

# LONGITUDINAL DYNAMICS STUDIES FOR ISIS UPGRADES

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## Abstract

ISIS is the pulsed neutron and muon source based at the Rutherford Appleton Laboratory in the UK. Operation is centred on a loss-limited 50 Hz proton synchrotron which accelerates  $\sim 3 \times 10^{13}$  protons per pulse from 70 MeV to 800 MeV, corresponding to mean beam powers of 0.2 MW.

A number of ISIS upgrades are currently under study. One option replaces the linac for higher energy injection into the existing ring, potentially increasing beam current through reduction in space charge. The other main option adds a new  $\sim 3$  GeV Rapid Cycling Synchrotron (RCS), boosting the energy of the beam to provide higher beam power. For both these upgrade routes, longitudinal dynamics of the existing and proposed new rings play a crucial role in achieving high intensity with low loss. This paper summarizes work underway to optimize the longitudinal dynamics of the upgrade rings, which is presently focused on development and checking of simulation codes. This includes comparison of different space charge algorithms and codes.

## INTRODUCTION

A number of ISIS upgrade options are currently under study [1]. The two main options are to replace the current linac for higher energy injection into the existing ring and/or to add a new  $\sim 3$  GeV RCS boosting the energy of the beam to provide higher beam power. Both options require careful examination of the longitudinal dynamics and the effect of space charge.

Circumference / m	408.4
Energy / GeV	0.8 – 3.2
Repetition Rate / Hz	50
Harmonic Number	5
Gamma Transition	7.2044
$\frac{\Delta p}{p}$ Acceptance	$\pm 0.02$
RF Frequency / MHz	3.0910 – 3.5749
Peak Volts per Turn / kV	730
Protons per Pulse	$2.8 \times 10^{13}$

Table 1. Longitudinal Parameters for 3.2 GeV RCS.

The second of these two options will be focused upon in this paper, and in particular a 4 superperiod lattice based upon an existing design [2] accelerating the beam from 0.8 – 3.2 GeV. The ring circumference is 2.5 times that of the present ISIS synchrotron and has a RF harmonic number 5. This allows fast injection with near lossless bucket-to-bucket transfer. The magnetic field ramp is sinusoidal and all simulations in this paper were performed with single harmonic RF. A summary of the main parameters are shown in Table 1. This design is one of a few possible options under consideration.

## Longitudinal Dynamics Code

A stand-alone longitudinal code has been written primarily to study and better understand the longitudinal parameters for ISIS and its possible upgrades. It incorporates two methods of space charge calculation allowing detailed checks of code accuracy and convergence. These are the ‘Difference Algorithm Method’ and the ‘Discrete Fourier Transform Method’. Each method is used to simulate the ring and the results compared. The ring is also simulated using the existing code ORBIT [3].

The longitudinal code was written in C++ to match with the existing 2D particle tracking code, Set [4], and runs on a single processor. The code has also been installed on a high performance computing cluster called SCARF [5] for more efficient running.

## SPACE CHARGE CALCULATION

The longitudinal space charge kicks on a particle bunch can be calculated using equation (1)

$$E_{sc} = -\frac{q}{4\pi\epsilon_0\gamma^2} \left(1 + 2 \ln \frac{a}{b}\right) \frac{\partial\lambda}{\partial z} \quad (1)$$

where  $E_{sc}$  is the induced electric field due to space charge;  $q$  is the charge of the particle;  $\epsilon_0$  is the permittivity of free space;  $\gamma$  is the relativistic factor;  $a$  and  $b$  are the vacuum chamber and beam radii respectively;  $\lambda$  is the line density and  $z$  the longitudinal co-ordinate [6].

An accelerated particle beam can be simulated by using a smaller number of macroparticles. These can be binned in the longitudinal plane. The change in charge per bin then becomes the derivative of line density,  $\partial\lambda/\partial z$ . For small numbers of macroparticles per bin statistical averaging of bin charge can improve kick calculations. There are two methods of calculating the derivative of line density. These are described below.

## Difference Algorithm Method

The simpler of the two methods, the difference algorithm, calculates the average difference between a given bin population and those either side giving a value for  $dN/d\phi$  for each bin ( $N$  is the number of macroparticles and  $\phi$  is the particle’s phase). With an appropriate conversion factor this gives the required derivative of the line density.

## Discrete Fourier Transform (DFT) Method

This method takes the binned macroparticles and performs a one dimensional complex discrete fourier transform using FFTW, a C subroutine library [7]. This allows for solving of forces in the frequency domain, such

as space charge and other impedances. Therefore the space charge impedance is calculated on each turn. The frequency distribution is then multiplied by this impedance and an inverse fourier transform performed. With an appropriate conversion factor this gives the required derivative of the line density.

### Comparisons

Theory shows that an elliptical density distribution in energy and phase has a parabolic phase projection. This produces a space charge kick that is linear in phase within the bunch and zero outside the bunch.

A comparison of space charge kick distributions for each method is shown in Figure 1. Both the DFT method and difference algorithm method result in very similar functions and follow theoretical predictions closely.

Difference Algorithm = Purple, long dash; FFT Method = Blue, dash;  
Theoretical = Green, solid; ORBIT = Red, dot dash

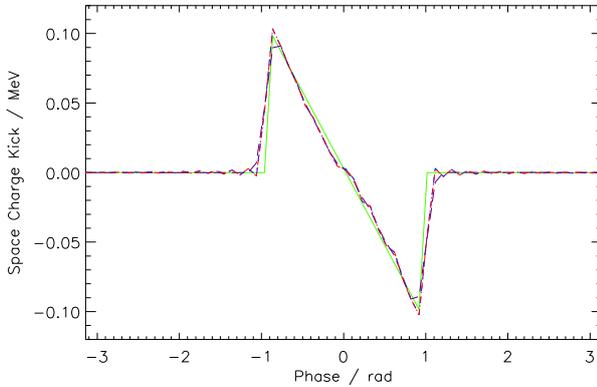


Figure 1. Space charge kick as a function of phase along the 2D elliptic distribution bunch for the theoretical (green), DFT (blue), difference algorithm (purple) and ORBIT (red) cases.

For comparison, ORBIT's longitudinal module was also used to calculate the space charge kick for the same distribution and is shown in Figure 1. ORBIT's result and the DFT lie on top of one another as ORBIT also uses this method of space charge calculation. The oscillations in kick outside the bunch are due to performing the DFT over a finite range, i.e.  $2\pi$  radians.

### CONVERGENCE TESTS

Most simulations here use the ring described in Table 1 as a working example. Although the longitudinal parameters used are reasonable, they are not yet optimized. It is beneficial for code testing purposes to have observable bunch oscillations and halo that would not be desirable in the final machine.

The parameters shown in Table 1 were used in the longitudinal code with a 2D elliptic macroparticle distribution and a non-optimized RF voltage law as a test case. Convergence tests were carried out by comparing loss patterns and bunch moments with varying numbers of macroparticles and different numbers of longitudinal bins. All convergence tests were carried out using the DFT space charge calculation method. The intensity was kept constant throughout all the simulation runs.

### Varying Macroparticle Number

The first two moments, average phase and bunch length, are shown in Figure 2 over the full 10 ms acceleration cycle (6815 turns) with different numbers of macroparticles used. All the plots from 10k – 90k ( $1 \times 10^4$  –  $9 \times 10^4$ ) are almost identical for average phase, or bunch centre with respect to synchronous phase. Periodical oscillations show that the RF voltage law is non-optimal. Bunch length oscillations are also very similar with a slight difference for the 10k macroparticle case.

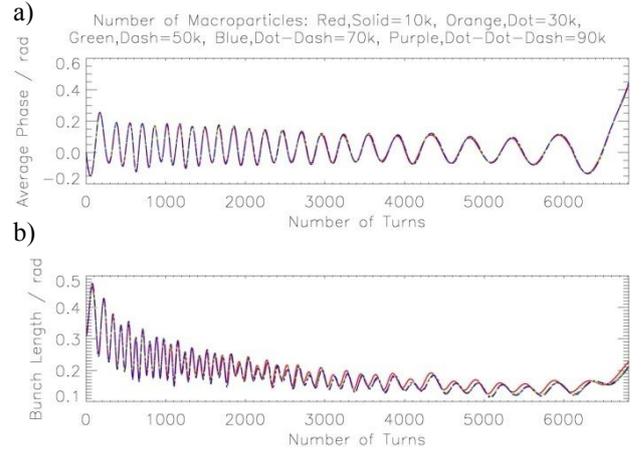


Figure 2. a) Average phase (above) and b) RMS bunch length (below) versus number of turns for different numbers of macroparticles (64 longitudinal bins).

Figure 3 shows how the loss profile with time varies with the number of macroparticles. All the plots from 10k to 90k have a total loss of  $0.22 \pm 0.06$  %. The loss distribution and peak positions are similar.

Number of Macroparticles: Red, Solid=10k, Orange, Dot=30k,  
Green, Dash=50k, Blue, Dot-Dash=70k, Purple, Dot-Dot-Dash=90k

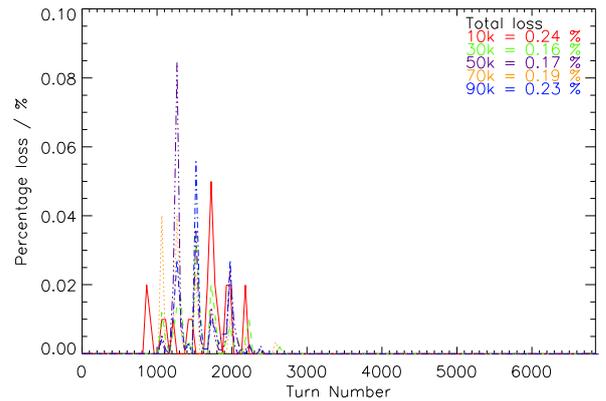


Figure 3. Percentage loss versus number of turns for different numbers of macroparticles, binned every 50 turns (64 longitudinal bins).

50k macroparticles was taken as an optimal value as it lies in the middle of the range and shortens computation time in comparison to larger bunches.

### Varying Number of Longitudinal Bins

The average phase of the macroparticle bunch with respect to the synchronous phase is shown for the full

10 ms (6815 turns) acceleration cycle, with different numbers of longitudinal bins, in Figure 4. Although not a requirement, all bin numbers are powers of two to improve FFTW's computation time for the space charge calculation. As expected the difference algorithm method is quicker the smaller the number of bins used.

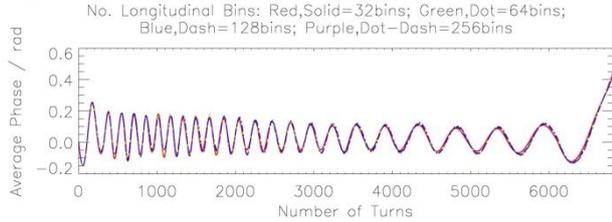


Figure 4. Average phase versus number of turns for different numbers of longitudinal bins (50k macroparticles).

Each run is very similar with minor variations in amplitude of oscillation. Once again it is noted that the oscillations are due to a non-optimal RF voltage law.

Percentage loss over the acceleration period differs depending upon the number of longitudinal bins chosen. The timing of loss is similar for all but the 32 bin case.

## COMPARISON WITH ORBIT

Simulations for comparison between the two space charge calculations and with ORBIT were done using 50k macroparticles and 64 longitudinal bins. The particle phase space distributions for the 6000<sup>th</sup> turn (8.86 ms) can be compared for each case. These are plotted in Figure 5.

Both the difference algorithm and DFT approaches have resulted in remarkably similar phase space distributions even after 6000 turns. All three figures show similar tail features although some significant differences require further investigation.

## SUMMARY

A new longitudinal code has been written with two methods for calculating the effect of space charge on particle dynamics. Comparisons have been made between the space charge induced kick calculated using both difference algorithm and DFT methods alongside theoretical calculations and ORBIT's treatment.

Initial convergence tests and benchmarking against ORBIT 1D simulations have shown promising results. Further benchmarking is planned before attaching to the existing Set code to create a 2.5D particle tracker to simulate ISIS and future upgrades.

Although the emphasis here has been on testing simulations, results do suggest that with appropriate optimization suitably low longitudinal losses will be possible in the proposed new rings.

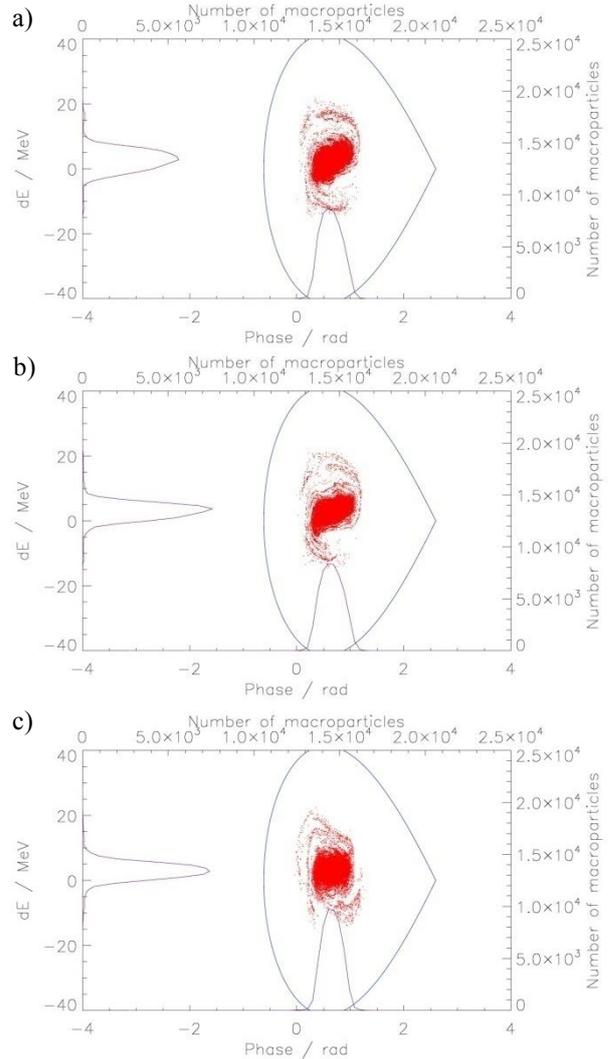


Figure 5. Longitudinal phase space for the 6000<sup>th</sup> turn using a) the new code with difference algorithm, b) the new code with DFT and c) ORBIT.

## REFERENCES

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