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THE GEOMETRICAL RECONSTRUCTION OF BUBBLE CHAMBER TRACKS

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ABSTRACT

The Geometrical Reconstruction Program of the Rutherford Laboratory bubble chamber analysis system is described in detail. The program uses a two-stage process to reconstruct the tracks in space. In the first stage a circle is fitted to the space points on the track found by the method of corresponding points. The results of this fit are used to fit a helix to the rays of all three views of the track. The helix is given a mass-dependent correction to allow for the slowing down of the particle in the chamber liquid.

The program is built into a library and book-keeping system, which is also described.

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1. INTRODUCTION

The analysis of the measurements of bubble chamber photographs is conveniently divided into three stages. In the first stage the spatial position and momentum, with errors, are determined for individual tracks by geometrical reconstruction. The second stage consists of testing a whole event against a given hypothesis of particle mass assignment. In the final stage statistics of interesting physical quantities are formed from identified events. A complete system of analysis programs to cover these three stages has been written at the Rutherford Laboratory. These programs have been written in the Fortran computer language and an outline of the system is given in reference 1. This present report contains a detailed description of the calculations involved in the first stage of the system, which we call the Geometry program.

The principal task of the geometry program is to calculate the momentum $\underline{p}$ at the middle of each track and an error matrix $<\delta p_i \delta p_j>$ on this momentum. In addition the program calculates the spatial co-ordinates of all vertices together with errors and also works out the topology of the event.

In order to isolate the program from changes in the form of the data produced by the measuring machines, the measurements of an event must be presented to the program in a standard form, called the INPUT Format. A small interpretive program must be written for each system of measurement to transform data to the Input format. The data, additional to co-ordinates, required by the Input format has been kept to a minimum, so that the number of control symbols that must be punched with the measurements is small.

At an early stage in the program, film co-ordinates are transformed into ray co-ordinates, where a ray is defined by a co-ordinate pair (X_G, Y_G) of the point at which the ray cuts the front glass of the chamber and (U_G, V_G) the direction ratios of the ray. Most of the calculation is carried out in terms of these rays and the final fit to obtain the momentum is made by fitting a helix to the rays of the three views. The fit is made to a projection onto a nominal film plane. In cases where the curvature of the track is significantly changed by slowing down in the chamber liquid a mass-dependent correction is made to the helix. This correction is made assuming for the track the masses of the pion, the kaon and the proton. In this way feed-back from the hypothesis testing program to the geometry program is eliminated at the expense of some extra computation.

The output of the Geometry program is in the library format (Tape B) and can be read by both the hypothesis testing and statistics programs.

2. INPUT FORMAT

The measurements of an event must be presented to the geometry program in a standard format. This format consists of five lists.

L1. Book-keeping List

Frame number
Event number on a given frame

Number of any completely missing view.
Measurement number (for indicating re-measurements).
Measurer.
Measuring machine.
Date of measurement.

L2. Fiducial List

Total number of fiducial points measured.
Total number for each view.
Index of first co-ordinate pair of each view.
Fiducial co-ordinates.

L3. Vertex List

Total number of vertices measured (This must be the same for all views).
Index for finding co-ordinates of each vertex in the point list.
Type of vertex (Elastic scatter, stopping etc).

L4. Track List

Total number of tracks measured (This must be the same for all views).

Then for each view and each track:

Total number of points measured.
Index of the first point on the track in the Point List.
Label at the beginning of the track. This is to equal 1 if the track has a vertex at the beginning, or otherwise to equal zero.
Label at the end of the track.
Direction of track. The program will in general find this from the topology of the event, in some circumstances however, this cannot be done and if the direction is known, say from a delta-ray, this can be indicated.
Type of track. Mass if known, ionization etc.

L5. Point List

Total number of points.
Co-ordinates.

A detailed description of the card format used by the IBM 7090 version of the program is given in Appendix II.

In addition to the above information, two measurement conventions are assumed by the program. Firstly, the program assumes that a given fiducial is measured first for each view. And secondly that the first or last point measured on a track are sufficient by near to any vertex on the track that this vertex can be unambiguously identified. Points on tracks must be measured in order along the track and only charged tracks are to be measured.

3. FIDUCIALS

For each view we define a standard frame of reference as having the point where the optic axis cuts the film plane as origin and x and y axes parallel to the x and y axes of the co-ordinate system of the chamber volume. For each view a set of expected fiducial position (X_j, Y_j) in the standard frame must be supplied. A particular fiducial on each view must always be measured first and a translation is made on the measured fiducials so that this first fiducial is in its correct position. Then, assuming that the axes of the measurement frame and the axes of the standard frame are only inclined at a small angle, the measured fiducials can be identified with the standard fiducials. From the two corresponding sets (x_i, y_i) of measured fiducials and (X_i, Y_i) the standard fiducials, we calculate θ the angle and x_r, y_r the translations between the measurement and standard axes and also f_1 and f_2 the x and y film shrinkages.

We have

$$X_i = (f_1 x_i - x_r)\cos\theta + (f_2 y_i - y_r)\sin\theta$$

$$Y_i = (f_1 x_i - x_r)\sin\theta + (f_2 y_i - y_r)\cos\theta$$

We write these equations as

$$X_i = ax_i + by_i + c$$

$$Y_i = dx_i + ey_i + f \qquad (1)$$

and solve for a, b, c, d, e and f by minimizing $\sum_{i=1}^{n} d_i^2$ where

$$d_i^2 = (X_i - ax_i - by_i - c)^2 + (Y_i - dx_i - ey_i - f)^2$$

and n = total number of fiducials measured.

Equations 1 are checked for each fiducial in turn using the values found for the transformation coefficients a, b ... f. Any badly measured fiducial is rejected and the process repeated. A check is also made on the identity $ab + ed = o$.

In the case of only two fiducials measured we take $f_1 = f_2 = f$ and then solve the 4 equations for the 4 unknowns θ, x_r, y_r and f.

The transformation coefficients are found for each view and these are then used to translate all the points of each view to the standard frame of each view. A further rotation common to all points can be made at this stage to change the standard orientation of the chamber axes.

4. TRACK END LABELS AND VERTEX IDENTIFICATION.

The vertices of each view must be measured as single points separate from measurements of the tracks at the vertex and these vertices will be numbered for each view according to the order of measurement. The track end labels are then the numbers of the vertices at the beginning and end of each track. The label 0 is used for non-vertex track-end, that is for tracks leaving the chamber. Tracks can be measured in either direction and the order of the two end-labels is defined by the direction of measurement. As a preliminary we ensure that all tracks begin with a vertex, reversing the direction of tracks as necessary.

From the Input track list we have an indication of which track-ends have a vertex and the next operation is to identify the number of the vertex at these track-ends. To do this we find for each view the minimum distance between vertices and then look for a vertex that is within half this distance from the end point measured on the track. Thus the vertex point itself need not be measured as part of the track, but a point must be measured which is sufficiently close to avoid ambiguity in the vertex identification.

5. STEREO SORTING

It is intended at a later stage to allow measurements of tracks and vertices to be made which are uncorrelated between views, and to program any necessary rearrangement of the order of the track and vertex lists. At present, tracks and vertices must be measured in the same order on each view. If for some reason a track cannot be measured on one view it must still appear in the track list for that view but can have the total number of points measured equal to zero. Tracks can still be measured in different directions on different views and the directions will be reversed to the direction on view 1. The track-end labels, direction indicator and type description are merged for the three views to just one set for the track and similarly for the type description of vertices.

6. CONVERSION OF POINTS TO RAYS

Assuming parallel glasses and film plane and no lens distortion for the bubble chamber optics, we derive the following relations for the transformation from film points (x, y) to rays (X, Y, U, V) in the chamber liquid.

$$X = -\frac{x}{a_o}\left[\sum_{i=1}^{n} \frac{a_i}{\mu_i}\left(1 + \frac{\mu_i^2 - 1}{\mu_i^2 a_o^2} r^2\right)^{-\frac{1}{2}} + A_T - A_G\right] + C_x$$

$$Y = -\frac{y}{a_o}\left[\sum_{i=1}^{n} \frac{a_i}{\mu_i}\left(1 + \frac{\mu_i^2 - 1}{\mu_i^2 a_o^2} r^2\right)^{-\frac{1}{2}} + A_T - A_G\right] + C_y$$

$$U = \frac{-x}{\mu_H a_o}\left[1 + \frac{\mu_H^2 - 1}{\mu_H^2 a_o^2} r^2\right]^{-\frac{1}{2}}$$

$$V = \frac{-y}{\mu_H a_o}\left[1 + \frac{\mu_H^2 - 1}{\mu_H^2 a_o^2} r^2\right]^{-\frac{1}{2}}$$

where $r^2 = x^2 + y^2$, a_o = lens to film distance

A_T = total distance from lens to liquid

n = number of glasses between lens and liquid

a_i = thickness of i^{th} glass

μ_i = refractive index of i^{th} glass

$A_G = \sum_{i=1}^{n} a_i$, total glass thickness

μ_H = refractive index of chamber liquid

$\left.\begin{array}{l} C_x \\ C_y \end{array}\right\}$ are co-ordinates of the appropriate camera axis.

X, Y are co-ordinates of the point where the ray cuts the plane z = o, which is taken as the liquid glass interface on the camera side of the chamber. The z-axis is taken positive into the chamber liquid giving a left-handed system of axes.

For small bubble chambers where the ray angle is small, that is $\tan\theta = \frac{r}{a_o}$ small, the square roots in the expressions for the ray co-ordinates may be expanded to speed the calculation.

For each image of a track we have a set of co-ordinates (X_1, Y_1) to (X_N, Y_N) on the chamber window. Finding the mid-point of the line joining the end points

$$X_M = \frac{X_1 + X_N}{z} \qquad Y_M = \frac{Y_1 + Y_M}{z}$$

we rotate and translate the co-ordinate as shown in Fig.1.

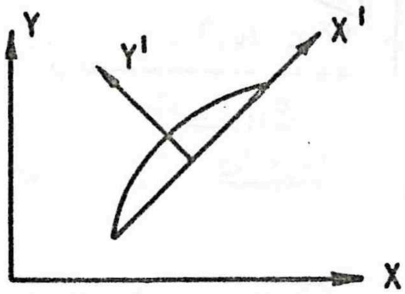

Fig. 1

by

$$X_i' = (X_i - X_M)\cos\theta + (Y_i - Y_M)\sin\theta$$

$$Y_i' = -(X_i - X_M)\sin\theta + (Y_i - Y_M)\cos\theta$$

with $\tan\theta = \frac{Y_N - Y_1}{X_N - X_1}$

A parabola $Y' = aX'^2 + bX' + c$ is fitted to these co-ordinates, unless the maximum Y' is less than a given constant in which case a straight line $Y' = aX' + b$ is fitted. A check is then made to find the point farthest from the fitted curve and if this point is at a greater distance than a given check the point is rejected as badly measured and the fit is redone on the remaining points. If a further point is out of line the event is failed.

From the coefficients a, b, c we calculate a radius and centre of a circle through the points

$$R = {}^{1}/_{2a}$$

$$X_c = -bR\cos\theta - (R + c - \tfrac{1}{2}b^2R)\sin\theta + X_M$$

$$Y_c = -bR\sin\theta + (R + c - \tfrac{1}{2}b^2R)\cos\theta + Y_M$$

A sign is given to R according to the rule

$$(X_N - X_1)(Y_c - Y_1) - (Y_N - Y_1)(X_c - X_1) \begin{cases} \geqslant 0 \text{ then R positive} \\ < 0 \text{ then R negative} \end{cases}$$

For the purpose of making checks later in program on whether the stereoscopic angle will allow the finding of corresponding rays, we calculate the angles that the radius vectors to the end points make with the y-axis (see Fig.2).

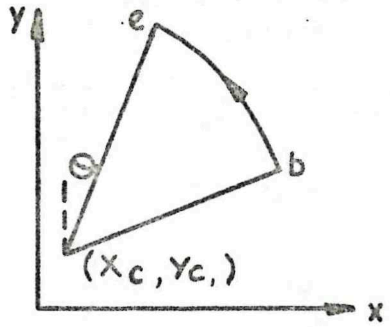

Fig. 2

$$\sin\theta_b = \frac{X_1 - X_c}{R} \qquad \sin\theta_e = \frac{X_N - X_c}{R}$$

$$\cos\theta_b = \frac{Y_1 - Y_c}{R} \qquad \cos\theta_e = \frac{Y_N - Y_c}{R}$$

For tracks where a straight line fit is made, R is set equal to a standard large number and

$$\sin\theta_b = \sin\theta_e = \frac{X_1 - X_N}{\left[(X_1 - X_N)^2 + (Y_1 - Y_N)^2\right]^{\frac{1}{2}}}$$

$$\cos\theta_b = \cos\theta_e = \frac{Y_1 - Y_N}{\left[(X_1 - X_N)^2 + (Y_1 - Y_N)\right]^{\frac{1}{2}}}$$

7. VERTEX CO-ORDINATES

For each vertex we have three rays (or possible only two if one view is missing) and we wish to find the point (x, y, z) in the chamber which is the point of nearest intersection of these rays. To do this we consider the projection of a point (x, y, z) onto the film and find the distance d from this projected point to the point where the ray cuts the film. We find that (see Section 11) on fitting a helix)

$$d^2 = w^2 [(x - X - Uz)^2 + (y - Y - Vz)^2]$$

where w = demagnification $= \dfrac{\mu_H a_o}{z + Z_o}$

and Z_o = optical distance from lens to top glass interface

$$= \mu_H \left[\sum_{i=1}^{n} \frac{a_i}{\mu_i} + A_T - A_G \right]$$

and the other symbols are defined as in Section 6.

To find the best values of (x, y, z) and errors on these co-ordinates, we minimize $\Sigma {d_i}^2$ for the rays. The equations to solve are

$$G \underline{x} = \underline{Y}$$

where $G = \begin{bmatrix} N & o & -\Sigma U_i \\ o & N & -\Sigma V_i \\ -\Sigma U_i & -\Sigma V_i & \Sigma {U_i}^2 + \Sigma {V_i}^2 \end{bmatrix}$ $\quad \underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and $\underline{Y} = \begin{bmatrix} \Sigma X_i \\ \Sigma Y_i \\ -\Sigma U_i X_i - \Sigma V_i Y_i \end{bmatrix}$

and N = total number of rays.

Then $\underline{x} = G^{-1}\underline{Y}$ and errors are given by

$$\langle x_i x_j \rangle = \frac{{\sigma_v}^2 G_{ij}^{-1}}{w^2}$$

where σ_V is the error in measurement on the film. In general σ_V is given a standard value, but the internal error of the fit is also calculated

$$\sigma^2 = \frac{w^2}{2N-3} \sum_{i=1}^{N} [(x - X_i - U_i z)^2 + (y - Y_i - V_i z)^2]$$

and if this value of σ_V is larger than a given check, this calculated value is used for σ_V.

8. RECONSTRUCTION OF SPACE POINTS

We make the first fit to the measurements of a track by finding space points on the track by the usual corresponding points method and then fitting a curve to these points(2). A 'best' view is selected for each track and for each ray on this view corresponding rays on each of the other views are found. These are used to find a weighted mean for the space point.

In finding corresponding rays between two views the ray co-ordinates are rotated into a system with x-axis along the direction between the two cameras. This will be called the stereo-axis. We shall call the best view the main view and the view on which corresponding rays are to be found the sub-view. Letting (X_M, Y_M, U_M, V_M) be the co-ordinates of the main view ray, then, ignoring optical constraints we would find the corresponding ray (X_s, Y_s, U_s, V_s) on the sub-view from the relation $Y_s = Y_M$. This would be done by finding the two rays on the sub-view with Y co-ordinates nearest to the value $Y_s = Y_M$ and then interpolating. From these two rays we can calculate the z-co-ordinate of the space point on the main view ray

$$Z = \frac{X_M - X_s}{U_s - U_M}$$

Assuming linear interpolation between the point co-ordinates on the sub-view and an equal measurement error σ on all the co-ordinates, it is easy to show that the mean square error on z is proportional to

$$< \delta z^2 > \propto \frac{\sigma^2}{\sin^2 \theta}$$

where θ is the angle that the tangent to the track on the sub-view makes with the stereo-axis. Thus if we calculate two z values from the corresponding rays of the two sub-views, we can find a weighted mean value of the z-co-ordinate of the space point

$$Z = \frac{\sin^2 \theta_1 z_1 + \sin^2 \theta_2 z_2}{\sin^2 \theta_1 + \sin^2 \theta_2}$$

In practice, because of the optical distortions, it is necessary to iterate to find corresponding rays and also to take account of the curvature of the track by interpolation

We take for the main view, the view for which

$$\frac{L}{L_{max}} + W\left[\frac{\overline{\sin^2\theta_1} + \overline{\sin^2\theta_2}}{\max\,(\overline{\sin^2\theta_1} + \overline{\sin^2\theta_2})}\right]$$

is a maximum. L is the length of the image of the track and L_{max} is the maximum value of L for the three views. $\overline{\sin^2\theta_1}$ and $\overline{\sin^2\theta_2}$ are the average values of $\sin^2\theta$ for the two sub-views. These are taken as the mean of the values of $\sin^2\theta$ at the ends of the image, calculated from the $\sin\theta_B$, $\sin\theta_E$ etc. (see Section 6). W is a specified weighting factor. We apply a cut-off to $\sin^2\theta$ and set $\sin^2\theta = 0$ if its value is less than a given check. However reconstruction of some points on the track is allowed even if points at the end of the track fail this check.

For each ray of the main view, we first find the two rays (X_1, Y_1, U_1, V_1) and (X_2, Y_2, U_2, V_2) of the sub-view with Y co-ordinates nearest in value to Y_M and set $Y_s = Y_M$. If Y_s lies outside the range of Y co-ordinates of the sub-view rays, we first make an extrapolation check

$$\left|\frac{Y_s - Y_1}{Y_1 - Y_2}\right| < C_E$$

If this check is not satisfied we set $\sin^2\theta = 0$ and proceed with the next ray. Otherwise we find the centre (X_c, Y_c) of a circle of radius R through the two points (X_1, Y_1) and (X_2, Y_2). R is the radius found from a parabolic fit to all the points of the sub-view (see Section 6). For (X_c, Y_c) we have

$$X_c = \tfrac{1}{2}(X_1 + X_2) \pm (Y_2 - Y_1)\left[\frac{R^2}{(X_1 - X_2)^2 + (Y_1 - Y_2)^2} - \tfrac{1}{4}\right]^{\frac{1}{2}}$$

$$Y_c = \tfrac{1}{2}(Y_1 + Y_2) \pm (X_2 - X_1)\left[\frac{R^2}{(X_1 - X_2) + (Y_1 - Y_2)^2} - \tfrac{1}{4}\right]^{\frac{1}{2}}$$

where the sign is given by the sign of R. We now interpolate on this circle to find the X_s corresponding to $Y = Y_s$. We find for X_s

$$X_s = X_c \pm [R^2 - (Y_s - Y_c)^2]^{\frac{1}{2}}$$

where the sign is positive if $X_1 > X_c$ or otherwise negative. We find U_s and V_s by linear interpolation between the nearest rays

$$U_s = U_1 + (U_1 - U_2)\left[\frac{Y_s - Y_1}{Y_1 - Y_2}\right]$$

$$V_s = V_1 + (V_1 - V_2)\left[\frac{Y_s - Y_1}{Y_1 - Y_2}\right]$$

The weighting factor $\sin^2\theta$ is given by

$$\sin^2\theta = \frac{(X_s - X_c)^2}{R^2}$$

For some cases with wide angle optics and highly curved tracks linear interpolation for U and V would not be quite good enough for the most accurate measurements. However as a helix fit to the rays is made to obtain the final results for a track, the complication of better interpolation would not seem worthwhile.

We now have a first approximation (X_s, Y_s, U_s, V_s) to the corresponding ray. In general, because of the optical distortions due to the chamber windows and liquid, these two rays will not meet. The condition that two rays meet is

$$Z = \frac{X_s - X_M}{U_M - U_S} = \frac{Y_s - Y_M}{V_M - V_s}$$

$$\text{or} \quad Y_s = Y_M + \frac{V_M - V_S}{U_M - U_s}(X_S - X_M) \qquad \text{......(1)}$$

We can use equation (1) to iterate for Y_s. Taking a starting value of Y_s, we calculate X_s, U_s and V_s as outlined above and then substitute these values into the right hand side of equation (1) to obtain an improved value Y_s'. We repeat the process until $|Y_s' - Y_s|$ is less than a specified value. In general one step of iteration is sufficient for convergance to the accuracy of measurement. After the first step of the iteration there is no need to recalculate the centre of the circle. Finally the z-co-ordinate is given by

$$Z = \frac{X_s - X_M}{U_M - U_s}$$

The z-co-ordinates from reconstruction with the two sub-views are weighted to give the z-co-ordinate of the space point on the main view ray

$$Z = \frac{\sin^2\theta_1 z_1 + \sin^2\theta_2 z_2}{\sin^2\theta_1 + \sin^2\theta_2}$$

The procedure is repeated for all the rays of the main view, any ray for which both $\sin^2\theta_s$ are zero being marked. If less than two space points can be found on the track, the program tries again with a second choice of main view. If this also fails the event is rejected. The case of two vertex tracks with only two measured points on each view is treated specially (see Section 9.d.)

9. FIRST SPACE FIT

From the space point reconstruction we have a set of n points (x_i, y_i, z_i) on each track where

$$(x_i, y_i, z_i) = (X_i + U_i z_i, Y_i + V_i z_i, z_i)$$

If n is greater than two, we calculate the magnetic field value for the track.

$$B_z = B_o [W_1 B(x_{n/2}, y_{n/2}, z_{n/2}) + W_2(B(x_1, y_1, z_1) + B(x_n, y_n, z_n))]$$

where B_o is the value of B_z at the origin, $B(x, y, z)$ is $\frac{B_z}{B_o}$ at point (x, y, z) and W_1 and W_2 are weighting factors. The points (x_i, y_i, z_i) are rotated into a frame with x-axis parallel to the line joining the end points of the track

$$x' = x \cos\theta_R + y \sin\theta_R$$

$$y' = x \sin\theta_R + y \cos\theta_R$$

where $\sin\theta_R = \frac{y_n - y_1}{s}$ and $\cos\theta_R = \frac{x_n - x_1}{s}$

with θ_R the angle of rotation and $s^2 = (x_n - x_1)^2 + (y_n - y_1)^2$.

The points are also translated to a frame (x'', y'') such that $\sum_{i=1}^{n} x_i'' = o$ and $\sum_{i=1}^{n} y_i'' = o$

$$x'' = x' - \frac{\Sigma x'}{n}$$

$$y'' = y' - \frac{\Sigma y'}{n}$$

These measures are taken to avoid numerical difficulties with the fits.

We make the fit in the usual two part manner, first fitting to the x, y projection of the track and then fitting the Z co-ordinates. For tracks turning through a large angle we make a circle fit to the (x, y) co-ordinates (from here on we leave off the primes from the co-ordinates). For tracks turning through a small angle this circle fit can give trouble through rounding errors and in this case we make a parabolic fit. Finally for tracks which are straight to the accuracy of the points, we make a straight line fit. To decide between a circle or parabolic fit we make a crude check.

$$\theta = \frac{\ell}{R} \sim \frac{s}{R} \sim \frac{8h}{s}$$

where θ is the angle turned through, ℓ is the length track, s is the length of the chord (see above) and h is the sagitta and is taken as $y''_{n/2} - y''_1$. For a circle fit to be tried both $\frac{8h}{s}$ and s must be greater than specified checks.

9a. <u>Circle fit</u>

To fit the circle $(x - \alpha)^2 + (y - \beta)^2 = \rho^2$ to the (x, y) co-ordinates of the points, it is most convenient to minimize[3].

$$F^2(\alpha, \beta, \rho^2) = \sum_{i=1}^{n} (d_i^2 - \rho^2)^2 \qquad (1)$$

where $d_i^2 = (x_i - \alpha)^2 + (y_i - \beta)^2$.

The normal least squares process would be to minimize $\sum\limits_{i=1}^{n} (d_i - \rho)^2$.

Factorizing equation (1) we have

$$F^2 = \sum_{i=1}^{n} (d_i + \rho)^2 (d_i - \rho)^2$$

and the first term $(d_i + \rho)^2$ is almost constant and equal to $(2\rho)^2$. Thus the two processes are almost equivalent.

Minimizing first with respect to ρ^2 we find

$$\rho^2 = \frac{1}{n} (\Sigma x_i^2 + \Sigma y_i^2) + \alpha^2 + \beta^2$$

(note we are in a frame of reference in which $\Sigma x_i = \Sigma y_i = 0$) and then for α and β we find

$$\alpha = \frac{1}{2|\Gamma|} \left[\Sigma y_i^2 \; (\Sigma x_i^3 + \Sigma x_i y_i^2) - \Sigma xy(\Sigma y_i^3 + \Sigma x_i^2 y_i) \right]$$

$$\beta = \frac{1}{2|\Gamma|} \left[\Sigma x_i^2 \; (\Sigma y_i^3 + \Sigma x_i^2 y_i) - \Sigma xy(\Sigma x_i^3 + \Sigma x_i y_i^2) \right]$$

where $|\Gamma| = \Sigma x_i^2 \Sigma y_i^2 - (\Sigma x_i y_i)^2$.

We also evaluate errors $<\delta\alpha^2>$, $<\delta\beta^2>$ and $<\delta\rho^2>$. The Σd_i^2 of Appendix 1 is equal to $F^2/4\rho^2$ and setting $\alpha_1 = \alpha$, $\alpha_2 = \beta$ and $\alpha_3 = \rho$ then

$$H_{\lambda\mu} = \frac{1}{8\rho^2} \; \frac{\partial^2 F^2}{\partial\alpha_\lambda \partial\alpha_\mu}$$

$$\text{and} \quad <\delta\alpha_\lambda^2> = \sigma_{xy}^2 \; H_{\lambda\mu}^{-1}$$

where σ is the R.M.S. error on the co-ordinates. Differentiating we find for $H_{\lambda\mu}$

$$H_{\lambda\mu} = \frac{1}{4\rho^2} \begin{pmatrix} 4(\Sigma x_i^2 + n\alpha^2) & 4(\Sigma x_i y_i + n\alpha\beta) & -2\,N\alpha \\ 4(\Sigma x_i y_i + n\alpha\beta) & 4(\Sigma y_i^2 + n\beta^2) & -2\,N\beta \\ -2\,N\alpha & -2\,N\beta & N \end{pmatrix}$$

The uncertainty in ρ is given by

$$\frac{<\delta\rho^2>}{\rho^2} = \frac{<\delta\alpha_3^2>}{4\rho^4} = \frac{\sigma_{xy}^2 H_{33}^{-1}}{4\rho^4}$$

that is

$$\frac{<\delta\rho^2>}{\rho^2} = \frac{\sigma_{xy}^2}{\rho^2}\left[\frac{1}{n} + \frac{\alpha^2 \Sigma y_i^2 + \beta^2 \Sigma x_i^2 - 2\alpha\beta\,\Sigma x_i y_i}{|\Gamma|}\right]$$

The azimuthal angle ϕ, that is the angle that the tangent to the x, y projection of the tracks makes with the x-axis, at the point (x, y) on the track is given by

$$\phi = -\tan^{-1}\frac{x-\alpha}{y-\beta}$$

Differentiating we obtain

$$<\delta\phi^2> = \frac{1}{\rho^2}[\sin^2\phi <\delta\beta^2> + \cos^2\phi <\delta\alpha^2> + 2\sin\phi\cos\phi <\delta\alpha\,\delta\beta>]$$

At the centre of the track we have

$$<\delta\phi^2> = \frac{<\delta\alpha^2>}{\rho^2} = \frac{\sigma_{xy}^2\,\Sigma y_i^2}{|\Gamma|}$$

9b. Parabola Fit

If the track turns through too small an angle or is too short for a circle fit, we fit the parabola $y = ax^2 + bx + c$ to the points. In the usual manner we find (note again $\Sigma x_i = \Sigma y_i = o$)

$$a = \frac{n}{|\Gamma|}[\Sigma x_i^2 y_i \;\; \Sigma x_i^2 - \Sigma x_i y_i \;\; \Sigma x_i^3]$$

$$b = \frac{n}{|\Gamma|}[\Sigma x_i^4 \;\; \Sigma x_i y_i - \Sigma x_i^3 \Sigma x_i^2 y_i - \frac{1}{n}\;\Sigma x_i y_i (\Sigma x_i^2)^2]$$

$$c = -\frac{a}{n}\;\Sigma x_i^2$$

$$\text{where} \quad |\Gamma| = n\Sigma x_i^4 \Sigma x_i^2 - n(\Sigma x_i^3)^2 - (\Sigma x_i^2)^3$$

and for $H_{\lambda\mu}$ with $\alpha_1 = a$, $\alpha_2 = b$ and $\alpha_3 = c$

$$H_{\lambda\mu} = \begin{pmatrix} \Sigma x_i^4 & \Sigma x_i^3 & \Sigma x_i^2 \\ \Sigma x_i^3 & \Sigma x_i^2 & 0 \\ \Sigma x_i^2 & 0 & n \end{pmatrix}$$

To obtain the parameters of the corresponding circle we expand the square-root in

$$y - \beta = (\rho^2 - (x-\alpha)^2)^{\frac{1}{2}}$$

$$\simeq \rho \left[1 - \frac{(x - \alpha)^2}{2\rho^2} \right]$$

and equate terms to give

$$\alpha = -\frac{b}{2a} \qquad \beta = c + \frac{1}{2a} - \frac{b^2}{4a} \quad \text{and} \quad \rho = \frac{1}{2|a|}$$

Again we find the errors

$$\frac{<\delta\rho^2>}{\rho^2} = \frac{<\delta a^2>}{a^2} \quad \text{with} <\delta a^2> = \sigma_{xy}^2 \frac{n \Sigma x_i^2}{|\Gamma|}$$

and

$$<\delta\phi^2> = \frac{<\delta\alpha^2>}{\rho^2} \quad \text{with} \quad <\delta\alpha^2> \simeq \rho^2 <\delta b^2> = \frac{\sigma_{xy}^2 \rho^2}{|\Gamma|} \left[n \Sigma x_i^4 - (\Sigma x_i^2)^2 \right]$$

A test is made that the sagitta $h = \frac{s^2}{8\rho}$ is greater than a given constant (say 2 or 3 times σ_{xy}) and if this is not satisfied a straight line fit is made to the track (see Section 9d).

9c. Z-Fit

For each point on the track we calculate θ_i where

$$\theta_i = -\tan^{-1}\left(\frac{x_i - \alpha}{y_i - \beta} \right)$$

and using the co-ordinates found for the vertex, we find θ_B the θ at the beginning of the track and θ_E the θ at the end. If there is no vertex at the end, we put $\theta_E = \theta_n$. From the sign of $\theta_B - \theta_E$ and the direction of the magnetic field, we can find the charge of the track. We also calculate the azimuth ϕ at the centre of the track

$$\phi = \theta_R + \tfrac{1}{2} (\theta_B + \theta_E)$$

θ_R is the angle of rotation of the co-ordinate frame (see above) and ϕ is kept in the range $0 < \phi \leqslant 2\pi$.

If we rotate about the z-axis by a further $\frac{\pi}{2}$ or $\frac{3\pi}{2}$, according to the charge of the track, the track will be in a position as shown in Fig.3.

The direction of the track along the curve is deterred by the charge. In this frame the track can be represented by the helix

$$x = \alpha + \rho \cos\theta$$

$$y = \beta + \rho \sin\theta$$

$$z = \gamma + \rho\theta \tan\lambda$$

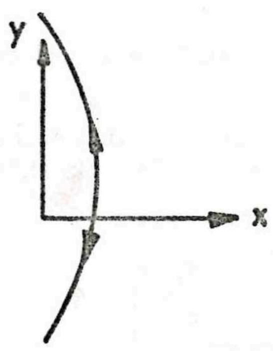

Fig.3.

where the Θ is the Θ calculated above for the points and α and β are in the new frame. To find γ and $\tan\lambda$ we make a linear fit to the z and Θ co-ordinates of the points. Least squares fitting $z = a\Theta + b$ to the points gives

$$\begin{bmatrix} \Sigma\Theta_i^2 & \Sigma\Theta_i \\ \Sigma\Theta_i & n \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \Sigma z_i\Theta_i \\ \Sigma z_i \end{bmatrix}$$

from which we find

$$a = \frac{\Sigma\Theta_i^2\,\Sigma z_i - \Sigma\Theta_i\,\Sigma z_i\Theta_i}{|\Gamma|}$$

$$b = \frac{n\Sigma z_i\Theta_i - \Sigma\Theta_i\,\Sigma z_i}{|\Gamma|}$$

$$\text{with } |\Gamma| = n\Sigma\Theta_i^2 - (\Sigma\Theta_i)^2$$

Finally we have

$$\gamma = a$$

$$\tan\lambda = \frac{b}{\rho}$$

and for the error on $\tan\lambda$

$$<\delta\tan\lambda^2> = \frac{n\sigma_z^2}{\rho^2|\Gamma|}$$

where σ_z is the root mean square error in space on the z co-ordinates. We can also calculate the momentum of the track

$$\frac{1}{p} = \frac{10^3\cos\lambda}{0.3\,B_z\rho}$$

and by differentiation an associated error

$$< \delta (\frac{1}{p})^2 > \; = \frac{1}{p^2} \left[\frac{<\delta \rho^2>}{\rho^2} + \sin^2\lambda \; \cos^2\lambda < \delta \tan\lambda^2 > \right]$$

p is in Gev/c and we use $\frac{1}{p}$ as a variable rather than p as this reciprocal is linearly related to the curvature of the track which is directly related to the measurements.

The length of the track is given by

$$L = \frac{\rho \left| \Theta_B - \Theta_E \right|}{\cos \lambda}$$

For two vertex tracks we also calculate an error on the length

$$<\delta L^2> \; = \; L^2 \left[\frac{<\delta \rho^2>}{\rho^2} \left(1 - \frac{\tan \Theta}{\Theta} \right)^2 + \; \sin^2\lambda \cos^2 \lambda <\delta \tan\lambda^2> \right] + 2 \sec^2\lambda \; \overline{<\delta x_v^2>}$$

where $\Theta = \frac{1}{2}(\Theta_B - \Theta_E)$ and $\overline{<\delta x_v^2>}$ is the average of the errors on the x and y co-ordinates of the two vertices.

We now have first approximations to the kinematical variables associated with each track, except for the case of tracks which are straight to the accuracy of measurement. We now deal with this case.

9d. Straight Tracks

For tracks that are too straight for a curvature to be found, we make least squares fits of $y = \alpha x$ and $z = \beta + \gamma x$ to the points (again in the rotated system $\Sigma x = \Sigma y = 0$). We find

$$\alpha = \frac{\Sigma x_i y_i}{\Sigma x_i^2}$$

$$\beta = \frac{\Sigma z_i}{n}$$

$$\gamma = \frac{\Sigma x_i z_i}{\Sigma x_i^2}$$

From these we calculate the angle variables for the track

$$\phi = \tan^{-1}\alpha$$

or in the unrotated system

$$\phi = \Theta_R + \tan^{-1}\alpha$$

$$\text{and } \tan\lambda = \frac{\gamma}{(1 + \alpha^2)^{\frac{1}{2}}}$$

The errors are given by

$$
\begin{aligned}
<\delta\phi^2> &= \cos^4\phi <\delta\alpha^2> \\
&= \cos^4\phi \, \frac{\sigma_{xy}^2}{\Sigma x_i^2}
\end{aligned}
$$

$$
\begin{aligned}
\text{and } <\delta\tan\lambda^2> &= \tan^2\lambda \left[\frac{<\delta\gamma^2>}{\gamma^2} + \sin^2\phi\cos^2\phi <\delta\alpha^2>\right] \\
&\simeq \tan^2\lambda \left[\frac{\sigma_z^2}{\gamma^2\Sigma x_i^2}\right]
\end{aligned}
$$

The second term in $<\delta\tan\lambda^2>$ and the correlation term $<\delta\phi\delta\tan\lambda>$ can be neglected as in this frame the azimuthal angle ϕ is very nearly equal to zero.

Finally we calculate the length L of the track, For tracks with only one vertex, we use for L

$$L = [(x_n - x_v)^2 + (y_n - y_v)^2 + (z_n - z_v)^2]^{\frac{1}{2}}$$

where (x_v, y_v, z_v) are the co-ordinates of the vertex. For two vertex tracks we find the distance along the straight line fit between the points on the fit nearest to the vertices. We find

$$L = \left[\frac{((x_1 - x_2) + \alpha(y_1 - y_2) + \gamma(z_1 - z_2))^2}{(1 + \alpha^2 + \gamma^2)}\right]^{\frac{1}{2}}$$

where (x_1, y_1, z_1) and (x_2, y_2, z_2) are the co-ordinates of the vertices rotated into the co-ordinate frame of the fit. In this case we also calculate the error on the length

$$
\begin{aligned}
<\delta L^2> = \frac{1}{L^2}\Big\{ & (x_1 - x_2)^2(<\delta x^2>_1 + <\delta x^2>_2) + (y_1 - y_2)^2(<\delta y^2>_1 + <\delta y^2>_2) + \\
& (z_1 - z_2)^2(<\delta z^2>_1 + <\delta z^2>_2) + 2\Big[(x_1 - x_2)(y_1 - y_2)(<\delta x\delta y>_1 + <\delta x\delta y>_2) + \\
& (x_1 - x_2)(z_1 - z_2)(<\delta x\delta z>_1 + <\delta x\delta z>_2) + (y_1 - y_2)(z_1 - z_2)(<\delta y\delta z>_1 + \\
& <\delta y\delta z>_2)\Big]\Big\}
\end{aligned}
$$

In the case of two vertex tracks with only two measured points on each view, we use the vertex co-ordinates and errors to find the parameters of the track. We have

$$\tan\phi = \frac{y_E - y_B}{x_E - x_B}$$

$$\text{and} \quad \tan\lambda = \frac{z_E - z_B}{L} \quad \text{where } \ell^2 = (x_E - x_B)^2 + (y_E - y_B)^2$$

and B and E stand for the vertex at the beginning and end of the track. The direction cosines are $(\cos\phi \cos\lambda, \sin\phi \cos\lambda, \sin\lambda)$ and

$$\sin\phi = \frac{y_E - y_B}{\ell} \quad \text{and} \quad \cos\phi = \frac{x_E - x_B}{\ell}$$

The errors are given by

$$<\delta\phi^2> = \frac{1}{\ell^2}[\cos^2\phi<\delta y^2> + \sin^2\phi<\delta x^2> - 2\sin\phi\cos\phi<\delta x\delta y>]$$

$$<\delta\tan\lambda^2> = \frac{1}{\ell^2}[<\delta z^2> + \tan^2\lambda\,(\cos^2\phi<\delta x^2> + \sin^2\phi<\delta y^2> + 2\sin\phi\cos\phi<\delta x\delta y>)$$

$$-2\tan\lambda\,(\cos\phi<\delta x\delta z> + \sin\phi<\delta y\delta z>)]$$

$$<\delta\phi\delta\tan\lambda> = \frac{1}{\ell^2}[\cos\phi<\delta y\delta z> - \sin\phi<\delta x\delta z> - \tan\lambda\{\sin\phi\cos\phi\,(<\delta y^2> - <\delta x^2>)$$

$$+ (\cos^2\phi - \sin^2\phi)<\delta x\delta y>\}]$$

where $<\delta x^2> = <\delta x^2>_B + <\delta x^2>_E$

$<\delta y^2> = <\delta y^2>_B + <\delta y^2>_E$ etc.

The length of the track is

$$L = [(x_E - x_B)^2 + (y_E - y_B)^2 + (z_E - z_B)^2]^{\frac{1}{2}}$$

and the error $<\delta L^2>$ is given by the same formula as for normal straight tracks.

10. BEAM TRACK IDENTIFICATION AND TRACK DIRECTIONS

At this stage in the analysis we have first appromations to the angles and in most cases the momentum associated with each track. We can now proceed to find the beam track and to determine where possible the directions of tracks. Each track is checked for the following properties and these must all be satisfied for the track to be identified as the beam track

1) Single vertex track

2) Correct charge

3) The co-ordinates of the non-vertex end of the track must have values appropriate for the position where the beam enters the chamber.

4) The azimuthal and dip angles at the non-vertex end should be within specified limits.

5) The momentum, if known, should be within specified limits about the beam momentum. If this condition is not satisfied it is tried again with plus or minus twice the error found from the first fit added to the momentum.

It would seem most unlikely that two tracks of the same event would both satisfy these conditions. For experiments with a neutral beam or in which no beam track is measured a parameter must be set, which indidates to the program that the first vertex measured is the beam vertex.

We can find the true directions of most tracks. We assume that all tracks, other than the beam track, with a zero end label are moving towards the non-vertex end (i.e. to the end with the zero label). The directions of tracks connected to the beam interaction vertex via charged tracks are also determined. This only leaves ambiguous the directions of two vertex tracks which are only connected to the beam vertex via a neutral track. This ambiguity arises because we do not know which vertices are joined by a neutral track. This ambiguity can be dealt with by the Hypotheses Testing program, but to save calculation and to avoid possible misinterpretation of the event, the direction of such tracks can, if known, be given with the measurements (see Section 2).

We are now in a position to make the main geometrical fit for each track, that is to fit a helix to the rays of the three views.

11. <u>FITTING A HELIX TO THE MEASUREMENTS OF TRACKS.</u>

There are several disadvantages to the method of reconstructing tracks by corresponding points described above. In the first place there is no entirely satisfactory method of doing the interpolations involved in finding corresponding points. Secondly, as the fits are made to space points, we must know the errors on the co-ordinates of these points to be able to calculate the errors on the fitted quantities. The space points are however related in a very complicated way to the measurements, thus the errors on space points are difficult to calculate and are also correlated. Thus there is no really satisfactory method of calculating errors. Finally the measurements from the three views cannot be treated symmetrically.

A method of overcoming these difficulties has been given by W.G. Moorhead(4) (C.E.R.N. 60-33). In this method the parameters of the helix which lies closest to the rays corresponding to the image points on the three views are found by iteration. This method overcomes nearly all the difficulties described above. However even with this procedure it is difficult to relate the error in measurement on the film to errors on the parameters. We shall follow a method of fitting a helix to the rays given by F.T. Solmitz(5) (Ecole Polytechnique, 1960) in which the fitting is done on the film, thus making the calculation of errors quite straightforward.

For each of the three views of the track we have a set of rays defined by co-ordinates (X, Y, U, V) where X, Y are co-ordinates on the front glass of the chamber and U, V are direction ratios for the ray. We find a best fit helix to these rays by projecting both the rays and the helix back onto a nominal film plane and then minimixing Σd_i^2 where d_i is the perpendicular distance from the point where the i^{th} ray cuts the film plane to the projected image of the helix.

11a. <u>Calculation of d</u>

The z = o plane is the hydrogen-glass interface on which the (X, Y) are defined and the z-axis points down into the hydrogen away from the camera lenses. Using information from the first space fit, the co-ordinates of the rays are rotated and translated in the (X, Y) plane so that the origin is at the 'centre of gravity' of the track and the track lies along the y-axis (see Fig. 4).

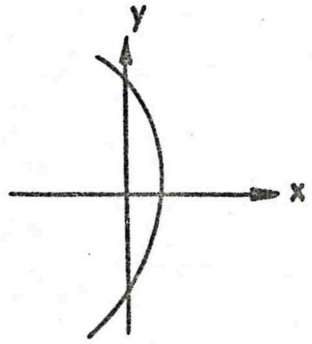

Fig. 4 Orientation of the helix

Note the central portion of the helix always has positive x and that opposite charges travel in opposite directions along the curve.

We first find the projection of a point (x, y, z) onto film. We assume that the camera lens is in the plane $z = -Z_o$ and the film in the plane $z = -Z_o - A_o$. The point (x, y, z) will be near some ray (X, Y, U, V) (we are of course only interested in points on the helix near to a ray). As the ray will go through the camera lens, we can write the lens co-ordinates as $(X - UZ_o, Y - VZ_o, -Z_o)$. Points on the line joining (x, y, z) to the lens are given by

$$(x', y', z') = (X - UZ_o,\ Y - VZ_o,\ -Z_o) + s(x - X + UZ_o,\ y - Y + VZ_o,\ z + Z_o)$$

The point where this line cuts the film has $z' = -(Z_o + A_o)$ giving

$$s = \frac{-A_o}{z + Z_o}$$

Thus the co-ordinates of the image of (x, y, z) on the film are

$$(X_F', Y_F') = \left(X - UZ_o - \frac{A_o}{z + Z_o}(x - X + UZ_o),\ Y - VZ_o - \frac{A_o}{z + Z_o}(y - Y + VZ_o)\right)$$

The ray cuts the film in the point

$$(X_F, Y_F) = (X - U(Z_o + A_o),\ Y - V(Z_o + A_o))$$

The distance d between the two points is given by

$$d^2 = \left[\frac{A_o}{z + Z_o}\right]^2 [(x - X - Uz)^2 + (y - Y - Vz)^2] \qquad (1)$$

We note that when (x, y, z) lies on the ray d = o.

The factor $\frac{A_o}{z + Z_o}$ can be regarded as a weighting factor to be kept constant in the minimizing process. Z_o and A_o could be made different for each ray to take account of the optical distortions due to the chamber window and the liquid in the chamber. We shall however take them as constant and equal to their optic axis values

$$A_o = \mu_H a_o$$

$$Z_o = \mu_H \left[\Sigma \frac{a_G}{\mu_G} - \Sigma a_G + Z_T \right]$$

where
μ_H = refractive index of chamber liquid
a_o = lens to film distance
a_G = glass thickness
μ_G = refractive index of glass
Z_T = total distance from lens to chamber liquid

For points (x, y, z) on a helix, we have

$$\left.\begin{aligned} x &= \alpha + \rho \cos\theta + C_x(\theta) \\ y &= \beta + \rho \sin\theta + C_y(\theta) \\ z &= \gamma + \rho\,\theta \tan\lambda \end{aligned}\right\} \qquad (2)$$

α, β specify the axis of the helix, ρ the radius and λ the pitch. γ is the height of the point on the helix with $\theta = 0$. C_x and C_y are correction to the helix to allow for slowing down of the particle by the liquid and also possibly for small changes in B_z along the track. C_x and C_y depend on p, the momentum at the centre of the track and also the particle mass assumed for the track. We shall give a calculation for C_x and C_y in a later section.

We have next to find the value of θ for the point at the foot of the normal to the image of the helix from the point (X_F, Y_F) where the ray cuts the film. If we let the tangent to the helix at the foot of the normal make an angle ϕ with the x-axis, then

$$\frac{X_F - X_F'}{Y_F - Y_F'} = -\frac{\sin\phi}{\cos\phi}$$

that is $$\cos\phi\,(x_H - X - Uz_H) + \sin\phi\,(y_H - Y - Vz_H) = 0. \qquad (3)$$

Equation 3 must be solved to find the θ corresponding to a given ray. To calculate the tangent angle we note that the tangent $\underline{t}$ to the helix is

$$\underline{t} = \left(-\sin\theta + \frac{1}{\rho}\frac{dC_x}{d\theta},\ \cos\theta + \frac{1}{\rho}\frac{dC_y}{d\theta},\ \tan\lambda\right)$$

and the ray $\underline{r}$ is

$$\underline{r} = (U, V, 1)$$

Then the projection of $\underline{r} \times \underline{t}$ will give the normal to the image of the helix in the neighbourhood of the ray. The x and y components of $\underline{r} \times \underline{t}$ are

$$\left.\begin{aligned} U_1 &= -\cos\Theta + V\tan\lambda - \frac{1}{\rho}\frac{dC_y}{d\Theta} \\ U_2 &= -\sin\Theta - U\tan\lambda + \frac{1}{\rho}\frac{dC_x}{d\Theta} \end{aligned}\right\} \tag{4}$$

Thus we have

$$\sin\phi = \frac{-U_1}{(U_1{}^2 + U_2{}^2)^{\frac{1}{2}}} \qquad \cos\phi = \frac{U_2}{(U_1{}^2 + U_2{}^2)^{\frac{1}{2}}}$$

and substituting into equation 3, the equation for Θ becomes

$$f(\Theta) \equiv U_2(x_H - X - Uz_H) - U_1(y_H - Y - Vz_H) = 0 \tag{5}$$

The distance d is given by equation (1) with the value of Θ found from equation (5). As equation 5 is implicit we must approximate and this we do, following the method of Solmitz, by regarding the helix as made up of small line elements in the neighbourhood of each ray. Thus if we have an approximate value Θ_o as a solution of equation (5), we take

$$\Theta = \Theta_o + \Delta\Theta$$

$$\text{where} \quad \Delta\Theta = -\frac{f(\Theta_o)}{f'(\Theta_o)}$$

Writing

$$\Delta x_o = (x_H - X - Uz_H) \text{ evaluated at } \Theta = \Theta_o$$

$$\text{and} \quad \Delta y_o = (y_H - Y - Vz_H) \text{ evaluated at } \Theta = \Theta_o$$

we find for $\Delta\Theta$

$$\Delta\Theta = \frac{-U_2\Delta x_o + U_1\Delta y_o}{\rho(U_1{}^2 + U_2{}^2) - \cos\Theta_o\,\Delta x_o - \sin\Theta_o\,\Delta y_o}$$

$$\simeq \frac{-U_2\Delta x_o + U_1\Delta y_o}{\rho(U_1{}^2 + U_2{}^2)}$$

where we keep only terms linear in Δx_o and Δy_o.

Substituting into equation (1), d^2 becomes

$$d^2 = w^2\left[(\Delta x_o + \rho U_2\Delta\Theta)^2 + (\Delta y_o - \rho U\,\Delta\Theta)^2\right] \text{ where } w = \frac{A_o}{z + Z_o}$$

That is

$$d^2 = \frac{w^2}{(U_1{}^2 + U_2{}^2)^2}\left[(U_1{}^2\Delta x_o + U_1U_2\Delta y_o)^2 + (U_1U_2\Delta x_o + U_2{}^2\Delta y_o)^2\right]$$

$$= \frac{w^2}{(U_1{}^2 + U_2{}^2)^2}\left[(U_1{}^2 + U_2{}^2)(U_1{}^2\Delta x_o{}^2 + 2U_1U_2\Delta x_o\Delta y_o + U_2{}^2\Delta y_o{}^2)\right]$$

giving finally

$$d = \frac{w}{(U_1^2 + U_2^2)^{\frac{1}{2}}} \left[U_1 \Delta x_o + U_2 \Delta y_o \right] \qquad (6)$$

d is a function of the helix parameters α, β, γ, ρ and $\tan\lambda$ and of θ_o. To obtain best values for the helix parameters we shall minimize Σd_i^2, where the summation is over the rays of all three views. The equations for the minimum are non-linear and we shall solve them by iterating linearized equations (see Appendix I). We use as starting values the values of the helix parameters fc md in the first space fit. Using these starting values we can calculate θ_o for each ray. We give the calculation used in our program in a later section. We note that, as we are solving the minimizing problem by iteration, we could at each stage of the iteration recalculate θ_o for each ray and thus if necessary eliminate the line segment approximation. In general, however, the curvature of the helix is not very great and we shall have good starting values for the helix parameters, so that this recalculation of the θ_o's is unnecessary.

11b. Least squares calculation

To perform the least squares calculation we require the derivatives of d with respect to the helix parameters. Writing

$$w' = \frac{w}{(U_1^2 + U_2^2)^{\frac{1}{2}}}$$

we find

$$\frac{\partial d}{\partial \alpha}, \frac{\partial d}{\partial \beta}, \frac{\partial d}{\partial \gamma} = w' (U_1, U_2, U_3)$$

where $U_3 = -UU_1 - VU_2$

and $$\frac{\partial d}{\partial \rho} = w'(U_1 \cos\theta + U_2 \sin\theta + \theta U_3 \tan\lambda)$$

$$\frac{\partial d}{\partial \tan\lambda} = w' U_3 \left[\rho\theta + \frac{U_1 \Delta y_o - U_2 \Delta x_o}{U_1^2 + U_2^2} \right]$$

The second term in the expression $\frac{\partial d}{\partial \tan\lambda}$ depends on Δx_o and Δy_o and will be small. We include this term to ensure that the iteration converges. We note that U_3 does not depend on the helix parameters and is thus constant throughout the iteration. Using the relation (see Section 11e).

$$\frac{\sin\theta}{\rho} \frac{dC_x}{d\theta} = \frac{\cos\theta}{\rho} \frac{dC_y}{d\theta}$$

we can simplify $\frac{\partial d}{\partial \rho}$ to become

$$\frac{\partial d}{\partial \rho} = w' \left[\tan\lambda \, (V \cos\theta - U \sin\theta + U_3 \theta) - 1 \right]$$

Writing $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5 = \alpha, \beta, \gamma, \rho, \tan\lambda$ and given starting values α_λ^o, we can calculate improved values α_λ given by (see Appendix I)

$$\alpha_\lambda = \alpha_\lambda^o + \delta\alpha_\lambda$$

where $$\delta\alpha_\lambda = -H_{\lambda\mu} Y_\mu$$

with $$H_{\lambda\mu} = \sum_{i=1}^{N} \left(\frac{\partial d_i}{\partial \alpha_\lambda}\right)_o \left(\frac{\partial d_i}{\partial \alpha_\mu}\right)_o$$

$$Y_\mu = \sum_{i=1}^{N} \left(\frac{\partial d_i}{\partial \alpha_\mu}\right)_o d_i^o$$

The summations are over all the rays and N is the total number of rays. The new value of Σd_i^2 is

$$\Sigma d_i^2 = \Sigma d_i^{2o} + \Delta\Sigma d_i^2$$

where $$\Delta\Sigma d_i^2 = \delta\alpha_\lambda \; Y_\lambda$$

In practice we find that $\delta\alpha \approx -\delta\rho$ and in order to keep accuracy in the calculation of $\delta\alpha_\lambda$, it is necessary to change variable to $\alpha + \rho$ instead of α. We use the values found by the first space fit for α_λ^o and iterate until

$$\frac{\Delta\Sigma d_i^2}{\Sigma d_i^2} < \varepsilon$$

We have taken $\varepsilon = 0.05$ and find that at most two iterations are necessary for convergance.

llc. Calculation of momentum and errors

From the parameters of the fitted helix, we calculate the momentum variables of the track used in the kinematic fits. These variables are all defined at the centre of the track and are ϕ, the aximuthal angle, $\tan\lambda$, where λ is the dip angle and $\frac{1}{p}$ where p is the momentum of the track in Gev/c. $\tan\lambda$ can be taken directly from the helix fit. The momentum p is given from the radius of the helix by

$$p = \frac{0.3 H\rho}{\cos\lambda} \times 10^{-3} \text{ Gev/c}$$

where H is the z-component of the magnetic field in kilogauss. We take this momentum to be the momentum of the track at the point $\theta = o$. The corresponding azimuth ϕ will then be

$$\phi = \theta_R \pm \frac{\pi}{2}$$

where θ_R is the angle that the measurements have been rotated through and the sign is taken according to the sense in which the particle is moving round the helix.

To calculate the length of the track, we first find the θ - value for a vertex on the track. From the first space fit we have an approximate value of θ for a vertex on the track. We use this value of θ to calculate the corrections C_x and C_y for the vertex, then

$$\theta_V = \tan^{-1}\left\{\frac{y_V - \beta - C_y}{x_V - \alpha - C_x}\right\} \qquad (8)$$

where x_V, y_V are the vertex co-ordinates rotated into the helix fit frame. The length of the track from its centre to this vertex is

$$= \left|\frac{\rho}{\cos\lambda}\ \theta_V\right|$$

For tracks with only one vertex, this is taken as the half length. If the track has a vertex at each end, the θ_V of the other vertex is found using formula (8) and the half length of the track is taken as

$$= 0.5\left|\frac{\rho}{\cos\lambda}\left(\theta_V^{(1)} - \theta_V^{(2)}\right)\right|$$

The ϕ and $\frac{1}{p}$ of the track are altered so that they apply to the centre of the track (by length).

The error matrix for the parameters of the helix is

$$<\delta a_\lambda\, \delta a_\mu> \;=\; \sigma_F^2\, H^{-1}_{\lambda\mu} \qquad (9)$$

where σ_F is the R.M.S. measurement error on the film. For errors on the track variables we have

$$\delta\phi = -\frac{\delta a_2}{\rho}$$

$$\delta\tan\lambda = \delta a_5$$

$$\delta\left(\frac{1}{p}\right) = -\frac{1}{p}\left[\frac{\delta a_4}{\rho} + \sin\lambda\cos\lambda\,\delta a_5\right] \qquad (10)$$

where $\rho = a_4$ and $\sin\lambda\cos\lambda = \dfrac{a_5}{(1 + a_5^2)}$. The expression for $\delta\phi$ applies to the centre of the track and is derived in section 9 (1st space fit). It may be necessary to include an additional term $\frac{\delta s}{\rho}$ where δs is the uncertainty in the position of a vertex on the track projected onto the horizontal plane.

From equations (10) we can work out the elements of the error matrix for the track variables

$$<\delta\phi^2> = \frac{<\delta a_2{}^2>}{\rho^2}$$

$$<\delta\tan\lambda^2> = <\delta a_5{}^2>$$

$$<\delta(\tfrac{1}{p})^2> = \frac{1}{p^2}\left[\frac{<\delta a_4{}^2>}{\rho^2} + \frac{2\sin\lambda\cos\lambda}{\rho}<\delta a_4\,\delta a_5> + \sin^2\lambda\cos^2\lambda<\delta a_5{}^2>\right]$$

$$<\delta\tan\lambda\,\delta(\tfrac{1}{p})> = -\frac{1}{p}\left[\frac{<\delta a_4\delta a_5>}{\rho} + \sin\lambda\cos\lambda<\delta a_5{}^2>\right]$$

$$<\delta\phi\,\delta(\tfrac{1}{p})> = \frac{1}{p\rho}\left[\frac{<\delta a_4\delta a_2>}{\rho} + \sin\lambda\cos\lambda<\delta a_2\,\delta a_5>\right]$$

$$<\delta\phi\,\delta\tan\lambda> = \frac{-<\delta a_2\,\delta a_5>}{\rho}$$

The value of χ^2 for the fit is

$$\chi^2 = \frac{1}{\sigma_F{}^2}\sum_{i=1}^{n} d_i{}^2(a_\lambda *)$$

where $a_\lambda = a_\lambda^*$ are the values at the minimum. The expected value of χ^2 would be $n-5$ (n = no. of rays), if the effects of coulomb scattering, errors in the optical constants and turbulence could be neglected. We define $\sigma_p{}^2$ as

$$\sigma_p{}^2 = \frac{1}{n-5}\sum_{i=1}^{n} d_i{}^2\,(a_\lambda^*)$$

then by histogramming $\sigma_p{}^2$ for a large number of tracks we can determine a suitable standard value of $\sigma_p{}^2$ for use in given circumstances. Fig.5 shows a histogram of $\sigma_p{}^2$ for tracks from the 30 cm hydrogen bubble chamber of the C.E.R.N. bubble chamber group measured on the Oxford University measuring machine. If $\sigma_p{}^2$ is larger than a given constant the program assumes that the track has been poorly measured and $\sigma_p{}^2$ is used in place of the standard $\sigma_F{}^2$ in the calculation of the error matrix for the track.

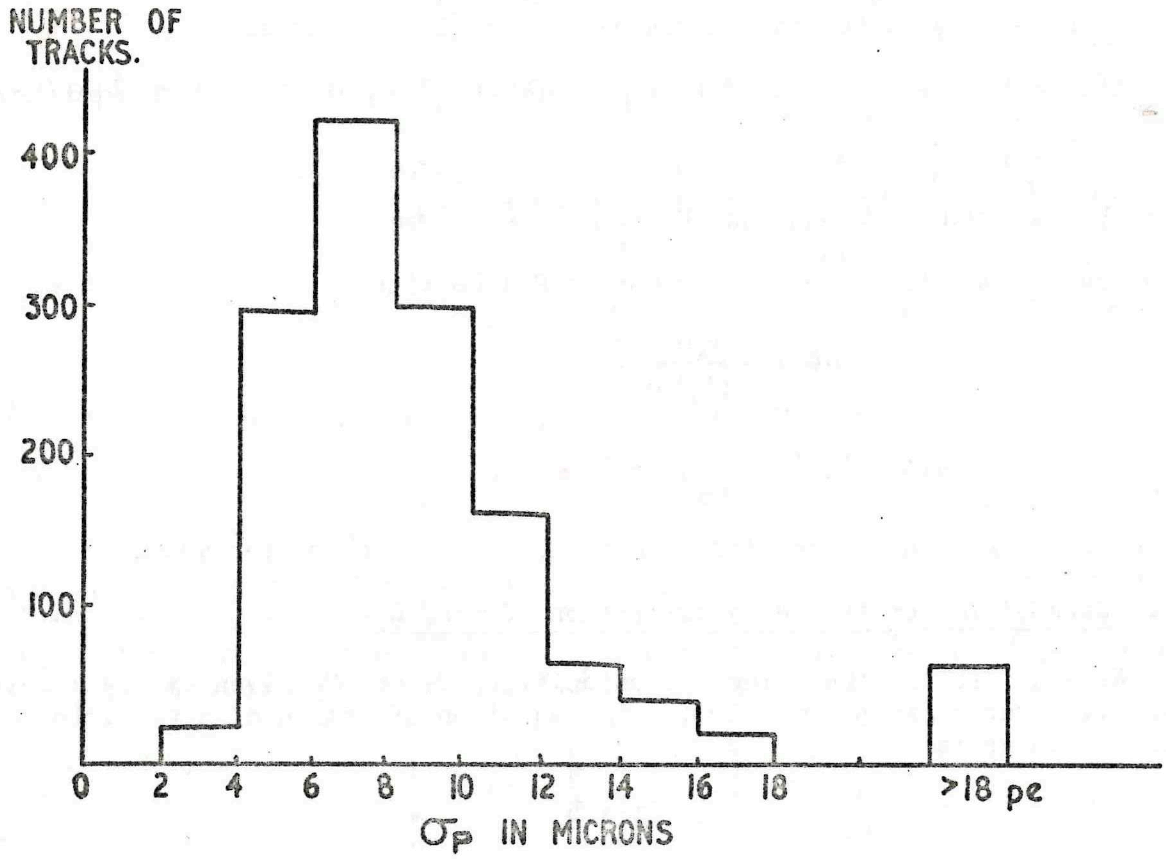

Fig.5 Frequency histogram of σp, the film measurement error.

11d. Calculation of θ_o for each ray

The rays of the main view, for which space points have been found, will have values of θ calculated from the first space fit. For the remaining rays we calculate an approximate θ

$$\theta = \theta_1 + \delta\theta$$

where $$\delta\theta = \frac{(\alpha + \rho\cos\theta_1) - (X + U(\gamma + \rho\theta_1\tan\lambda))}{\rho\,(U\tan\lambda + \sin\theta_1)} \quad \text{for } \theta_1 < \frac{\pi}{4}$$

or $$\delta\theta = \frac{(\beta + \rho\sin\theta_1) - (Y + V(\gamma + \rho\theta_1\tan\lambda))}{\rho\,(V\tan\lambda - \cos\theta_1)} \quad \text{for } \theta_1 > \frac{\pi}{4}$$

The helix parameters α, β etc. are those found from the first space fit. For the first ray on any view θ is taken equql to θ_V, the θ value of the vertex at the beginning of the track. These expressions are found by linearizing the equation giving the point on the helix with x and z co-ordinates (or y and z co-ordinates) equal to those of a point on the ray. In this way approximate Θ-values are found for all the rays.

Each Θ-value is further improved by iteration to satisfy

$$f(\Theta) \equiv U_2(\alpha + \rho\cos\Theta - X - U(\gamma + \rho\Theta\tan\lambda) - U_1(\beta + \rho\sin\Theta - Y - V(\gamma + \rho\Theta\tan\lambda))$$

$$= 0$$

where $U_1 = -\cos\Theta + V\tan\lambda$ and $U_2 = -\sin\Theta - U\tan\lambda$

An approximate solution Θ is improved to $\Theta + \delta\Theta$ with

$$\delta\Theta = -\frac{f(\Theta)}{f'(\Theta)}$$

$$\text{with } f'(\Theta) = \rho(U_1^2 + U_2^2)$$

In general one step of the iteration is sufficient for convergance.

11e. Calculation of the helix corrections C_x and C_y

We shall follow the method of calculating C_x and C_y given by Frank Solmitz[3] (L.R.L., Alvarez group memo 220). The equation of motion of a particle in a bubble chamber is

$$p(s)\frac{d\underline{t}}{ds} = -0.3e\,(\underline{t} \times \underline{B}) \qquad (1)$$

s is distance along the track, p(s) is the momentum of the particle in Mev/c, $\underline{t}$ is the unit tangent vector to the track, $\underline{B}$ the magnetic field in kilogauss and $e = \pm 1$ the charge of the particle. The − sign is necessary because we use left-handed axes. The momentum p decreases with s due to the slowing down effect of the chamber liquid. We shall consider the case in which $\underline{B}$ is constant and equal to (o, o, B). We make a change of variable to Θ where

$$s = \frac{e\rho_o\Theta}{\cos\lambda} \qquad (2)$$

where ρ_o is the radius of curvature of the track at s = o, given by

$$\rho_o = \left|\frac{p_o\cos\lambda}{0.3B}\right| \quad \text{with } p_o = p(o)$$

Then $\frac{dz}{d\Theta}$ = constant = $\rho_o\tan\lambda$.

The azimuthal angle ϕ, that is the angle that the projection of the tangent of the track makes with the x-axis, is given by

$$\tan\phi = \frac{dy}{dx} = \frac{dy}{d\Theta}\Big/\frac{dx}{d\Theta} \qquad (3)$$

Differentiating $$\frac{d\phi}{d\theta} = \frac{\cos^2\phi}{\left(\frac{dx}{d\theta}\right)^2}\left[\frac{dx}{d\theta}\frac{d^2y}{d\theta^2} - \frac{dy}{d\theta}\frac{d^2x}{d\theta^2}\right]$$

and substituting from equation (1)

$$\frac{d\phi}{d\theta} = \frac{1}{\left[\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2\right]}\left[\frac{0.3\, e^2 B\rho_o}{p\cos\lambda}\left(\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2\right)\right]$$

Giving finally

$$\frac{d\phi}{d\theta} = \frac{p_o}{p}\,. \tag{4}$$

We now approximate by expanding $\frac{p_o}{p}$ as a polynominal in θ

$$\frac{p_o}{p} = 1 + 2b\theta + 3c\theta^2 \tag{5}$$

and then determine b and c so that the formula is correct at 2 points $\pm$ s on the track (it is also correct at s = o). The half-length L of the track is given by the first fit and we take $s = \alpha L$ where α is a constant less than unity. Writing $p_+ = p(+s)$ and $p_- = p(-s)$ and solving for b and c we find

$$b = \frac{1}{4\theta_\alpha}\left(\frac{p_o}{p_+} - \frac{p_o}{p_-}\right)$$

and $$c = \frac{1}{6\theta_\alpha^2}\left(\frac{p_o}{p_+} + \frac{p_o}{p_-} - 2\right) \tag{6}$$

where $\theta_\alpha = \frac{0.3B}{ep_o}\alpha L$.

We take for p_o the value of momentum for the track given by the first fit and then p_+ and p_- can be found using the range-momentum table. If the value of p_o is such that the range R is less than L, then we take for p_o a value of momentum that would give a range equal to L.

Integration of (4) and (5) gives

$$\phi = \phi_o + \theta + b\theta^2 + c\theta^3 \tag{7}$$

and from equations (2) and (3) we have

$$\frac{dx}{d\theta} = \rho\cos\phi$$

$$\frac{dy}{d\theta} = \rho\sin\phi \tag{8}$$

To integrate these equations for C_x and C_y we make the assumption that $(b\theta^2 + c\theta^3)$

is a small angle and linearize $\sin\phi$ and $\cos\phi$ in this angle. If the track is in a frame such that it is moving along the y-direction at the point $s = \Theta = o$, then $\phi_o = \frac{\pi}{2}$ and

$$\cos\phi \simeq -\sin\Theta - \cos\Theta\,(b\Theta^2 + c\Theta^3)$$

$$\sin\phi \simeq \cos\Theta - \sin\Theta\,(b\Theta^2 + c\Theta^3)$$

Then

$$\int \cos\phi\, d\Theta = \cos\Theta + \sin\Theta\,(-c\Theta^3 - 6\Theta^2 + 6c\Theta - + 2b) + \cos\Theta\,(-3c\Theta^2 - 2b\Theta + 6c)$$

$$\int \sin\phi\, d\Theta = \sin\Theta + \sin\Theta\,(-3c\Theta^2 - 2b\Theta + 6c) + \cos\Theta\,(c\Theta^3 + b\Theta^2 - 6c\Theta - 2b)$$

and finally

$$x = \alpha + \rho\cos\Theta + C_x$$

$$y = \beta + \rho\sin\Theta + C_y$$

where

$$C_x = \rho[\,\sin\Theta\,(-c\Theta^3 - b\Theta^2 + 6c\Theta + 2b) + \cos\Theta\,(-3c\Theta^2 - 2b\Theta + 6c) - 6c\,]$$

and $C_y = \rho[\,\sin\Theta\,(-3c\Theta^2 - 2b\Theta + 6c) + \cos\Theta\,(c\Theta^3 + b\Theta^2 - 6c\Theta - 2b) + 2b\,]$

We also require the derivative of C_x and C_y and these are

$$\frac{1}{\rho}\,\frac{dC_x}{d\Theta} = \cos\Theta\,(-c\Theta^3 - b\Theta^2)$$

$$\frac{1}{\rho}\,\frac{dC_y}{d\Theta} = \sin\Theta\,(-c\Theta^3 - b\Theta^2)$$

12. COULOMB ERRORS

To the measurement errors calculated from the helix fit, we must add the uncertainties in the momentum variables caused by multiple Coulomb scattering. At present we use the formulae found by R. Böck from a Monte Carlo calculation(6). The mean angle of scattering is given by

$$<\Theta_{sc}^2> = f^2 L$$

$$\text{where } f^2 = \frac{K^2}{p^2\beta^2} = \frac{K^2}{p^2}\left(1 + \frac{m^2}{p^2}\right)$$

with L = length of the track, K a constant determined by the properties of the chamber liquid and m equal to the particle mass. In terms of f^2 we have

$$<\delta\phi^2>_{Coul} = C_2 f^2 L$$

$$<\delta\tan\lambda^2>_{Coul} = C_1 f^2 L \sec^4\lambda$$

$$\langle \delta(\tfrac{1}{p})^2 \rangle_{Coul} = \frac{f^2}{p^2}\left[\frac{C_o\rho^2}{L} + \tan^2\lambda\, C_1 L\right]$$

where C_o, C_1 and C_2 are constants.

The current values of the constants used in the program are

$$K^2 = \frac{(21.2 \times 10^{-3})^2}{L_{rad}} \quad \text{with } L_{rad} = 990 \text{ cms.}$$

giving $$K^2 = 0.45 \times 10^{-6}\ \text{Gev}^2/\text{cm.}$$

and $C_o = 1.14$, $C_1 = 0.405$ and $C_2 = 0.23$.

13. CHARGE BALANCE

The charge associated with each track has been found from the curvature of the track, except in cases where the track is straight to within the accuracy of measurement. We can use charge balance at each vertex to check the assignment of charges and where there is one straight track at a vertex, to find the charge of this track. If N is the number of tracks at a given vertex, then

$$\sum_{\substack{\text{outgoing}\\\text{tracks}}} e_i - \underset{\substack{\text{incoming}\\\text{track}}}{e} = \begin{cases} 0 & N \text{ even} \\ 1 & N \text{ odd.}\end{cases}$$

This relation is checked for each vertex (except in the case $N = 1$ where the track may be a stopped proton recombining with an electron) or used to find a missing charge. If a missing charge is found and if there is any track remaining with undetermined charge after the charge balance, the process is repeated. It is worth noting that charge balance gives no information about the direction of tracks, but equally the charge balance can be carried out even if some track directions are unknown.

14. BEAM TRACKS, STOPPING TRACKS AND ELASTIC SCATTERS

In this section of the program we deal with what might be called special cases.

14a. Beam tracks

We allow for three different ways of treating beam tracks. Firstly nothing special can be done, secondly the momentum and the error on the momentum can be replaced by fixed standard values given by the engineering of the beam. In this case the correlation errors involving the momentum are set to zero. Finally the results from the helix fit and the beam engineering values can be used together to give a weighted mean value of $\frac{1}{p}$. For the weighted mean we have

$$\frac{1}{p} = \frac{\langle\delta(\frac{1}{p})^2\rangle_{ENG}\ \frac{1}{p_{HFIT}} + \langle\delta(\frac{1}{p})^2\rangle_{HFIT}\ \frac{1}{p_{ENG}}}{\left(\langle\delta(\frac{1}{p})^2\rangle_{ENG} + \langle\delta(\frac{1}{p})^2\rangle_{HFIT}\right)}$$

In this case the errors $\langle\delta(\frac{1}{p})^2\rangle$, $\langle\delta\phi\delta(\frac{1}{p})\rangle$ and $\langle\delta\tan\lambda\delta(\frac{1}{p})\rangle$ must be multiplied by the factor

$$\frac{\langle\delta(\frac{1}{p})^2\rangle_{ENG}}{\left(\langle\delta(\frac{1}{p})^2\rangle_{ENG} + \langle\delta(\frac{1}{p})^2\rangle_{HFIT}\right)}$$

14b. Stopping tracks

There are two ways in which the program can take a vertex as a stopping vertex. First if this is indicated with the measurements (see Section 2) and secondly if the vertex has only one track associated with it and if this track is also a two vertex track (note this rules out the beam interaction vertex). Before accepting the vertex as a stopping vertex, the co-ordinates of the vertex are checked to see that the vertex is in the liquid of the chamber and not on one of the windows of the chamber.

For stopping tracks we calculate the momentum at the centre of the track by taking the range equal to the half-length and finding the corresponding momentum from the range-momentum table. The momentum error is given by

$$\langle\delta(\frac{1}{p})^2\rangle = \frac{1}{p^4}\left[(\langle\delta L_m{}^2\rangle + \langle\delta L_s{}^2\rangle)\ \frac{dp}{ds}^2 + p^2\ \frac{\delta d^2}{d^2}\right]$$

where $\langle\delta L_m{}^2\rangle$ is the measurement error on the length, $\langle\delta L_s{}^2\rangle$ is the uncertainty in the range for a given momentum caused by the statistical nature of the energy loss process. It is a function of $\frac{T}{m}$ where T is the kinetic energy and m the mass is found in the program from a table.

d is the density of the liquid

$\langle\delta d^2\rangle$ is the uncertainty in the density

$\frac{dp}{ds}$ is the gradient of the range-momentum table at the momentum p.

The momentum and error are calculated for the three masses π, K and proton, and a check is made that the momentum from curvature and the momentum from range agree within the errors. Disagreement causes a print-out, but at present the momentum from range is always taken. The correlation errors involving the momentum are set to zero.

The present method of dealing with stopping tracks is somewhat arbitrary. A better method would be to leave the decision whether to take momentum from range or curvature, where these agree within errors, as a further ambiguity to be treated by the hypothesis testing program.

14c. Elastic Scatters

All vertices where there are three charged tracks are tested for coplanarity. If $\underline{t}_1$ is the tangent vector to the incoming track at the vertex and $\underline{t}_2$ and $\underline{t}_3$ the tangents to the outgoing tracks, then Θ the angle between the vector $\underline{t}_1$ and the normal to the plane containing $\underline{t}_2$ and $\underline{t}_3$ is given by

$$\cos\Theta = \frac{\underline{t}_1.(\underline{t}_2 \times \underline{t}_3)}{|\underline{t}_2 \times \underline{t}_3|}$$

or in terms of the azimuthal and dip angles of the tracks

$$\cos\Theta = \frac{\tan\lambda_1\sin(\phi_3-\phi_2)+\tan\lambda_2\sin(\phi_1-\phi_3)+\tan\lambda_3\sin(\phi_2-\phi_1)}{[(1+\tan^2\lambda_1)(\tan^2\lambda_2+\tan^2\lambda_3-2\tan\lambda_1\tan\lambda_2\cos(\phi_3-\phi_2)+\sin^2(\phi_3-\phi_2))]^{\frac{1}{2}}}$$

The azimuthal angles at the vertex are found from the angles at the centre of the track by

$$\phi_{\text{vertex}} = \phi_{\text{centre}} \pm \frac{0.3\,B_z L}{p}$$

where L is the half length of the track and the sign is taken according to the charge and the direction of the track.

For tracks to be coplanar Θ must equal $\frac{\pi}{2}$, that is $\cos\Theta$ equal to zero. To test for coplanarity we test that $\cos\Theta$ differs from zero by less than some given multiple, say three, of the error on $\cos\Theta$. To simplify to the calculation of $\langle\delta\cos\Theta^2\rangle$ we assume that the errors on the dip angles dominate and as we are only interested in $\cos\Theta$ small we ignore errors in the denominator. With these approximations we find

$$\langle\delta\cos\Theta^2\rangle = \frac{\langle\delta\tan\lambda_1{}^2\rangle\sin^2(\phi_3-\phi_2)+\langle\delta\tan\lambda_2{}^2\rangle\sin^2(\phi_1-\phi_3)+\langle\delta\tan\lambda_3{}^2\rangle\sin^2(\phi_2-\phi_1)}{[(1+\tan^2\lambda_1)(\tan^2\lambda_2+\tan^2\lambda_3-2\tan\lambda_2\tan\lambda_3\cos(\phi_3-\phi_2)+\sin^2(\phi_3-\phi_2))]}$$

Vertices with three coplanar tracks are taken to be elastic scatters and are marked for special treatment by the hupothesis testing program. In addition we assume that there are no neutrals at the vertex and if possible use this information to resolve a track direction ambiguity.

This is the last calculation performed by the geometry program and the results are now written onto a Library tape. A summary of the results is printed out.

15. OPERATION OF THE LIBRARY TAPE AND BOOK-KEEPING ARRANGEMENTS

All the programs of the bubble chamber analysis system are built round a Library system. The results of the main calculations performed on each event are kept on magnetic tape as a Library of events. Events are stored in frame number order and the format of the Library is the same for all programs.

In general programs will be adding new results to a Library tape and to avoid writing on tapes containing valuable information the programs all operate in the following manner. First the results of previous calculations for an event are read into the computer from an 'old' Library tape. If no calculation is to be done on the event, then the results are simply copied onto a new magnetic tape. If calculations are performed, then the new results are either added to or used to overwrite the old results and then written onto the new magnetic tape. Thus no writing operations are made onto the old tape and these can be prevented from happening by accident by 'file protecting' the old tape. Finally both tapes can be kept until further computer runs have established that the new Library tape is a good replacement for the old.

For this method of operation to be possible programs must be able to read Library tapes that include events that have been processed any number of times by any of the programs of the system. This is possible as the Library format is the same for all programs. This Library format consists of five lists

(1) Book-keeping List
(2) Vertex List
(3) Track List
(4) Helix fit List
(5) Kinematic fit List.

The book-keeping list contains the frame number, the event number on the frame; information about scanning and re-scanning, information about measurement and any re-measurement and details of the calculations done on the event. The vertex list contains the space co-ordinates with errors of each vertex, and the total number of charged tracks at each vertex and the numbers of these tracks. The track list contains all the information about each track other than the results of the various least squares fits for the momenta and angles. It includes the end labels, the direction label, the charge, the magnetic field associated with the track and an index for finding the helix fit results in the helix fit list. The helix fit list contains the results of the various fits made to find the momentum variables of each track. For each fit it contains the azimuthal angle ϕ, the tangent of the dip angle $\tan\lambda$, and the reciprocal of momentum $\frac{1}{p}$ (all defined at the centre of the track) together with the diagonal and correlation errors on the quantities. It also includes the length of the track and the mass associated with the fit. The results of the first space fits and the mass-dependant helix fits are stored in this list together with the results for successful kinematics fits. Finally the kinematic fit list gives details of all the kinematic fits made on the event.

Full details of the Library format are given in Appendix V. The book-keeping list must always be present, but any or all of the other lists can be empty. All Library tapes start with a title record and this is checked by the program before processing starts.

Events to be processed by the Geometry program must always be presented in frame number order. There are two modes of operation of the program, operation with and without a 'Master List' of events. In the 'master list' mode a book-keeping program is first used to write book-keeping lists for a set of events found at the scanning stage on a new magnetic tape. This program also writes a title record at the front of the tape with the name of the tape and the total and highest and lowest frame numbers of the events. Subsequently this tape is used as the 'old Library tape of a run with the Geometry program. Used in this mode the Geometry program only deals with measurements of events which have book-keeping records and rejects any other events. When an event is successfully processed by the Geometry program, vertex, track and helix fit lists are added to the book-keeping and written on the new Library tape. In the case of a re-measurement the old results are overwritten. If the event fails in the Geometry program only the book-keeping record is written on the new tape. In all cases information about the measurement and the computer run are added into the book-kee ing list.

In the non 'master list' mode events that go successfully through the Geometry program are either written on a new tape or merged in with old results.

In the 'master list' mode additional book-keeping programs can be used to read a Library tape and to find the progress of events through the system.

One of the main problems of keeping a library of events in frame number order is that inserting information involves 'pushing down' all subsequent information. The Geometry program is always used to process the whole of a tape so the only problem is to avoid pushing information off the end of the tape. Using the 'master list' we shall try to limit the number of events put on a tape so that this is unlikely to happen. In the case of accidental overflow, we shall use a continuation tape and will not alter the other tapes of the experiment.

16. SUMMARY

The main purpose of the program is to calculate the momentum with errors for each track of a bubble chamber event. We choose as variables to specify the momentum the azimuthal angle ϕ, the tangent of the dip angle $\tan\lambda$, and the reciprocal of the absolute value of the momentum in Gev/c $\frac{1}{p}$. The values given by the program are for the centre of each track. To find these quantities the program makes a first fit to space co-ordinates found on the track by the method of corresponding points. This part of the program follows the methods of the PANG program[(2)] with the differences that rays rather than film points are used and also that all three views are used to find the space points. A second mass-dependant fit of a helix to the rays of all three views is made next. This fit is made essentially by projection onto the film plane and thus makes for an easy calculation of errors in momentum due to errors in measurement. Tracks are either fitted assuming the K-mass or three fits are made with the masses K, π and proton. The fit is an iterative process and uses values of the helix parameters found from the first fit as starting values for the iteration. The procedure almost always converges in one or two iterations.

Further information about the event is worked out for use by the Hypothesis Testing program. Where possible the connections between tracks, the directions of tracks and the charge are found by the program. The feature here of special interest is that the system has been designed so that there is no rigid connection between the way in which an event is measured and the way in which the kimematic analysis of the event is made. In addition an attempt has been made to keep the symbolic information that has to be supplied with the measurements to a minimum. Further developments in this direction will be to improve vertex identification and to introduce a stereo-matching routine to relax the restriction that the event must be measured in the same order on each view.

The program is part of a complete bubble chamber analysis system that is being written at the Rutherford Laboratory. It is built into a Library system and includes book-keeping facilities. The program is written in the Fortran language and has been used to process about a thousand events so far. It is hoped that few program errors remain to be found.

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Appendix I Least Squares Method

We have a set of measurements x_i of a function $f(\alpha_\lambda, \theta)$ at the points $\theta = \theta_i$. The α_λ are parameters defining the function and we write $f_i(\alpha_\lambda) \equiv f(\alpha_\lambda, \theta_i)$. We wish to find the best values of the α_λ by minimizing

$$\sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} (x_i - f_i(\alpha_\lambda))^2 \tag{1}$$

where n is the total number of measurements, that is by solving the equations

$$\frac{\partial}{\partial\alpha_\lambda} \sum_{i=1}^{n} d_i^2(\alpha_\lambda) = 0 \tag{2}$$

Starting from an approximate solution of equations (2), $\alpha_\lambda = \alpha_\lambda^{o}$, we can improve this solution. Expanding Σd_i^2 in a Taylor series about α_λ^{o}, we have

$$\Sigma d_i^2(\alpha_\lambda) = \Sigma d_i^2(\alpha_\lambda^{o}) + \delta\alpha_\lambda\left(\frac{\partial\Sigma d_i^2}{\partial\alpha_\lambda}\right)_o + \tfrac{1}{2}\,\delta\alpha_\mu\delta\alpha_\lambda\frac{\partial^2\Sigma d_i^2}{\partial\alpha_\mu\partial\alpha_\lambda} + O(\delta\alpha_\lambda^3) \tag{3}$$

Substituting into equations (2), we have

$$\frac{\partial\Sigma d_i^2}{\partial\alpha_\lambda} = \left(\frac{\partial\Sigma d_i^2}{\partial\alpha_\lambda}\right)_o + \delta\alpha_\mu \frac{\partial^2\Sigma d_i^2}{\partial\alpha_\lambda\partial\alpha_\mu}$$

$$= 0$$

The improved solution is $\alpha_\lambda = \alpha_\lambda^{o} + \delta\alpha_\lambda$ where $\delta\alpha_\lambda$ is given by

$$\delta\alpha_\lambda = -H^{-1}_{\lambda\mu} Y_\mu \tag{4}$$

$$\text{where}\quad H_{\lambda\mu} = \tfrac{1}{2}\sum_i\left(\frac{\partial^2 d_i^2}{\partial\alpha_\lambda\partial\alpha_\mu}\right)_o \quad\text{and}\quad Y_\mu = \sum_i d_i^{o}\left(\frac{\partial d_i}{\partial\alpha_\mu}\right)_o$$

We can rewrite the second derivative

$$\tfrac{1}{2}\frac{\partial^2 d^2}{\partial\alpha_\lambda\partial\alpha_\mu} = \frac{\partial}{\partial\alpha_\lambda}\left(d\,\frac{\partial d}{\partial\alpha_\mu}\right) = \left(\frac{\partial d}{\partial\alpha_\lambda}\right)\left(\frac{\partial d}{\partial\alpha_\mu}\right) + d\frac{\partial^2 d}{\partial\alpha_\lambda\partial\alpha_\mu}$$

The second term can be neglected as it contains the factor d, which is small. (Note. The derivatives of d do not involve x_i and are the same as the derivatives of f.) Thus we have finally

$$H_{\lambda\mu} = \sum_{i=1}^{n}\left(\frac{\partial d_i}{\partial\alpha_\lambda}\right)_o\left(\frac{\partial d_i}{\partial\alpha_\mu}\right)_o$$

Substituting back into equation (3) we have

$$\Sigma d_i^2(\alpha_\lambda^o + \delta\alpha_\lambda) = \Sigma d_i^2(\alpha_\lambda^o) + \delta\alpha_\lambda \ \Sigma 2d_i^o \left(\frac{\partial d_i}{\partial\alpha_\lambda}\right) - \delta\alpha_\lambda \ \Sigma d_i^o \left(\frac{\partial d_i}{\partial\alpha_\lambda}\right)_o$$

$$= \Sigma d_i^2(\alpha_\lambda^o) + \delta\alpha_\lambda \ Y_\lambda$$

Thus given close enough starting values, we can iterate to find the values $\alpha_\lambda = \alpha_\lambda^*$ at the minimum.

Assuming that the true values are α_λ and that the expectation value of x_i is given by

$$< x_i > = f_i(\alpha_\lambda)$$

we can calculate $<\alpha_\lambda^*>$. Let $\alpha_\lambda = \alpha_\lambda^o + \delta\alpha_\lambda$ and $\alpha_\lambda^* = \alpha_\lambda^o + \delta\alpha_\lambda^*$, then from equation (4)

$$\delta\alpha_\mu^* \sum_i \left(\frac{\partial d_i}{\partial\alpha_\mu}\right)_o \left(\frac{\partial d_i}{\partial\alpha_\lambda}\right)_o = -\sum_i d_i^o \left(\frac{\partial d_i}{\partial\alpha_\lambda}\right)_o$$

$$= -\sum_i (x_i - f_i(\alpha_\lambda^o)) \left(\frac{\partial d_i}{\partial\alpha_\lambda}\right)_o$$

Taking expectation values

$$<\delta\alpha_\mu^*> \sum_i \left(\frac{\partial d_i}{\partial\alpha_\mu}\right)_o \left(\frac{\partial d_i}{\partial\alpha_\lambda}\right)_o = -\sum_i (f_i(\alpha_\lambda) - f_i(\alpha_\lambda^o)) \left(\frac{\partial d_i}{\partial\alpha_\lambda}\right)_o$$

however

$$f_i(\alpha_\lambda) - f_i(\alpha_\lambda^o) = \delta\alpha_\lambda \left(\frac{\partial f_i}{\partial\alpha_\lambda}\right)_o = -\delta\alpha_\lambda \left(\frac{\partial d_i}{\partial\alpha_\lambda}\right)_o$$

and on substitution we have

$$<\delta\alpha_\mu^*> = \delta\alpha_\mu$$

Thus the expectation value $<\alpha_\lambda^*>$ equals α_λ and we have an unbiased estimate of the α_λ.

Taking the expectation value $< d_i^2(\alpha_\lambda) > = \sigma^2$, and assuring that there is no correlation between measurements, we calculate the error matrix for the fit $<(\alpha_\lambda^* - \alpha_\lambda)(\alpha_\mu^* - \alpha_\mu)> = <(\delta\alpha_\lambda^* - \delta\alpha_\lambda)(\delta\alpha_\mu^* - \delta\alpha_\mu)>$. We have

$$\delta\alpha_\lambda^* = -H_{\gamma\lambda}^{-1} \sum_i (x_i - f_i(\alpha_\lambda^o)) \left(\frac{\partial d_i}{\partial\alpha_\gamma}\right)_o$$

and

$$\delta\alpha_\lambda = -H^{-1}_{\gamma\lambda} \sum_i \left(f_i(\alpha_\lambda) - f_i(\alpha_\lambda{}^o)\right)\left(\frac{\partial d_i}{\partial\alpha_\gamma}\right)_o$$

Thus

$$\delta\alpha_\lambda * - \delta\alpha_\lambda = -H^{-1}_{\gamma\lambda} \sum_i d_i(\alpha\lambda)\left(\frac{\partial d_i}{\partial\alpha_\gamma}\right)_o$$

and

$$<(\delta\alpha_\lambda * - \delta\alpha_\lambda)(\delta\alpha_\mu * - \delta\alpha_\mu)> = H^{-1}_{\gamma\lambda} H^{-1}_{\delta\mu} \sum_i \sigma^2 \left(\frac{\partial d_i}{\partial\alpha_\gamma}\right)_o \left(\frac{\partial d_i}{\partial\alpha_\delta}\right)_o$$

$$= H^{-1}_{\gamma\lambda} H^{-1}_{\delta\mu} H_{\gamma\delta}\sigma^2$$

$$= \sigma^2 H^{-1}_{\lambda\mu}$$

APPENDIX II

Form of data for each event input into the Geometry program

Card	Format	Symbol	Description
1	F 10.0	FRAME	Frame number
	F 3.0	EVENT	Event number
	16I4	NOSORT	+1 No track or vertex sorting required (Must be +1 in present program) 0 otherwise
		MISVW.	Number of any missing view, or zero if all 3 views present.
		NRL	Total number of co-ordinate pairs measured.
		NCTL	Total number of charged tracks in the event.
		NCVL	Total number of charged vertices in the event.
		MEAS	Measurer) These can be left blank if information is not available
		MEASM	Measuring machine)
		NMEAS	Measurement number)
		MDAY) MMONTH) MYEAR)	Date of measurement)
			The remaining 5 I4's are spare (left blank)
2 NCTL cards	18I4	NRT(I,J)	Starting location for measured points on the I^{th} view of J^{th} track (List 3)
		NTT(I,J)	The total number of measured points on the I^{th} view of J^{th} track
		LLB(I,J)	Beginning label of the track) +1 if vertex, 0 otherwise
		LLE(I,J)	End label of the track)
		LLD(I,J)	Direction of the track - +1 if measured in correct direction -1 if measured in wrong direction 0 if unknown (left to program to decide)
		IIMASS(I,J)	Mass of particle if known (will be used for ionization measure)

cont'd

Card	Format	Symbol	Description
2 NCTL contd.	18I4	I = 1, 3 J = 1,NCTL	The 18 quantities for a given J appear on 1 card, the six quantities for view I first, followed by the six for view II etc.
3 2NRL/13 cards	13F6.4 (no decimal points)	XG(I) YG(I) I = 1,NRL	X, Y co-ordinate pairs of all measured points. The 3 views of a vertex must be stored together in I, I+1, I+2.
4	13I4	NTF(I)	Number of fiducials measured on the Ith view
		NRF(I) I = 1, 3	Starting location for fiducial co-ordinates in List 5
		NRV(J) J = 1, 7	Starting location in point list (List 3) for vertex co-ordinates
	21I1	DUMV(I,J) J = 1, 3 I = 1, 7	Vertex comments: no comment = blank or zero; stopping = 1; elastic scatter = 2
5 2NFL/13 cards	13F6.4 (no decimal points)	XF(I) YF(I) I = 1,NFL	X, Y co-ordinates of all the fiducials measured. NFL $= \sum$ NTF(I) is not read in.

Appendix III

List of constants for the Geometry program

Common Geometry Data

Card	Format	Symbol	Meaning
1	I3, 3F10.4	NGLASS	No. of optical components (other than air) including window
		DLENFM	Lens to film distance.
		DLENBC	Lens to front glass/liquid interface distance.
		REFLIQ	Refractive index of bubble chamber liquid.
2 (may be more than 1 card)	8F10.4	W(I + 3) & W(I + 8) (I = 1, NGLASS)	Thickness of optical component) Refractive index of optical component) NGLASS pairs
		C(15) C(16) C(17) C(18) C(19) C(20)	X co-ordinate of camera 1) Y co-ordinate of camera 1) X co-ordinate of camera 2) in y x co-ordinate system Y co-ordinate of camera 2) X co-ordinate of camers 3) Y co-ordinate of camera 3)
3	8F10.4	C(9)	Minimum distance between vertices allowed for sorting.
		C(10)	Sagitta check for fitting parabola to points projected onto front glass.
		C(11)	Check for badly measured points.
		C(12)	Maximum radius of curvature allowed.
		C(14)	Angle to rotate film measurement system to obtain y x co-ordinate system on the film.
		SIGMAV	Standard error on vertex measurements (on film).
		$C(87)^{\frac{1}{2}}$	Standard error on x and y co-ordinates in space.
		$C(88)^{\frac{1}{2}}$	Standard error on z co-ordinate in space.
4	4I3,	NC(1)	+ or -1 (+1 for R.H.S.) of x, y co-ordinate (-1 for L.H.S.) measurements on film.
		NC(2)) NC(3)) NC(4))	Total No.of fiducals that could be measured view 1) 2) 3) (must be $\leqslant$ 10)
		C(13) C(21) C(22)	Check for identifying fiducials. Check for fiducial condition ab+ed = o in L.S.F. Check for individual fiducial accuracy.

Card	Format	Symbol	Meaning
5 (more than 1 card)	8F10.4	C(138)→ C(197)	Expected X, Y co-ordinate pairs of fiducials on the film with respect to the optic axis in the R.H.S. which agrees with a R.H. measurement system. All X, Y pairs for view 1, then view 2, then view 3.
6	8F10.4	W(1)	Stereo angle check in radians.
		C(74)	Extrapolation ratio limit.
		C(75)	Accuracy for Z iteration in finding corresponding rays
		W(2) W(3) W(4)	Rotation angles (in radians) between cameras 1 & 2, 1 & 3, 2 & 3 such that the new X axis lies along the line joining these cameras.
7	I3, 3F10.4	NC(6)	Check on number of iterations for theta (θ)
		C(89)	Sagitta check for straight line fit in space.
		C(90)	Angle check for parabola fit in space.
		$C(91)\times 10^3$	0.3 times the mean magnetic field in Kgauss.
8	8F10.4		(Information about the beam track on entering the bubble chamber, used to identify the beam track)
		C(23)	Tangent of dip angle.
		C(24)	Error on tangent of dip angle.
		C(25)	Azimuthal angle + π (in radians).
		C(26)	Error on azimuthal angle.
		C(27)	X co-ordinate.
		C(28)	Error on X co-ordinate.
		C(29)	Y co-ordinate.
		C(30)	Error on Y co-ordinate.
9	I3, 6F10.4		(Further information about beam track)
		NC(5)	Charge.
		C(31)	Z co-ordinate.
		C(32)	Error on Z co-ordinate.
		C(33)	1/momentum $(\text{Gev}/c)^{-1}$ (-ve if beamtrack is missing)
		C(34)	Error on 1/momentum allowed in curvature determination

cont'd

Card	Format	Symbol	Meaning
9 contd.	I3, 6F10.4	$C(36)^{\frac{1}{2}}$	True error on 1/beam momentum = $\frac{\Delta p}{p^2}$ (Δp and p^2 are in Gev/c).
10	8F10.4	C(93)	$\Delta\theta$ check for convergence in θ iterations.
		C(94)	Parameter α between 0.5 and 1 used in slowing down correction.
		C(95)	Magnetic field weighting for middle of track.
		C(96)	Magnetic field weighting for end of track.
		C(97)	Test on $\frac{\Delta\Sigma d2}{\Sigma d^2}$ for helix fit convergence.
		C(98)	1/KAON mass.
		C(99)	1/PION mass.
		C(100)	1/PROTON mass.
11	8F10.4	$C(101)^{\frac{1}{2}}$	Standard measurement error on film.
		$C(102) \times 10^6$	$C_0{}^2$(parameter used in calculating coulomb errors).
		C(103)	$C_1{}^2$ " " " " " "
		C(104)	$C_2{}^2$ " " " " " "
		C(105)	$C_3{}^2$ " " " " " "
		C(106)	Check for calculated measurement error of ordinary points on film.
		C(107)	Check for calculated vertex measurement error on film.
		C(108)	Weighting factor on stereo check for selecting mainview.
12	8F10.4	C(109)	$\pm$ 1.0000 (+ve if magnetic field is in +ve direction, -ve if magnetic field is in -ve direction)
		C(110)	[error in density of liquid/density]^2
		C(111)	Constant for testing scatters.
		C(112)	Statistics constant.
		C(113)	Check on [length of track]^2 for circle fit in space.
		C(114)) C(115)) C(116))	Unused.
13	18I3	NC(7)	Number of iterations allowed in helix fitting for Kaon mass.
		NC(8)	Number of iterations allowed in helix fitting for pion or proton mass

cont'd

Card	Format	Symbol	Meaning
13 contd.	18I3	NC(9)	Number of entries in the range/momentum table.
		NC(10)	±1 (+1 Carry on if helix fit fails, -1 Reject event if helix fit fails).
		NC(11)	±1 or 0 (-1 Use weighted average of momentum from curvature given. 0 Use given beam momentum and error. +1 Use momentum and error found from curvature) for beam track.
		NC(12)	±1 = sign of magnetic field (+ve in same direction as Z (-ve if in opposite direction).
		NC(13)	±1 or 0 (-1 for an old tape with master list expected on channel 9. 0 for an old tape with NO master list expected on channel 9. +1 No old tape exists)
		NC(14)	Number of the constants used in the geometry run (+ve integer).
		NC(15)	Number of field values in table.
		NC(16)	±1 (+1 tries scatter test on beam vertex and any others. -1 tries scatter test only on others)
		NC(17) NC(18) NC(19) NC(20)	Unused.
14 (many cards)	6F10.3	AMOM(I) ARANGE(I) (I = 1,NC(9))	Range/momentum table.
15 (can be many cards)	6E13.5	FIELD(I) (I = 1, NC(15))	Values of magnetic field to be used in BMAG (If NC(15) = 0, no card is expected.)
16	3I3	IDAY MONTH IYEAR	Day Month Year (final 2 digits only) } of geometry run

Card	Format	Symbol	Meaning
17	F10.0	C(69)	Maximum frame no. expected in a given set of events to be run.
18	10A6	W(1) ↓ W(10)	Title associated with the events (up to 60 alphanumeric quantities).

Appendix IV

List of Fault Numbers

Fault No.	Routine	Reason	Print
1	RADANG	Track goes through more than 180°.	Track No. View No. Track List. Ray List for this track.
2	RADANG	Two points out of line.	Track No. View No. Point. No. Distance off. Track List. Ray List for this track.
3	VIDENT	Vertices too close for stereo sorting.	View No. Vertex co-ordinates for all 3 views.
4	FIDUC	Less than two fiducials measured.	View No.
5	FIDUC	Less than two of fiducials identified.	View No.
6	FIDUC	Fiducial condition ab + ed = 0 not satisfied.	View No. a, b, c, d, e, f, for the view.
7	BEAMID	More than one track found.	Track List. Helix Fit List. Vertex co-ordinates.
8	BEAMID	No beam track found.	Track List. Helix Fit List. Vertex co-ordinates.
9	CHBAL	No charge balance at vertex.	Vertex No. Track List.
10	VIDENT	Same end labels on track.	Track No. View No. End Label.
51	SPACEZ	Two views with no points measured.	Track No. Track List.
52	SPACEZ	Insufficient points reconstructed on track.	Track No. Track List. Ray List for this track.
53	SPANG	Too many iterations for theta.	Track No. Track List. Ray List for this track.
54	HFIT	Too many helix fit iterations on track.	Track No. Track List. Helix Fit List.

APPENDIX V

Library Lists and Format

The Library tape entry consists of five records for each event.

1. Book-keeping List

Variable	Type	Description	Source
GEN(1) GEN(2)	FL.PT FL.PT	Frame number Event number	set up by MASTER LIST and checked by GEOMETRY PROGRAM.
GEN(3)	FL.PT	No Sort (see Geometry input)	set up by GEOMETRY PROGRAM.
GEN(4)	FL.PT	Missing view number	set up by GEOMETRY PROGRAM.
GEN(5)	12 bit	1) Experiment number 2) Type number 3) File number	set up by MASTER LIST.
GEN(6)	6 bit	1) Scanner 2) Scan table 3) Day 4) Month — date of scanning 5), 6) Year	set up by MASTER LIST.
GEN(7)	6 bit	As for GEN(6) for re-scan	set up by MASTER LIST
GEN(9) to GEN(16) (one for each measurement)	6 bit	1) Measurer 2) Measuring machine 3) Day 4) Month 5), 6) Year	set up by GEOMETRY PROGRAM.
GEN(17)	FL.PT.	Total number of GEOMETRY tries.	Set up by GEOMETRY PROGRAM.
GEN(18) to GEN(27) (one for each GEOMETRY try)	6 bit	1), 2) Pass or Fault No. 3) Day 4) Month — date of GEOMETRY 5), 6) Year	set up by GEOMETRY PROGRAM.
GEN(28)	FL.PT.	-1 if book-keeping record only NC(14) if GEOMETRY run is successful. NC(14) = title number of constants used.	set up by GEOMETRY PROGRAM.

Variable	Type	Description	Source
GEN(29)	12 bit	1) Roll number 2) Spare 3)	set up by MASTER LIST.
GEN(30)	FL.PT	Total number of hypothesis tried.	set up by HYPOTHESIS TEST.
GEN(31) to GEN(99)	6 bit	1) Success or failure 2) 3) 4) Hypothesis name or number 5) 6)	set up by HYPOTHESIS TEST.

2. Vertex List

Variable	Description	Source	Comments
I	Number of words in record after first five	GEOM. or H.T.*	
NCTL	Number of charged tracks	GEOM	i.e. number of tracks measured.
NCV	Number of vertices with charged tracks.	GEOM	
NNTL	Total number of tracks	GEOM & H.T	i.e. including
NNVL	Total number of vertices	GEOM & H.T	neutral tracks.
For I=1, NCVL or NNVL as appropriate	number of vertex.		
SVERT(I)	-1 if vertex is a possible elastic scatter. -2 if tried as a scatter but failed coplanarity test +1 if stopping vertex	GEOM	
NRV(I)	Index giving position of vertex co-ordinates.	GEOM	
NCTV(I)	Total number of charged tracks at vertex.	GEOM	

Variable	Description	Source	Comments
NCT(I,J) J = 1,NCTV(I)	Track number of J^{th} track at the I^{th} vertex.	GEOM	
UG(K) UG(K+1) UG(K+2)	x) y) space co-ordinates of vertex z)	GEOM	
VG(K) VG(K+1) VG(K+2)	) diagonal squared errors on) co-ordinates)	GEOM	K = NRV(I)
ZS(K) ZS(K+1) ZS(K+2)	) correlated errors on vertex) co-ordinates.)	GEOM	

* GEOM GEOMETRY PROGRAM

H.T. HYPOTHESIS TESTING PROGRAM

3. Track List

Storage Location	Quantities stores	Source	Comments
I	Number of words in record after itself.	GEOM	
For i = 1, NCTL			
DELTAL(i)	Square of error in length	GEOM	
STOP(i)	+1 if track stops		
BZ(i)	Magnetic field Kg($\times 0.3 \times 10^{-3}$)		
DUMT(i)	R.M.S. error from helix fit.		
LB(i)	Vertex label at beginning of track.		) Labels are zero) if no vertex at) an end.
LE(i)	Vertex label at end of track.		
LD(i)	+1 if direction of track known 0 if unknown		

Storage Location	Quantities stored	Source	Comments
IMASS(i)	Mass of track if known.		
NCHAR(i)	Charge		
NHF(i)	Number of first entry in helix list for this track.		i.e. position of geometry results.
MAINVW(i)	Main view used in reconstructing track.		
For i = NCTL + 1, NVTL			
LB(i) LE(i) NHF(i)	As for charged track.	H.T.	it is only necessary to store these for neutrals.

4. Helix Fit List

This contains the results of both geometry and hypothesis testing programs. They are made available by means of indices in the track list and kinematical fit lists.

Storage Location	Quantities stored	Source	Comments
NHL	Number of entries in list.	GEOM & H.T.	
For i = 1, NHL			
MASS(i)	The mass used for the track when these results were obtained (Geom. or Kin).		
XMEAS(i,j)	j = 1 gives ϕ, azimuthal angle j = 2 gives $\tan\lambda$, λ = dip angle j = 3 gives 1/p, p = momentum.		
EMEAS(i,j,j)	j = 1, 3. Diagonal error elements.		
EMEAS(i,j,k)	j = 1, k = 2 j = 1, k = 3 Correlation terms j = 2, k = 3		
TLEN(i)	Track length		

5. Kinematical Fit List

This contains enough information to characterise the fits carried out on the event. One entry is made each time a fit is carried out.

Storage Location	Quantities stored	Source	Comments
NKF	Number of entries.	H.T.	
NNKF	Number of words in Kinfit List [KFL(i), i = 1, NNKF]		
For i = 1, NNKF			
KFL(i)	Vector of Kinfit list entries Given in detail below**.		
For i = 1, NKF			
NSTK(i)	Starting point in KFL vector for i^{th} entry.		
NKFEL(i)	Number of words in the i^{th} entry.		
NKFR(i)	If > 0, starting position of results in helix fit list. If -1, indicates failure of fit.		
CHI(i)	X^2 for successful fit.		

**Kinfit List entry for one fit. KFL(j), j = NSTK(i), NSTK(i) + NKFEL(i)

JVERS(NFFS)	Number of vertices being fitted.

For each vertex in numerical order

JAT(i)	Vertex number.
MTV(i) - MCTV(i)	Number of neutrals.
KAT(i)	Neutral track nos. at vertex in numerical order.
NTMASS(i)	Mass assignments for each track at the vertex, in numerical order of track numbers.
MINDEX(i)	Locations in helix fit list where data for fit has come from, in numerical order of track numbers.

Appendix VI

Suggested symbols for measuring machine

The following symbols (buttons) are sufficient for interpreting measurements in order to produce the Input format for the GEOMETRY program.

			Teleprinter code
1.	New Event.	To separate events	→
2.	New view.	To indicate change of view	+
3.	New track.	To separate tracks (and vertices)	*
4.	Co-ordinate.	For ordinary co-ordinates	none
5.	Vertex co-ordinate.	For vertex co-ordinates to give a symbol followed by co-ordinates.	/
6.	Missing track.	To indicate an unmeasured track.	η
7.	Erase co-ordinate.	To delete last measured co-ordinate pair.	π
8.	Erase track.	To delete measurements back to last new track symbol.	-
9.	Erase event.	To delete measurements back to last new event symbol.	.
10.	Tape terminating.	End of paper tape.	x