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Design for Additive Manufacture (DfAM): The “Equivalent Continuum Material” for cellular structures analysis

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ABSTRACT

Additive manufacturing (AM) offers many advantages, including material savings, lightening, design freedom, function integration, etc. In the case of cellular materials, regular structures (lattice and honeycomb) are particularly important due to their ability to reduce weight. However, the design process and FEM analysis of this type of structure is very high time-consuming. In order to mitigate this problem, we propose a modelling method, called "Equivalent Continuum Material", based on the treatment of a cellular material as a continuous mass. This document describes the method and presents examples of applications, to facilitate and understand its use.

Keywords: AM, additive manufacturing, DfAM, design for additive manufacturing, cellular material, lattice, finite element analysis, lightweight structures, multiscale modeling, equivalent continuum

1. INTRODUCTION

Additive manufacturing (AM) enables parts to be obtained by a layer-by-layer conforming process from a 3D CAD file. Some of the advantages offered, and for which it is widely used in sectors such as aeronautics or the automotive industry, are material savings, lightweighting, parts almost in their final shape, design freedom, integration of functions, reduction of time to market, etc.

However, these capabilities must take account some features associated with the AM process itself, known as "Design for Additive Manufacturing" (DfAM). It considers specific structures implementable through AM such as technological limits (in terms of manufacturing processes), recommendations (regarding the orientation of the parts) and a long etcetera, which even today they have not been standardized.

In the specific case of cellular materials, in which weight reduction is important and even the integration of other functions (such as heat exchange), regular structures (lattice and honeycomb) take on special relevance since they are easily executable by means of AM and provide large advantages to the designs (multi-scale structures, in this case mesoscale) (Anón s. f.).

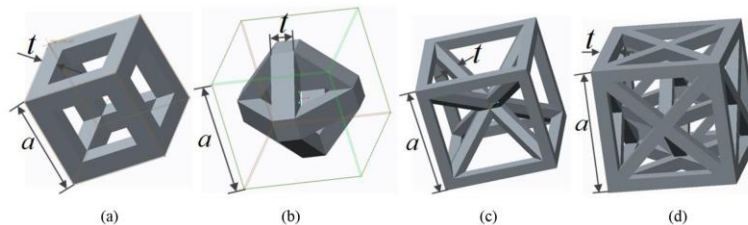


Figure 1.- Some samples of cubic lattice cellular structures (a- edge, b- face centered, c- body centered and d- octet).

Cellular structures have simple unit geometry, in most cases, and are repeated over a part of the volume of the component to be manufactured. However, the calculation time, associated both with the modelling of the geometry and with the FEM analysis, can be very high (Azman s. f.). This has become one of the major disadvantages of the use of cellular structures, the computational cost associated with designing and analyzing a part in the case of comparing several topologies and with different design parameters.

In order to provide a solution to the above problem, this document describes the “Equivalent Continuum Material” method, essentially consisting of modelling the lightweight structure (by means of cell structures) as a continuum material with equivalent mechanical properties to those of the cell structure (Equivalent Continuum Material) (Noor 1988).

2. METHODOLOGY

The method is based on the treatment of a cellular material as a continuous mass. The cellular unit is analyzed and its behavior is associated with an equivalent continuous orthotropic material. The process has basically the following steps (Jin, Li, y Zhang 2018):

- Modelling: Definition of the starting problem
- Typology: Designation of the type of the unit cell
- Simulation: of cells under known conditions
- Adjustment: of the coefficients to the appropriate material model
- Mapping: Application of the results obtained to the starting problem

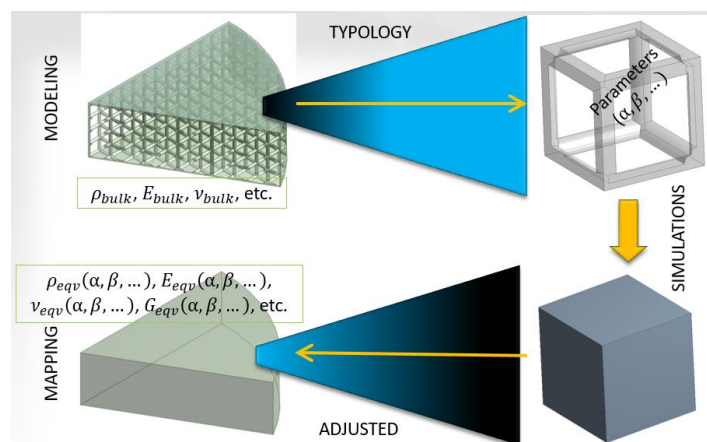


Figure 2.- Schematic diagram of the “Equivalent Continuum Material” method.

2.1 Modelling: Definition of the “geometric model” for our problem

It must be clear at all times that this modelling method can be applied to any type of structure or substructure that needs to be lightweighting, although for the sake of simplicity, this document presents as an example the case of a circular mirror sample for astronomical instrumentation. The effect of material change can also be analyzed, although again, in order to make it easier to understand, a single material is chosen.

To describe the method, we are focused on a flat circular mirror sample, of cylindrical substrate, with an external diameter of $\phi = 100$ mm and a height of $h = 16.667$ mm ($h/\phi = 1/6$). The mirror is held by three circular supports of 6 mm diameter, equispaced at 120° on a diameter of 64.54 mm (Arnold 1995) and with a height of 1 mm.

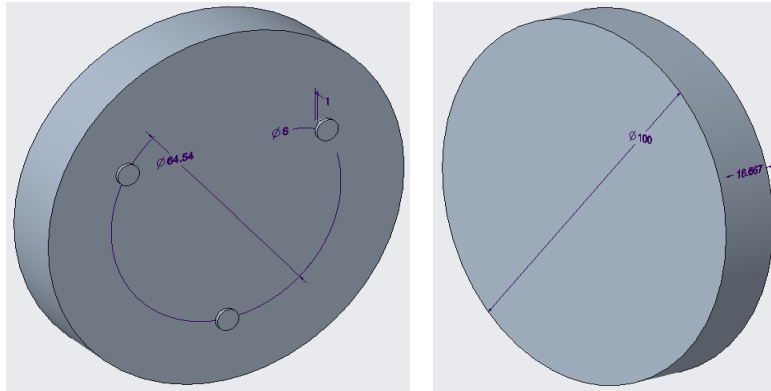


Figure 3.- General mirror geometry (right) and supporting points (left).

Although the method can be applied to any lightweight structure, its “raison d’être” is linked to making the most of the potential advantages of AM, so that the chosen materials are those that can be processed using AM technologies, and within these, mainly those that are applicable to the specific case under consideration (SLM, EBM and SLA).

From a previous trade-off to the establishment of the equivalent continuous material method, a series of aluminum alloys, titanium and ceramic materials were chosen as the most suitable for successful processing, amongst which we can highlight:

- Si-SiC (silicon carbide doped with silica) and Al_2O_3 (alumina) for SLA.
- AlSi10Mg, Al339, Scalmalloy® and Ti6Al4V for SLM.
- Ti6Al4V for EBM (Al alloys are not considered due to the high volatility of some of their constituent elements, which makes them difficult to process with this technology).

Special attention should be given to the inclusion of Zerodur as a consideration material, not because of its applicability for AM, but because there are many well documented examples that can be used as method control elements.

Material	E (GPa)	ν	ρ (g/cm ³)	YS (MPa)
Zerodur (reference)	90,3	0,24	2,53	31
Si-SiC	420,0	0,16	3,21	21
Al_2O_3	320,0	0,23	3,80	320
AlSi10Mg	71,0	0,33	2,67	248
Al339	78,8	0,33	2,69	317
Scalmalloy®	65,0	0,33	2,67	450
Ti6Al4V	113,8	0,34	4,43	880

Table 1.- Suitable materials for Astronomical Instrumentation AM mirrors. In Green, chosen material for the example in this document.

The potential of the method lies in its simple application to a wide variety of materials, unit cell topologies and cell parameters (basically aspect ratio), but in order to facilitate understanding and to illustrate an application, the aluminum alloy Al339 (see table 1) has been chosen as the material, with the following main mechanical properties of the base (raw) material:

- Density, $\rho = 2.69 \text{ g/cm}^3$
- Young’s Modulus, $E = 78.8 \text{ GPa}$
- Poisson’s Ratio, $\nu = 0.33$
- Yield Strength, $YS = 317 \text{ MPa}$

The load case used for the analysis and comparative study of results is the mirror sample under its own weight. For this, gravity ($g = 9.80665 \text{ m/s}^2$) is applied in the plane perpendicular to the optical surface (point A in figure 4).

It should be noted that, although other loading hypotheses such as polishing pressure have been applied, the results obtained with the equivalent model are in the order of magnitude of those of the detailed model. Moreover, with this method, it is not possible to duplicate the local behavior of the optical surface (quilting pattern), something that is reasonable since with the equivalent model a continuous material is used that does not replicate the local stiffness of the cellular structures. Consequently, it is preferred to focus the decision-making on the behavior under its own weight. Regarding the boundary conditions at the component supports (three circular supports B, C and D highlighted in figure 4), radial displacements have been left free, restricting both tangential and vertical displacements (perpendicular to the plane of the optical surface), $u_\theta = u_z = 0$.

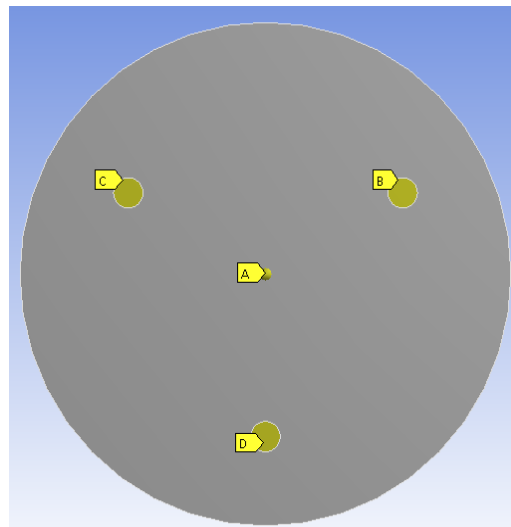


Figure 4.- Gravity Load application point (A) and supporting areas (B, C y D) where to establish the boundary conditions for the mirror sample.

2.2 Typology: Setting of the unit cell to be used, and its parameters

The next step consists of defining the unit cell to be used and the characteristic parameters that will allow us to generalize this cell. For the selection, we will take into account the conditions of use that our system will have. In our case, we have considered appropriate to use the cell in lattice with cubic envelope edge structure, and square section (figure 5), since its extrapolation to the extreme maximum fits the solid model, allowing the comparison of the results obtained (control case).

In order to generate any geometry, for a specific topology, the parameters that we are going to use to standardize the cell are: the cube main dimension (a) and the semi-side of the column section (t). In this way, we can define the aspect ratio (t/a), which, as will be seen below, will be fundamental in the characterization of the properties.

For the illustrative example, the three unit cell topologies (with square cross-section) shown in figure 5 have been used: simple cubic, face centered cubic (FCC) and octet.

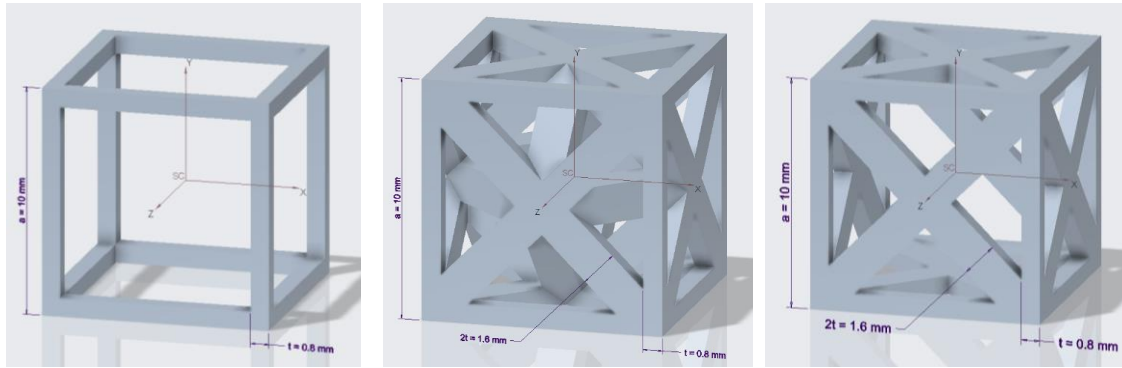


Figure 5.- Unit cells chosen for the example: cubic (left), FCC (center) and octet (right).

2.3 Simulation: Simulation of the unit cell to obtain the equivalent properties

Once the unit cell has been defined, and its fundamental parameters (in our case, t/a), as well as the mechanical properties of the bulk material, we can proceed to perform the simulations necessary to adequately characterize our orthotropic material. In order to do this, we are going to consider a unitary cell, under different load conditions and, varying its fundamental parameters, obtain the equivalent mechanical properties. In order to generalize the procedure, we have carried out all the simulations using ANSYS 2020 R1.

The description of the equivalent mechanical properties with respect to the bulk mechanical properties shall be as follows:

- Density; $\rho_{eq}(\tau) = \rho \cdot s(\tau)$
- Young's Module; $E_{eq}(\tau) = E \cdot \varepsilon(\tau)$
- Poisson's Ratio; $\nu_{eq}(\tau) = \nu \cdot \eta(\tau)$
- Shear Modulus; $G_{ref}(\tau) = \frac{E_{eq}(\tau)}{2(1+\nu_{eq}(\tau))}$ $G_{eq}(\tau) = G_{ref}(\tau) \cdot \gamma(\tau)$

Values for ρ , E and ν are those corresponding to the base material chosen for the study. For the sake of simplicity in the descriptive example, Al339 has been chosen, but for a more general case a wide variety of materials can be swept away (such as those presented previously in Table 1). Functions $s(\tau)$, $\varepsilon(\tau)$, $\eta(\tau)$ and $\gamma(\tau)$ are dependent on the aspect ratio of the unit cell ($\tau = t/a$) and are unique for each cell topology.

The main objective of the equivalent continuum method, and its subsequent uses, is to obtain the equivalent mechanical properties of the lattice structures. In the case of simple and regular cellular structures, the analytical forms of these properties can be obtained, according to the condition of deformation compatibility. However, for more complex structures, the calculation is complicated and finite element programming is needed.

Using energetic methods, in which it is assured that the deformation energy of the equivalent model is equal to that of the detailed model, the constants of the equivalent elastic constitutive relations of an orthotropic material can be obtained through nine different deformation states. If we consider the symmetry conditions of the cell, there are relations between these constants that allow us to reduce the calculation to three constants, Young's Module equivalent, Poisson's Coefficient equivalent and Shear Module equivalent. For the specific case of cubic envelope, these expressions can be associated to the aspect ratio (t/a).

For a given geometry, we can establish (analytically or numerically) the relative density as volume ratio, $s = \frac{\rho_{equ}}{\rho} = \frac{V}{V_{equ}}$.

To obtain Young's modulus and Poisson's coefficient we are going to test the cell for a known deformation in one direction (e.g. x-direction, 0.1mm). Bearing in mind the constitutive relations and the symmetry conditions, the constants mentioned can be obtained with the following expressions.

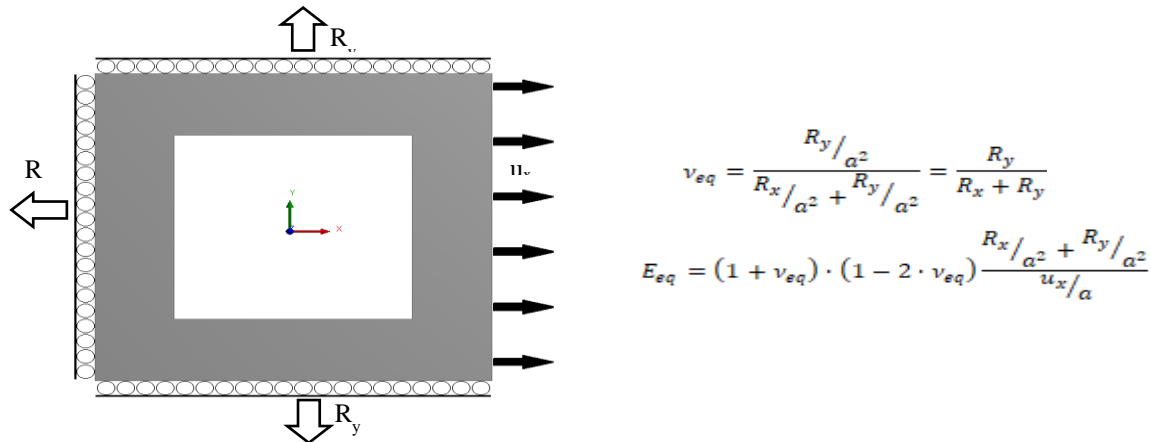


Figure 6.- Simulation and equations to obtain the equivalent Young's modulus and Poisson's coefficient

To get the shear module, we are going to test the cell for a well-known displacement in its own plane (of a free face), and fixing the opposite face (as shown below). Considering these conditions, the calculated shear modulus can be obtained by the expression showed:

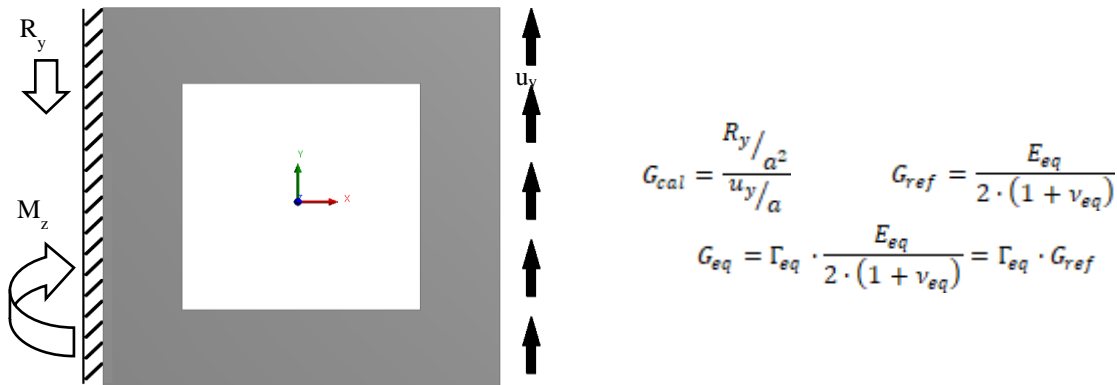


Figure 7.- Simulation and equations to obtain the equivalent Shear Module

For non-symmetric or rhombohedral cells, more complex equations can be developed to obtain the equivalent properties, but this point will not be further developed in this document.

2.4 Adjustment: Adjustment of the considered properties to suitable functions

Once the simulation is done and the data is stored, we can proceed with the adjustment. We can use multiple mathematical tools to adjust the mechanical properties with many types of functions. However, it seems that the most useful functions could be covered with:

- Potential functions, $f(x) = A \cdot x^B$
- Polynomial functions, $f(x) = A_0 + A_1 \cdot x + A_2 \cdot x^2 + A_3 \cdot x^3 + \dots$
- Rational functions, $f(x) = (A_0 + A_1 \cdot x + A_2 \cdot x^2 + \dots) / (1 + B_1 \cdot x + B_2 \cdot x^2 + \dots)$
- Piecewise functions, $f(x) = \begin{cases} f_1(x) & \text{if } (x < x_0) \\ f_2(x) & \text{if } (x \geq x_0) \end{cases}$

For comparing the detailed model with the equivalent one, the adjustment error in deformation energy has been considered to be less than 15%.

2.5 Mapping: Application of the parametric functions to our model

After obtaining the mechanical properties of the “Equivalent Continuum Material”, we will now study the response of our starting model. While there are a lot of magnitudes (results) that provide valuable information for drawing conclusions on the utility of the equivalent continuous material method, the parameters to be used for decision-making between the different models will be the mass (rather the percentage of lightweight) and the shape error of the optical surface (Doyle, Genberg, y Michels 2012). Other magnitudes such as strain energy, reactions or simulation/computation time are used to validate the reliability of the models, as well as to highlight their great potential in terms of time savings during the design and modelling phases of the parts (in this case, the mirrors for astronomical instrumentation).

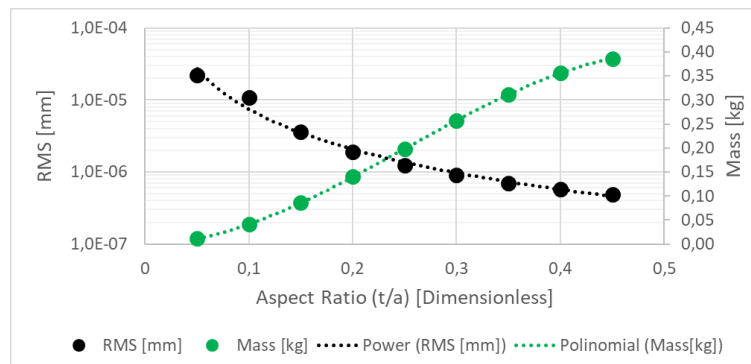


Figure 8.- Mapping the results of the mirror under its own weigh loading, for a cubic lattice structure.

3. RESULTS

As described throughout the document, the aim is to analyze the weight reduction of a circular flat mirror for astronomical instruments. The problem presents the following constraints:

- Geometry: $\phi = 100$ mm, $h = 16.667$ mm ($h/\phi = 1/6$).
- Supports: 3 of 6 mm, equispaced at 120° on a diameter of 64.54 mm.
- Material: Al339.
- Boundary conditions: free radial displacement and restricted tangential and vertical displacement.
- Load case: self-weight.
- Objective: to optimize the mass (M) and shape error of the optical surface (δ_{rms}).
- Compare the detailed models with the models obtained by the “Equivalent Continuum Material” method.
- Unit cells considered: simple cubic, FCC and octet.

In a general case, one could vary both the material and widen the unit cells to be considered, and even perform a scan of the aspect ratio, but for the sake of understanding, the problem has been limited in the example in order to simplify the analysis.

The potential of the method lies in the possibility to save having to do detailed models of all cell structure options in the preliminary design phases, which can be replaced by equivalent models with much more reduced computational time. After analyzing a wide range of structure options (varying material, unit cell and aspect ratio), the 2 or 3 most promising options would be chosen, which are the ones on which the detailed model would finally be made, having saved days, and even weeks for the final decision. In this case, detailed models must be made in order to be able to compare and check

the reliability of the method, and its advantages, as well as the limitations. For the aforementioned detailed models, the following surface thicknesses are to be defined, taking into account the cells considered (simple cubic, FCC and octet):

- Nominal aspect ratio, $\tau = 0.08$ (a single value is chosen for simplicity)
- Optical surface thickness, $t_{\text{opsu}} = 0.5 \text{ mm}$
- Outer perimeter thickness, $t_{\text{peri}} = 0.5 \text{ mm}$
- Rear cover thickness, $t_{\text{back}} = 0.5 \text{ mm}$
- Core height, $h_{\text{core}} = h - t_{\text{opsu}} - t_{\text{back}} = 15.667 \text{ mm}$
- Cell height, considering a stack of 3 unit cells, $a = h_{\text{core}}/3 = 5.222 \text{ mm}$
- Half width of the bars, $t = \tau a = 0.418 \text{ mm}$ (width $\Rightarrow 0.836 \text{ mm}$)

Values of these parameters are properly fixed, so there is as much similarity as possible between the detailed FEM and the equivalent FEM.

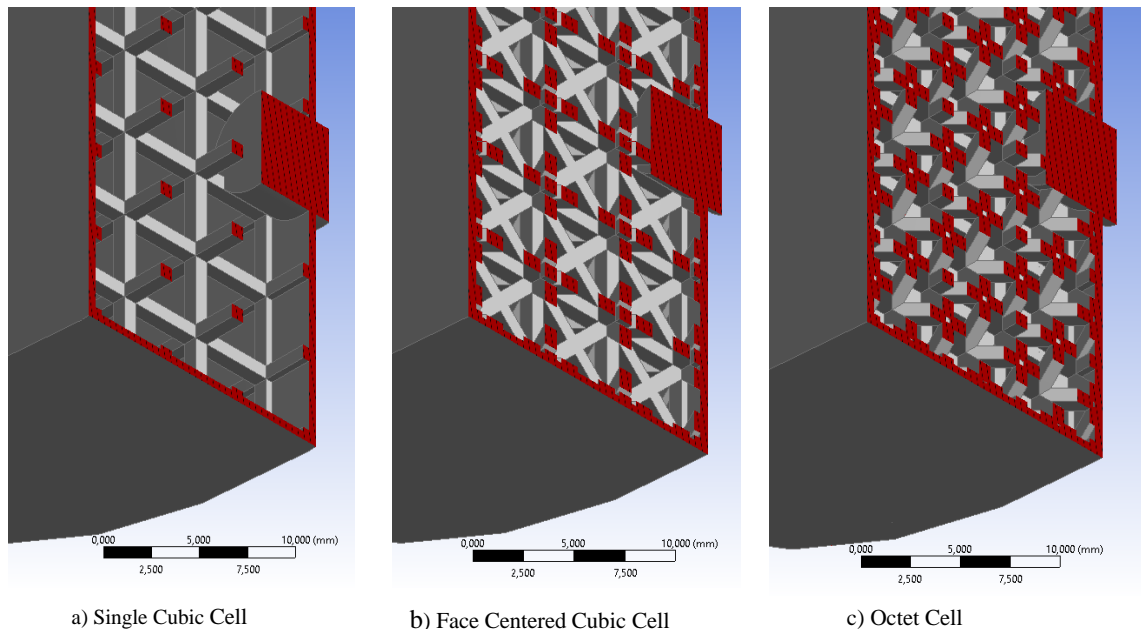


Figure 9.- Cross-section of the detailed models, where you can see the interior of the mirror with: Single Cubic (left), FCC (center) and Octet (right).

As has been commented, in order to limit the problem and simplify the explanation, only three unit cell topologies have been chosen (simple cubic, FCC and octet), a single material (Al339), and we will adopt a single value for the aspect ratio.

Steps to be followed are:

1. Establishment of the value for $\tau = t/a = 0.08$.
2. With this value for the aspect ratio, obtain the corresponding values of the functions $s(\tau)$, $\varepsilon(\tau)$, $\eta(\tau)$ and $\gamma(\tau)$ for the three unit cell topologies (example for the simple cubic cell in figure 10).
3. Knowing that the material is Al339, we have the values of ρ , E and ν (see table 1).
4. The values obtained in points 2 and 3 are introduced into the equations of equivalent mechanical properties presented previously, obtaining in this case the values presented in table 2 for each of the topologies analyzed.
5. With the equivalent property values resulting from 4, FEM models are made as a solid part (see figure 11), so that both the generation, and the computation and extraction times of the model results are significantly reduced.

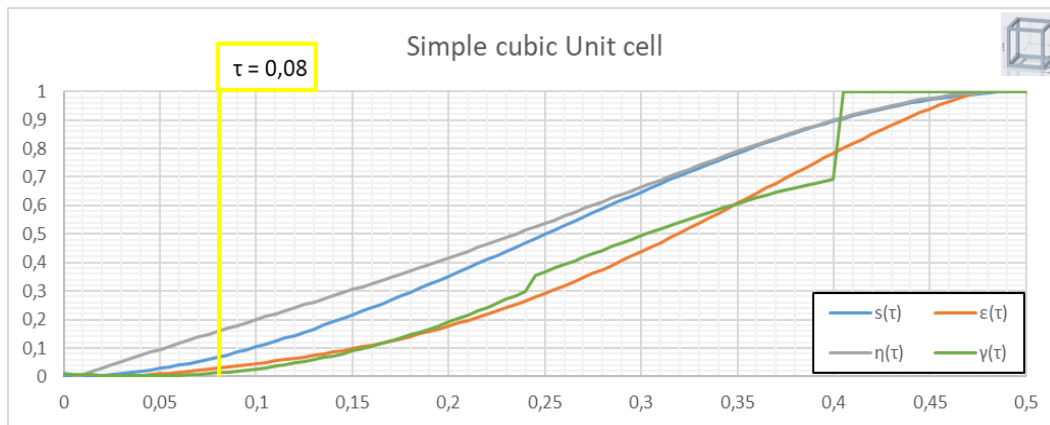


Figure 10.- Example for obtaining values of the functions $s(\tau)$, $\varepsilon(\tau)$, $\eta(\tau)$ and $\gamma(\tau)$ for $\tau = 0.08$.

It should be noted that more precise values are obtained from the $s(\tau)$, $\varepsilon(\tau)$, $\eta(\tau)$ and $\gamma(\tau)$ functions if, instead of entering the graphs, the value of τ is entered in the corresponding functions defined in parts, as presented below. Here it has been decided to use the graphical representation in order to ease understanding.




Material	$\rho_{eq} \text{ (g/cm}^3\text{)}$	$E_{eq} \text{ (GPa)}$	ν_{eq}	$G_{eq} \text{ (GPa)}$	
Simple cubic	0,185	2400,1	0,0523	15,2	
FCC	0,609	5578,1	0,2947	2126,7	
Octet	0,986	9259,8	0,3427	3894,5	

Table 2.- Equivalent mechanical properties of continuous material for unit cells: cubic, FCC and octet.

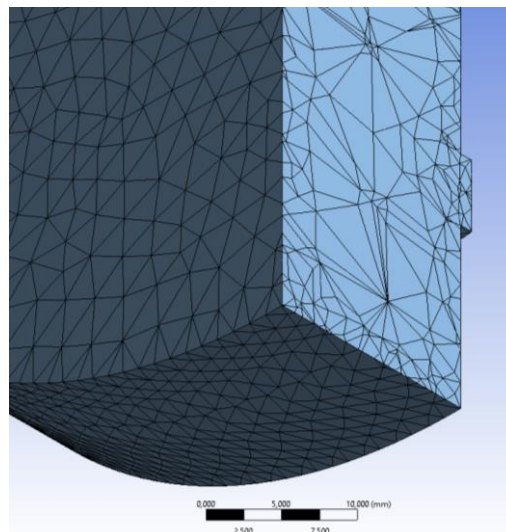


Figure 11.- Continuous Solid Meshing

Once the FEM analyses have been carried out, taking into account the mentioned aspects for the detailed and equivalent models, the results presented in table 3 are obtained.

Model	Type	Mass (g)	δ_{ave} (nm)	δ_{rms} (nm)	$M \cdot \delta_{rms}$ (g.nm)	Time (s)	ED (mJ)	$\frac{Time_{det}}{Time_{eq}}$
Cubic	Detailed	50,51	39,43	10,560	533,41	114	1,2492E-05	-
	Equivalent	24,17	96,87	30,379	734,40	67	3,9652E-04	2
FCC	Detailed	101,02	5,85	1,149	116,02	2072	3,1722E-06	-
	Equivalent	79,81	9,52	2,064	164,73	40	2,0990E-05	50
Octet	Detailed	139,51	4,51	0,997	139,13	3455	3,3862E-06	-
	Equivalent	129,15	6,59	1,864	240,68	29	1,9444E-05	120

Table 3 Detailed and equivalent model results for mirror on Al339 with $\tau = 0.08$.

Next figures show the displacements in z of the optical surface, comparing in each of them the detailed model with the equivalent one, for the simple cubic cells, FCC and octet respectively.

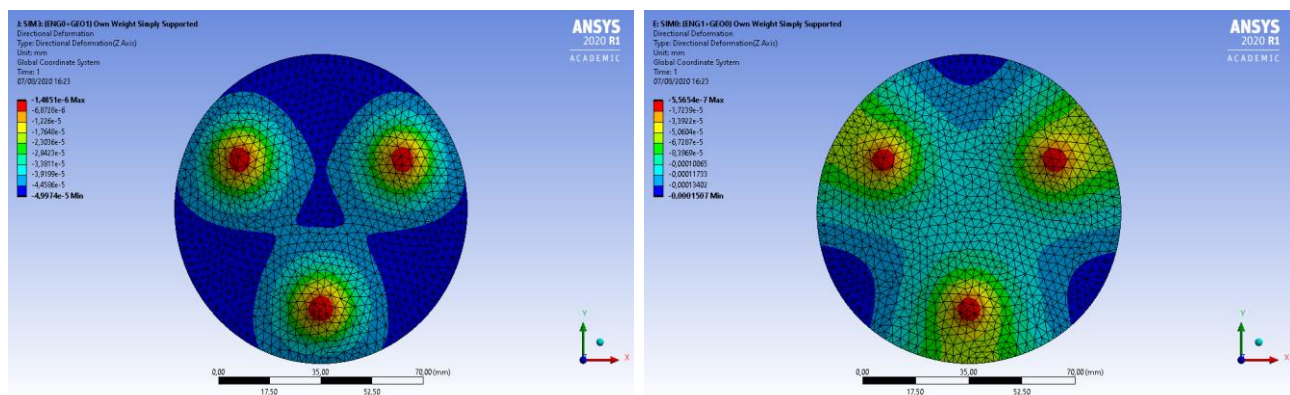


Figure 12.- Displacement in z (mm) of the detailed model optical surface (left) vs. equivalent (right). Simple cubic cell.

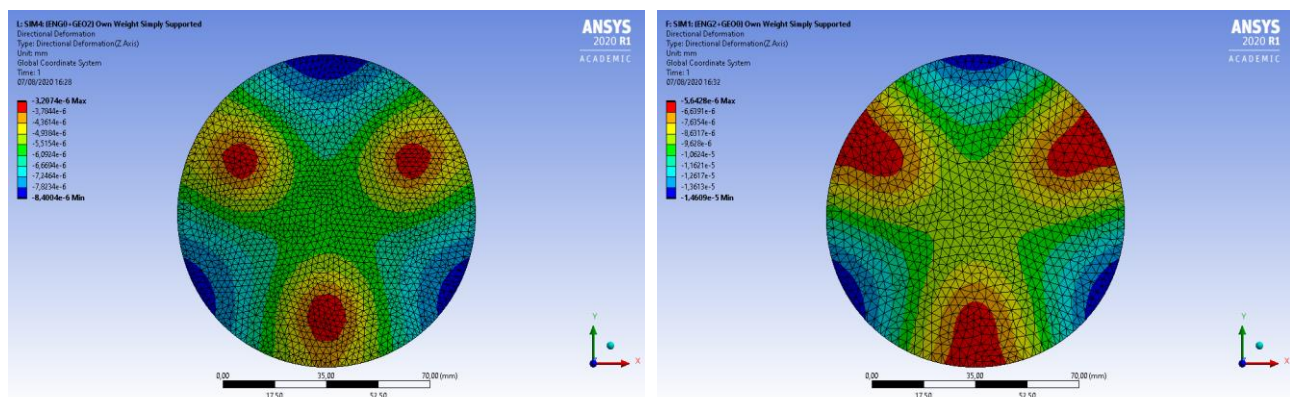


Figure 13.- Displacement in z (mm) of the detailed model optical surface (left) vs. equivalent (right). FCC cell.

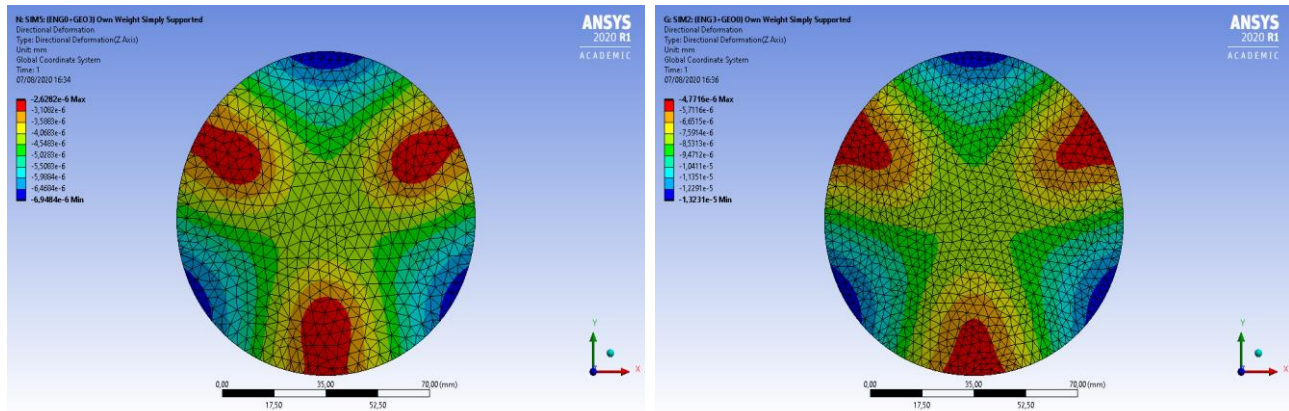


Figure 14.- Displacement in z (mm) of the detailed model optical surface (left) vs. equivalent (right). Octet cell.

Some of the conclusions that can be drawn from the analysis of the results presented in table 3 are the following:

1. It is a rather extreme lightweighting (aspect ratio closer to 0 than 0.5).
2. Obviously, the simpler the unit cell (in this case the fewer the bars) the lower the mass. The difference in values between the detailed model and the equivalent one is due to the fact that the solid surfaces of the former are not considered in the latter (because of the own definition of the cell). In this case, the volumes that are reduced in the equivalent are those of the optical surface, the lateral face and the rear cover.
3. The values of δ_{ave} and δ_{rms} are of the same order of magnitude comparing detailed and equivalent models. The behavior of the FCC and octet cells is similar in terms of optical surface error.
4. Analyzing the parameter $M \cdot \delta_{rms}$ (which we should remember that it evaluates both the light-weighting level and the optical surface shape error), we can see that the order from best to worst is FCC < Octet < Simple cubic, both for the detailed model and for the equivalent one, which is of vital importance for decision-making.
5. Strain energies are of the same order of magnitude comparing detailed and equivalent models.
6. The more complex the cell, the longer the computation time. It is important to be clear about the fact that this time is simply the resolution time of the FEM analysis, without including the additional time to generate the 3D part and export it to the FEM program. It is to say, in the case of the equivalent solid, the computation time is almost zero (without taking account the number of cell analyzed) while for detailed models, the time is very high and sometimes even impossible to perform. This makes the time difference even greater, making the use of the method presented in this paper even more meaningful.
7. The design conclusion of the study would be to use the FCC as a unit cell, adequately propagated along the corresponding volume of the area to be light-weighted of the mirror sample for astronomical instrumentation.

4. SUMMARY AND FUTURE WORKS

Following the above, we can conclude that the used method:

- reduces the complexity of the model, and with it,
 - reduces computation time
 - and increases the number of analyses available,
- in addition, it allows the use, partially or totally, in other DfAM techniques, such as
 - Topological Optimization (non-uniform distribution of lattice structures)
 - or Reverse Engineering (obtaining equivalent properties of manufactured parts),
- finally, it allows the qualitative study of the model, according to the parameters considered, easing the design decision.

Likewise, the framework proposed in the method makes it scalable, that is to say, it has the capacity to adapt to improvements and extensions in the method:

- Inclusion of complex typologies and combinations (stacking)
- Increase in simulations, associated with equivalent properties under study
- New definitions of adjustment functions, appropriate to the properties under study
- Expansion of results to be investigated.

Although this method can be applied to other fields of engineering that can be weight-relieved, the biggest potential is in the case of systems that require considerable lightening, together with the need for a channelling or internal circulation system that promotes heat exchange, in our specific case, cooled mirrors for astronomical instrumentation operating at any wavelength.

However, this method also has its disadvantages:

- do not include the effects of cell stacking,
 - although it could be included as a parameter in a complex typology
- local effects of lattice structures are not included (buckling or large deformations)
 - nevertheless, it could be dealt further, once the design decision has been made,
- in the simulations presented, the behaviour in tensions is not captured
 - meanwhile, equivalent properties could be implemented, with criteria to take into account behaviour in tensions,
 - or studies could be carried out on the raw cell

Taking into account the conclusions, the most interesting future lines could be,

- the application of this method to topological optimisation
- the use of other typologies and their parametric study
- inclusion of other equivalent properties, thermal properties (conductivity or expansion)

In summary, the great potential of the method consists in allowing a preliminary analysis of a model, varying both the material and the typology of the cell and the aspect ratio. As the computation time is much shorter than that associated with a detailed model, it allows fast decisions to be made and two or three candidates to be selected for the final solution, saving valuable engineering time (which can be in the order of days). The candidate models for the final solution should then be analyzed in detail in order to obtain the quantitative results of both strains and stresses.

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7. ANNEX I

Next figures present the graphics and the expressions of the corresponding functions $s(\tau)$, $\varepsilon(\tau)$, $\eta(\tau)$ and $\gamma(\tau)$ of the three types of unit cells used in this paper: simple cubic, FCC and octet.

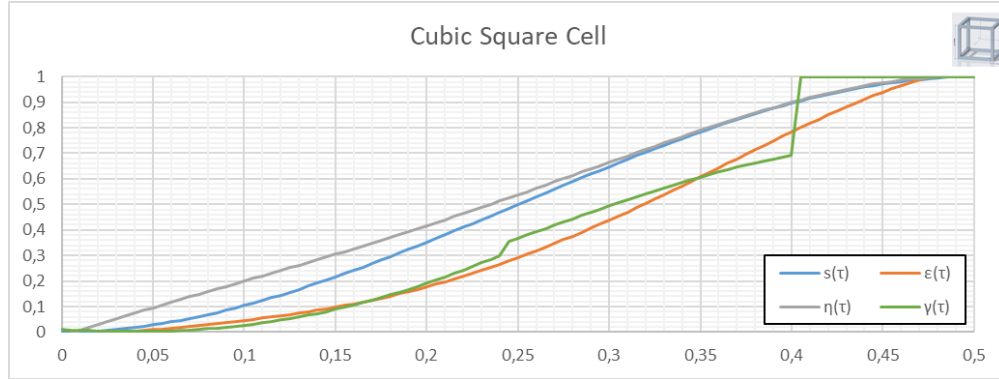


Figure 15.- Functions for obtaining equivalent mechanical properties. Simple cubic cell.

$$\begin{aligned}
 s(\tau) &= -16 \cdot \tau^3 + 12 \cdot \tau^2 \\
 \eta(\tau) &= -32,084 \cdot \tau^4 + 24,537 \cdot \tau^3 - 5,13 \cdot \tau^2 + 2,4725 \cdot \tau - 0,0177 \\
 \varepsilon(\tau) &= -51,842 \cdot \tau^4 + 44,267 \cdot \tau^3 - 7,1193 \cdot \tau^2 + 1,1285 \cdot \tau - 0,0348 \\
 \gamma(\tau) &= \begin{cases} CF(\tau) \cdot \Gamma(\tau) & \tau \leq 0,24 \\ \Gamma(\tau) & 0,24 < \tau \leq 0,40 \\ 1 & \tau > 0,40 \end{cases} \\
 CF(\tau) &= -15,121 \cdot \tau^3 + 9,1907 \cdot \tau^2 + 0,2568 \cdot \tau + 0,4964 \\
 \Gamma(\tau) &= -0,0664 \cdot \tau^4 - 18,8718 \cdot \tau^3 + 14,2584 \cdot \tau^2 - 1,0044 \cdot \tau + 0,0222
 \end{aligned}$$

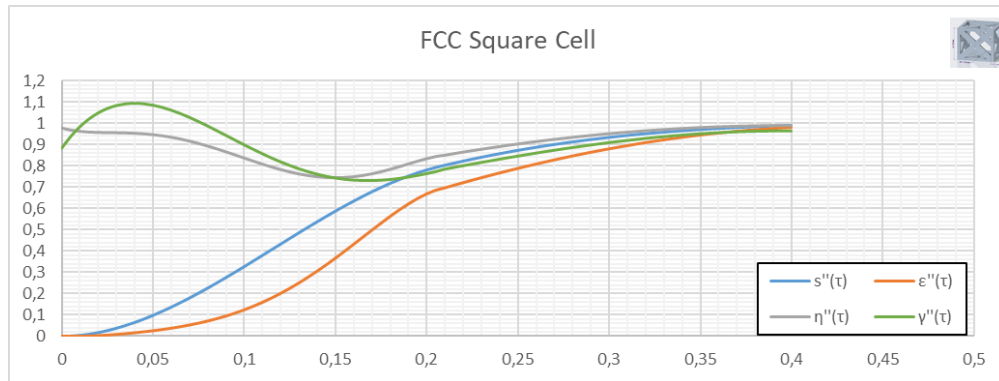


Figure 16.- Functions for obtaining equivalent mechanical properties. FCC cell.

$$\begin{aligned}
 s(\tau) &= \begin{cases} -131,88 \cdot \tau^3 + 45,941 \cdot \tau^2 & \tau \leq 0,2071 \\ 8 \cdot \tau^3 - 12 \cdot \tau^2 + 6 \cdot \tau & \tau > 0,2071 \end{cases} \\
 \varepsilon(\tau) &= \begin{cases} -8937,9 \cdot \tau^5 + 3418,9 \cdot \tau^4 - 371,05 \cdot \tau^3 + 27,206 \cdot \tau^2 - 0,30129 \cdot \tau & \tau \leq 0,2071 \\ -2,8863 \cdot \tau^3 - 2,8063 \cdot \tau^2 + 4,042 \cdot \tau & \tau > 0,2071 \end{cases} \\
 \eta(\tau) &= \begin{cases} -8937,9 \cdot \tau^5 + 3418,9 \cdot \tau^4 - 371,05 \cdot \tau^3 + 27,206 \cdot \tau^2 - 0,30129 \cdot \tau + 0,97847 & \tau \leq 0,2071 \\ -3,9063 \cdot \tau^2 + 3,1239 \cdot \tau + 0,36583 & \tau > 0,2071 \end{cases} \\
 \gamma(\tau) &= \begin{cases} -1812 \cdot \tau^4 + 1102,5 \cdot \tau^3 - 211,52 \cdot \tau^2 + 12,097 \cdot \tau + 0,88297 & \tau \leq 0,2071 \\ -9,8154 \cdot \tau^3 + 4,5004 \cdot \tau^2 + 1,0288 \cdot \tau + 0,46002 & \tau > 0,2071 \end{cases}
 \end{aligned}$$

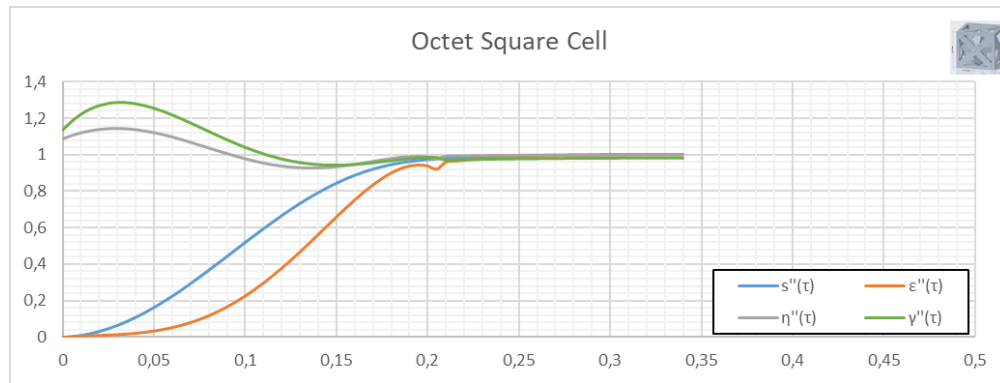


Figure 17.- Functions for obtaining equivalent mechanical properties. Octet cell.

$$\begin{aligned}
 s(\tau) &= \begin{cases} 6280 \cdot \tau^5 - 2727,3 \cdot \tau^4 + 125,43 \cdot \tau^3 + 55,818 \cdot \tau^2 + 0,45246 \cdot \tau & \tau \leq 0,2071 \\ -15,836 \cdot \tau^5 - 85,722 \cdot \tau^4 + 119,92 \cdot \tau^3 - 58,025 \cdot \tau^2 + 12,388 \cdot \tau & 0,2071 < \tau \leq 0,34 \\ 1 & \tau > 0,34 \end{cases} \\
 \varepsilon(\tau) &= \begin{cases} -8635,7 \cdot \tau^5 + 1143,2 \cdot \tau^4 + 307,38 \cdot \tau^3 - 18,382 \cdot \tau^2 + 0,7406 \cdot \tau & \tau \leq 0,2071 \\ -632,59 \cdot \tau^5 + 675,05 \cdot \tau^4 - 234,58 \cdot \tau^3 + 16,081 \cdot \tau^2 + 6,5119 \cdot \tau & 0,2071 < \tau \leq 0,34 \\ 1 & \tau > 0,34 \end{cases} \\
 \eta(\tau) &= \begin{cases} -11533 \cdot \tau^5 + 3916,7 \cdot \tau^4 - 86,754 \cdot \tau^3 - 69,181 \cdot \tau^2 + 3,9284 \cdot \tau + 1,0889 & \tau \leq 0,2071 \\ -62,345 \cdot \tau^4 + 74,621 \cdot \tau^3 - 33,567 \cdot \tau^2 + 6,7381 \cdot \tau + 0,48963 & 0,2071 < \tau \leq 0,34 \\ 1 & \tau > 0,34 \end{cases} \\
 \gamma(\tau) &= \begin{cases} -2824,6 \cdot \tau^4 + 1443,3 \cdot \tau^3 - 233,36 \cdot \tau^2 + 10,79 \cdot \tau + 1,1365 & \tau \leq 0,2071 \\ 4,4557 \cdot \tau^3 - 4,5583 \cdot \tau^2 + 1,5616 \cdot \tau + 0,80505 & 0,2071 < \tau \leq 0,34 \\ 1 & \tau > 0,34 \end{cases}
 \end{aligned}$$