100637-1002

DL/SCI/TM29A

technical memorandum Daresbury Laboratory

DL/SCI/TM 29 A

PMU3D - A PROGRAM FOR THREE DIMENSIONAL FIELD CALCULATIONS ON PERIODIC PERMANENT MAGNET SYSTEMS.

by

M.W. FAN, Atomic Energy Institute, Peking[†]

and

M.W. POOLE and R.P. WALKER, Daresbury Laboratory

NOVEMBER, 1981

[†]Visiting Scientist

LENDING BOX

Science & Engineering Research Council

Daresbury Laboratory

Daresbury, Warrington WA4 4AD

1. INTRODUCTION

A special purpose computer program has been written for designing periodic magnets composed of an array of permanent magnet blocks such as the one illustrated in fig.1. Such magnets are the basis of devices known as undulators and free electron lasers which are being studied at Daresbury[1], the former for possible inclusion in the Synchrotron Radiation Source and the latter in connection with the UK free electron laser proposal (a Daresbury/Glasgow/Heriot-Watt collaboration).

Permanent magnets offer many advantages over electromagnets for these types of device, particularly when short periods are required. This is largely a result of the availability of rare earth cobalt material with high remanent field, typically in the range 0.8–1.0 T. This material has an easy axis of magnetisation and permeability very close to unity both parallel and perpendicular to this direction and as a result can be represented mathematically by a set of surface currents around a volume in which there is uniform magnetisation. This property means that linear superposition applies and that the magnetic field distribution can be calculated much more simply than for electromagnets or other types of permanent magnet, and it is for this reason that a new program has been written rather than making use of existing codes.

In this report the mathematical basis of the program is described and the results of various tests to confirm its correct operation with a realistic magnet geometry are presented, including comparison with an earlier 2D program which had been checked against a 2D analytic formula[1,2]. A comprehensive user guide is also included.

2. MATHEMATICAL BASIS OF PMU3D

The magnetic field generated by a current I flowing in an element $\underline{dl}$

can be expressed as follows (in SI units):

$$\underline{dB} = \frac{\mu_o}{4\pi} I \frac{\underline{dl} \wedge \underline{r}}{r^3} \qquad (1)$$

where $\underline{r}$ is the vector from the current element to the field point. In the case of a permanent magnet the magnetisation, $\underline{M}$, produces an equivalent current density, $\underline{J}$, given by

$$\underline{J} = \underline{\nabla} \wedge \underline{M}$$

and so,

$$I\ \underline{dl} = \underline{J}\ dV = (\underline{\nabla} \wedge \underline{M})\ dV$$

where dV is an element of volume. In the present study the magnetisation is assumed to be uniform within each block so that the calculation involves surface currents only. In addition it is also assumed in the following that each block has its faces parallel to axes of a cartesian coordinate system, which is the usual case in the type of periodic magnet that will be studied using PMU3D. In this case

$$I\ \underline{dl} = -(\underline{M} \wedge \underline{dA}) \qquad (2)$$

where $\underline{dA}$ = (dydz, dxdz, dxdy) is a vector normal to a surface of the block and pointing inwards, and hence from (1) and (2)

$$\underline{dB} = -\frac{\mu_o}{4\pi} \frac{(\underline{M} \wedge \underline{dA}) \wedge \underline{r}}{r^3}$$

Using a standard vector identity the field due to a single surface of a block of material becomes

$$\underline{B} = -\frac{\mu_o}{4\pi} \iint \frac{(\underline{r} \cdot \underline{M})\ \underline{dA} - (\underline{r} \cdot \underline{dA})\ \underline{M}}{r^3}$$

For example, the vertical component of field due to a block of material in which the magnetisation has components only in the y and z directions can be written

$$B_y = -\frac{\mu_o}{4\pi} \left[M_z \iint \frac{z\ dx\ dz}{r^3} - M_y \iint \frac{x\ dy\ dz}{r^3} - M_y \iint \frac{z\ dx\ dy}{r^3} \right] \qquad (3)$$

In the above formula only those faces with positive components of $\underline{dA}$ have been included; a similar set of integrals must be evaluated for the three opposite faces for which the components of $\underline{dA}$ are negative.

Equation(3) can be generalised to give each component of field due to a block of material with arbitrary magnetisation direction:-

$$B_i = -\frac{\mu_o}{4\pi}\sum_{\substack{j=1\\ j\neq i}}^{3}\left[M_j\iint\frac{r_j\,dA_i}{r^3} - M_i\iint\frac{r_j\,dA_j}{r^3}\right] \qquad (4)$$

where again only three of the faces of the block have been included in the summation.

In general there are a total of eight double integrals to be evaluated for each component of field and these are of two types, which can be solved by use of standard integrals:-

Type 1
$$\iint\frac{r_j\,dA_i}{r^3} \equiv \int_{r_{j1}}^{r_{j2}}\int_{r_{k1}}^{r_{k2}}\frac{r_j\,dr_j\,dr_k}{r^3} \qquad \text{where } i\neq k\neq j$$

$$= \int_{r_{k1}}^{r_{k2}}\left[-\frac{1}{r}\right]_{r_{j1}}^{r_{j2}} dr_k = -\Big[\ln(r_k + r)\Big]_{r_{j1}r_{k1}}^{r_{j2}r_{k2}}$$

noting that $r = (r_i^2 + r_j^2 + r_k^2)^{1/2}$. The limits in the last expression are evaluated as follows:

$$\Big[F(x,y)\Big]_{x_1y_1}^{x_2y_2} \equiv F(x_2,y_2) + F(x_1,y_1) - F(x_1,y_2) - F(x_2,y_1)$$

Type 2
$$\iint\frac{r_j\,dA_j}{r^3} \equiv \int_{r_{i1}}^{r_{i2}}\int_{r_{k1}}^{r_{k2}}\frac{r_j\,dr_i\,dr_k}{r^3} \qquad \text{where } i\neq k\neq j$$

$$= r_j\int_{r_{k1}}^{r_{k2}}\left[\frac{r_i}{(r_j^2 + r_k^2)r}\right]_{r_{i1}}^{r_{i2}} dr_k = \left[\tan^{-1}\frac{r_i r_k}{r_j\,r}\right]_{r_{i1}r_{k1}}^{r_{i2}r_{k2}}$$

The final expression is therefore

$$B_i = \frac{\mu_o}{4\pi}\sum_{\substack{j=1\\ j\neq i}}^{3}\left\{M_j\Big[\ln(r_k + r)\Big]_{r_{j1}r_{k1}}^{r_{j2}r_{k2}} + M_i\left[\tan^{-1}\frac{r_i r_k}{r_j\,r}\right]_{r_{i1}r_{k1}}^{r_{i2}r_{k2}}\right\} \qquad (5)$$

In an initial version of the program the integrals were approximated by dividing each surface into a number of strips, allowing an easy numerical solution. This alternative gave identical results to the method described above when five or more strips were used and so provided a test of the satisfactory operation of the program, in addition to the study of a realistic magnet geometry described in the next section.

3. RESULTS OF TESTS USING PMU3D

As a first test that the program generates accurate solutions in three dimensions calculations were carried out on a single square block. The block geometry and the results for the vertical component of field, B_y, are shown in fig.2. The field is symmetrical with respect to the x and z axes and has the expected distribution, confirming that the contributions from all the block faces have been correctly computed.

The geometry chosen for the tests on an array of blocks is shown in fig.1. This is adequate to demonstrate the operation of the program but in any practical magnet there would be many more periods. In all the calculations the blocks have a cross section of 20 × 20 mm^2 and the gap is

25 mm. The directions of magnetisation are as indicated. The results from PMU3D were first compared with those from the 2D program: it was shown in the limit of very large widths, w, that the programs gave identical results. The tests described below were then carried out for smaller widths. Results are presented for the vertical component of field (B_y) only, since this component determines the major features of the trajectory of an electron beam passing along the axis of the magnet.

Figure 3 shows the field distribution along the y axis for different block widths. It can be seen that the field is roughly sinusoidal as expected and that the field amplitude increases as the width is increased, approaching the limiting value for infinite width given by the 2D program. Figure 4 shows the field amplitude, B_o, as a function of width in more detail and it can be seen that there are small but significant differences between the 3D and 2D results even at large widths, for example the difference in field being 2% at w=170 mm.

In the next two figures the field variation along the x axis is examined. It is this variation which is of major concern in determining the width required for a practical magnet. Figure 5 shows how the field homogeneity directly under a pole (z=0) improves as the block width increases. In fig.6 the field variation is shown for different z positions along the magnet. The distributions are similar, indicating that the homogeneity for z=0 is characteristic of the magnet as a whole.

These results demonstrate that good correspondence with the 2D program can be achieved in the limit of large widths and that consistent and believable solntions are obtained for realistic 3D magnet geometries. Therefore the program is judged to be a reliable tool for the design of such magnet systems. The amount of computer storage and execution time required is

minimal, enabling a design to be optimized very easily and efficiently. More comprehensive results with variation of magnet parameters such as period, gap and block height will be described in a future report.

4. USER GUIDE

The program is intended for the design of plane periodic permanent magnet systems so a number of simplifying assumptions have been made, for example that the blocks are all of the same dimension and that the magnetisation directions rotate from block to block in the usual fashion (see fig.1). In the version of the program described only the vertical component of field is calculated and details only of the top array of blocks need be specified. A feature of the program is the ability to divide the blocks into sections of different height along the x direction, as in fig.7, for the purpose of improving the field quality. This novel shimming technique will be described in a subsequent report.

4.1 Table of Input Parameters

The following shows the organisation of the input data on different cards. The data are given in free format. All dimensions are in millimetres.

NB	NP	NS	NSYM			1 card
AM	ZMINI	ZMAXI	ANG			1 card
XMINI	XMAXI	YMINI	YMAXI			NS cards
NOUT	NX	NY	NZ			1 card
XMIN	XMAX	YMIN	YMAX	ZMIN	ZMAX	1 card

The meaning of the various parameters is as follows:

NB total number of blocks (top array only, maximum = 100)

NP number of blocks per period

NS number of sections in the x direction (maximum = 5)

NSYM calculate field due to: NSYM=1 top array of blocks only

NSYM=2 top and bottom arrays of blocks

AM remanent field of the material (Tesla)

ZMINI, ZMAXI minimum and maximum positions occupied by the magnet in the z direction.

ANG angle between the magnetisation direction (in the y-z plane) and the y axis for the first block (face at ZMINI).

XMINI, XMAXI, YMINI, YMAXI define the size of each section of the blocks.

NOUT determines the format in which the results are printed:

=1 prints results for x-y plane at different z values.

=2 prints results for y-z plane at different x values.

=3 prints results for x-z plane at different y values.

For each plane a maximum array size of 50 × 15 points is permitted.

NX, NY, NZ number of equi-spaced field points to be calculated in each direction.

XMIN, XMAX, YMIN, YMAX, ZMIN, ZMAX defines the region in which the magnetic field is calculated.

4.2 Example

The following are the input cards needed for the geometry of fig.7, assuming two arrays of blocks and a remanent field of 0.95 T:-

```
 18      4       3      2
  0.95 -120.0  120.0   0.0
-80.0   -40.0   12.5  25.0
-40.0    30.0   12.5  15.0
 30.0    50.0   12.5  20.0
  1      10      5      6
-100.0  100.0  -50.0  50.0  0.0  10.0
```

The program will print 50 field values for the x-y plane at six different z values. An accompanying map of field homogeneity is also given in each case.

5. CONCLUSIONS

A program for designing periodic permanent magnet systems consisting of rare earth cobalt blocks has been developed and fully tested on a range of magnet geometries. Accurate three dimensional field calculations as required for practical designs of magnets for undulators and free electron lasers can be performed simply and very efficiently. The program has already been used in the study of an undulator suitable for the Synchrotron Radiation Source and of a possible magnet system for a proposed free electron laser project. Although the program was designed specifically for use with periodic magnets the field computation routines have been written in a completely general way allowing a more general purpose program to be written easily if required, and some development work is continuing on this topic.

6. ACKNOWLEDGEMENTS

One of the authors (MWF) wishes to thank Dr. D.J. Thompson and his staff in the SRS Division for their hospitality and assistance during his stay at the Laboratory.

7. REFERENCES

1. M.W. Poole and R.P. Walker, Proc. 7th Int. Conf. on Magnet Technology (MT-7), Karlsruhe, 1981 published in IEEE Trans. on Magnetics, Volume MAG-17, No.5, (1981) 1978.

2. R.P. Walker, PMU2D program (unpublished).

FIGURE CAPTIONS

Fig.1 Basic geometry of a periodic magnet.

Fig.2 Relative field distribution in x-y plane for a single square magnet block.

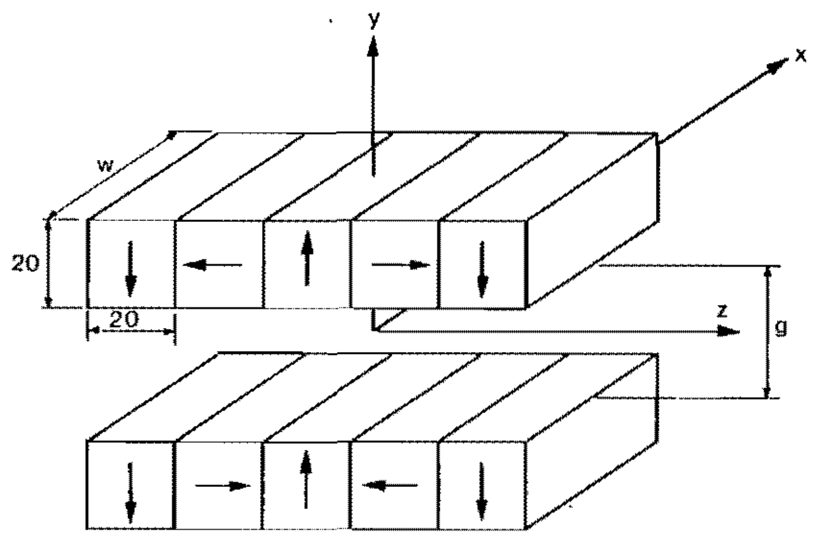
Fig.3 Field distribution along y axis for different block widths.

Fig.4 Variation of maximum field with block width.

Fig.5 Relative field distributions for different block widths.

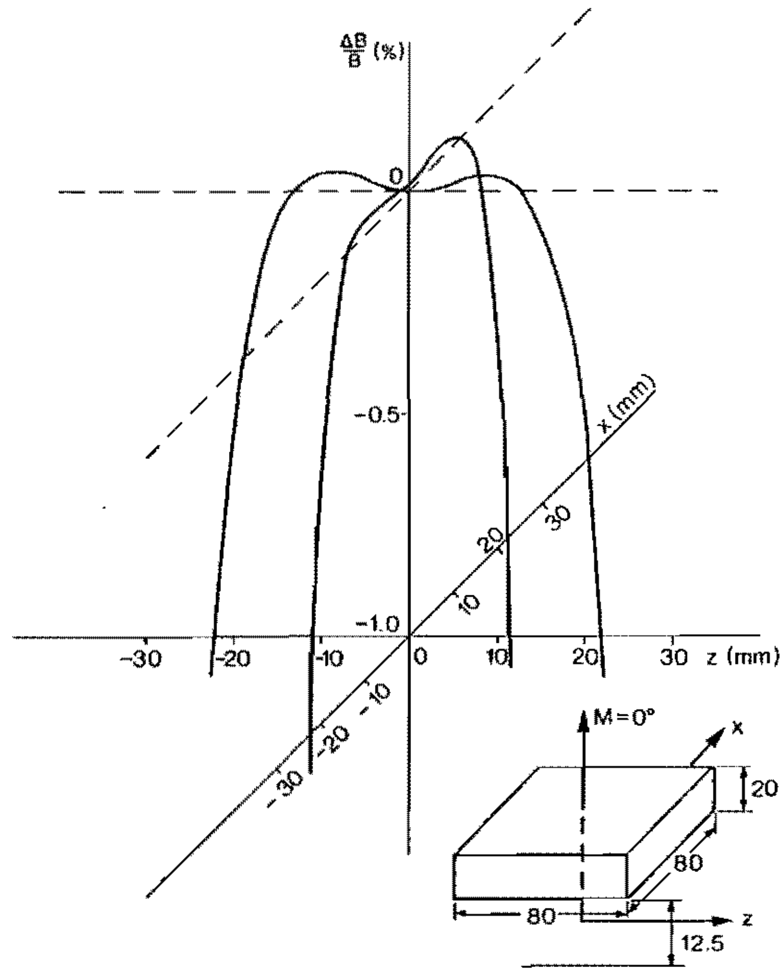
Fig.6 Field distributions for different y-coordinates.

Fig.7 Example of periodic magnet geometry than can be handled by PMU3D.



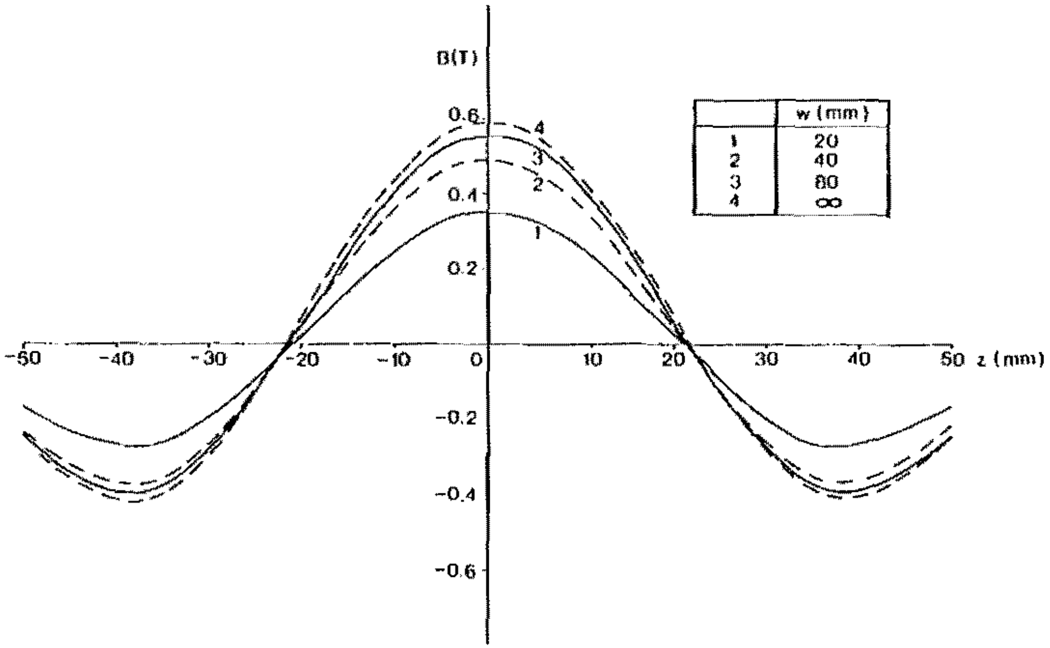
Basic geometry of a periodic magnet

Fig.1



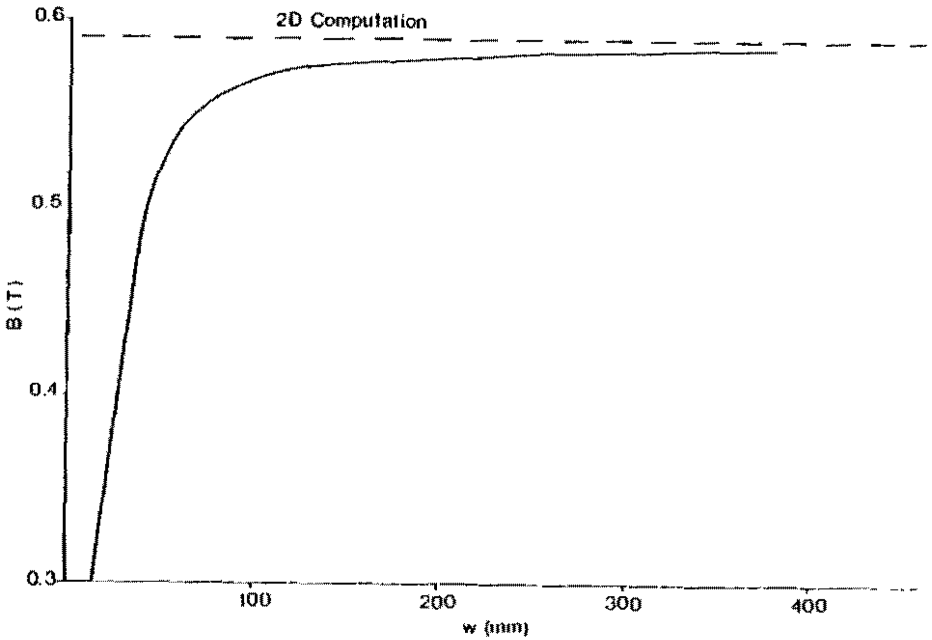
Relative field distribution in x - z plane for a single square magnet block

Fig. 2



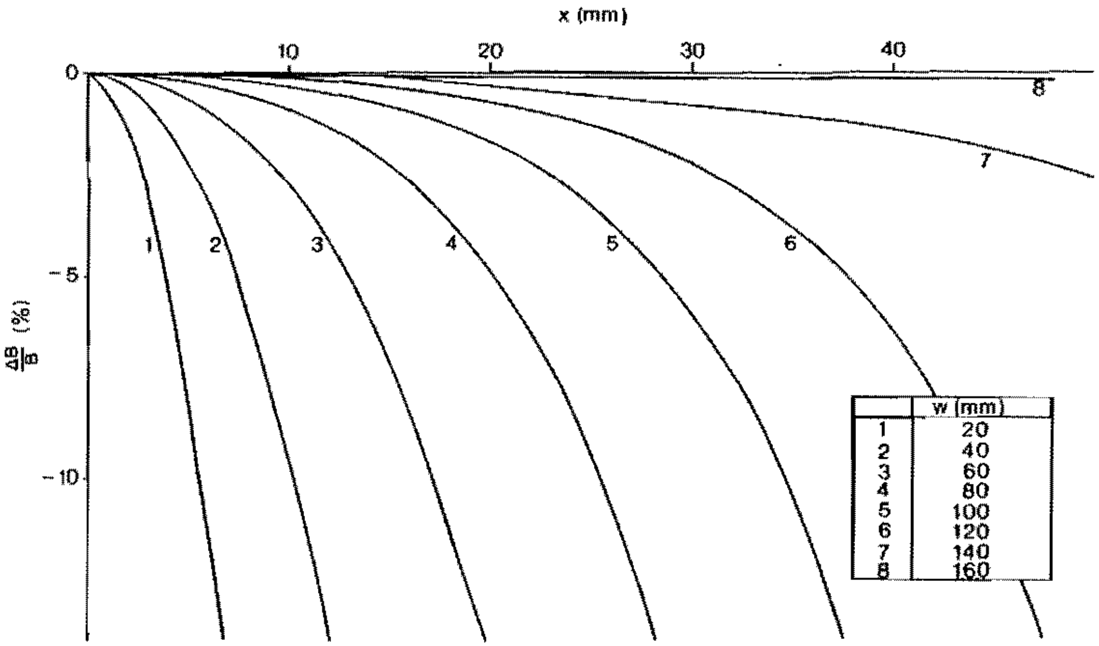
Field distribution along z axis for different block widths

Fig. 3



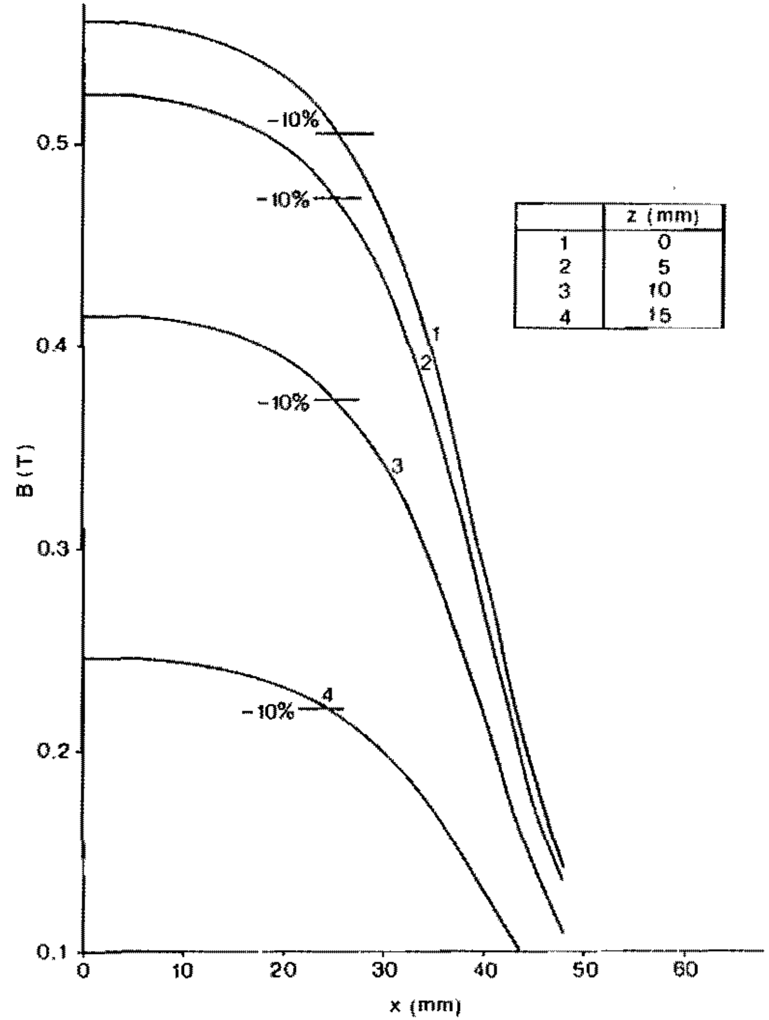
Variation of maximum field with block width

Fig. 4



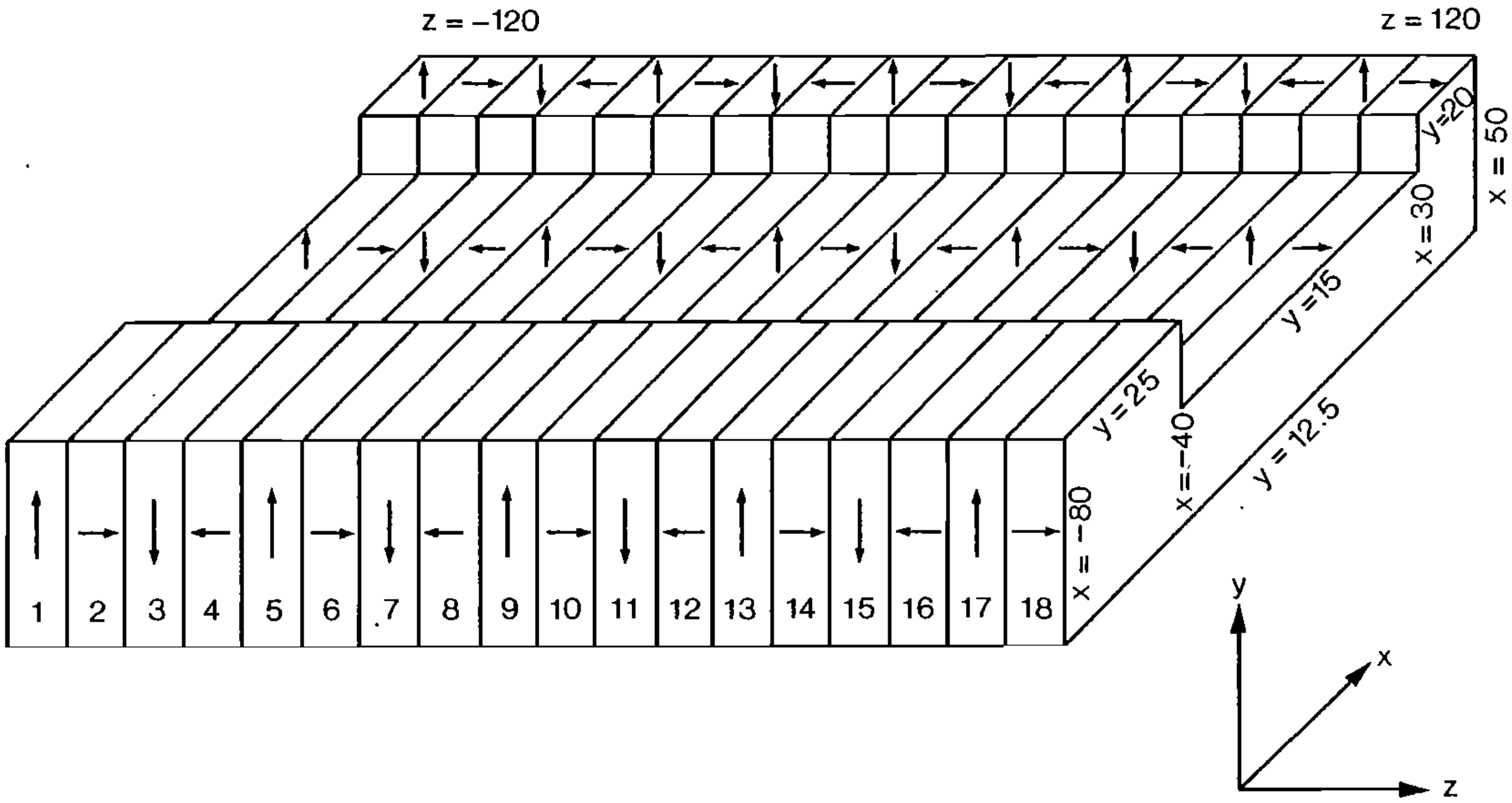
Relative field distributions for different block widths

Fig. 5



Field distributions for different z co-ordinates

Fig. 6



Example of periodic magnet geometry that can be handled by PMU3D

Fig.7

DL 43