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Rutherford Appleton Laboratory
CHILTON, DIDCOT, OXON, OX11 0QX

Crosscorrelation Techniques in Neutron Polarisation Analysis Time-of-Flight Studies

R Cywinski

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CROSSCORRELATION TECHNIQUES IN NEUTRON POLARISATION ANALYSIS TIME-OF-FLIGHT STUDIES

R Cywinski

Neutron Division, SERC Rutherford Appleton Laboratory,
Chilton, Didcot, Oxon OX11 OQX, UK

Abstract

A method of performing time-of-flight measurements on a neutron polarisation analysis instrument is described. The method involves a crosscorrelation technique in which the neutron beam polarisation prior to the scattering sample is modulated by a neutron spin flipper according to a predetermined pseudorandom sequence. It is shown that the final crosscorrelation of the measured countrate at the detector with the pseudorandom sequence yields a time-of-flight spectrum which is proportional to the difference between the spin flip and non spin flip scattering cross-sections of the sample. Implementation of the technique and an example of its use are discussed.

It is also shown that in a simple form the crosscorrelation technique is useful in providing discrimination against inelastic scattering.

CONTENTS

1. INTRODUCTION

The principles and applications of the crosscorrelation technique in thermal neutron scattering time-of-flight studies have been extensively discussed in the literature during the last fifteen years (see, for example, [1]). Briefly the technique involves the modulation of the intensity of a monochromatic neutron beam according to a predetermined pseudorandom pulse sequence. Each resulting neutron burst is modified in shape by the transfer function (ie time-of-flight, or energy, spectrum) of the sample by which it is scattered. The times of arrival of scattered neutrons at the detector are recorded in the appropriate channels of a multichannel analyser and at the end of the experiment the transfer function of the sample is recovered by crosscorrelation of the stored countrate with the modulating pulse sequence.

Mezei and Pellionisz [2] introduced a modification to the conventional cross-correlation procedure wherein the incident monochromatic neutron beam is polarised and the net spin direction of the beam, rather than its intensity, is modulated according to the pseudorandom sequence. The application of the modified crosscorrelation technique to the case of a polarised neutron spectrometer was briefly discussed by Mezei and Pellionisz and it was indicated that a time-of-flight spectrum representing the difference between the transfer functions of a magnetically saturated sample for spin up and spin down neutrons is obtained from the final crosscorrelation.

The conventional and modified crosscorrelation techniques differ in that the former allows detected neutrons to be associated only with an "on" pulse of the pseudorandom sequence whereas the latter has detected neutrons arising from both "on" and "off" pulses. It is the purpose of this report to examine in detail the principles of the modified crosscorrelation technique and to consider its application in neutron polarisation analysis instrumentation.

2. THE MODULATING SEQUENCE

Before proceeding to a detailed discussion of the mathematical principles of the modified crosscorrelation technique it will be useful to examine first the statistical and physical properties required of a modulating sequence.

(i) The statistical nature of the sequence

For any time dependent function, $s(t)$, to be considered as a random sequence over the period T it must have an autocorrelation function, $A_{ss}(\tau)$, which satisfies the criterion

$$A_{ss}(\tau) = \frac{1}{T} \int_0^T s(t)\, s(t-\tau)dt = a_1\delta(\tau) + a_2 \tag{1}$$

where a_1 and a_2 are constants. While the output sequences of white noise and random number generators generally satisfy this relationship as $T \to \infty$, marked deviations from the required behaviour are found when T becomes short. Consequently, should a finite and reproducible modulating sequence with autocorrelative properties of a random sequence be required, great care must be exercised in selecting $s(t)$.

The statistical base of a finite sequence can be defined as a binary string of L bits, s_i, which repeats such that $s_{i+L} = s_i$. The s_i can take only the values 0 or 1, and of the L bits of the string K are 1 and L-K are 0. The autocorrelation function of such a binary string can be written as

$$A_k = \sum_i^L s_i s_{i+k} \tag{2}$$

The binary string is pseudorandom providing that this autocorrelation function is of the same form as that given by eq (1), ie if

$$A_k = (A_o - A)\ \delta(k) + A \qquad 2(a)$$

with

$$A_k = \begin{cases} A_o & \text{if } k = 0, L, 2L \text{ etc} \\ A & \text{otherwise} \end{cases}$$

From equation 2

$$A_o = (\sum_i^L s_i{}^2) = K$$

and

$$\sum_{k=1}^{L} A_k = (\sum_{i=1}^{L} s_i)^2 = K^2$$

As

$$\sum_{k=1}^{L} A_k = A_o + (L-1)A$$

we have

$$K(K-1) = A(L-1) \qquad (3)$$

As A must be integer, this equation provides a selection rule for permitted values of L and K. Also 2(a) can be written

$$A_k = K(1-c)\ \delta(k) + Kc \qquad 2(b)$$

with

$$c = \frac{K-1}{L-1}$$

c represents the "duty cycle" of the binary string.

It has been shown [3] that for a finite binary string, s_i, to have the appropriate autocorrelation function (eq (2a)) and satisfy the selection rule of eq (3) it must belong to the class of binary sequences known as Cyclic Difference sets [4], and for the purpose of crosscorrelation measurements a cyclic difference set can thus be used as the statistical base for a finite pseudorandom modulating sequence.

Unfortunately most cyclic difference sets are difficult to construct, although the appendix to ref [4] contains a very useful list of all known difference sets for which $K \leqslant 225$ catalogued according to the expression (L, K, A). There is however one class of cyclic differece sets, the sub-group known as Singer sets, which may be readily generated electronically from shift register sequences (see eg [3]). Such sets have a length given by $L = 2^n - 1$, where n is any integer, and a duty cycle of $c = \frac{1}{2}$.

The main points of this section are illustrated in Figure 1 where three binary strings with L = 127 are shown together with their respective autocorrelation functions. 1(a) shows a sequence generated by the random number intrinsic of a computer, while 1(b) shows the shift register sequence (127, 64, 32). Only the latter has an autocorrelation function of the correct form. The importance of maintaining the order of elements within a pseudorandom binary string is demonstrated in (c) where the sequence shown in (b) has had only two elements, $s_{11} = 0$ and $s_{63} = 1$, interchanged. The resulting autocorrelation function is clearly no longer that of a pseudorandom sequence.

(ii) <u>The physical characteristics of the modulating sequence</u>

From the string of L bits with the statistical properties outlined above a modulating sequence of pulses with precisely the same statistical properties must be obtained. Ideally pulses should have as rapid rise and fall times as possible and also be symmetrical about their centres (eg a rectangular pulse shape). If pulses of shape $\Gamma(t)$ are centred at times t_i the required modulating sequence M(t) can therefore be constructed by convoluting the statistical and physical characteristics, ie

$$M(t) = \sum_i s_i \Gamma(t-t_i) \qquad 0 \leqslant M(t) \leqslant 1 \tag{4}$$

the condition of pulse shape symmetry being expressed by

$$\sum_i \Gamma(t-t_i) = 1 \qquad \text{for all } t \tag{5}$$

3. PRINCIPLES OF THE MODIFIED CROSSCORRELATION TECHNIQUE

In a conventional crosscorrelation experiment one usually wants to obtain the form of the TOF or energy spectrum, $F(\tau)$, of the specimen. As incident neutrons arise only from "on" pulses of the pseudorandom modulating sequence the time dependent countrate at the detector will reflect the modulation of the incident beam. In fact the observed countrate is simply a convolution of $F(\tau)$ with $M(t)$, thus

$$\begin{aligned} Z(t) &= \int_0^{T_F} F(\tau)M(t-\tau)d\tau \\ &= \int_0^{T_F} F(\tau) \sum_i s_i \Gamma(t-t_i-\tau)d\tau \end{aligned} \tag{6}$$

Here $Z(t)$ is the observed countrate and T_F is a time interval outside which $F(\tau)$ is always zero. The function $F(\tau)$ can then be recovered from the countrate by performing the crosscorrelation

$$C'(\tau) = \int_0^T Z(t)M(t-\tau)dt \tag{7}$$

where T is the total time of the experiment

In the modified crosscorrelation experiment, however, two countrates are measured simultaneously, one arising from the "on" pulses, the associated neutrons having sampled a TOF spectrum $S(\tau)$, and the other arising from the "off" pulses with the associated neutrons sampling a different spectrum $N(\tau)$. The countrates at time t are thus

$$\begin{aligned} Z_{ON}(t) &= \int S(\tau') \sum_i s_i \Gamma(t-t_i-\tau')d\tau' \\ Z_{OFF}(t) &= \int N(\tau') \sum_i (1-s_i) \Gamma(t-t_i-\tau')d\tau' \end{aligned} \tag{8}$$

In a real experiment, of course, there will be an additional countrate due to a background, b', which is correlated with neither "on" nor "off" pulses. The total countrate at a time t will therefore be

$$Z(t) = Z_{ON}(t) + Z_{OFF}(t) + b' \tag{9}$$

One could now perform the crosscorrelation of $Z(t)$ with $M(t)$ described in equation 7, as is generally done in crosscorrelation experiments, but instead a crosscorrelation procedure first suggested by Von Jan and Scherm [5] will be used in which the countrate $Z(t)$ is crosscorrelated with the statistical base of the pseudorandom sequence only, ie with $s(t)$, such that

$$C(\tau) = \int_0^T Z(t)s(t-\tau)dt \tag{10}$$

This is equivalent, for the purposes of the crosscorrelation only, to treating the modulating sequence as a series of δ-functions located at the centre of each pulse. This procedure not only simplifies the mathematics which follow, but was shown by Von Jan and Scherm to provide for increased resolution in the final recovered spectrum.

If the total time of the experiment T is an integer multiple, r, of the length of the finite pseudorandom sequence T_m such that $T=rT_m$ equation 10 becomes

$$C(\tau) = \sum_{j=1}^{rL} s_j Z(\tau+t_j) \tag{11}$$

from which we obtain, using equations 8 and 9

$$C(\tau)=\sum_{j=1}^{rL} s_j (\int S(\tau') \sum_i s_i \Gamma(\tau-t_i-\tau'+t_j)d\tau' + \int N(\tau') \sum_i (1-s_i) \Gamma(\tau-t_i-\tau'+t_j)d\tau' + b')$$

$$= r\sum_{i,j=1}^{L}\{s_j s_i \int S(\tau')\Gamma(\tau-t_i+t_j-\tau')d\tau' + s_j\int N(\tau')\Gamma(\tau-t_i+t_j-\tau')d\tau'$$

$$- \quad s_i s_j \int N(\tau')\Gamma(\tau-t+t-\tau')d\tau'\} + rKb'$$

thus

$$C(\tau)=r\sum_{i,j=1}^{L}\{s_i s_j\int D(\tau')\Gamma(\tau-t_i+t_j-\tau')d\tau' + s_j\int N(\tau')\Gamma(\tau-t_i+t_j-\tau')d\tau'\} + rKb' \qquad (12)$$

where $D(\tau') = S(\tau')-N(\tau')$, ie $D(\tau')$ represents the difference between the transfer functions of the sample measured by neutrons associated with the "on" and "off" pulses of the sequence respectively. The significance of this in an experiment in which the neutron beam spin direction is being modulated will be discussed in detail in a later section.

Taking the first term in brackets in equation 12

$$r\sum_{i,j=1}^{L} s_i s_j \int D(\tau')\Gamma(\tau-t_i+t_j-\tau')d\tau'$$

$$= \quad r\{s_1 s_1 \int D(\tau')\Gamma(\tau-\tau')d\tau' + s_1 s_2\int D(\tau')\Gamma(\tau-\tau'-t_1)d\tau' + \ldots$$

$$+ \quad s_2 s_2 \int D(\tau')\Gamma(\tau-\tau')d\tau' + s_2 s_3\int D(\tau')\Gamma(\tau-\tau'-t_1)d\tau' + \ldots.$$

$$\ldots\ldots \text{etc}\}$$

$$= \quad r\sum_{k=1}^{L} A_k\int D(\tau')\Gamma(\tau+t_k-\tau')d\tau'$$

A_k being defined in equation 3

we therefore have

$$= r\sum_{k=1}^{L}\{K(1-c)\delta(k)\int D(\tau')\Gamma(\tau+t_k-\tau')d\tau' + Kc\int D(\tau')\Gamma(\tau+t_k-\tau')d\tau'$$

Finally using the pulse symmetry condition of equation 5 this reduces to

$$= rK(1-c)\int D(\tau')\Gamma(\tau-\tau')d\tau' + rKc\int D(\tau')d\tau' \qquad 12(a)$$

While the first term of equation 12(a) is clearly a convolution of $D(\tau')$ with the pulse shape, the second term is proportional to the difference between the total number of "on" counts and the total number of "off" counts recorded in the experiment. Turning now to the second expression in equation 12

$$r\sum_{i,j=1}^{L} s_j\int N(\tau')\Gamma(\tau-t_i+t_j-\tau')d\tau'$$

$$= r\{s_1\int N(\tau')\Gamma(\tau-\tau')d\tau' + s_1\int N(\tau')\Gamma(\tau-\tau'+t_1)d\tau' + \ldots\ldots$$

$$+ \quad s_2\int N(\tau')\Gamma(\tau-\tau'-t_{L-1})d\tau' + s_2\int N(\tau')\Gamma(\tau-\tau')d\tau' + \ldots\ldots$$

$$+ \quad \text{etc}$$

$$= rK\int N(\tau')\Gamma(\tau-\tau')d\tau' + rK\int N(\tau')\Gamma(\tau-\tau'+t_1)d\tau' + \ldots.\ \text{etc}$$

$$= rK\sum_{k}\int N(\tau')\Gamma(\tau-\tau'+t_k)d\tau'$$

and again, using the symmetry condition, this becomes

$$rK\int N(\tau')d\tau' \qquad 12(b)$$

Equations 12(a) and 12(b) can now be combined to give, for equation 12,

$$C(\tau) = rK(1-c)\int D(\tau')\Gamma(\tau-\tau')d\tau' + rKc\int D(\tau')d\tau' + rK\int N(\tau')d\tau' + rKb'$$

which finally reduces to

$$C(\tau)=rK(1-c)\int D(\tau')\Gamma(\tau-\tau')d\tau' + rKc\int S(\tau')d\tau' + rK(1-c)\int N(\tau')d\tau' + rKb' \qquad (13)$$

In an actual experiment the countrate $Z(t)$ and ultimately $C(\tau)$ will be contained in a finite number of channels, N, of a multichannel analyser. The time width of each MCA channel, δ, should be chosen such that $N\delta$ is somewhat greater than the overall width of both $S(\tau)$ and $N(\tau)$. The background per channel is thus $b=\delta b'$. Using a subscript notation rather than arguments to denote the quantities measured in an MCA channel of width δ, and also adopting a convention in which asterisk superscripts indicate that a quantity has been convoluted <u>once</u> with the pulse shape $\Gamma(t)$ eg

$$D^*_\tau = \int_{\tau-\delta/2}^{\tau+\delta/2} \int D(\tau')\Gamma(\tau-\tau')d\tau' \, d\tau$$

equation 13 can be written as

$$C_\tau = rK(1-c)D^*_\tau + \frac{rKc}{p}\sum_{\tau=1}^{N} S^*_\tau + \frac{rK(1-c)}{p}\sum_{\tau=1}^{N} N^*_\tau + rKb \qquad (14)$$

Where p is an integer representing the ratio of the modulating pulse width to the channel width, δ.

Equation 14 is extremely important as it contains all the information embodied in a modified crosscorrelation experiment. It can now be seen that the result of crosscorrelating the measured countrate with the statistical part of the modulating sequence yields, in a particular channel τ, a term proportional to the difference between the two transfer functions S^*_τ and N^*_τ, superimposed upon a "background of ignorance" which, within statistical accuracy, should be completely independent of channel.

4. APPLICATIONS OF THE MODIFIED CROSS-SECTION TECHNIQUE IN NEUTRON POLARISATION ANALYSIS

The expression "polarisation analysis" is somewhat of a misnomer; experimentally it is the partial cross-sections connecting the two neutron spin states that are measured rather than the final polarisation of the scattered beam. Furthermore it is usual to measure cross-sections for the spin flip (SF) and non spin flip (NSF) processes without differentiating between +− and −+ or between ++ and −− scattering events. The SF and NSF cross-sections are related to the magnetic, nuclear and nuclear spin incoherent scattering cross-

sections, ie $\left(\frac{d\sigma}{d\Omega}\right)^M$, $\left(\frac{d\sigma}{d\Omega}\right)^N$ and $\left(\frac{d\sigma}{d\Omega}\right)^{NSI}$ respectively, of the sample and ultimately it is these cross-sections that are of interest [6].

For systems in which, averaged over the whole sample, the atomic spins are randomly oriented (eg multidomain antiferromagnets and para-magnets) the SF and NSF cross-sections can be expressed as [6].

$$\left(\frac{d\sigma}{d\Omega}\right)^{SF}_\tau = \frac{2}{3}\left(\frac{d\sigma}{d\Omega}\right)^{NSI}_\tau + \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)^{M}_\tau (1 + \cos^2\psi)$$
$$\left(\frac{d\sigma}{d\Omega}\right)^{NSF}_\tau = \frac{1}{3}\left(\frac{d\sigma}{d\Omega}\right)^{NSI}_\tau + \left(\frac{d\sigma}{d\Omega}\right)^{N}_\tau + \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)^{M}_\tau (1 - \cos^2\psi) \qquad (15)$$

Here ψ is the angle between the neutron polarisation direction and the scattering vector, $\underline{K}$. The subscript τ has the same meaning as in the previous section.

In a neutron polarisation analysis experiment where the neutron polariser and analyser have polarising efficiencies of the same sign it is normal to associate the SF and NSF cross-sections with the countrates obtained with a neutron spin flipper placed before the sample in the ON and OFF conditions respectively. If the spin flipper is therefore switched according to a pseudorandom sequence we can write, in the nomenclature of the previous section

$$S_\tau \propto \left(\frac{d\sigma}{d\Omega}\right)^{SF}_\tau, \quad N_\tau \propto \left(\frac{d\sigma}{d\Omega}\right)^{NSF}_\tau \qquad 16(a)$$

where the constants of proportionality are related to instrumental and sample characteristics. This is only strictly correct, however, in the ideal situation of perfect neutron polarisation and spin analysis, and for a perfectly efficient neutron spin flipper. Although the latter condition is not difficult to realise [eg 7] the efficiencies of neutron polarsers and analysers often vary greatly with neutron energy.

Assuming a 100% efficiency for the spin flipper, equation 16(a) can be rewritten

$$S_\tau \; \alpha \; \frac{1}{2}\left\{\left(\frac{d\sigma}{d\Omega}\right)^{SF}_{\tau} \cdot (1 + P_\tau^2) + \left(\frac{d\sigma}{d\Omega}\right)^{NSF}_{\tau} \cdot (1 - P_\tau^2)\right\}$$

$$N_\tau \; \alpha \; \frac{1}{2}\left\{\left(\frac{d\sigma}{d\Omega}\right)^{NSF}_{\tau} \cdot (1 + P_\tau^2) + \left(\frac{d\sigma}{d\Omega}\right)^{SF}_{\tau} \cdot (1 - P_\tau^2)\right\} \qquad 16(b)$$

similarly

$$D_\tau \; \alpha \; P_\tau^2\left\{\left(\frac{d\sigma}{d\Omega}\right)^{SF}_{\tau} - \left(\frac{d\sigma}{d\Omega}\right)^{NSF}_{\tau}\right\} \qquad 16(c)$$

where P_τ is the effective beam polarisation for time-of-flight τ.

Although it is not immediately clear why a difference spectrum such as that given in equation 16(c) should be of interest a great deal of information can be obtained from it. To illustrate this point two specific situations will be considered; (i) that in which the neutron beam is polarised in a direction perpendicular to the scattering plane and (ii) that in which the polarisation within the scattering plane.

(i) If the scattered neutrons are detected in the horizontal plane, then vertical polarisation of the neutrons at the sample position ensures that whatever the energy change of the scattered neutron the polarisation remains perpendicular to the scattering vector, ie $\psi = 90^o$. The measured difference spectrum D_τ is therefore

$$D_\tau \; \alpha \; P_\tau^2\left\{\frac{1}{3}\left(\frac{d\sigma}{d\Omega}\right)^{NSI}_{\tau} - \left(\frac{d\sigma}{d\Omega}\right)^{N}_{\tau}\right\}$$

For most elements $\left(\frac{d\sigma}{d\Omega}\right)^{NSI}$ is relatively small, and in general

$$D_\tau \; \alpha - P_\tau^2\left(\frac{d\sigma}{d\Omega}\right)^{N}_{\tau} \qquad 17(b)$$

In otherwords the cross correlation spectrum consists of a purely <u>nuclear</u> time-of-flight spectrum superimposed upon the background of ignorance. The <u>magnetic</u> scattering has been totally and unambiguously removed from the time-of-flight spectrum, and appears only as a contribution to the flat background of ignorance.

(ii) In situations where the neutron polarisation at the sample is in the scattering plane of the detected neutrons it is usual to arrange the polarisation direction to be parallel to the scattering vector for elastic scattering (Fig 2). The angle ψ therefore varies as a function of energy transfer of the neutron according to the expression

$$\cos^2\psi = \frac{(1 - \cos 2\theta)\,(K_I + K_F)^2}{2(K_I^2 + K_F^2 - 2K_I\,K_F\cos 2\theta)} \qquad (18)$$

where 2θ is the scattering angle and K_I and K_F are the incident and scattered neutron wavevectors respectively. D_τ therefore can be written as

$$D_\tau \; \alpha \; P_\tau^2\left\{\frac{1}{3}\left(\frac{d\sigma}{d\Omega}\right)^{NSI}_{\tau} - \left(\frac{d\sigma}{d\Omega}\right)^{N}_{\tau} + \left(\frac{d\sigma}{d\Omega}\right)^{M}_{\tau}\cos^2\psi\right\} \qquad (19)$$

From the expressions 17(a) and 19 it can be seen that magnetic scattering will always occur as a positive going peak in the cross correlation function C_τ, while nuclear scattering appears as a dip below the background of ignorance. Furthermore the subtraction of C_τ obtained in case (ii) from that obtained in case (i) isolates the magnetic scattering ie

$$D_\tau^{(i)} - D_\tau^{(ii)} \; \alpha \; P_\tau^2\left\{\cos^2\psi\left(\frac{d\sigma}{d\Omega}\right)^{M}_{\tau}\right\} \qquad (20)$$

with the background of ignorance removed by the subtraction.

5. IMPLEMENTATION OF THE POLARISATION MODULATED CROSSCORRELATION TECHNIQUE

The flexibility of the polarisation modulated crosscorrelation technique is such that it can be used to perform either complete energy analysis of the scattered neutrons or alternatively, using much simpler equipment, to perform discrimination against inelastically scattered neutrons:

5(i) Full energy analysis can be accomplished by performing the crosscorrelation in one of two ways.

The "direct" method involves counting the scattered neutrons in the time channels of an MCA appropriate to their time of arrival at the detector. At the end of the experiment this stored countrate is finally cross correlated with the statistical base of the pseudorandom sequence used to switch the flipper.

The second method involves the crosscorrelation being performed on-line in a manner similar to the "inverse time-of-flight" method described by Hiismaki [8]. In this "inverse" method a detected neutron triggers a sweep of the MCA during which elements of the delayed pseudorandom sequence (ie previous states of the spin flipper) are stored in the appropriate MCA channels. In this way a cross-correlation pattern, mathematically identical to C_τ in equation (14) is constructed dynamically. The "inverse" method is somewhat simpler to implement than the "direct" method as it does not require a single final computation of the crosscorrelation function, and allows the time-of-flight spectrum to be viewed continuously during data acquisition. However its use is restricted to experiments with low count rates, as only one neutron event can be stored per MCA sweep.

5(ii) Discrimination against inelastic scattering: To discriminate against inelastic scattering events the pseudorandom spinflipper switching sequence is delayed by the time-of-flight of elastically

scattered neutrons, τ_{el}, and used to trigger an electronic gate routing detected neutrons to one of two scalers depending upon the state of the switching sequence. This is equivalent to evaluating the cross correlation function, C_τ, at a single channel corresponding to $\tau=\tau_{el}$, the channel width being the same as the switching sequence pulse width (ie. p=1). Assuming that the duty cycle of the pseudorandom sequence is $\frac{1}{2}$, inspection of the expression for C_τ shows that for the scaler associated with the flipper-on condition

$$C^{on}_{\tau_{el}} = rK\left[\tfrac{1}{2}(S^*_{\tau_{el}} - N^*_{\tau_{el}}) + \tfrac{1}{2}\sum S^*_\tau + \tfrac{1}{2}\sum N^*_\tau + b\right]$$

$$\text{ie. } C^{on}_{\tau_{el}} = rK\left[S^*_{\tau_{el}} + \tfrac{1}{2}\left\{\sum S^*_\tau - S^*_{\tau_{el}} + \sum N^*_\tau - N^*_{\tau_{el}}\right\} + b\right]$$

As the total, flipper-on plus flipper-off, inelastic scattering I* can be expressed as

$$I^* = \sum(S^*_\tau + N^*_\tau) - S^*_{\tau_{el}} - N^*_{\tau_{el}}$$

the flipper-on scaler records

$$C^{on}_{\tau_{el}} = rK\left[S^*_{\tau_{el}} + \tfrac{1}{2}I^* + b\right]$$

In this mode, the situation for the flipper-off state is symmetrical with that for the flipper-on and in the other scaler

$$C^{off}_{\tau_{el}} = rK\left[N^*_{\tau_{el}} + \tfrac{1}{2}I^* + b\right]$$

is measured. In other words the elastic flipper-on or flipper-off scattering is measured in the appropriate scaler, while the flipper-on plus flipper-off inelastic scattering is, within statistical accuracy, equally distributed between the two scalers. The difference between the two scalers is, of course, independent of any inelastic scattering processes. The switching frequency of the flipper governs the resolution of the elastic discrimination.

The LONGPOL neutron polarisation instrument at AAEC Research Establishment, Lucas Heights, Australia has recently been modified to allow the polarisation modulated cross correlation technique to be implemented in either the full time-of-flight analysis mode (employing the "inverse" method) or the inelastic discrimination mode. A full description of the LONGPOL instrumentation and implementation of the cross correlation procedures described in this report can be found in Ref [9]. A schematic diagram of LONGPOL is given in Figure 3.

An illustration of the application of the LONGPOL time-of-flight facility is given in Figure 4 where a spectrum obtained from an antiferromagnetic γ-MnNi single crystal is shown. The crystal, maintained at room temperature, was oriented such that a low wavevector magnon could be observed. Details of the experimental parameters are given in the figure. The illustrated spectrum was collected in 90 hours, but the position of the magnon was quite clear after 40 hours. For clarity the "background of ignorance" has been subtracted from C_τ leaving a difference spectrum proportional to $D^*_\tau = (S^*_\tau - N^*_\tau)$. No correction has been made for imperfect beam polarisation.

As the polarisation direction of the beam is closely parallel to the scattering vector for small energy changes S^*_τ represents predominantly magnetic scattering whereas N^*_τ is predominantly nuclear scattering. It is therefore to be expected that D^*_τ is negative, as observed, for elastic scattering as for the present orientation of the crystal the elastic scattering consists of diffuse atomic disorder and magnetic defect scattering in the ratio ~ 3:1 [10]. The positive peak at an energy transfer of 10 meV, on the other hand, unambiguously shows that at this energy transfer the magnetic scattering is far greater than any nuclear contribution. This peak can therefore be identified as resulting from magnon annihilation.

Figures 3 and 4 have been reproduced from Ref [9].

6. SUMMARY

In this report the mathematical principles and the methods of implementing a polarisation modulated crosscorrelation technique in neutron polarisation analysis time-of-flight studies have been discussed. It has been shown that crosscorrelation of the observed neutron countrate with the statistical base of the pseudorandom sequence used to switch the neutron spin flipper provides a time-of-flight spectrum which is proportional to the difference between the spin flip and non spin flip scattering cross-sections of the sample superimposed on a background which, within statistical accuracy, is independent of time-of-flight. It has further been shown that changing the relative orientations of the neutron polarisation direction and scattering vector at the sample position enables the time-of-flight spectra for nuclear and magnetic scattering to be unambiguously separated.

A method for using the crosscorrelation procedure to simply discriminate against inelastic neutron scattering has also been described.

Although the crosscorrelation technique as described in this report is strictly valid only for steady state neutron sources, with a monochromatic polarised neutron beam incident on the scattering sample, the application of a similar technique to pulsed neutron sources and polychromatic polarised beams is currently under investigation and details will be reported shortly.

Acknowledgements

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REFERENCES

1. Proceedings of IAEA 'Neutron Inelastic Scattering' Symposium from 1968 onwards

2. Mezei F and Pellionisz P, Neutron Inelastic Scattering, IAEA-SM-155 (1972), 797

3. Wilhelmi G and Gompf F, NIM 81 (1970) 36

4. Baumert L D. "Cyclic Difference Sets", Lecture Notes in Mathematics 182, Springer-Verlag, (1971)

5. Von Jan R and Scherm R, NIM 80 (1970) 69

6. Moon R M, Riste T and Koehler W C, Phys Rev 181 (1969) 920

7. Jones T J L and Williams W G, RL-77-079/A (1977)

8. Hiismaki P, Neutron Inelastic Scattering IAEA-SM-155 (1972), 803

9. Davis J R, Cywinski R, Moze O and Hicks T J, Int Conf on "Impact of Polarised Neutrons on Solid-State Chemistry and Physics", Grenoble 1982, to be published in Journal de Physique

10. Moze O and Hicks T J, J Phys F12 (1982) 1

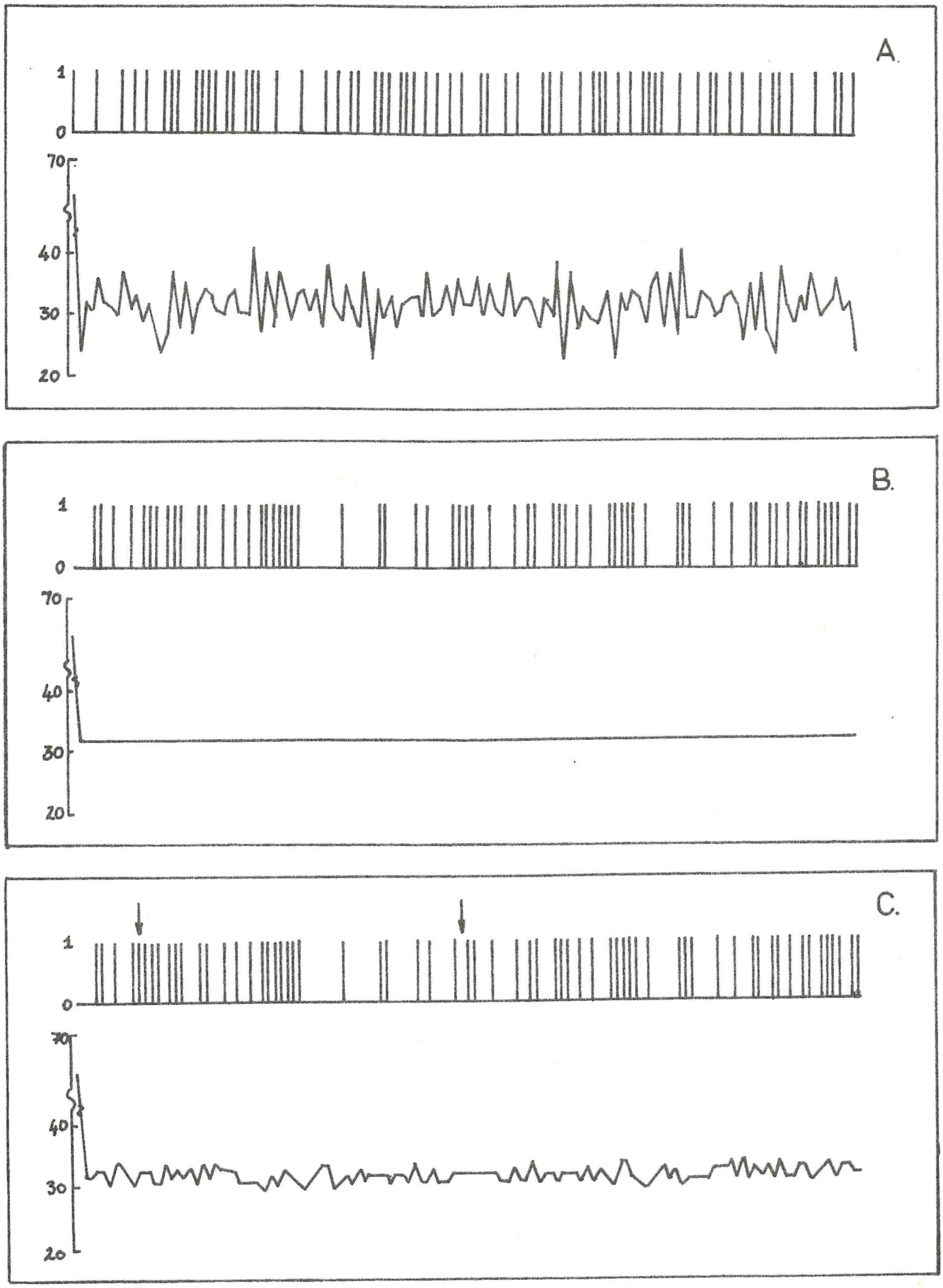

Figure 1 A. 127 bit binary sequence with 64 pulses randomly distributed by computer random number intrinsic, together with autocorrelation function.
B. Shift register sequence (127, 64, 32) with autocorrelation.
C. Sequence B with indicated elements interchanged together with autocorrelation function.

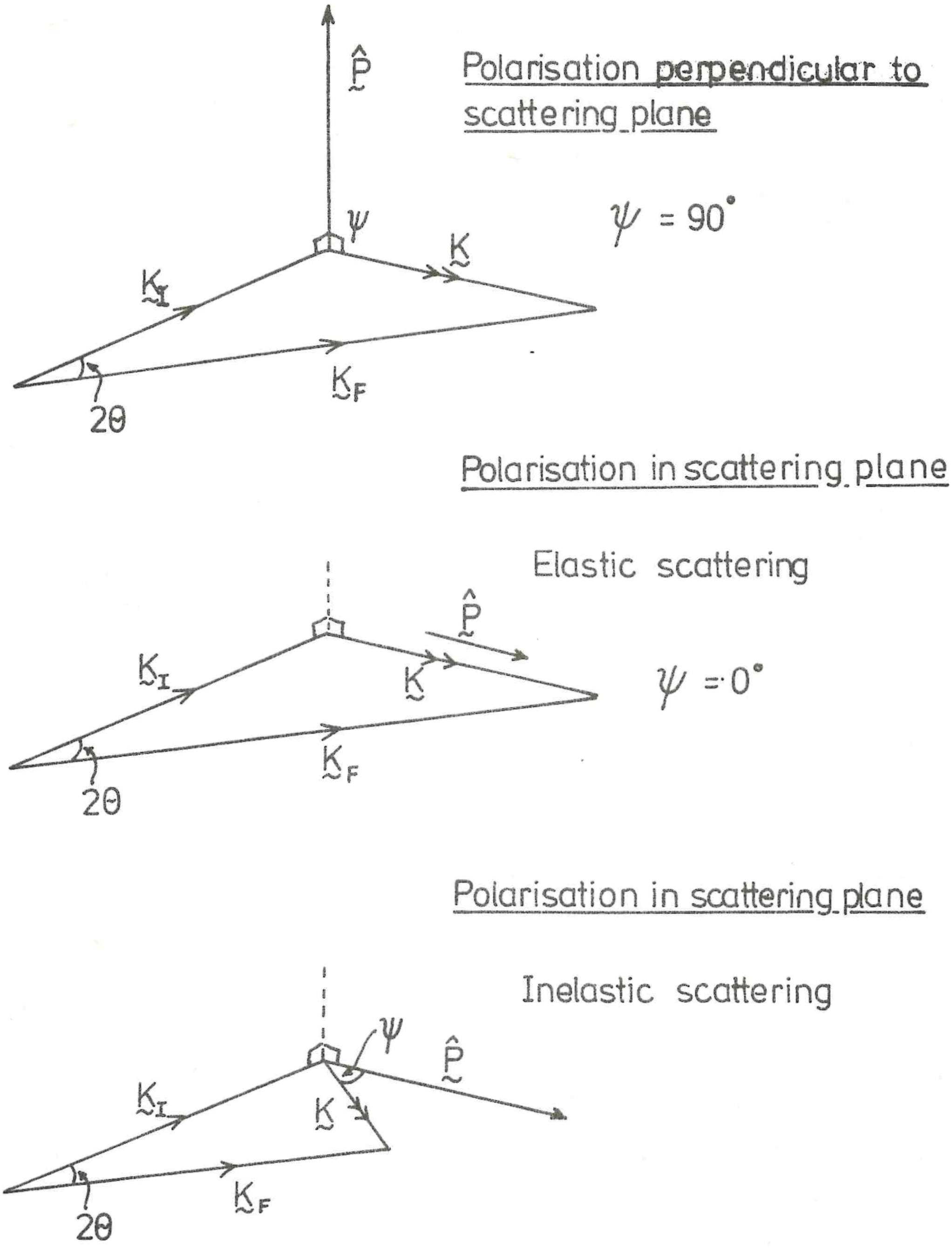

Figure 2. Relative orientations of the vectors K_I, K_F, K and $\hat{P}$ for the scattering geometries discussed in section 4

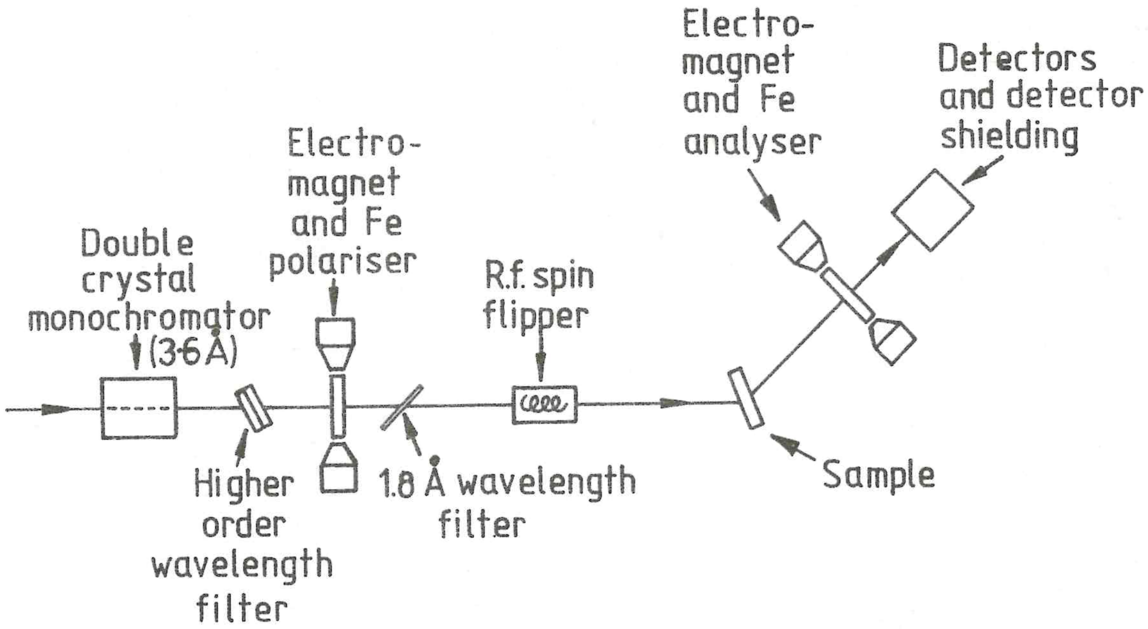

Figure 3: Schematic diagram of the LONGPOL instrument

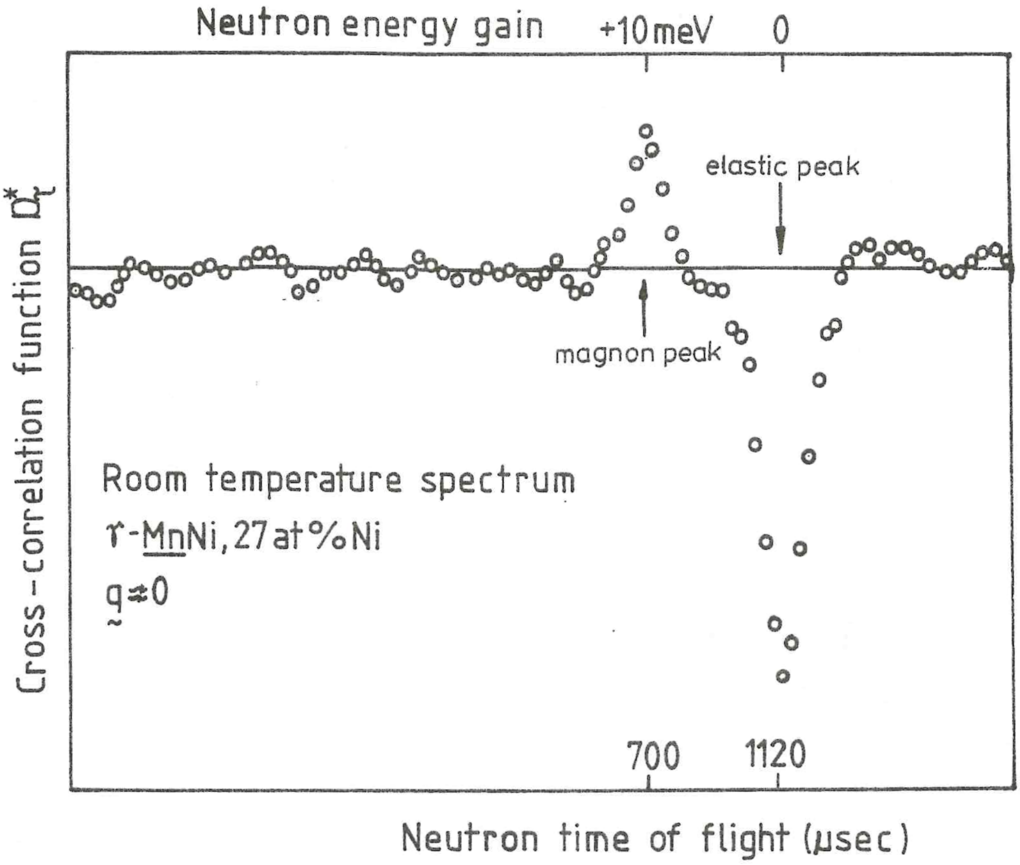

Figure 4: Time of flight spectrum from γ-MnNi

Channel width = 25 μsec

Sequence length L = 4096

Duty cycle C = 0.5