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SOME ASPECTS OF ION PRODUCTION IN THE SRS

by

S.A. Armitage
Daresbury Laboratory

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Science Research Council

Daresbury Laboratory

Daresbury, Warrington WA4 4AD

where K(s) is the magnetic focussing function of the storage ring and k(s) represents the electrostatic interactions with the other charged particles in the vicinity of the test particle. Due to the relativistic velocities of the electrons there is cancellation of the mutual coupling forces and hence no contribution to k(s) from the (n_e-1) electrons of the beam. However the presence of N_i ions trapped in the electron beam vicinity by the beam space charge does provide a contribution to k(s) producing a Q shift away from the value that is characterised by K(s) alone.

Making the assumption that the trapped ions will have the same transverse distribution as that of the electrons in the beam means that eqn.(6b), giving the electric field within the electron beam, also describes the electric field due to the ions with N_e becoming N_i, the number of ions trapped around the ring. Equation (6b) shows a linear dependence of electric field upon transverse displacement so that k(s) can be readily evaluated in terms of this electric field and eqn.(8) is valid for these small perturbations of the focussing function.

Courant and Snyder[4] give an expression for the Q shift due to a small perturbation of K(s) as:

$$\Delta Q = \frac{1}{4.\pi} \int_0^C \beta(s)\; k(s)\; ds \tag{9}$$

where β(s) is the amplitude function in the region where the perturbation occurs and the integral sums the effect of such perturbations k(s) distributed around a ring of circumference, C. Following usual accelerator theory, the perturbation function can be written as:

$$k(s) = \frac{1}{m_o c^2 \gamma} \frac{\partial F(s)}{\partial x} = -\frac{e}{m_o c^2 \gamma} \cdot \frac{\partial E_x(s)}{\partial x} \tag{10}$$

where γ is the electron beam energy in rest mass units. The electric field depends on the electron beam size at the point, s, so that k(s) can be expressed in terms of the amplitude function and the average electron beam size:

$$k(s) = \frac{e}{m_o c^2 \gamma} \cdot \frac{N_i e}{4\pi\varepsilon_o R} \; \frac{\bar{\beta}}{\sigma_x^2\, \beta_x(s)} \left[1 + \frac{\sigma_y}{\sigma_x}\sqrt{\frac{\beta_y(s)}{\beta_x(s)}}\right]^{-1}$$

$$= \frac{N_i\, \bar{\beta}\quad r_e}{\gamma\;\; R\; \sigma_x^2 \beta_x(s)} \left[1 + \frac{\sigma_y}{\sigma_x}\sqrt{\frac{\beta_y(s)}{\beta_x(s)}}\right]^{-1} \tag{11}$$

with r_e as the classical electron radius (2.81×10^{-15}m) and $\bar{\beta}$ as the average value of the amplitude function. Substituting for k(s) in eqn.(9) and setting $\bar{\beta} = R/Q$ gives:

$$\Delta Q_x = \frac{N_i . r_e}{4\pi\gamma\; Q_x \sigma_x^2} \int_0^C \left[1 + \frac{\sigma_y}{\sigma_x}\sqrt{\frac{\beta_y(s)}{\beta_x(s)}}\right]^{-1} ds \tag{12}$$

From eqn.(12), it is evident that the operating Q shifts to higher values as ions accumulate in the space charge region. The possibility then exists of a Q shift sufficiently great to reach a fractional integer value for which a driving force exists and beam loss can be expected. Taking the working point in the SRS for the horizontal plane as $Q_x = 3.25$, then the $3^1/_3$ resonance is the nearest higher point at which beam blow-up might be expected, although the required superperiodicity of ten for the driving error is not obvious in the storage ring lattice. Computations[5] suggest that the $3^1/_3$ resonance has a width of < 0.02 in the horizontal plane so that the maximum allowable Q shift is $\Delta Q_x \sim (3.33 - 0.02 - 3.25) \simeq 0.05$.

It is perhaps more convenient to consider a neutralisation ratio, $\eta = N_i/N_e$, and re-arrangement of eqn.(12) allows an upper limit to be set where K(s) is the magnetic focussing function of the storage ring and k(s) represents the electrostatic interactions with the other charged particles in the vicinity of the test particle. Due to the relativistic velocities of the electrons there is cancellation of the mutual coupling forces and hence no contribution to k(s) from the (n_e-1) electrons of the beam. However the presence of N_i ions trapped in the electron beam vicinity by the beam space charge does provide a contribution to k(s) producing a Q shift away from the value that is characterised by K(s) alone.

Making the assumption that the trapped ions will have the same transverse distribution as that of the electrons in the beam means that eqn.(6b), giving the electric field within the electron beam, also describes the electric field due to the ions with N_e becoming N_i, the number of ions trapped around the ring. Equation (6b) shows a linear dependence of electric field upon transverse displacement so that k(s) can be readily evaluated in terms of this electric field and eqn.(8) is valid for these small perturbations of the focussing function.

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where γ is the electron beam energy in rest mass units. The electric field depends on the electron beam size at the point, s, so that k(s) can be expressed in terms of the amplitude function and the average electron beam size:

$$\begin{aligned} k(s) &= \frac{e}{m_o c^2 \gamma} \cdot \frac{N_i e}{4\pi\varepsilon_o R} \cdot \frac{\bar{\beta}}{\sigma_x^2\, \beta_x(s)} \left[1 + \frac{\sigma_y}{\sigma_x}\sqrt{\frac{\beta_y(s)}{\beta_x(s)}}\right]^{-1} \\ &= \frac{N_i\, \bar{\beta}\, r_e}{\gamma \cdot R\, \sigma_x^2 \beta_x(s)} \left[1 + \frac{\sigma_y}{\sigma_x}\sqrt{\frac{\beta_y(s)}{\beta_x(s)}}\right]^{-1} \end{aligned} \tag{11}$$

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It is perhaps more convenient to consider a neutralisation ratio, $\eta = N_i/N_e$, and re-arrangement of eqn.(12) allows an upper limit to be set

At 10^{-9} torr, $N_g = 7 \times 10^{22} \times 10^{-9}$ atoms/m^3 and $\sigma = 10^{-22}$ m^2 for a mass 28 gas molecule. Thus eqn.(15) gives a production time of 0.47 sec.

If some external field of sufficient magnitude to counteract the beam potential acts upon the trapped ions, then ion loss will occur in this 'sink' and the neutralisation ratio will become very small. It is not usually practical to maintain such clearing fields over the full length of the drift region, and an alternative approach provides compact clearing field regions spaced so that the neutralisation ratio does not build up to its critical value (in terms of Q shift).

A characteristic transit or loss time due to ion migration can be established assuming that the clearing regions are spaced at regular intervals, ℓ, around the ring so that:

$$\tau_{loss} = \frac{\ell}{2\bar{v}} \qquad (16)$$

where $\bar{v}$ is the longitudinal component of the r.m.s. thermal drift velocity of the gas molecule, approximately 300 m/s at 300°K for a mass 28 molecule.

These two characteristic times allow a simple representation of the neutralisation ratio by:

$$\eta = \tau_{loss} / \tau_p \qquad (17)$$

Equations (16) and (17) allow an estimate to be made of the spacing required between ion clearing stations:

$$\ell < \eta \; \tau_p \; . \; 2\bar{v}_s \qquad (18)$$

where the neutralisation ratio that should be maintained is given by eqn.(13). This spacing applies to sections of the ring where there is no

restraint imposed on the longitudinal drift of the ions. Interaction between the magnetic guide field and the charged gas molecules may produce trajectories resulting in non-thermal, effective drift velocities, and the following section takes up this point in greater detail. To maintain values of $\eta \sim 10^{-3}$ requires the distribution of clearing stations at intervals of approximately 0.3m intervals. A ring vacuum of 10^{-9} torr has been assumed, and any improvement in this figure, particularly through a reduction in the partial pressure of the heavier constituents such as nitrogen, water vapour and carbon monoxide, would permit an increase in the separation between clearing stations through the τ_p term in eqn.(18).

6. ION MOTION IN THE DIPOLE MAGNET REGIONS

Consider an ion moving along the s-axis in the co-ordinate system of Fig.4. The force on the particle will be given by:

$$\underline{F} = e(\underline{v} \wedge \underline{B} + \underline{E}) \qquad (19)$$

The velocity vector has components along all three axes due to thermal motion, only $-B_y$ exists in the dipole magnet, and the electric field vector has two components $E_y(y)$ and $E_x(x)$ due to the radial distribution of the transverse space-charge (eqn.(6b). From eqn.(19), three equations of motion can be written:

$$F_x = M\ddot{x} = eB_y\,\dot{s} + eE_x(x)$$

$$F_y = M\ddot{y} = eE_y(y)$$

$$F_s = M\ddot{s} = -\,e.B_y\,\dot{x}$$

where M is the mass of the ion and $E_x(x)$ is given by eqn.(6b). Further re-arrangement produces:

$$\ddot{x} - \omega\dot{s} + \xi_x x = o \quad (20)$$

$$\ddot{y} + \xi_y y = o$$

$$\ddot{s} + \omega\dot{x} = o \quad (21)$$

where $\omega = \frac{eB_y}{M}$ from consideration of the radial motion of a charged particle in a magnetic field and

$$\xi_x = \left(\frac{e}{M}\right) \cdot \frac{N_e}{4\pi\epsilon_o R\sigma_x(\sigma_x+\sigma_y)}$$

Integrating eqn.(21) and eliminating $\dot{s}$ from eqn.(20) gives:

$$\ddot{x} + \alpha^2 x + \omega k = o \quad (22)$$

where $\alpha^2 = \xi_x + \omega^2$ and k is a constant of integration. Equation (22) has the form of a second order linear equation with a solution of the type:

$$x = A \cos \alpha t + D \sin \alpha t - \frac{k\omega}{\alpha^2} \quad (23)$$

Putting in boundary conditions of $x = o$, $\dot{x} = \bar{v}_x$, i,e. the ion is assumed to be within the beam and most probably at the bottom of the potential well moving with thermal velocity, $\bar{v}_x$, at $t = o$ gives:

$$x = \frac{k\omega}{\alpha^2} (\cos \alpha t - 1) + \frac{\bar{v}_x}{\alpha} \sin \alpha t \quad (24)$$

Integrating and substituting for x gives:

$$\dot{s} - k\frac{\omega^2}{\alpha^2} (\cos\alpha t - 1) + \bar{v}_x \frac{\omega_o}{\alpha} \sin\alpha t + k = o \quad (25)$$

with boundary conditions of $\dot{s} = \bar{v}_s$ at $t = o$ gives

$$k = -\bar{v}_s \quad (26)$$

so that eqn.(25) becomes:

$$\dot{s} = \bar{v}_s \left(1 - \frac{\omega^2}{\alpha^2}\right) - \frac{\bar{v}_x \cdot \omega}{\alpha} \sin\alpha t + \frac{\bar{v}_s \omega^2}{\alpha^2} \cos\alpha t \quad (27)$$

Equations (24) and (27) indicate a cycloidal trajectory in the x - s plane with the amplitude of this motion in the transverse (x) direction determined by the beam intensity through the $^1/\alpha^2$ term. For low beam intensities the cycloid has a large amplitude, decreasing as the beam intensity increases. This change in amplitude is reflected in a variable rate of progression of the cycloid along the s axis; as shown in eqn.(27), the longitudinal velocity is modified by the beam intensity dependant term $(1 - \omega^2/\alpha^2)$. The saturation characteristics of this term are depicted in fig.5.

The spacing of clearing stations in the dipole magnet is thus a more complex function than eqn.(18) suggests due to the presence of a crossed magnetic and electric field term. An energy dependence might be expected in the $(1 - \omega^2/\alpha^2)$ term though an increase in magnetic field with energy is offset by the change in beam size so, as Table 1 shows, the longitudinal velocity is substantially independent of beam energy and closely approaches the thermal velocity value at beam currents of 1A.

Table 1 Ion Longitudinal Velocity in the dipole regions of the SRS for 1A stored beam.

Energy (GeV)	Type 1	Type 2
0.6	$1.0\ \bar{v}_s$	$\sim 1.0\ \bar{v}_s$
2.0	$0.81\ \bar{v}_s$	$0.9\ \bar{v}_s$

From eqn.(18) it might be expected that a decreasing ion longitudinal velocity with decreasing current would be offset by the greater neutralisation ratio that could be tolerated at the lower beam currents. In practice the overall dependence of electrode spacing on beam intensity is a complex one and calculations show that the most critical situation still occurs for vertical Q shift at injection energies, requiring clearing electrode spacings of $\ell \sim 0.3$m. For mechanical reasons, the most likely solution is a continuous electrode system fitted along the full length of the dipole chambers. Detailed calculations of maximum tolerable spacings within a dipole region are therefore not too important.

7. SUMMARY

The potential well associated with the stored electron beam space charge will trap gas ions produced in the vicinity of the beam. The number of ions trapped by this means is defined in terms of a neutralisation ratio, and an upper limit on this ratio is set by the uncorrected Q shift that can be tolerated before resonant blow-up might occur. In order to maintain a specified neutralisation ratio, electrostatic ion clearing stations can be provided at intervals along the orbit path. Such clearing stations should maintain an electric field gradient sufficient to counteract the beam potential for the period of time that allows ion drift to the collecting plates.

For the Storage Ring, we have assumed a working point of 3¼ in both planes and anticipate an uncorrected positive tune shift of $\sim$0.05 before beam loss at the $3^1/_3$ resonance becomes a problem. The dependence of the neutralisation ratio on energy makes the injection period more critical than higher energies when ratios of $\eta \lesssim 10^{-3}$ should be maintained around the ring. This neutralisation ratio can be achieved by clearing electrode systems providing an electric field of $\gtrsim$ 120V/mm at the equilibrium orbit (the exact minimum value will depend on the beam size at the location of the electrodes) spaced at intervals of 0.3m. The electrode size in the beam direction need not be large particularly where electric fields well in excess of beam potential requirements are applied so that ions are accelerated out of beam region. At electrode potentials just sufficient to neutralise the beam potential, electrode lengths equivalent to the electrode-to-orbit spacing would be adequate to permit ion clearing at thermal drift velocities. Should the total length of the ion clearing electrode system become appreciable, then the integral in eqn.(9) need only be considered over the uncleared fraction of the ring circumference. This would lead to larger values for the permissible neutralisation ratio with a consequent increase in electrode spacing. If voltage breakdown considerations limit the clearing field to a value just sufficient to neutralise the beam potential or other criteria dictate long electrodes, then the spacing argument should be modified to take account of the ion-free regions along the beam orbit.

It should be emphasised that these recommendations are based on a tolerable, uncorrected Q shift of $\frac{+}{}$ 0.05 in either plane, a Q shift that allows little margin for error. Any decrease in this Q shift figure would require a pro rata decrease in the separation of ion clearing stations.

No attention has been devoted to the longitudinal structure of the beam and its possible effect on the ion drift velocity. Qualitatively, the bunching of the electrons would be expected to superimpose an oscillatory component on the ion velocity but with a uniform bunch structure no average velocity change is expected.

Finally, the assumption of a "hard-edged" beam of width 4σ should not be overlooked since this model neglects second-order effects that might be of greater importance in machine operation. In principle, the Q shift that the model predicts can be corrected by straightforward Q control though the second order effects resulting from a 'fuzzy' beam edge will probably lead to a broadening of the operating point in Q-space with evident repercussions on chromaticity.

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FIGURE CAPTIONS

Fig. 1 Co-ordinate system for an electron moving with velocity V

Fig. 2 Space charge potential due to a column of charge at the centre of a circular pipe of radius A

Fig. 3(a) Transverse section of an electron beam in a dipole chamber (not to scale)

(b) Equipotential curves for a potential of ±V volts applied to the electrode configuration of (a)

(c) Equipotential curves for a potential of 2V volts applied to the internal screen of (a)

Fig. 4 Co-ordinate system in dipole magnet

Fig. 5 Longitudinal ion drift velocity in dipole magnet region

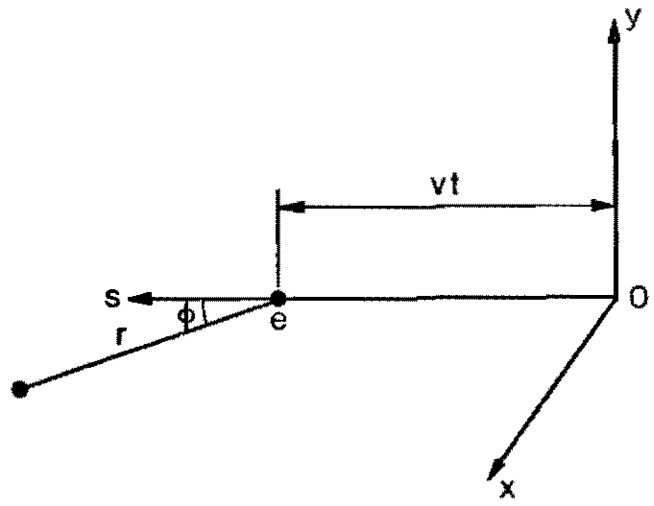

Fig. 1

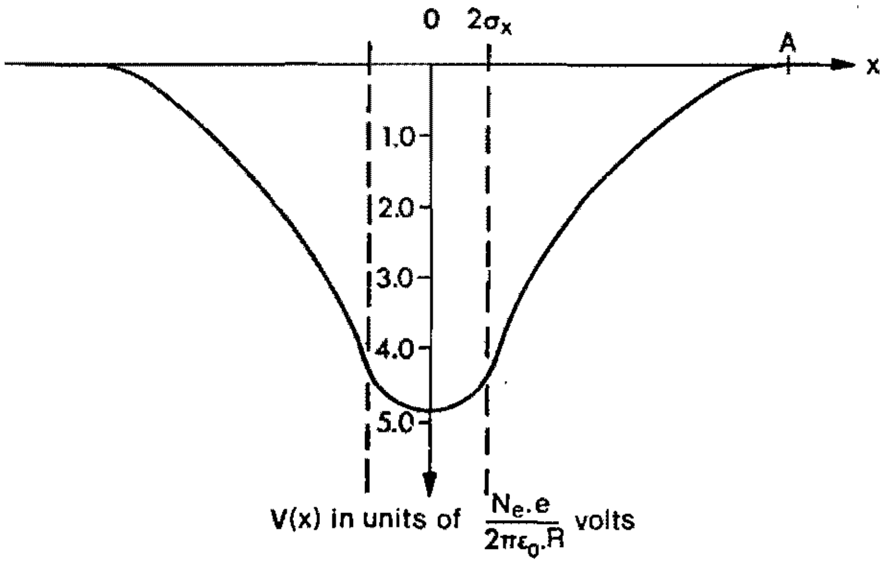
0
$2\sigma_x$
A
x
1.0
2.0
3.0
4.0
5.0
V(x) in units of $\frac{N_e.e}{2\pi\varepsilon_0.R}$ volts

Fig.2

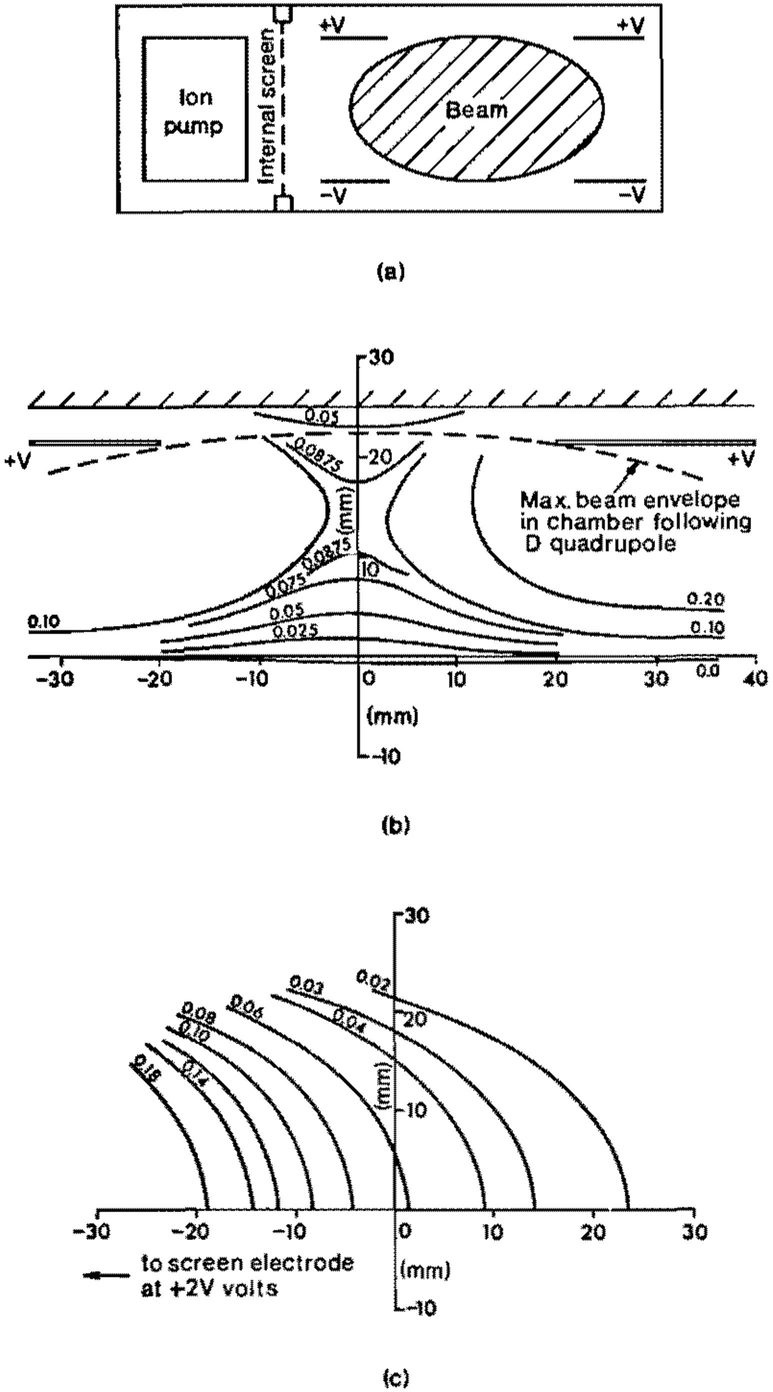
Ion pump
Internal screen
Beam
+V
+V
−V
−V
(a)
30
0.05
0.0875
20
+V
+V
Max. beam envelope in chamber following D quadrupole
(mm)
0.0875
10
0.075
0.05
0.025
0.10
0.20
0.10
0.0
−30
−20
−10
0
10
20
30
40
(mm)
−10
(b)
30
0.02
0.03
0.04
0.06
0.08
0.10
0.14
0.18
20
(mm)
10
−30
−20
−10
0
10
20
30
to screen electrode at +2V volts
(mm)
−10
(c)

Fig. 3

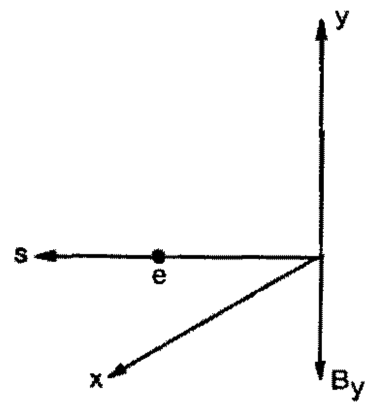

Fig. 4

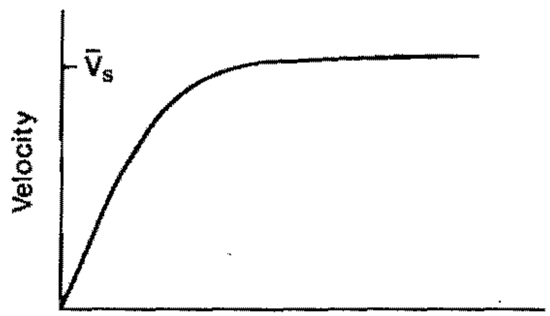

Fig.5