A SIMPLIFIED MODEL FOR PREDICTING THE POLLUTION EXCHANGE COEFFICIENT OF SMALL TIDAL EMBAYMENTS

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Abstract. In recent years there has been a significant increase in the development and application of ever more sophisticated multi-dimensional models for solving the hydrodynamic and constituent transport equations which govern the flushing of pollution. However, advanced numerical modelling techniques can sometimes be augmented by alternative mathematical approaches which use simplified analytical solutions to predict the dispersion of contaminants. In the present article, a novel analytical tidal prism model is described for predicting the pollution flushing characteristics of small tidal embayments. The model relates the water quality response of the basin to the external forcing effects of the tide, the initial pollutant loading and the freshwater inflow rate. In general, the pollution flushing not only depends upon the geometry of the embayment and the tidal range but also on the proportion of effluent water which leaves the basin on an ebb tide, mixes with the surrounding coastal water and returns on subsequent flood tides. This effect has been taken into account in the mathematical model by the inclusion of a 'pollution-return' parameter. The analytical approach offers a viable and computationally inexpensive alternative to conventional multi-dimensional pollutant transport simulations and, more importantly, provides an increased understanding of the flushing characteristics of semi-enclosed tidal basins. The efficiency of the tidal flushing can be expressed in terms of the *pollution exchange coefficient* which measures the proportion of water exchanged with the sea each tidal cycle. To demonstrate the accuracy of the proposed mathematical formulation, analytical water quality predictions are compared against experimental pollution data from a 1:400 scale laboratory model of a generic square harbour. The results demonstrate that the analytical approach provides a simple and robust method of determining the water quality response of well-mixed tidal embayments.

Keywords: mathematical modelling, pollution flushing, tidal embayment, tidal prism, water quality

1. Introduction

The determination of the flushing characteristics of small tidal embayments and the associated water quality implications are particularly important when conducting environmental impact assessments of marina or harbour developments. One of the most important criteria when assessing the release of any pollutant into a semi-enclosed tidal embayment is to ensure there is adequate flushing between the basin and sea in order to prevent excessive deterioration of the water quality beyond pre-defined limits such as those laid down in the EC bathing water directive (European Community, 1976).

The residence time of the water in a tidal basin is of paramount importance in determining the overall water quality of the system. However, residence times are

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extremely difficult to measure in the field and estimates are usually obtained via mathematical modelling. In recent years there has been an increase in the use of sophisticated multi-dimensional models for solving the hydrodynamic and constituent transport equations which govern the flushing of pollution (Koutitas, 1988; Casulli and Cheng, 1992; Falconer, 1993; Wu and Tsanis, 1994; Li and Falconer, 1995; Rajar and Sirca, 1996; Fuentes *et al*., 1999). However, such models are time consuming to develop and may require high spatial resolution and small time-steps for stability. Consequently, advanced numerical modelling techniques are sometimes augmented by alternative mathematical approaches which use simplified analytical techniques to predict the dispersion of pollution.

One of the most popular methods of simplifying the pollution flushing problem is to adopt a 'zero-dimensional' mathematical approach based on the concept of repeated exchange of the inter-tidal volume. Zero-dimensional or tidal prism models have been used for many years to determine the pollutant levels in tidal systems. For example, Harleman (1966) referred to 'classical tidal prism theory' whilst Dyer (1973) presented the method without further references, inferring general familiarity with the technique. Callaway (1981), van de Kreeke (1983), Ozhan (1989) and Sanford *et al*. (1992) have proposed similar concepts for estimating the residence time of marinas whilst Hinwood and McLean (1996) presented a method of simulating complex estuaries using a cost-effective parametric modelling approach. The analytical model described in the present paper is based upon the Simplified Tidal Embayment Assessment Model (STEAM) developed by DiLorenzo *et al*. (1989, 1994) for predicting the water quality response and pollution susceptibility of small tidal basins. The STEAM tidal prism method was subsequently improved by DiLorenzo *et al*. (1995) to account for pollution which leaves the basin on an ebb tide and returns on the subsequent flood tide.

Rajar and Sajovic (1995) increased the flexibility of the tidal prism approach by extending the method to account for additional features such as pollution decay, background pollution levels in the exterior water body and discharges of polluted water into the embayment from the surrounding land. In addition, a novel analytical tidal prism method has recently been formulated by Wearing and Barber (1999) for estimating the water quality response of tidal embayments subjected to a continuous release of pollution. The present paper describes a useful extension of the analytical tidal prism approach for predicting the pollution exchange coefficient which can be used to identify tidal embayments at risk from potential water quality problems.

The most predictable mechanism governing the flushing of pollution from a tidal basin is the periodic rise and fall of the tide, although other factors such as wind, wave and density-driven circulation patterns may also play an important role. For example, the wind acting on the surface can force a three-dimensional circulation pattern within the embayment, or it can act on a larger scale by raising or lowering the coastal sea level beyond that of the normal astronomical tide. In addition, freshwater may enter the tidal basin from the land as either direct surface runoff or groundwater seepage. To simplify the model so that the water quality response can be predicted using an analytical technique, the present formulation considers the tide-induced and freshwater flushing effects in isolation. Although this assumption appears to be a serious omission, the mathematical model will underestimate the flushing capabilities of real situations and consequently overestimate the predicted pollution levels. In other words, the model is likely to err ‘safely’ on the conservative side.

2. Tidal Prism Model

The water quality response of a tidal embayment can be found by solving a set of interrelated time-dependent equations for volume of water and pollutant conservation on both the flood and ebb tides.

A primary consideration when applying a tidal prism model is to ensure the basin exhibits strong internal mixing to induce a uniform concentration of pollutant throughout the tidal cycle. Although this assumption may not be valid in large tidal embayments, well-mixed conditions can usually be assumed if the characteristic dimension of the basin does not exceed one tidal excursion (DiLorenzo *et al.*, 1994). Consequently, the maximum size of embayment that can be analysed using tidal prism theory is generally considered to be of the order of a few kilometres. A second but usually less restrictive constraint on the size of the basin arises from the fact that the tidal embayment is assumed to exhibit a spatially uniform sea-level response.

Following DiLorenzo *et al.* (1995), a number of important assumptions are invoked in formulating the governing pollution flushing equations, namely:

- the pollutant is conservative,
- complete internal mixing of the contaminant each tidal cycle,
- zero pollution in the exterior water body,
- no vertical stratification due to thermal or density effects,
- the initial pollutant distribution is uniform within the basin, and
- the freshwater/groundwater inflow from the surrounding land is constant with time.

DiLorenzo *et al.* (1995) and Wearing (2000) have described the development of the tidal prism model in detail. The governing water quality equations for the flood and ebb tides are generated separately and then combined to give an overall equation for the pollutant concentration after n complete tidal cycles. In order to obtain a tractable set of equations that can be solved analytically, the basin is assumed to experience repetitive sinusoidal tides of constant amplitude. Thus, the volume of water within the basin, V, is given by

$$V = V_{\mathrm{m}} + V_{\mathrm{t}} \cos(\omega t) \tag{1}$$

where V_{m} is the mean volume of the basin, V_{t} the amplitude of the oscillatory component of the tidal volume, t the time and ω the tidal angular frequency given by $\omega = 2\pi/T$, where T is the period of the tide. Equation (1) assumes the mathematical model commences from high water at $t = 0$ although it is a trivial task to reformulate the analysis to start from low water. The mixing processes within the basin are markedly different during the flood and ebb cycles, and therefore, the analysis is partitioned into separate intervals depending upon the direction of the tide. To obtain an analytical solution, the freshwater discharge from the surrounding land is assumed to be sufficiently small to have little effect on the duration of the flood and ebb tides. Since the mathematical model is considered to start from high water, the analysis first considers the pollution flux balance on the ebb tide.

2.1. EBB TIDE INTERVAL

$$2\pi(n-1) < \omega t < \pi + 2\pi(n-1) \quad n \in N$$

During the ebb tide interval, the pollutant concentration, C, and volume of water, V, inside the basin satisfy the following equations:

$$\frac{d}{dt}(CV) = C\frac{dV}{dt} + V\frac{dC}{dt} = QC \tag{2}$$

and

$$\frac{dV}{dt} = Q + Q_{\mathrm{f}} \tag{3}$$

where Q is the discharge through the entrance ($Q > 0$ on the flood tide, $Q < 0$ on the ebb tide) and Q_{f} is the steady freshwater discharge into the basin from the surrounding land. Combining Equations (2) and (3) and integrating over the first ebb interval yields:

$$\int_{C_0}^{C_{\mathrm{e}(1)}} \frac{dC}{C} = -Q_{\mathrm{f}} \int_0^{\pi/\omega} \frac{dt}{V_{\mathrm{m}} + V_{\mathrm{t}}\cos(\omega t)} \tag{4}$$

where C_0 is the initial concentration at the start of the simulation ($t = 0$) and $C_{\mathrm{e}(1)}$ the pollutant concentration at the end of the first ebb tide. It can readily be shown that

$$\int_0^{\pi/\omega} \frac{dt}{V_{\mathrm{m}} + V_{\mathrm{t}}\cos(\omega t)} = \frac{\pi}{\omega\sqrt{V_{\mathrm{m}}^2 - V_{\mathrm{t}}^2}} \tag{5}$$

and therefore, Equation (4) can be integrated and rearranged to give

$$C_{\mathrm{e(1)}} = C_0 \exp\left\{ \frac{-\pi Q_{\mathrm{f}}}{\omega\sqrt{V_{\mathrm{m}}^2 - V_{\mathrm{t}}^2}} \right\}. \tag{6}$$

Since the tidal basin is assumed to experience repetitive harmonic tides of constant amplitude, Equation (6) can be rewritten for the generalised case after *n* tidal cycles as

$$C_{\mathrm{e(n)}} = C_{\mathrm{f(n-1)}} \exp\left\{ \frac{-\pi Q_{\mathrm{f}}}{\omega\sqrt{V_{\mathrm{m}}^2 - V_{\mathrm{t}}^2}} \right\} \tag{7}$$

where $C_{\mathrm{e(n)}}$ is the pollutant concentration at the end of *n* ebb tides and $C_{\mathrm{f(n-1)}}$ is the corresponding concentration at the end of the previous flood cycle.

2.2. FLOOD TIDE INTERVAL

$$\pi + 2\pi(n-1) < \omega t < 2\pi n \quad n \in N$$

During the flood tide, the pollutant concentration and volume of water inside the basin satisfy the following equations:

$$\frac{d}{dt}(CV) = C\frac{dV}{dt} + V\frac{dC}{dt} = 0 \tag{8}$$

and

$$\frac{dV}{dt} = Q + Q_{\mathrm{f}}. \tag{9}$$

The volume of water in the basin at low tide is $V_m - V_t$ whilst the corresponding volume at high tide is $V_m + V_t$. Consequently, Equation (8) can be integrated to give

$$\int_{C_{\mathrm{e(1)}}}^{C_{\mathrm{f(1)}}} \frac{dC}{C} = -\int_{V_{\mathrm{m}}-V_{\mathrm{t}}}^{V_{\mathrm{m}}+V_{\mathrm{t}}} \frac{dV}{V} \tag{10}$$

where $C_{\mathrm{f(1)}}$ is the pollutant concentration at the end of the first flood cycle. Integrating and rearranging yields

$$C_{\mathrm{f(1)}} = C_{\mathrm{e(1)}}\left[\frac{V_{\mathrm{m}} - V_{\mathrm{t}}}{V_{\mathrm{m}} + V_{\mathrm{t}}}\right] \tag{11}$$

and generalising the result for the nth flood tide gives

$$C_{\mathrm{f(n)}} = C_{\mathrm{e(n)}}\left[\frac{V_{\mathrm{m}} - V_{\mathrm{t}}}{V_{\mathrm{m}} + V_{\mathrm{t}}}\right]. \tag{12}$$

Substituting for $C_{\mathrm{e(n)}}$ from Equation (7) enables the water quality response of the tidal embayment to be determined using a recurrence relationship:

$$C_{\mathrm{f(n)}} = C_{\mathrm{f(n-1)}}\left[\frac{V_{\mathrm{m}} - V_{\mathrm{t}}}{V_{\mathrm{m}} + V_{\mathrm{t}}}\right]\exp\left\{\frac{-\pi Q_{\mathrm{f}}}{\omega\sqrt{V_{\mathrm{m}}^2 - V_{\mathrm{t}}^2}}\right\}. \tag{13}$$

Repeated application of Equation (13) from the initial conditions at $t = 0$ yields the pollutant concentration after n tidal cycles in terms of the initial concentration, C_0:

$$C_{\mathrm{f(n)}} = C_0\prod_{i=1}^{n}\left[\frac{V_{\mathrm{m}} - V_{\mathrm{t}}}{V_{\mathrm{m}} + V_{\mathrm{t}}}\right]\exp\left\{\frac{-\pi Q_{\mathrm{f}}}{\omega\sqrt{V_{\mathrm{m}}^2 - V_{\mathrm{t}}^2}}\right\} \tag{14}$$

which may be rewritten as

$$C_{\mathrm{f(n)}} = C_0\left[\frac{V_{\mathrm{m}} - V_{\mathrm{t}}}{V_{\mathrm{m}} + V_{\mathrm{t}}}\right]^n\exp\left\{\frac{-\pi Q_{\mathrm{f}} n}{\omega\sqrt{V_{\mathrm{m}}^2 - V_{\mathrm{t}}^2}}\right\}. \tag{15}$$

The pollutant concentration at the end of the nth ebb tide can be obtained by substituting $C_{\mathrm{f(n-1)}}$ into Equation (7):

$$C_{\mathrm{e(n)}} = C_0\left[\frac{V_{\mathrm{m}} - V_{\mathrm{t}}}{V_{\mathrm{m}} + V_{\mathrm{t}}}\right]^{(n-1)}\exp\left\{\frac{-\pi Q_{\mathrm{f}} n}{\omega\sqrt{V_{\mathrm{m}}^2 - V_{\mathrm{t}}^2}}\right\}. \tag{16}$$

2.3. POLLUTION RETURN-FLOW FACTOR

The predicted water quality response has to be adjusted to account for pollution which leaves the embayment on an ebb tide and returns on the subsequent flood tide – the so-called ‘return-flow’ effect. In the present model, a convenient (although approximate) method of accounting for the return flow is to reduce the amplitude of the oscillatory tidal prism. Thus, the effective amplitude of the oscillatory tidal volume can be expressed as:

$$V_t^* = (1 - b)V_{\mathrm{t}}. \tag{17}$$

where b is the pollution-return parameter between 0 and 1 defining the proportion of pollutant which returns to the embayment on the flood tide after being expelled during the previous ebb cycle. The 'return-flow' parameter, therefore, expresses the fate of the tidal prism after it leaves the embayment and its value depends upon the level of mixing between the effluent plume and the surrounding coastal waters.

The reduced oscillatory tidal volume, V_t^*, is substituted into all previous water quality equations. Thus, the pollutant concentration at the end of the nth flood tide can be rewritten as:

$$C_\mathrm{f(n)} = C_0\left[\frac{V_\mathrm{m} - V_\mathrm{t}^*}{V_\mathrm{m} + V_\mathrm{t}^*}\right]^n \exp\left\{\frac{-\pi Q_\mathrm{f} n}{\omega\sqrt{V_\mathrm{m}^2 - V_\mathrm{t}^{*2}}}\right\} \tag{18}$$

whilst the pollutant concentration at the end of the nth ebb tide can be rewritten as:

$$C_\mathrm{e(n)} = C_0\left[\frac{V_\mathrm{m} - V_\mathrm{t}^*}{V_\mathrm{m} + V_\mathrm{t}^*}\right]^{(n-1)} \exp\left\{\frac{-\pi Q_\mathrm{f} n}{\omega\sqrt{V_\mathrm{m}^2 - V_\mathrm{t}^{*2}}}\right\}. \tag{19}$$

2.4. POLLUTION EXCHANGE COEFFICIENT

The efficiency of the tidal flushing can be expressed in terms of the *pollution exchange coefficient*, E, which measures the proportion of water exchanged with the sea during each tidal cycle. Specifically, Falconer and Yu (1991) defined the pollution exchange coefficient as the fraction of water removed during a tidal cycle and replaced by unpolluted water. The coefficient can thus be expressed in generalised form as

$$E = 1 - \left(\frac{C_\mathrm{f(n)}}{C_0}\right)^{1/n}. \tag{20}$$

Substituting $C_\mathrm{f(n)}$ from Equation (18) into (20) and simplifying yields

$$E = 1 - \left[\frac{V_\mathrm{m} - V_\mathrm{t}^*}{V_\mathrm{m} + V_\mathrm{t}^*}\right] \exp\left\{\frac{-\pi Q_\mathrm{f}}{\omega\sqrt{V_\mathrm{m}^2 - V_\mathrm{t}^{*2}}}\right\}. \tag{21}$$

In the absence of freshwater flushing ($Q_\mathrm{f} = 0$), the pollution exchange coefficient, E, is given by

$$E = 1 - \left[\frac{V_\mathrm{m} - V_\mathrm{t}^*}{V_\mathrm{m} + V_\mathrm{t}^*}\right]. \tag{22}$$

High values of pollution exchange coefficient are indicative of efficient tidal flushing. The analytical model demonstrates that the flushing rate of a tidal embayment depends crucially on the ratio of the volume of water in the basin at low tide to the volume of water at high tide, $(V_m - V_t^*)/(V_m + V_t^*)$. Small values of this ratio lead to high pollution exchange coefficients and rapid flushing. Conversely, values of $(V_m - V_t^*)/(V_m + V_t^*)$ which approach unity suggest that the basin may be at risk from water quality problems.

3. Results

In the present work, the tidal prism model has been validated by comparing predictions against experimentally observed pollution flushing data from a physical model of a generic square-shaped tidal basin. Conducting the pollution flushing experiments in a laboratory model offered greater flexibility than field experiments and enabled identical tests to be repeated several times to ensure the accuracy of the experimental data. The laboratory tests were conducted on a vertically distorted model of an idealised flat-bottomed tidal embayment having a square plan-form area of 432 m × 432 m (18.7 ha), a single asymmetric entrance 48 m wide and vertical side walls. These dimensions are representative of small to medium sized marinas in the USA and larger marinas in the UK (Nece and Falconer, 1989; Falconer and Yu, 1991). The laboratory experiments were conducted in a 7.5 m × 3.5 m tidal testing facility at the University of Salford as detailed by Barber and Wearing (2000) and Wearing (2000). The model was scaled according to Froude number similarity and was constructed at a horizontal scale of 1:400. The experimental tidal basin was, therefore, 1.08 m × 1.08 m in plan with an entrance 0.12 m wide. Three sets of vertical scale ratios were used in the preliminary tests to investigate the effects of vertical distortion. However, the experiments soon revealed that the vertical distortion ratio had a marginal effect on the pollution flushing characteristics of the laboratory embayment. The results presented in this paper were obtained from experiments conducted at a vertical scale of 1:40.

Sinusoidal water level variations in the testing facility were generated using a computer-controlled tidal generator. Water was supplied to the laboratory model at a constant rate with the level being determined by a computer-controlled moveable weir which allowed excess water to flow back to a collecting sump. The system avoided the use of complex feedback control and was able to deliver tidal levels in the testing facility to within an accuracy of ±0.5 mm.

The laboratory tests considered an instantaneous release of contaminant at high water. At the start of each test, the entrance to the tidal embayment was temporarily sealed and a small quantity of sodium fluorescein dye thoroughly mixed with the water inside the basin. A distributed water quality sample was then taken to determine the initial pollutant concentration, C_0. Once the turbulence created by the mixing had decayed, the barrier across the entrance was removed and the tidal generator started. Sampling was carried out simultaneously at nine equispaced points within the embayment every high tide for six complete tidal cycles. At the end of the tests the water samples were analysed using a Perkin-Elmer LS-5B luminescence spectrometer to determine the pollution flushing characteristics.

The analytical formulation demonstrates that the water quality response depends crucially on the value of $\alpha = (V_m - V_t)/(V_m + V_t)$, the ratio of the volume of water in the embayment at low tide to the volume of water at high tide. The tests employed repetitive sinusoidal tides having a prototypical semi-diurnal period of 12.42 h, a maximum water depth of 8 m, and tidal ranges of 2, 4 and 6 m (corresponding to values of α of 0.75, 0.50 and 0.25, respectively). The tidal regime in the laboratory model had a period of 708 s, a maximum water depth at high tide of 200 mm and tidal ranges of 50, 100 and 150 mm. Initially, the tidal flushing characteristics were considered in isolation and so the freshwater inflow, Q_f, was set to zero. The pollution-return parameter, b, was estimated by applying a least-squares analysis to the observed water quality data and was found to equal 0.135 which is relatively low and indicates efficient mixing between the effluent plume and the surrounding water just outside the embayment. Figure 1 presents the results of the validation tests and shows very good correlation between the analytical predictions and observed flushing characteristics. It should be noted that the pollution flushing data have been non-dimensionalised and presented as a log-linear graph of relative concentration (C/C_0) against the number of tidal cycles (n).

The numerical formulation was found to be sensitive to the chosen value of the pollution-return parameter, b. Early tidal prism models (e.g., Dyer, 1973; Callaway,

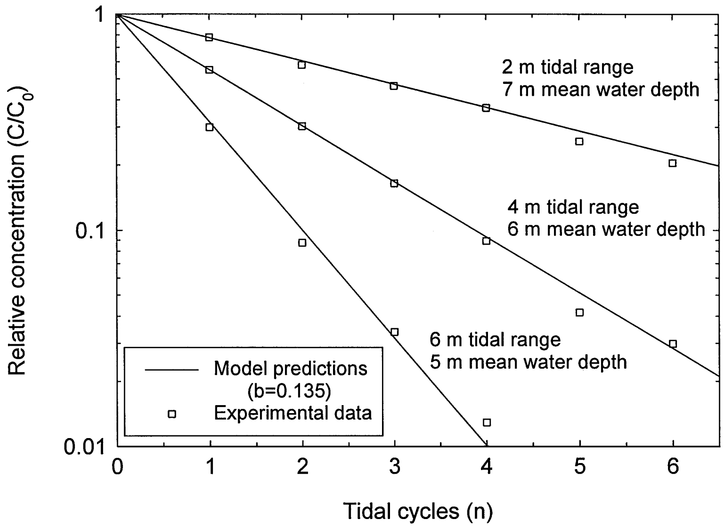

Figure 1. Experimental validation of proposed pollution flushing model (432 m × 432 m square embayment; varying tidal range).

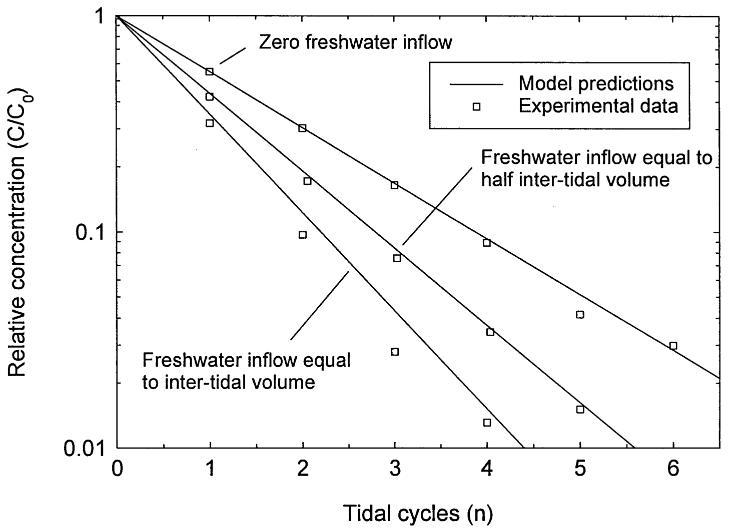

Figure 2. Experimental validation of proposed pollution flushing model (432 m × 432 m square embayment; varying freshwater inflow).

1981) often ignored the pollution-return effect completely, while other studies have suggested that the return-flow parameter should be specified as 0.5 in the absence of additional information (U.S. EPA, 1985). The present results using a pollution-return parameter of 0.135 indicate that both these previous scenarios would yield erroneous pollution flushing rates. The laboratory tests reveal that the pollution flushing characteristics can be predicted using a single return-flow parameter, b, despite the variation in the tidal range.

A second set of pollution flushing experiments (Figure 2) assessed the accuracy of the analytical technique for simulating the combined effect of tide-induced and freshwater flushing. In this case, a prototypical tidal range of 4 m was considered which corresponds to the middle tidal regime of the previous validation tests ($\alpha = 0.5$). The freshwater was fed into the tidal embayment at the corner diametrically opposite the entrance and was varied from zero up to a maximum discharge of P/T where P is the volume of the inter-tidal prism ($P = 2V_t$) and T is the tidal period. The freshwater inflow was non-dimensionalised with respect to the inter-tidal volume to assist in the interpretation of the results. In the laboratory tests, the maximum freshwater inflow was specified to be $Q_f = 0.165$ l/s.

The pollution-return parameter depends on the level of mixing between the effluent plume and the surrounding coastal water and is therefore dependent on the magnitude of the freshwater inflow, Q_f. The pollution-return parameter, b, was estimated by applying a least-squares analysis to the observed water quality data and was found to equal 0.06 and 0.0 for non-dimensionalised freshwater inflows of $0.5P/T$ and P/T, respectively. It can be seen that the accuracy of the mathematical

TABLE I

Summary of pollution exchange coefficients for a 432 m × 432 m square embayment

Prototype tidal range (m)[a]	Freshwater inflow rate	Pollution exchange coefficient, E	
		Observed	Predicted
2	0	0.221	0.220
4	0	0.454	0.448
6	0	0.670	0.683
4	$0.5P/T$	0.571	0.561
4	P/T	0.696	0.649

[a] Prototype maximum water depth = 8 m.

model is reduced as the freshwater discharge is increased, but this is to be expected since the analytical solution is derived from a basic assumption that the flood and ebb tides are of the same duration. As the freshwater discharge is increased, the duration of the flood tide is shortened whilst the duration of the ebb tide is lengthened. Consequently, large freshwater inflow rates invalidate the assumption of partitioning the tidal cycle into equal intervals of $0.5T$ and reduce the accuracy of the analytical solution.

To demonstrate the effectiveness of the pollution flushing model, Table I compares the observed and predicted pollution exchange coefficients for each validation test. Following Falconer and Yu (1991), the experimental pollution exchange coefficients were determined by applying Equation (20) over *four* complete tidal cycles whilst the predicted pollution exchange coefficients were evaluated using Equation (21). Table I demonstrates that the proposed analytical model offers a viable and accurate method of predicting the pollution exchange coefficient under a variety of tidal regimes.

To test whether the proposed analytical approach can be used for other geometries of tidal embayment, experiments were conducted using rectangular-shaped basins with different aspect ratios of length and breadth (Wearing, 2000). For example, Figure 3 presents the results of a tidal flushing test on a rectangular embayment having an aspect ratio of 2:1. The experiment considered a prototype tidal basin having a rectangular area of 864 m × 432 m (37.3 ha) and an entrance 48 m wide half-way along the longest side wall. The maximum water depth in the prototype basin was assumed to be 8 m, the tidal range was chosen to be 4 m (giving a value of α of 0.50) and the tidal flushing characteristics were considered without any freshwater inflow ($Q_f = 0$). Once again, the close correlation between the experimental pollution data and the analytical predictions demonstrates that the mathematical formulation offers a simple yet surprisingly accurate method of predicting the water quality response in well-mixed tidal embayments. Additional tests using other

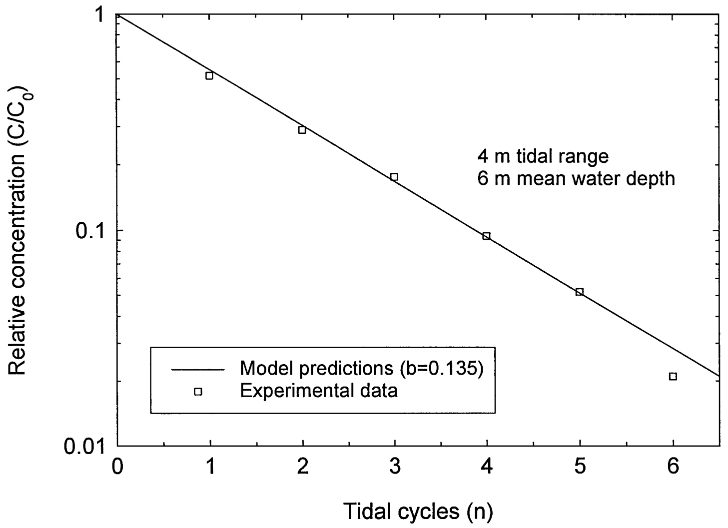

Figure 3. Experimental validation of proposed pollution flushing model (864 m × 432 m rectangular embayment).

geometries confirm that the analytical technique is equally valid for other shapes of tidal embayment.

4. Conclusions

This paper describes a tidal prism model for predicting the water quality response of well-mixed tidal embayments. Two separate regimes have been considered in the study, namely a basin subjected to vigorous tidal mixing and secondly a basin where the freshwater inflow from the surrounding land is equally important to the pollution flushing characteristics. Predictions from the mathematical model are compared with experimental water quality data from a 1:400 scale laboratory study of a generic square tidal embayment. The results from the validation tests are very encouraging and show that the mathematical formulation is able to replicate the observed flushing characteristics irrespective of the tidal range or freshwater inflow. The proposed model is shown to provide accurate estimates of the pollution exchange coefficient and offers a computationally inexpensive alternative to conventional multi-dimensional pollutant transport simulations.

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