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HALL PLATE CALIBRATION FOR THE
DARESBURY MAGNET TEST FACILITY

by

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1. INTRODUCTION

The requirement for magnetic field measurements on the SRS Booster combined function magnets and Storage Ring dipole, quadrupole and multipole magnets has led to the setting up of an improved Laboratory Magnet Test Facility[1]. The main features of the Facility are its flexibility, allowing measurement of a variety of magnets over a wide field range, and the high degree of automation provided by computer control.

The magnetic field sensor that is used is a Hall plate (Siemens FC32). Hall plates are well suited to such a general purpose system and have the advantage that the electronics associated with them are simple and do not require much attention. The main drawback however is that Hall plates cannot be used for high precision measurements unless they are calibrated and subsidiary effects such as the variation of output with temperature are taken into account.

2. HALL PLATE CHARACTERISTICS

A Hall plate produces a voltage which is roughly proportional to the magnetic field and to the current passed through it and which is large enough to be read directly on a DVM. The value of 1.3 V/TA for the FC32 is typical of many plates. The main process by which the voltage is produced is called the Hall effect. It is the production of an electric field in a crystal which is carrying a current when the crystal is acted on by a magnetic field, the electric field, current and magnetic field being mutually orthogonal. The Hall plate is typically a small thin square of semiconductor with connections on the input, or control side, for the current and on the output, or Hall side, for measuring the voltage, as shown in Fig.1.

Several other effects must be taken into account before a Hall plate can be used to obtain accurate measurements of magnetic field:

(a) The planar Hall effect[2]

In addition to the Hall voltage produced by the component of field perpendicular to the plate, Bn, there is a voltage produced by the component of field in the plane of the plate, Bp, which varies as $\sin(2\phi)$ with the orientation of the plate, ϕ (see fig.1). This planar Hall voltage is relatively small ($\approx 10^{-2}$ V/T^{2}A) but must be taken into account when measuring strong fields with components acting along the X and Y axes of the plate. Hall plates can be calibrated for this effect enabling the true field components to be calculated from measurements taken with three orthogonal plates[3,4]. In the present case where a single plate is used measurements will usually be taken in a symmetry plane with the field either perpendicular to the plate or in the X-Z or Y-Z planes so that this effect may be neglected. Alternatively, measurements can be taken with both polarities of field; the planar Hall voltage is independent of polarity and is eliminated when an average is taken of the two sets of measurements.

(b) Temperature effects

The Hall voltage is proportional to temperature with a coefficient of about -0.07% per °C for the FC32 plate. Changes in the temperature of the plate are brought about by changes of the ambient temperature or by self-heating due to varying power dissipation in the control side. Methods of overcoming this effect employing a temperature controlled oven[5], a compensating network involving a thermistor[6], or both[7] have been reported. The temperature-controlled probes are large in size and are only suitable for measuring very large magnets and the compensating network suffers from the disadvantage that it can only be accurate over a narrow range of temperature and for a certain range of field. In this system a new approach has been adopted. A thermistor is mounted in contact with the Hall plate on the probe assembly. The measuring system automatically records the voltage across the thermistor, which is proportioned to temperature, whenever the Hall plate voltage is read. Readings are then normalised to a certain temperature using the temperature coefficients of the Hall voltage. The experiments which have been carried out to determine the coefficients and the method of correction are described in the Appendix.

The control side internal resistance is a function of both temperature and field. The effect on the current and hence the Hall plate voltage is eliminated when, as in this system, the plate is operated with a stabilized constant current supply. The effect of the varying power dissipation however does cause varying self-heating as mentioned above.

(c) Non-linearity of the Hall voltage

If the Hall plate is operated with a constant current supply and if temperature variations are adequately corrected there still remains a non-linear variation of voltage with magnetic field. It has been found that the field values obtained from Hall plate readings using the best straight line calibration vary above and below the true field values by as much as 10mT in the range ± 1.4T for this FC32 plate.

The dominant effect is therefore the non-linearity and because of this it is usual to calibrate a Hall plate against an NMR probe, which gives an absolute measurement of field to high accuracy, about 1 in 10^5. A particular Hall plate reading is calibrated either by evaluating the polynomial which has been fitted to the data or by interpolating directly between the data points.

For the measurement of SRS magnets an accuracy of 0.1mT is required over the range ± 1.4T and so clearly a calibration must be performed. The usual laboratory standard, however, the NMR probe can typically only be used to measure fields greater than 0.2T and so if the Hall plate is required to operate at lower field values, as in the present case, another method of calibration must be used. This problem has not been discussed in the literature and so the rest of this report deals in detail with the measurement and data analysis techniques that have been used in order to obtain a sufficiently accurate calibration over the entire range -1.4 to 1.4T.

3. CALIBRATION METHOD

A coil and integrator was chosen as a secondary calibration standard as it is inherently linear and can operate over the required field range. The device used was a magnetometer type-J manufactured by Newport Instruments Ltd. The magnetometer had first to be calibrated against the absolute standard, the NMR probe, in order to obtain an accurate calibration constant.

The method used was to take measurements of Hall plate voltage, NMR frequency and magnetometer voltage together in a homogeneous field. The magnetometer was calibrated against the NMR using the readings taken above 0.2T and this calibration was used to convert the magnetometer readings taken below the cut-off of the NMR into field values. Thus field values corresponding to each Hall plate reading were obtained either directly from the NMR frequency or indirectly from the magnetometer voltage and its calibration against field. These results were then fitted using a least-squares fitting program to obtain a calibration curve for the Hall plate.

4. MEASUREMENTS

The Hall probe assembly was removed from its normal mounting in the test area and positioned inside a nearby W17 magnet which can provide a suitable homogeneous field over the required range. The Hall plate and thermistor remained electrically connected to the measuring system and so the voltage across each could be read on a DVM in the usual way. With the magnet powered the Hall plate was aligned perpendicular to the field by adjusting its orientation to obtain the maximum voltage.

Readings were taken at 50 field values in the range -1.4T to 1.4T with proportionately more at low field to compensate for the fact that the magnetometer readings were less accurate than those from the NMR probe, and also because large deviations from linearity have been reported with some Hall plates at low field. At each field level the magnet was first allowed to stabilize for about two minutes. Even after this interval a slow drift in field was noticeable. Readings with the NMR probe were taken at fields above 0.2T. Six probe heads were used to cover the range up to 1.4T. To reduce errors resulting from the field drift all the readings were taken as quickly as possible with one operator alternately taking readings with the NMR probe and magnetometer while a second recorded the Hall plate and thermistor voltages.

To measure the field with the magnetometer the search coil was first placed in a zero field chamber and the integrator zeroed. Then the coil was inserted into the magnet close to the Hall probe and the orientation adjusted to obtain the maximum voltage, which was read on a DVM. This movement had to be carried out swiftly as the magnetometer itself had a relatively large drift rate which made it difficult to observe the peak voltage. The peak voltage was recorded and the procedure repeated with the coil rotated through 180° so that a reading of opposite sign was obtained. Two pairs of readings were taken at each field setting

At frequent intervals during the measurements the zero setting and the drift rate of the integrator were checked. The latter was obtained by allowing the integrator to run while the coil was located in the zero field chamber. If the drift rate was found to be outside a limit corresponding to 2×10^{-4} T/minute then adjustments were made to the integrator. This was required approximately every hour. Adjustment of the drift rate was found to affect the zero setting slightly but it was not found to vary beyond the limits $\pm 0.2 \times 10^{-4}$ T. Failure to make adjustments to the integrator to reduce the drift rate would have resulted in discontinuities between data taken at different times.

5. ANALYSIS OF DATA AND RESULTS

The readings of Hall plate, thermistor and magnetometer voltage, and NMR probe frequency at each field level were first averaged and then placed in a data set on the Laboratory's central computer. A calibration program was written which read in the data and performed all the necessary computation. Firstly, each Hall plate voltage was normalised to a particular temperature using the thermistor voltage and the temperature coefficients of the plate, and the NMR frequencies were converted into field values. A least-squares straight line fit was made to the field and magnetometer voltage data for the range 1.4-0.2T for both polarities of field. This gave a calibration constant for the magnetometer, 0.16023 ± 0.00001 T/V, which was used to convert all the readings taken below 0.2T into field values. A polynomial fit was made to the corrected Hall plate readings and field data, with the Hall plate voltage treated as the independent variable. The coefficients of the polynomial could then be used to convert any Hall plate voltage into a field value.

The main problem in performing a least-squares polynomial fit to a set of data lies in determining where to terminate the infinite series. Statistical tests such as the χ^2 test or the F-test can only be applied when reasonably accurate estimates of the error on each measurement are available, in order to determine the weight given to each point. First attempts were made with each point given an equal weight. In this case the quality of fit is indicated by a quantity which is very similar to the r.m.s. deviation of the points about the fitted curve. As a function

of the number of terms of the fitting polynomial this quantity decreased rapidly at first, then remained roughly constant between 8 and 17 terms before increasing. This behaviour is typical of polynomial fits. It is usual to take the start of the "plateau" region to indicate the best fit, in this case an 8 term polynomial. This function also gave the lowest value of the estimated average calibration error over the range ± 1.4T, which was $\pm 0.55 \times 10^{-4}$ T.

In the present case where the measurement errors are not all similar, mainly because of the difference in accuracy between the magnetometer and the NMR probe, it was felt that a weighted fit would be more realistic. Several approximations were made in the calculation of weights. It was assumed that:

i) the only Hall plate voltage error was due to rounding error on the DVM,

ii) each field value obtained from the magnetometer voltage contained an error due to rounding error and drift, estimated from the data and a calibration error estimated from the straight line calibration against the NMR field values.

iii) the NMR probe readings contributed no errors.

The quality of fit for weighted fits is indicated by the value of the reduced chi-square (χ^2_ν). There is some ambiguity in interpreting this quantity since it not only measures how well the true data values can be fitted by the given curve but is affected by the values of the weights chosen for the data. In general if the value of χ^2_ν is close to 1.0 then the fitting function is considered appropriate for describing the data. This test alone is not sufficient to indicate the best order of fit of a polynomial. Another test which should be applied in conjunction with this is the F-test, which gives a measure of how much the addition of another term in the fitting function improves the value of χ_ν. Conversely, corresponding to a value of this statistic ($F\chi$) for a given number of terms of a fitting function there is a probability of being correct in terminating the series at that point.

Table 1 gives the values of χ^2_ν and $F\chi$ as a function of the number of terms of the polynomial (N) for the weighted fit of field against Hall plate voltage. It can be seen that between N=11 and N=17 the values of

χ_ν^2 are all close to 1.0 and so indicate reasonable fits to the data. The value of F_χ for N=13 shows that very little improvement has been made to the fit by including the 13th term. The value corresponds to a probability of 85% of being correct in terminating the series at N=12. The value of χ_ν^2 for this function is sufficiently close to 1.0 to indicate that higher order terms in the series may be neglected. The table also gives values of the average calibration error i.e. the average error in the fitting function, in the range ± 1.4T, which were calculated using the error matrix produced by the least-squares fit. The error is a minimum for a 12 term polynomial, giving additional support to the choice of this function as the best fit. The calibration error is ± 0.17 x 10^{-4}T.

Figure 2 shows the data and the calibration curve plotted over the full range. The deviation from linearity is hardly noticeable on this scale. This is shown better in fig.3 where the residuals from a least-squares straight line fit to the data are plotted. The curve in the figure is the calibration curve with the straight line subtracted so it is effectively a polynomial fit to the residuals. It can be seen that the Hall plate is not symmetrical between fields of different direction. A plot of the data and the calibration curve in the range ± 0.01T is shown in fig.4. This illustrates the fact that even at low fields the Hall plate is essentially linear.

Figure 5 shows the variation of calibration error with field for the 8-term polynomial produced by a non-weighted fit and also for the 12-term polynomial produced by a weighted fit. It can be seen that for field strengths less than about 0.1T the two functions give roughly the same error. For the non-weighted fit the error increases up to 1.0 x 10^{-4}T at the highest fields, whereas in the case of the weighted fit the error stays roughly constant. A probable explanation of this behaviour is that the large number of data points at low field dominate the non-weighted fit producing large errors at high field. The inclusion of weighting factors counteracts this effect giving better smoothing of the data and an almost constant calibration error. The undulations in the curves are most probably caused by there being insufficient data points in the high field region.

5. CONCLUSION

A calibration curve has been obtained for the Hall plate that will allow a reading to be converted into a field value with an average error of ± 0.2 x 10^{-4}T over the range ± 1.4T. It is now appropriate to consider other factors contributing to the total absolute error on a field value obtained from a Hall plate reading. The DVM and constant current source are considered sufficiently stable that they do not introduce any significant error. There are two remaining contributions:

(a) The rounding error on the reading of Hall plate voltage on the DVM. With the present equipment this is:

± (0.02 ± 10^{-4})T in the range 0-160mT
± (0.2 ± 10^{-4})T in the range 160mT-1.6T

(b) The error introduced by temperature correction. If there are no varying thermal gradients between Hall plate and thermistor the error is due to uncertainties in the temperature coefficients of the Hall voltage (see Appendix). The error is in the range ± (0.1-0.3) x 10^{-4}T per $^\circ$C of temperature variation, depending on the field.

The total absolute error of a field value obtained from a single Hall plate reading is therefore in the range ± (0.2-0.4) x 10^{-4}T for a 1°C temperature variation. For larger variations it is the error in the temperature correction which dominates. To date the largest temperature deviation that has been observed is 4°C which gives rise to a 1.2 x 10^{-4}T error on an absolute field measurement at 0.8T.

For many measurements it is the relative accuracy only that is important. In this case above 160mT relative errors are dominated by rounding errors, provided that the variations in field and temperature are not large. Below 160mT the rounding error is insignificant and relative errors are due to systematic temperature correction and calibration errors.

The present calibration and correction method should be adequate for all of the measurements on the SRS magnets. It is hoped however to repeat or at least check the calibration at regular intervals in order to detect any ageing effects of the Hall plate. The opportunity will be taken to extend the calibration to higher fields and to increase the density of data points above 1.0T. It has been shown that the temperature correction can contribute as much to the total absolute error as the calibration error and that this contribution dominates for large temperature variations. Further measurements are planned in order to reduce the error introduced by the temperature correction.

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APPENDIX

DETERMINATION OF THE TEMPERATURE COEFFICIENTS OF THE HALL VOLTAGE AND THE TEMPERATURE CORRECTION METHOD

The experiment was carried out with the Hall plate located in the SRS Booster prototype magnet under normal measuring conditions, apart from there being a plastic pipe over the probe. This had been fitted so that the temperature of the air around the probe could be varied. The magnet current was adjusted to obtain the desired field at the Hall plate. Hot air was then blown along the pipe until the temperature of the probe rose by about 20°C as measured by the change in thermistor voltage. Readings of the Hall plate voltage and thermistor voltage were taken as the probe slowly returned to normal temperature. The probe was then cooled by about 10°C by blowing air along the pipe which had passed over liquid nitrogen. Readings were again taken as the probe was allowed to return to normal temperature. This procedure was carried out at 5 field settings between ± 0.8T.

Graphs of Hall plate voltage against temperature were drawn for each current setting and these showed a linear variation with variable slope. This can be expressed as:

$$V_H = V_{Ho} + \alpha\Delta T$$

where,

V_H = Hall plate voltage at temperature T

V_{Ho} = Hall plate voltage at a standard temperature T_o (=21°C)

$\Delta T = T - T_o$

The slope α varies with field and hence with the Hall plate voltage itself. A graph of α against V_{Ho} was drawn and this showed a linear variation:

$$\alpha = \alpha_o + \alpha_1 V_{Ho}$$

A straight line was fitted to the points to obtain the following values for the intercept and slope:

$$\alpha_o = (2.6 \pm 1.4) \times 10^{-6}\ V/^{o}C$$

$$\alpha_1 = (-7.4 \pm 0.2) \times 10^{-2}\ \%/^{o}C$$

The overall variation of the Hall plate voltage with temperature may thus be written:

$$V_H = \alpha_o \Delta T + (1 + \alpha_1 \Delta T)\ V_{Ho}$$

This can be rearranged to give V_{Ho} in terms of the other quantities:

$$V_{Ho} = \frac{V_H - \alpha_o \Delta T}{1 + \alpha_1 \Delta T}$$

Thus knowing α_o and α_1 enables any given Hall plate reading (V_H) at temperature T to be converted to a value V_{Ho} at the standard temperature T_o.

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Table 1

Results of weighted fits of field against Hall plate voltage

N	χ_ν^2	$F\chi$	Error (mT)
2	0.70×10^5	0.40×10^6	0.19×10^1
3	0.41×10^5	0.35×10^2	0.17×10^1
4	0.27×10^4	0.66×10^3	0.50
5	0.26×10^4	0.15×10^1	0.55
6	0.18×10^3	0.61×10^3	0.16
7	0.17×10^3	0.32×10^1	0.17
8	0.81×10^1	0.84×10^3	0.39×10^{-1}
9	0.79×10^1	0.21×10^1	0.43×10^{-1}
10	0.18×10^1	0.13×10^3	0.22×10^{-1}
11	0.10×10^1	0.31×10^2	0.18×10^{-1}
12	0.76	0.14×10^2	0.17×10^{-1}
13	0.78	0.39×10^{-1}	0.20×10^{-1}
14	0.74	0.29×10^1	0.26×10^{-1}
15	0.71	0.25×10^1	0.40×10^{-1}
16	0.69	0.20×10^1	0.87×10^{-1}
17	0.66	0.28×10^1	0.21
18	0.34×10^4	-0.31×10^2	0.66×10^3

FIGURE CAPTIONS

Fig.1 Hall plate

Fig.2 Field and Hall plate voltage data in the range ± 1.4T. The curve is a 12-term polynomial produced by a weighted least-squares fit to the data.

Fig.3 Residuals from a least-squares straight line fit to the data of fig.2. The curve is effectively a 12-term polynomial fit to this data.

Fig.4 Field and Hall plate voltage data in the range ± 10mT. The curve is the 12-term polynomial shown in fig.2.

Fig.5 Calibration error against field for the non-weighted 8-term polynomial fit (curve A) and for the weighted 12-term polynomial fit (curve B).

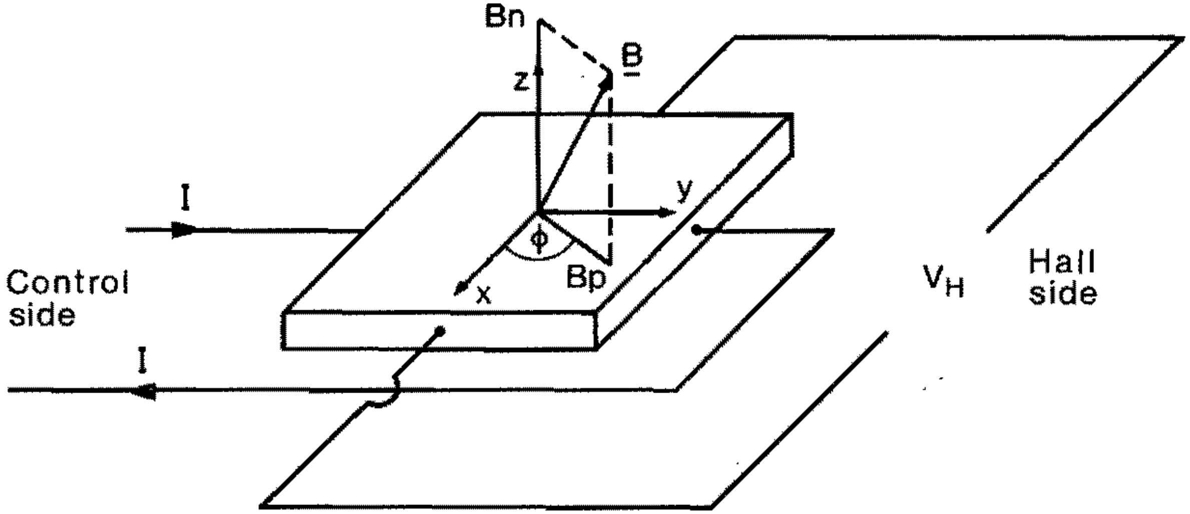
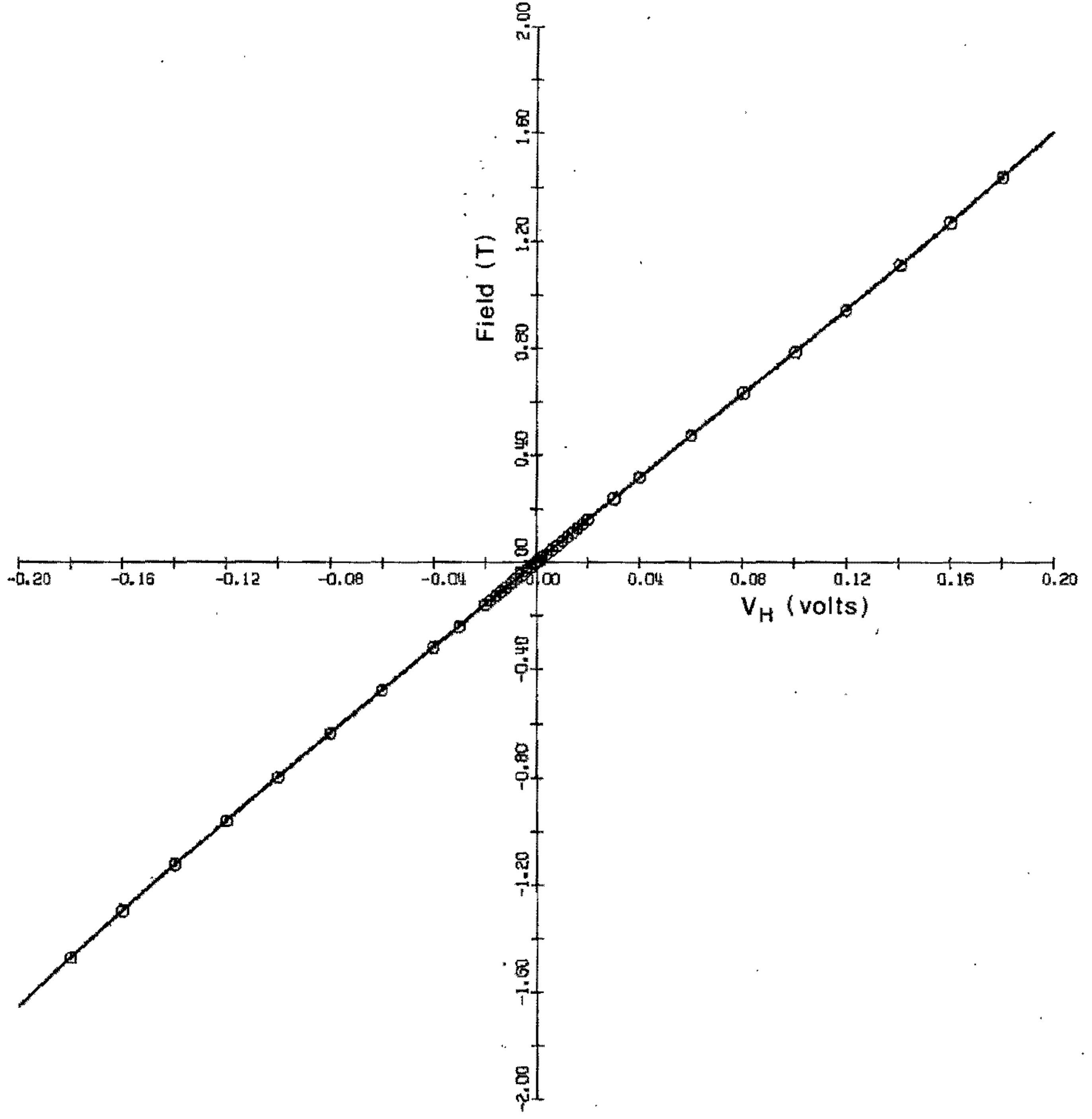


Fig. 1



Field (T)
2.00
1.60
1.20
0.80
0.40
.00
-0.40
-0.80
-1.20
-1.60
-2.00
-0.20
-0.16
-0.12
-0.08
-0.04
0.00
0.04
0.08
0.12
0.16
0.20
V$_H$ (volts)

Fig. 2

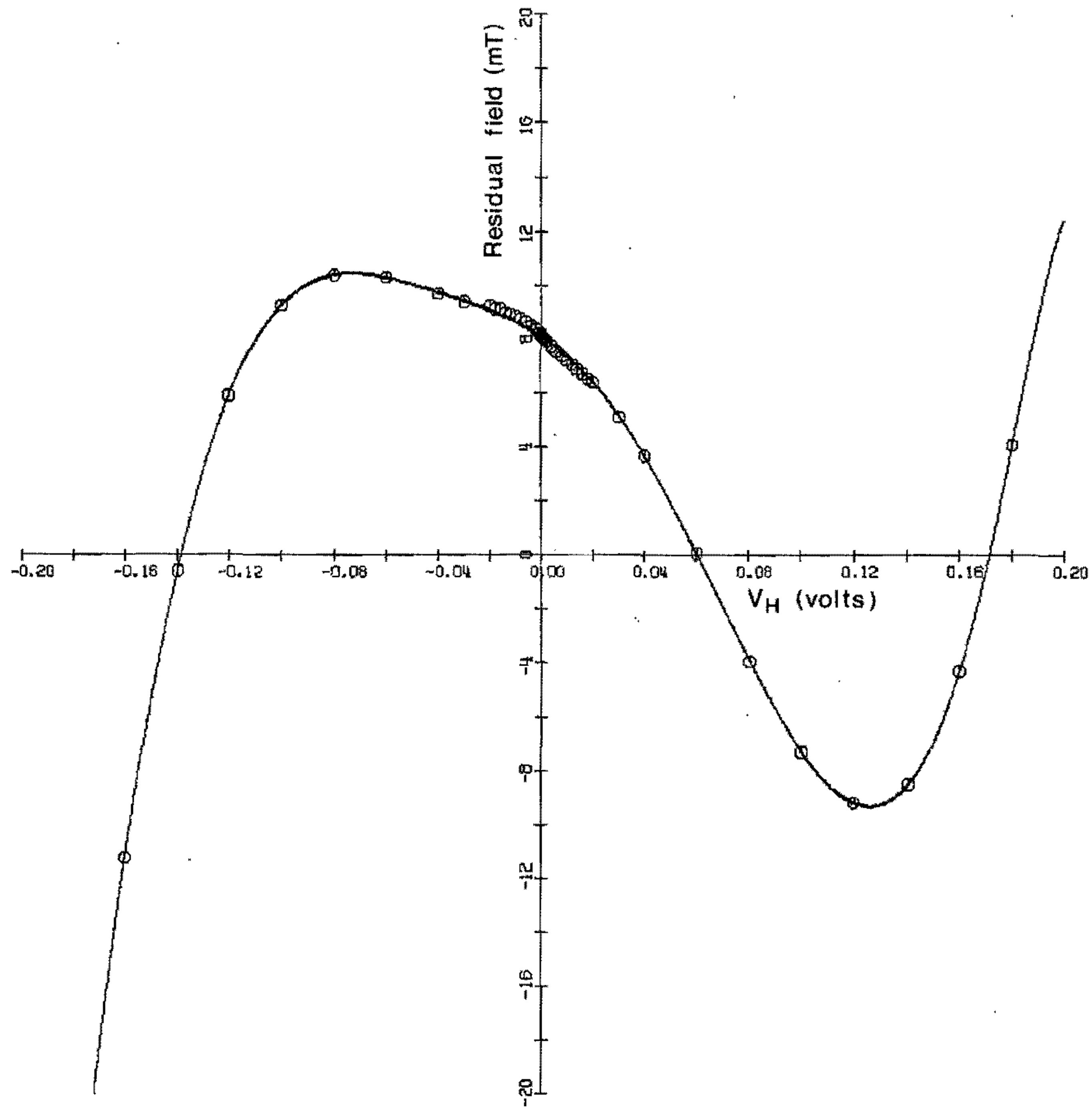

Residual field (mT)
20
16
12
8
4
0
-4
-8
-12
-16
-20
-0.20
-0.16
-0.12
-0.08
-0.04
0.00
0.04
0.08
0.12
0.16
0.20
V_H (volts)

Fig. 3

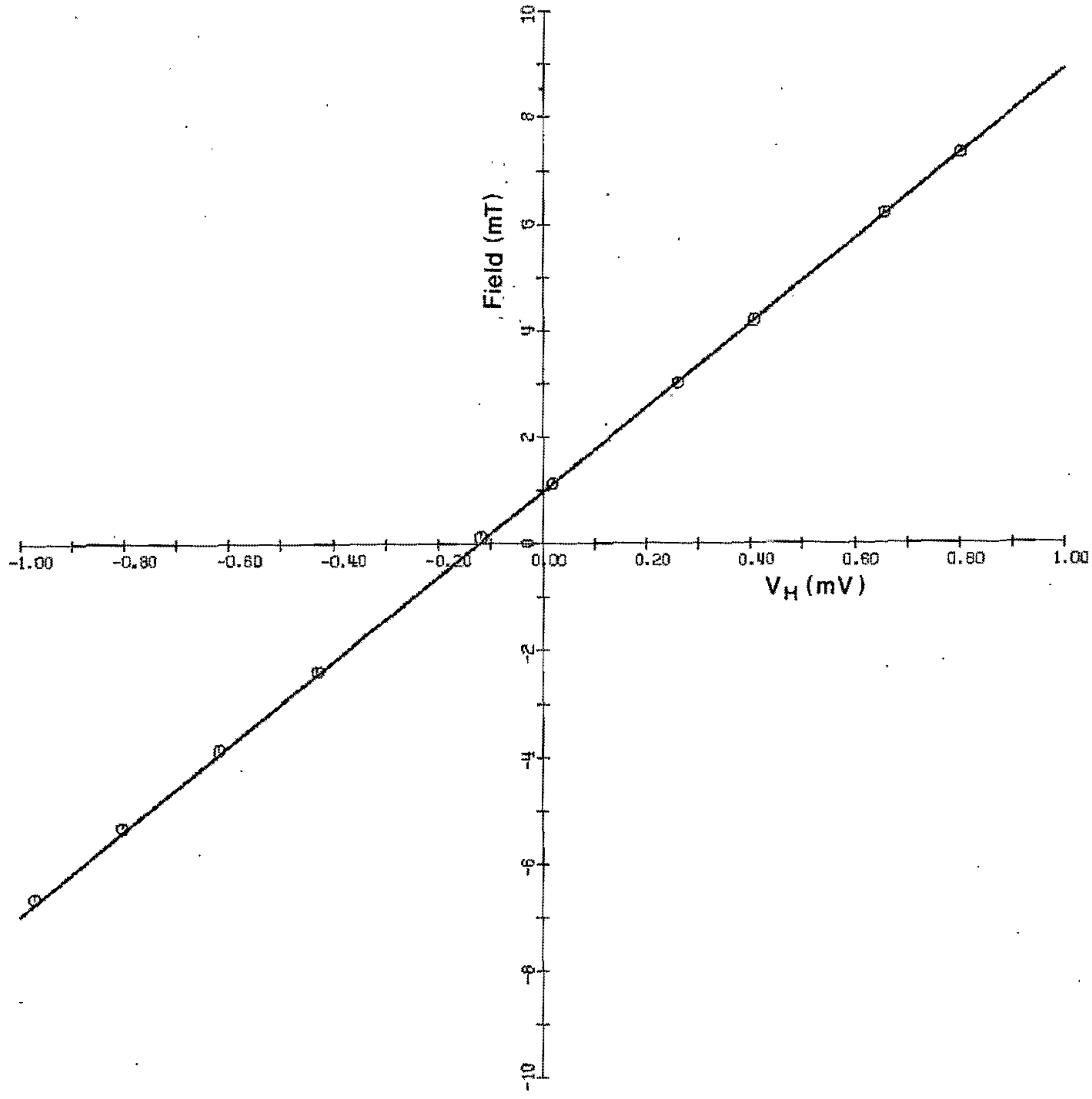
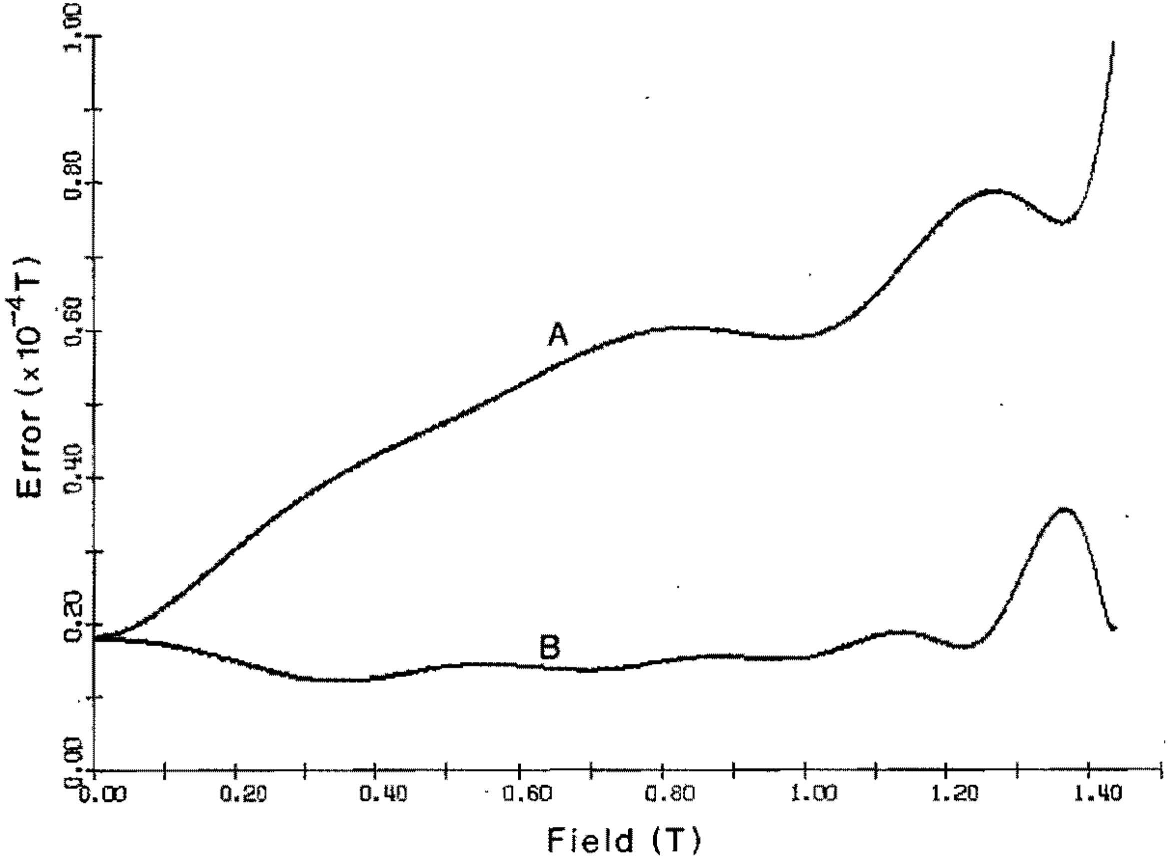

Field (mT)
10
8
6
4
2
0
-2
-4
-6
-8
-10
-1.00
-0.80
-0.60
-0.40
-0.20
0.00
0.20
0.40
0.60
0.80
1.00
V_H (mV)

Fig. 4



1.00
0.80
0.60
0.40
0.20
0.00
Error (×10⁻⁴T)
A
B
0.00
0.20
0.40
0.60
0.80
1.00
1.20
1.40
Field (T)

Fig. 5