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A Practical Procedure for Evolving Heavy Flavour Structure Functions

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Abstract

The transition from the low Q^2 region in deep inelastic scattering where charm production is described by photon-gluon fusion to a region where the charm structure function F_2^c is largely generated by a charm quark density should be a smooth passage. The all orders prescription for matching between the two schemes does not uniquely determine all the relevant coefficient functions (CF's) or guarantee the correct threshold behaviour at fixed order in α_s. The constraints of matching not only the value but also the *evolution* of F_2^c order by order allows *all* CF's to be determined at any order, each of which reduces to the appropriate massless $\overline{MS}$ expression as $Q^2 \to \infty$, and leads to a smooth transition in the threshold region. This procedure gives an excellent description of the F_2^c data for all Q^2.

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1 Introduction

Recent measurements of charm production at HERA [1, 2] emphasise the importance of having a consistent theoretical framework for heavy flavour production in deep inelastic scattering (DIS). The charm structure function F_2^c could be 20% or more of the total F_2 and a modern global analysis of structure functions must necessarily include a satisfactory description of F_2^c. Recently [3] we have developed a general order-by-order procedure which solves the problem of how to calculate F_2^c in the region close to $Q^2 = m_c^2$ where m_c is the charm quark mass. Below $Q^2 = m_c^2$ the conventional description in terms of order-by-order in α_s boson-gluon fusion is perfectly adequate, but for $Q^2 > m_c^2$ this description becomes increasingly unreliable due to the presence of potentially large logarithms at all orders in α_s which should be resummed. By changing to the alternative description where the charm quark is treated as a parton, at the point $Q^2 = m^2$, this resummation is automatically performed and, at the same time, a complete set of parton densities needed to calculate other processes involving nucleons is obtained. While this idea of transferring from one description to another is not new, until now there has not been a satisfactory procedure for preserving threshold behaviour across the transition point whilst systematically presenting the calculation to any well–defined order.

In this paper we present our results at leading order (LO) and next-to-leading order (NLO) to demonstrate the key features of our method, the practical advantages it offers and how it improves upon previous attempts to treat heavy flavours. We demonstrate that the requirement of all orders equivalence between the two descriptions does not actually provide unique expressions for every relevant CF. The order-by-order in α_s matching of the structure function itself between the two descriptions at the transition point does not remove the ambiguity. Extra information is required to do this and, in our case, this is provided by the further requirement that the order-by-order *evolution* of the structure function should also be continuous across the transition point. This added constraint then allows each CF to be determined with the practical benefit that a very smooth transition is ensured, the correct threshold behaviour being maintained across the transition point and above. Here we only sketch how our results are derived, the emphasis here being on how the procedure works in practice; the detailed theoretical justification is presented in full in [3].

2 The FFN, ZM-VFN and VFN schemes

Consider the case where charm is produced by the neutral current. At low Q^2, the number of active flavours, n_f, is 3 and the charm cross section is generated by photon-gluon fusion (PGF) [4, 5]. This corresponds to the so-called fixed flavour number scheme (FFNS) and provides an acceptable description provided Q^2 is not large and one is not interested in the concept of a charm quark density.

In the FFNS at order $\mathcal{O}(\alpha_s)$ the charm structure function is given by

$$F_2^c(x, Q^2, m_c^2) \;=\; \frac{\alpha_s}{2\pi}\, C_g^{(1)\,FF}(Q^2/m_c^2) \otimes g_{n_f=3}(\mu^2), \tag{1}$$

where the CF which is convoluted with LO evolved gluon density $g_{n_f=3}$ is

$$C_g^{(1)\,FF}(z,\epsilon) = \Big[(P_{qg}^0(z) + 4\epsilon z(1-3z) - 8\epsilon^2 z^2) \ln\Big(\frac{1+v}{1-v}\Big) + (8z(1-z) - 1 - 4\epsilon z(1-z))v \Big] \theta(\hat{W}^2 - 4m_c^2) \quad (2)$$

where $\epsilon = m_c^2/Q^2$, $\hat{W}^2 = Q^2(1/z-1)$, the gluon quark centre of mass energy, v is the velocity of the charm quark or antiquark in the photon–gluon centre–of–mass frame, defined by $v^2 = 1 - 4m_c^2/\hat{W}^2$, and $P_{qg}^0(z) = z^2 + (1-z)^2$, the LO quark-gluon splitting function. These v–dependent terms ensure that the coefficient function tends to zero smoothly as $\hat{W}^2 = 4m_c^2$ is approached from below, and hence the structure function has a smooth threshold in $W^2 = Q^2(1/x - 1)$.

In the limit $Q^2 \to \infty$ the gluon CF reduces to

$$C_g^{(1)\,FF}(z,\epsilon) \to P_{qg}^0(z)\ln(\epsilon) + P_{qg}^0(z)\ln\Big(\frac{(1-z)}{z}\Big) + 8z(1-z) - 1 \quad (3)$$

from which we can see the potentially large logarithm $\ln\epsilon = \ln(Q^2/m_c^2)$. The natural way to sum this contribution together with higher powers of such logs is to describe charm production through a charm density which evolves according to the standard (light quark) renormalisation group equations.

This leads us onto an alternative description which is simple (and therefore practical) and which is commonly used in global analyses of DIS data [6, 7] where the aim is to extract parton densities for both light and heavy flavours. In this scheme, which we label the zero mass variable flavour scheme (ZM-VFNS), a value of μ_c^2 ($\sim m_c^2$) is chosen below which m_c is taken to be infinite and above which m_c is taken to be zero. While this is intended to be a rough description only it has proved successful in describing the then existing (pre-HERA) charm structure function data well above the threshold region with $\mu_c^2 \approx 3GeV^2$. However it is bound to be unrealistic at low Q^2 because of the abrupt variation in Q^2 – in contrast to the proper smooth threshold behaviour in W^2. As Q^2 increases the ZM-VFNS becomes more reliable, the $\mathcal{O}(\alpha_s)$, i.e. the NLO expression in this case, being given by

$$F_2^c(x,Q^2,m_c^2) = C_c^{n_f=4}(Q^2/\mu^2) \otimes c_+(\mu^2, m_c^2/\mu^2) + C_g^{n_f=4}(Q^2/\mu^2) \otimes g_{n_f=4}(\mu^2, m_c^2/\mu^2) \quad (4)$$

where the charm density $c_+ \equiv \bar{c} + c$ and the CF's are expanded as

$$\begin{aligned} C_c^{n_f=4}(z,1) &= C_c^{(0)}(z) + \frac{\alpha_s}{2\pi} C_c^{(1)\,\overline{MS}}(z) \\ C_g^{n_f=4}(z,1) &= \frac{\alpha_s}{2\pi} C_g^{(1)\,\overline{MS}}(z) \end{aligned} \quad (5)$$

where

$$C_c^{(0)}(z) = z\,\delta(1-z) \quad \text{and} \quad C_g^{(1)\,\overline{MS}} = P_{qg}^0(z)\,\ln\Big(\frac{1-z}{z}\Big) + 8z(1-z) - 1. \quad (6)$$

Our aim is to develop an approach which extrapolates smoothly from the FFNS at low Q^2 to the ZM-VFNS at high Q^2, maintaining the correct ordering in both schemes. First we note

that in the FFNS eq(1) is valid up to corrections of $\mathcal{O}(\Lambda^2/m_c^2)$ while in the ZM-VFNS eq(4) is valid only up to corrections of $\mathcal{O}(m_c^2/\mu^2)$. In order to improve the accuracy of eq(4) we need to examine the connection between the parton densities in the two schemes. Eq(4) actually has an apparent extra degree of freedom over eq(1), namely the charm density evaluated at its starting scale. This is of course not really true in practice and the connection between the parton densities for 3 and 4 flavours takes the form

$$\begin{aligned} c_+(z,\mu^2,m_c^2/\mu^2) &= A^{cg}(\mu^2/m_c^2) \otimes g_{n_f=3}(\mu^2) \\ g_{n_f=4}(z,\mu^2,m_c^2/\mu^2) &= A^{gg}(\mu^2/m_c^2) \otimes g_{n_f=3}(\mu^2) \end{aligned} \tag{7}$$

at leading order, where the elements A^{ba} which contain $\ln(\mu^2/m_c^2)$ terms, are, in general, part of a full 5×4 matrix which also connects the light quark flavours.

For $Q^2 \gg m_c^2$, the equivalence of the FFNS and the ZM–VFNS at all orders then lead to the connections between the CF's in the two schemes up to $\mathcal{O}(m_c^2/\mu^2)$ [8], in particular up to $\mathcal{O}(\alpha_s^2)$

$$\begin{aligned} C_g^{FF}(z,Q^2/\mu^2,Q^2/m_c^2) &= C_c^{n_f=4}(Q^2/\mu^2) \otimes A^{cg}(\mu^2/m_c^2) \\ &\quad + C_g^{n_f=4}(Q^2/\mu^2) \otimes A^{gg}(\mu^2/m_c^2) + \mathcal{O}(m_c^2/\mu^2). \end{aligned} \tag{8}$$

The details of the connection are fully worked out in [8]. To improve the accuracy of eq(8), where the uncertainty is reduced to $\mathcal{O}(\Lambda^2/m_c^2)$, requires defining 'corrected' CF's, C_b^{VF}, in another $n_f = 4$ scheme – the variable flavour number scheme (VFNS) – where one can write

$$\begin{aligned} F_2^c(x,Q^2,m_c^2) &= C_c^{VF}(Q^2/\mu^2,m_c^2/\mu^2) \otimes c_+(\mu^2,m_c^2/\mu^2) \\ &\quad + C_g^{VF}(Q^2/\mu^2,m_c^2/\mu^2) \otimes g_{n_f=4}(\mu^2,m_c^2/\mu^2) + \mathcal{O}(\Lambda^2/m_c^2), \end{aligned} \tag{9}$$

where the corrected CF's are related to the FFNS CF's by

$$C_b^{VF}(z,Q^2/\mu^2,m_c^2/\mu^2) = C_a^{FF}(Q^2/\mu^2,m_c^2/\mu^2) \otimes \left[A^{ba}(\mu^2/m_c^2)\right]^{-1}, \tag{10}$$

the new n_f=4 CF's now being *exact* at all values of Q^2.

In the spirit of ref[9] the procedure we adopt is to use the FFNS for $Q^2 \leq m_c^2$ and to switch to the VFNS for $Q^2 \geq m_c^2$. The precise choice of the transition point is undetermined, though is naturally of order m_c^2. However, there are matching conditions between the partons in the two schemes in order to ensure consistency with the usual asymptotic expressions, e.g. eq(7), and taking $\mu^2 = m_c^2$ removes complications arising from $\ln(\mu^2/m_c^2)$ terms in these matching conditions. To make practical headway in the VFNS one must solve eq(10) for the C_b^{VF}. However one can see that the all orders matching of F_2^c in the two schemes, from which eq(10) arose, is not sufficient since, for example, at low orders the *single* quantity C_g^{FF} is expressed in terms of the *two* quantities C_c^{VF} and C_g^{VF}. We stress that any choice satisfying eq(8) is "correct" in the sense that it leads to the same all orders expression. Nevertheless, each choice leads to a different expression if one uses the usual rules of combining coefficient functions and parton

distributions of a given order to obtain a fixed order in α_s expression for the structure function. In principle a completely consistent theoretical approach would lead to a completely seamless transition between the two schemes at every order in perturbation theory. In practice this seems to be, at the least, incredibly difficult and is probably not even possible in any unique manner. However, in order to obtain as well–defined a theoretical procedure as possible we impose not only continuity of the structure function but also demand, in addition, order-by-order matching of the evolution of F_2^c in the two schemes, as we shall see below.

3 The practical solution

The explicit form of eq(7) for $\mu^2 = Q^2$ at $\mathcal{O}(\alpha_s)$ is

$$\begin{aligned} c_+(z,Q^2) &= \frac{\alpha_s}{2\pi}\ln\left(\frac{Q^2}{m_c^2}\right) P_{qg}^0 \otimes g_{n_f=3} \\ g_{n_f=4}(z,Q^2) &= g_{n_f=3}(z,Q^2) - \frac{\alpha_s}{6\pi}\ln\left(\frac{Q^2}{m_c^2}\right) g_{n_f=3}. \end{aligned} \tag{11}$$

Inserting the implied expressions for the matrix elements $A^{cg}(z,\mu^2/m_c^2)$ and $A^{gg}(z,\mu^2/m_c^2)$ into eq(8) gives the familiar relation

$$C_g^{(1)\,FF}(z,Q^2/m_c^2) = C_g^{(1)\,VF}(z,Q^2/m_c^2) + C_c^{(0)\,VF}(Q^2/m_c^2) \otimes P_{qg}^0 \ln\left(\frac{Q^2}{m_c^2}\right) \tag{12}$$

connecting the gluonic CF's in the FFNS and VFNS. In previous implementations of a VFNS [10], eq(12) served as the definition for $C_g^{(1)\,VF}$ in terms of the PGF CF (eq(2)) with an assumed form of $C_c^{(0)\,VF}$ given by

$$\hat{C}_c^{(0)\,VF}(z,Q^2/M^2) = z\,\delta(\hat{x}_0 - z)\left(1 + \frac{4m_c^2}{Q^2}\right), \qquad \hat{x}_0 = \left(1 + \frac{m_c^2}{Q^2}\right)^{-1} \tag{13}$$

where the delta-function describes the tree-level diagram for a massive quark scattering from a photon and the modified argument of the delta-function follows from demanding that the massive quark is on-shell. We note that the same definition of the zeroth order coefficient function is adopted in [11] though there are differences between this and [10], notably a mass dependent evolution.

We do not believe that it is appropriate to go suddenly from a scheme in which there is no charm parton distribution to one where the evolution and zeroth order coefficient function are such that the charm parton behaves much like any other parton, but that the zeroth order coefficient function should reflect the true physics near threshold. Indeed, there are several reasons why the above prescription is unsatisfactory. Usually at leading order in the expressions for structure functions involving only light quarks we include just the zeroth order coefficient function, and thus the expression is the summation of $\alpha_s(Q^2)\ln(Q^2)$ terms only. Combining both $C_c^{(0)\,VF}(Q^2/m_c^2)$ and $C_g^{(1)\,VF}(z,Q^2/m_c^2)$ with lowest order parton distributions we see that only the former really contributes to the LO expression, the first being down by a power of

$\ln(Q^2)$ at large Q^2. However, keeping the simple form of $\hat{C}_c^{(0)\,VF}$ above leads to an abrupt rise of the charm structure function as soon as $Q^2 > m_c^2$, which is totally incompatible with the smooth behaviour in W^2 required physically. Thus, previous implementations [10] of a VFNS have included both both $C_c^{(0)\,VF}(Q^2/m_c^2)$ and $C_g^{(1)\,VF}(z, Q^2/m_c^2)$ in their LO definition of the heavy quark structure function, and improved the smoothness by also changing the scale from $\mu^2 = Q^2$ to something like $\mu^2 = m_c^2 + 0.5\, Q^2\, (1 - m_c^2/Q^2)^2$. Thus, we seem to have the choice of either a complete lack of smoothness in the charm structure function near threshold or an unconventional, and strictly incorrect ordering of the expression. We also have the disquieting feature that none of the heavy quark CF's contain the correct threshold behaviour in W^2 for F_2^c. The correct threshold behavior can only be obtained overall by a cancellation of sometimes large incorrect contributions between different terms. All this just emphasises that the above procedure is not sufficient to yield a unique expression for the structure function in the VFNS.

In order to better reflect the true physics in our choice of coefficient functions let us now consider the evolution of F_2^c. From eq(1) the LO expression in the FFNS for the $\ln Q^2$ derivative is simply

$$\frac{dF_2^c(x, Q^2, m_c^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi} \frac{dC_g^{(1)\,FF}(Q^2/m_c^2)}{d\ln Q^2} \otimes g_{n_f=3}(Q^2). \tag{14}$$

The corresponding expression obtained by differentiating the LO expression in the VFNS, for Q^2 just above m_c^2, is

$$\begin{aligned} \frac{dF_2^c(x, Q^2, m_c^2)}{d\ln Q^2} &= \frac{dC_c^{(0)\,VF}(Q^2/m_c^2)}{d\ln Q^2} \otimes c_+(Q^2) \\ &+ \frac{\alpha_s}{2\pi} C_c^{(0)\,VF}(Q^2/m_c^2) \otimes \Big(P_{qg}^0 \otimes g_{n_f=4}(Q^2) + P_{qq}^0 \otimes c_+(Q^2) \Big). \end{aligned} \tag{15}$$

Throughout we are dropping the n_f label on α_s to simplify the notation. The question of how to treat the change in α_s from $n_f = 3$ to $n_f = 4$ is discussed in our long paper [3]. Now for $Q^2 = m_c^2$, the terms in eq(15) involving c_+ vanish because of eq(11) and so demanding continuity of the evolution across the transition point immediately leads, from eqs(14,15), to

$$C_c^{(0)\,VF}(Q^2/m_c^2) \otimes P_{qg}^0 = \frac{dC_g^{(1)\,FF}(z, Q^2/m_c^2)}{d\ln Q^2}. \tag{16}$$

Generalising this relation to be the definition of $C_c^{(0)\,VF}(z, Q^2/m_c^2)$ at *all* Q^2 offers a series of real advantages. Firstly, from the phenomenological point of view, it guarantees a smooth passage for the charm structure function from $Q^2 < m_c^2$ to $Q^2 > m_c^2$, by definition. Explicitly taking the derivative of eq(2) is straightforward, giving

$$\begin{aligned} \frac{dC_g^{(1)\,FF}(z, \epsilon)}{d\ln Q^2} &= \Big[(P_{qg}^0(z) + 2\epsilon \frac{z(1-2z^2)}{1-z} - 16\epsilon^2 z^2) \frac{1}{v} \\ &+ (-4\epsilon z(1-3z) + 16\epsilon^2 z^2) \ln\Big(\frac{1+v}{1-v}\Big) + (4\epsilon z(1-z))v \Big] \theta(\hat{W}^2 - 4m_c^2) \end{aligned} \tag{17}$$

and it is easy to see that in the limit $Q^2 \to \infty$,

$$\frac{dC_g^{(1)\,FF}(z,\epsilon)}{d\ln Q^2} \to P_{qg}^0(z). \tag{18}$$

Hence, from eq(16), we see that $C_c^{(0)\,VF}(z,\epsilon)$ must indeed tend to the simple form $z\,\delta(1-z)$ in this limit, consistent with eq(6). Also, since $C_g^{(1)\,FF}(z,\epsilon)$ contains the factor $\theta(\hat{W}^2 - 4m_c^2)$ so does its $\ln Q^2$ derivative, thus ensuring the correct threshold behaviour in W^2 for $C_c^{(0)\,VF}$ and in turn for F_2^c at LO. Futhermore eq(16) allows the gluonic CF in the VFNS to be written as

$$C_g^{(1)\,VF}(z,Q^2/m_c^2) = C_g^{(1)\,FF}(z,Q^2/m_c^2) - \frac{dC_g^{(1)\,FF}(z,Q^2/m_c^2)}{d\ln Q^2}\,\ln\Big(\frac{Q^2}{m_c^2}\Big), \tag{19}$$

and so now $C_g^{(1)\,VF}$ has the same threshold behaviour as $C_g^{(1)\,FF}$.

Finally, from eqs(3,12,18) we see that as $Q^2 \to \infty$, $C_g^{(1)\,VF}(z,Q^2/m_c^2)$ does indeed tend to the correct asymptotic $\overline{MS}$ limit given by eq(6).

The extension of this procedure to any arbitrary order, i.e. continuity of the derivative in the gluon sector, is described in full in [3]. (Using our general approach continuity of the derivative in the singlet quark sector is not possible, but this has negligible effect in practice.)

The only price that may seem to have been paid in exchange for obtaining a method which is far closer to true theoretical consistency and a much improved phenomenological description is the task of solving eq(16) for the LO charm CF, $C_c^{(0)\,VF}(z,Q^2/m_c^2)$. Note however that rather than $C_c^{(0)\,VF}$ appearing explicitly in the second term of eq(12), that term is now replaced by the simpler term in eq(19) and so we are just left with the term where $C_c^{(0)\,VF}$ is convoluted with the charm density c_+. Calculating this expression turns out to be easier than than deriving the form of $C_c^{(0)\,VF}$ itself, the convolution being given by

$$\begin{aligned} C_c^{(0)\,FF}(\epsilon) \otimes c_+(Q^2) \;=\; & -\int_x^{x_0} dz\,\frac{dC_g^{(1)\,FF}(z,\epsilon)}{d\ln Q^2}\Big(\frac{x}{z}\Big)^2\,\frac{dc_+(x/z,Q^2)}{d(x/z)} \\ & +\; 3\int_x^{x_0} dz\,\frac{dC_g^{(1)\,FF}(z,\epsilon)}{d\ln Q^2}\,\frac{x}{z}\,c_+(x/z,Q^2) \\ & -\; 2\int_x^{x_0} dz\,\frac{dC_g^{(1)\,FF}(z,\epsilon)}{d\ln Q^2}\int_{x/z}^1 dz'\,r(z')\,\frac{x}{zz'}\,c_+(x/zz',Q^2) \end{aligned} \tag{20}$$

where $x_0 = (1+4\epsilon)^{-1}$ and $r(z)$ is given by

$$r(z) = z^{\frac{1}{2}}\Big[\cos\Big(\frac{\sqrt{7}}{2}\ln\frac{1}{z}\Big) + \frac{3}{\sqrt{7}}\sin\Big(\frac{\sqrt{7}}{2}\ln\frac{1}{z}\Big)\Big]. \tag{21}$$

Thus we can calculate the charm structure function at LO, taking for $Q^2 < m_c^2$

$$F_2^{c\,(0)\,FF}(x,Q^2,m_c^2) = \frac{\alpha_s}{2\pi}\,C_g^{(1)\,FF}(Q^2/m_c^2) \otimes g_{n_f=3}(Q^2), \tag{22}$$

and for $Q^2 > m_c^2$

$$F_2^{c\,(0)\,VF}(x,Q^2) = F_2^{c\,(0)\,FF}(x,m_c^2) + C_c^{(0)\,VF}(Q^2/m_c^2) \otimes c_+(Q^2), \tag{23}$$

where the constant term becomes almost negligible for $Q^2 > 4m_c^2$. In the above, the partons are evolved only via the LO DGLAP equations.

For the NLO case, the partons are evolved via the full NLO equations. The calculation of $F_2^{c\,FF}$ for $Q^2 < m_c^2$ adds in the $\mathcal{O}(\alpha_s^2)$ contributions evaluated by Riemersma, Smith and van Neerven [12][1] and in our VFNS we include the contributions from $C_g^{(1)\,VF}(Q^2/m_c^2) \otimes g_{n_f=4}$ and the extremely small contribution from $C_q^{(1)\,VF}(Q^2/m_c^2) \otimes \Sigma_{n_f=4}$, where the coefficient function in the latter is determined by continuity of the evolution in the gluon sector at NLO.

We also briefly discuss the treatment of the longitudinal structure function F_L^c since it is an important example. The δ-function CF for F_2^c of eq(13), which previous procedures used, contains a contribution from F_L^c – corresponding to the $4m_c^2/Q^2$ term. Thus in those descriptions, F_L^c in the VFNS contains a zero-order part, leading to a mis-match with the FFNS which starts at order α_s with no $\ln(Q^2)$ enhancement. This implies rather odd Q^2 behaviour near threshold, see [11]. In our approach we do not require this zeroth-order contribution to F_L^c and so this problem is avoided. Our prescription ensures a smooth transition as in the F_2^c case, see [3].

4 Results

The results we present use a set of partons which are obtained from an analysis of DIS data in which charm and bottom flavours are treated by the new procedure we have discussed here. Only data on the full F_2 structure function are included in the fit, not the data on F_2^c though we shall compare with those data below. The fits obtained are slightly sensitive to the value of m_c and we find $m_c \sim 1.35$ GeV gives the best fit. The value of m_b was taken to be 4.3 GeV. We have evidence already that our more theoretically consistent treatment of heavy flavours improves the overall quality of the fits to F_2 [3]. We plan to incorporate this procedure into a new (MRS type) global analysis where new data from hadronic collisions will be included and together provide updated parton distributions.

In fig. 1 we show the result of the LO calculation of F_2^c for two values of x. This demonstrates the success of our key result, namely the new form of the charm quark CF, which ensures a smooth transition from the description at low Q^2 in terms of the FFNS to high Q^2 in terms of the ZM-VFNS. In contrast to previous approaches [10] this has been achieved without resorting to an unusual definition of LO structure functions (i.e. one in which the LO ZM–VFNS limit is not actually reached because $\mathcal{O}(\alpha_s)$ coefficients functions are included) or to any cancellation between large individual contributions.

Fig.2 shows the similar situation for the NLO case compared with the NLO ZM-VFNS result and the continuation of the NLO FFNS expression. In the NLO case, the new description

[1]We are grateful to them for providing the program to compute these corrections

remains close to the continuation of the FFNS result to larger Q^2 than in the LO case. This is what is expected since, at the higher order, the FFNS includes more $\ln(Q^2/m_c^2)$ terms which are resummed in the VFNS.

In fig.3 we show the comparison of the charm structure function $F_2^c(x,Q^2)$ resulting from our NLO analysis with all available data. The data at intermediate x values comes from EMC [13] measurements of inclusive muons, the new data from HERA is by the H1 [1] and ZEUS [2] collaborations measuring D and D^* cross sections. We show curves for three values of the charm mass and, as expected, the sensitivity to m_c is strongest at low Q^2. The values of F_2^c there from the EMC measurements suggest the higher value $m_c = 1.5$ GeV is favoured. Clearly, precise measurements of F_2^c in this region would provide very accurate estimates of the charm mass.

5 Summary

In this paper we draw attention to a new formulation for describing heavy flavour structure functions in DIS. While the concept of the various schemes discussed in section 2 is not new, the realisation (discussed in section 3) that beyond all orders consistency and order-by–order matching of the charm structure function across the transition from the FFNS to the VFNS extra constraints are required to uniquely determine all the relevant CF's is new. In fact, previous attempts to describe charm evolution suffered from both theoretical and phenomenological deficiencies. By demanding that the order-by-order evolution of the charm structure function should be continuous as well as F_2^c itself, we improve both the theoretical and phenomenological description of charm (and bottom) production in DIS. This successful procedure, based on an examination of the structure function expansion in a systematic order-by-order way, means that the procedure has a natural generalisation to any given order and also to any given physical process.

We have demonstrated that the observed charm structure function is very well described over a wide range of x and Q^2 including especially the threshold region and that, consistent with the relatively large importance of charm at low values of x, fits to the total F_2 are improved. For full details of our analysis for the general situation of heavy flavour production in DIS we refer the reader to ref[3].

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Figure captions

Fig. 1 Charm quark structure function, $F_{2,c}(x, Q^2)$ for $x = 0.05$ and $x = 0.005$ calculated using our LO prescription, our input parton distributions evolved at LO and renormalization scale $\mu^2 = Q^2$. Also shown are the continuation of the LO FFNS expression and the ZM–VFNS expression both calculated using the same parton distributions and same choice of scale.

Fig. 2 Same as fig.1 but with NLO prescriptions and NLO parton distributions.

Fig. 3 Our prediction for $F_2^c(x, Q^2)$ using our NLO prescription, the NLO partons obtained from our global fit and three different values of m_c compared with data from EMC [13], H1 [1], ZEUS [2] (from the 1994 run) and preliminary data from ZEUS (1995 run) [14].

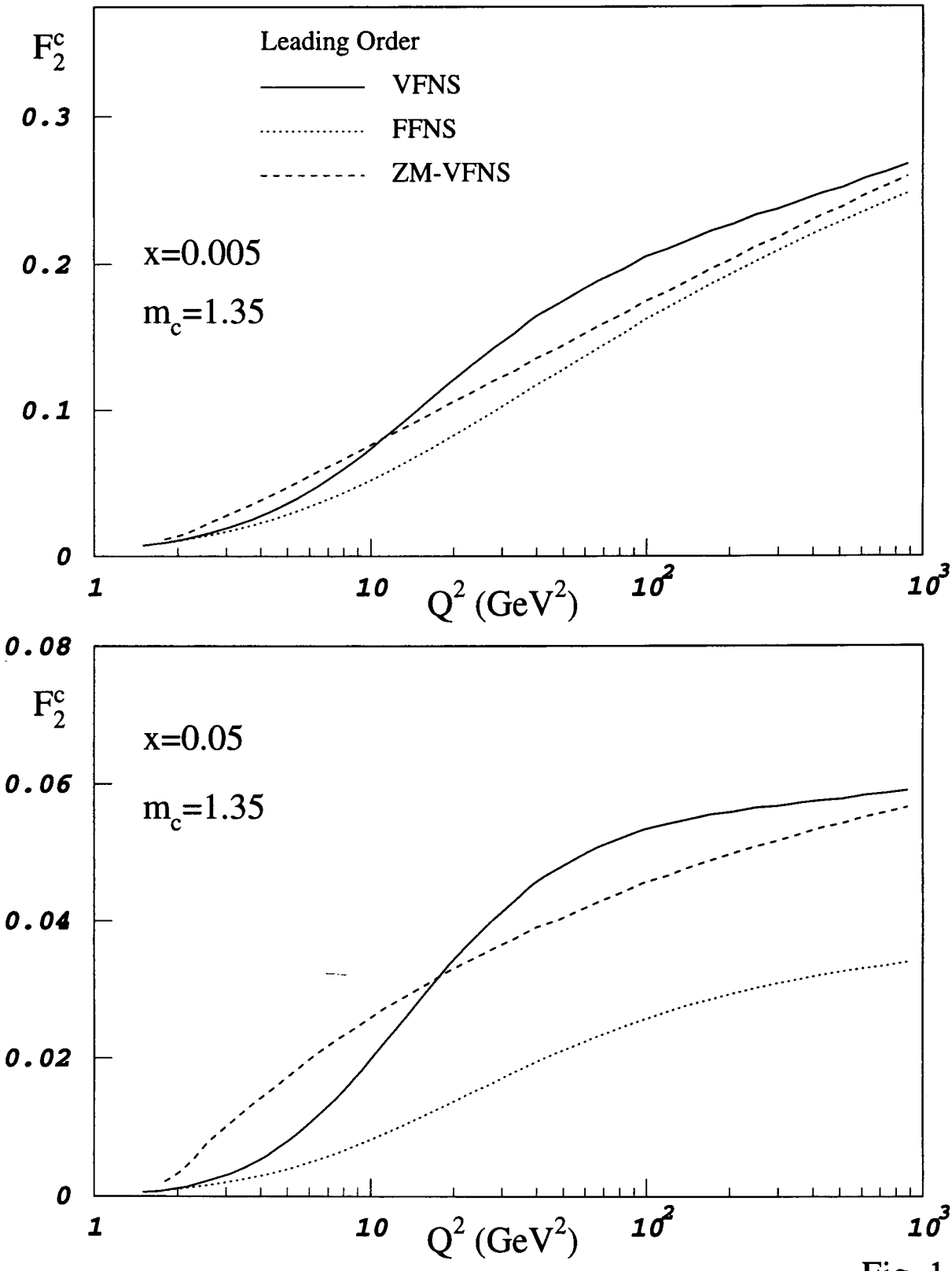
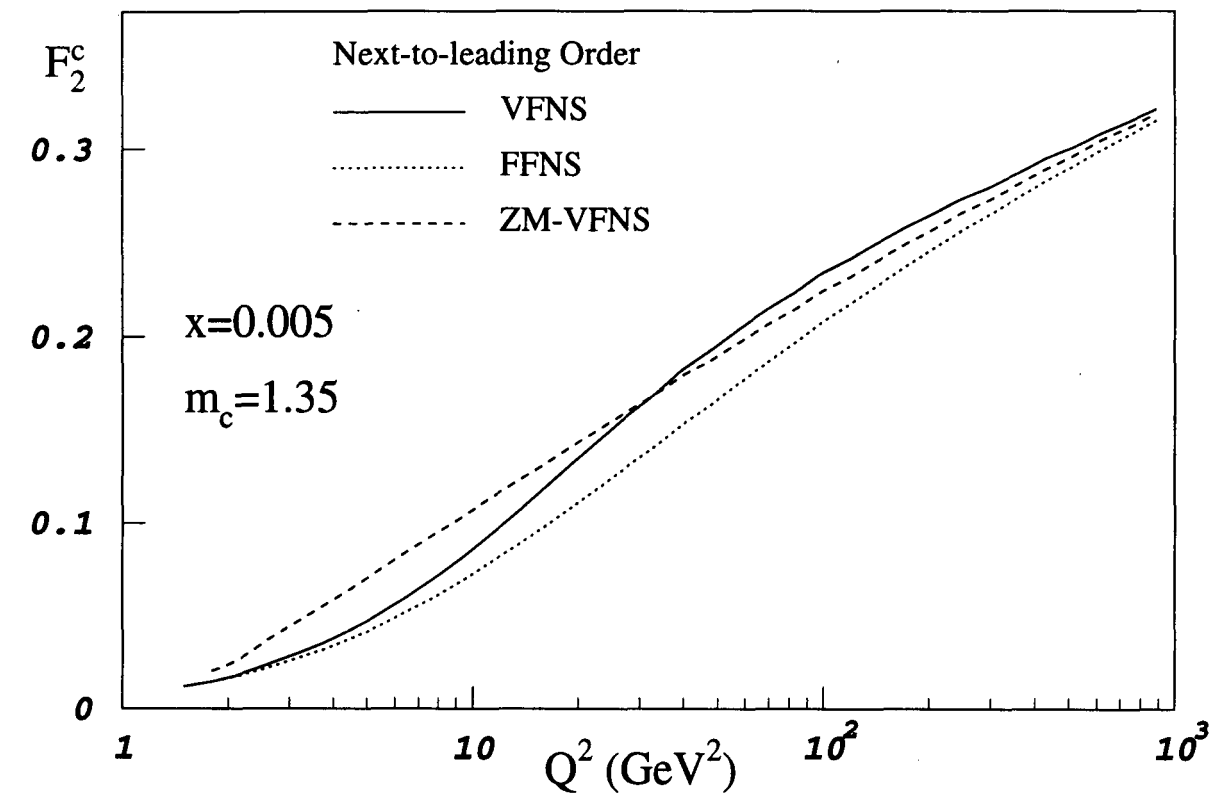

F_2^c
Leading Order
VFNS
FFNS
ZM-VFNS
x=0.005
m_c=1.35
0.3
0.2
0.1
0
1
10
Q^2 (GeV^2)
10^2
10^3
0.08
F_2^c
x=0.05
m_c=1.35
0.06
0.04
0.02
0
1
10
Q^2 (GeV^2)
10^2
10^3

Fig. 1


F_2^c
Next-to-leading Order
VFNS
FFNS
ZM-VFNS
x=0.005
m_c=1.35
0.3
0.2
0.1
0
1
10
Q^2 (GeV^2)
10^2
10^3

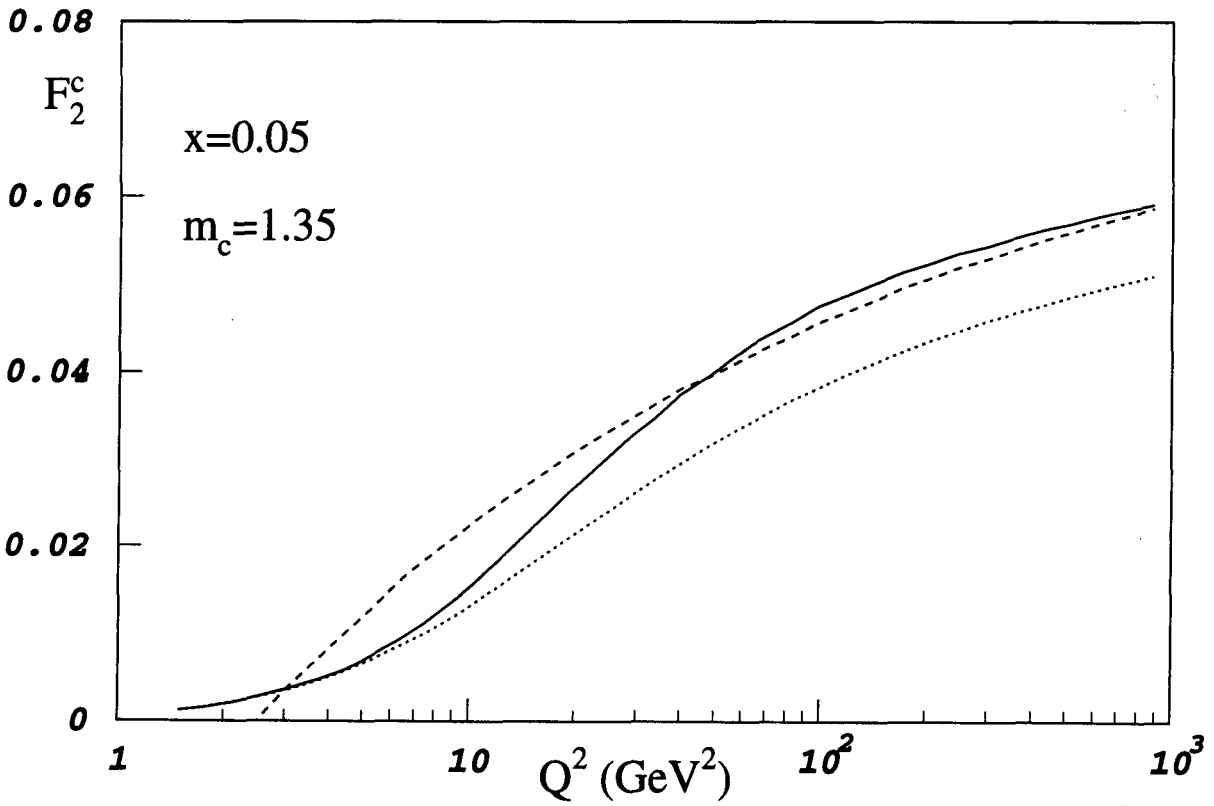

F_2^c
x=0.05
m_c=1.35
0.08
0.06
0.04
0.02
0
1
10
Q^2 (GeV^2)
10^2
10^3

Fig. 2

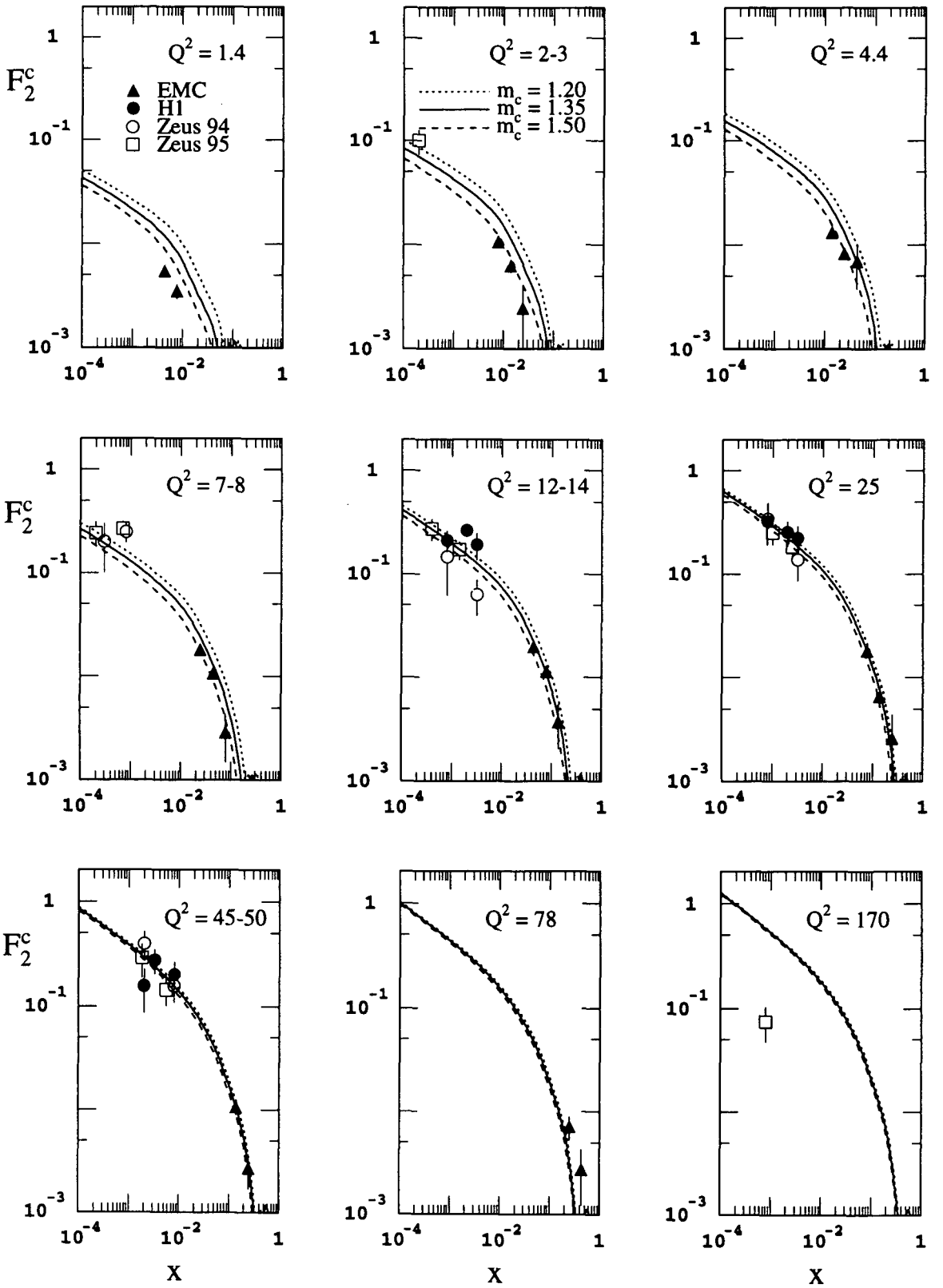

F_2^c
Q^2 = 1.4
EMC
H1
Zeus 94
Zeus 95
Q^2 = 2-3
m_c^c = 1.20
m_c^c = 1.35
m_c^c = 1.50
Q^2 = 4.4
Q^2 = 7-8
Q^2 = 12-14
Q^2 = 25
Q^2 = 45-50
Q^2 = 78
Q^2 = 170
1
10^{-1}
10^{-3}
10^{-4}
10^{-2}
x

Fig. 3