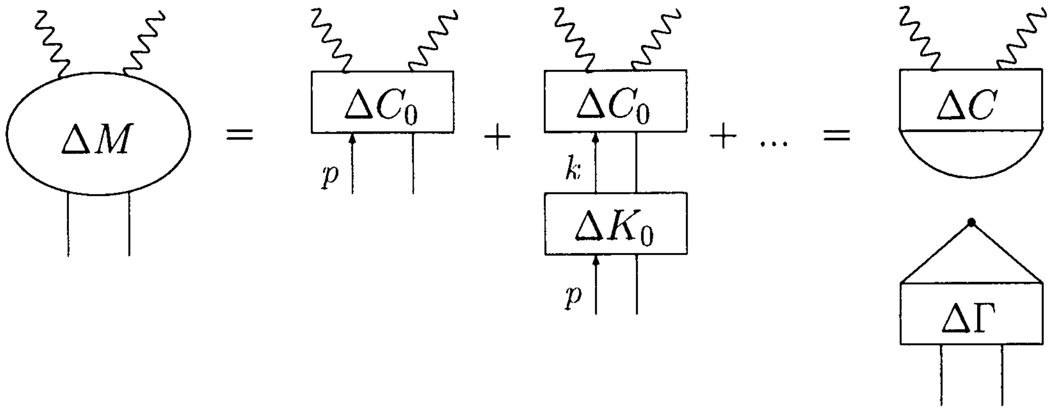

Fig.1

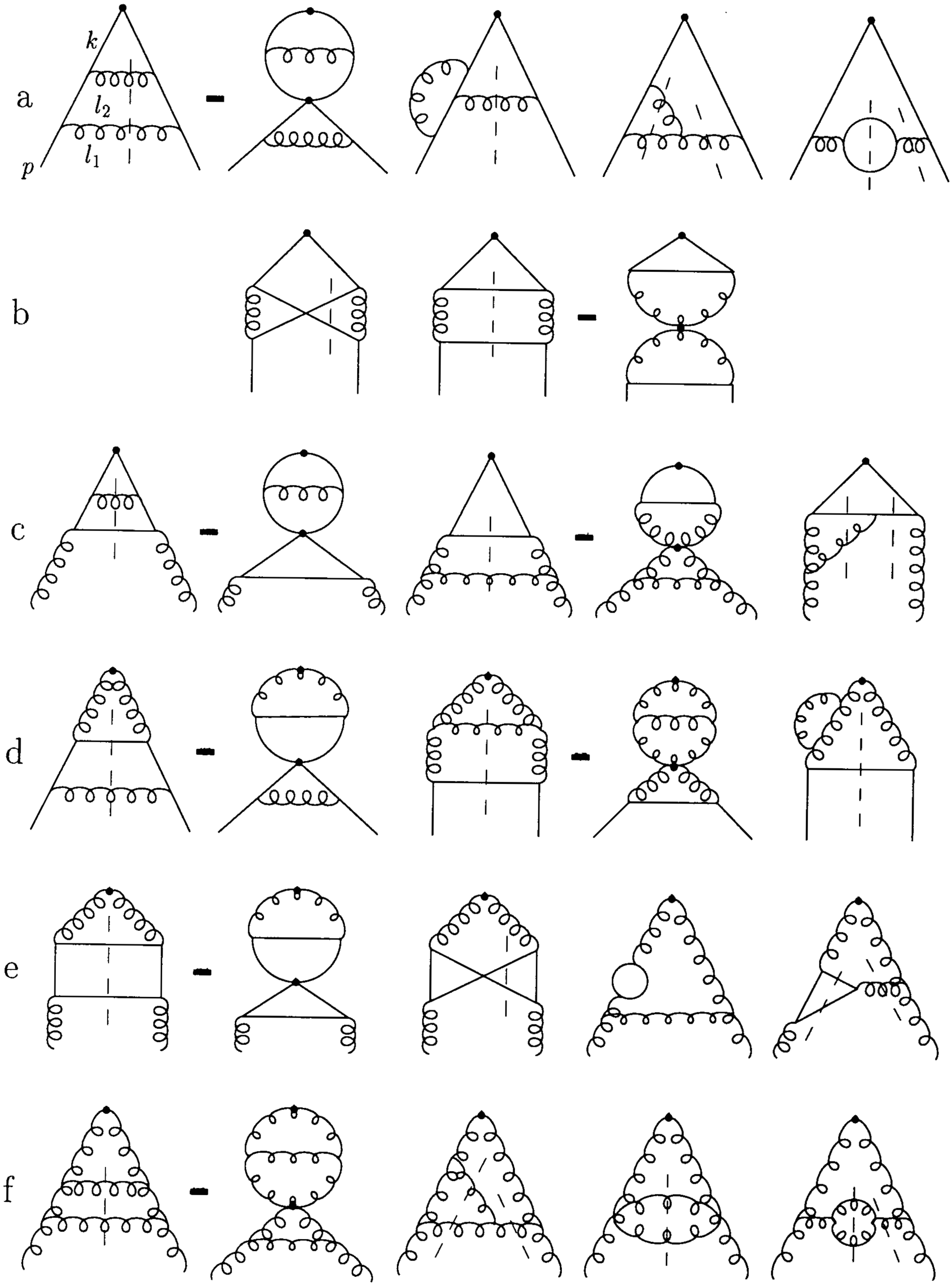

Fig.2