



**Technical Report**  
RAL-TR-95-0350

# **Do $\psi$ (4040), $\psi$ (4160) Signal Hybrid Charmonium?**

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September 1995

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**ISSN 1358-6254**

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## DO $\psi(4040)$ , $\psi(4160)$ SIGNAL HYBRID CHARMONIUM?<sup>1</sup>

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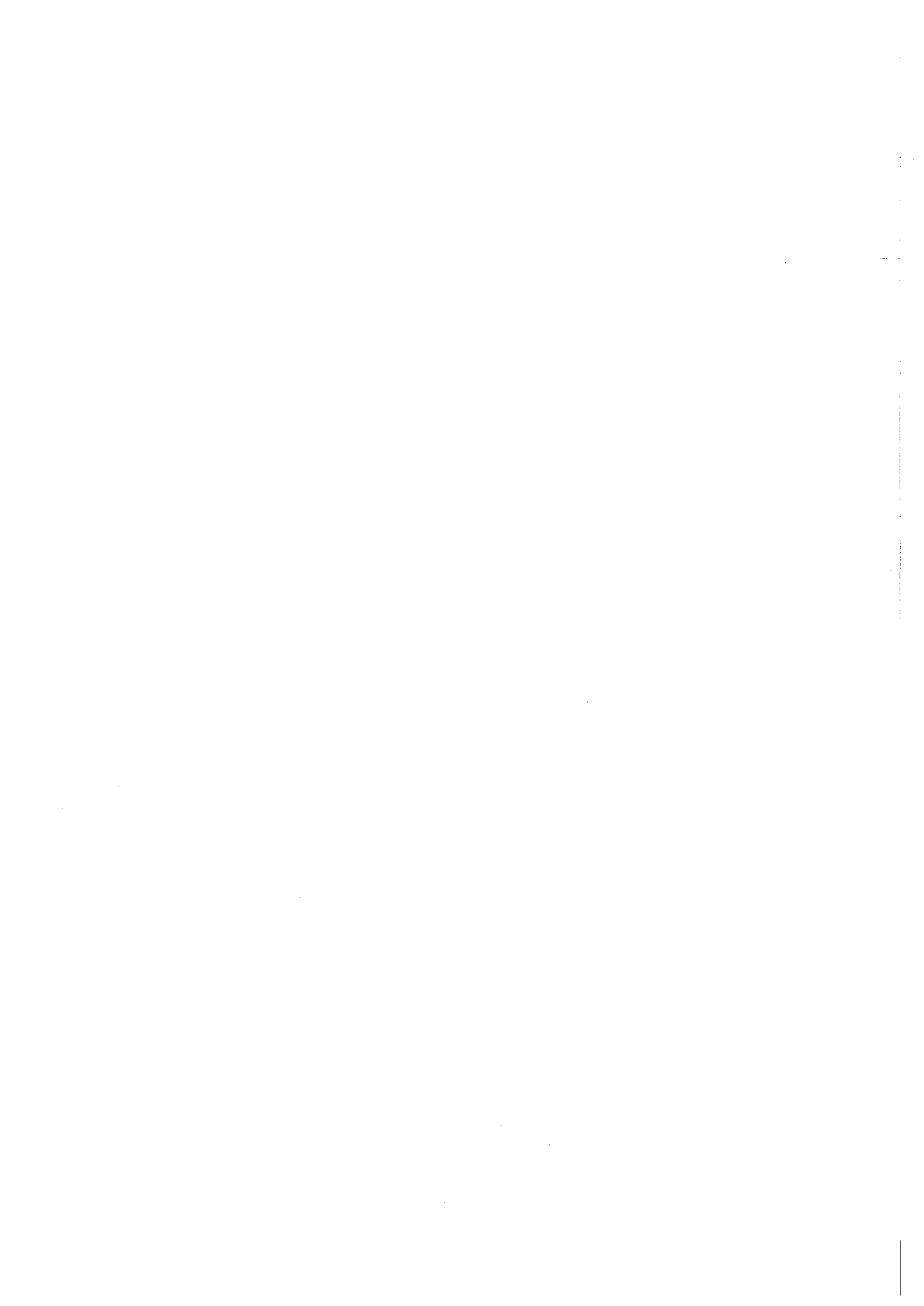
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### Abstract

We suggest that  $\psi(4040)$  and  $\psi(4160)$  are strong mixtures of ground state hybrid charmonium at  $\sim 4.1$  GeV and the  $\psi(3S)$  of conventional charmonium. The  $\Gamma^{e^+e^-}$ , masses and total widths of the  $\psi(4040)$  and  $\psi(4160)$  are in accord with this hypothesis. Their hadronic decays are predicted to be dominated by the  $\psi(3S)$  component and hence are correlated. In particular we find a spin counting relation  $\Gamma(4160 \rightarrow D_s D_s^*) \sim 4\Gamma(4040 \rightarrow D_s D_s)$  due to their common  $\psi(3S)$  component. For  $D$  and  $D^*$  production, using  $\psi(4040)$  branching ratios as input, we predict that the decay pattern of the  $\psi(4160)$  will be very different from that of the  $\psi(4040)$ . These predictions may be tested in historical data from SPEAR, BES or at future Tau-Charm Factories.

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<sup>1</sup>Work supported in part by the European Community Human Mobility program "Eurodafne" contract CHRX-CT92-0026 and the University of Cape Town.



The QCD sector of the Standard Model cannot be regarded as finally established while the question of the existence of glueballs and hybrids (hadrons where the gluonic modes are excited in the presence of quarks) remains unresolved. This is now becoming rather critical. On the theory side a consensus on spectroscopy and dynamics is emerging from lattice QCD and model simulations thereof whereas on the experimental side, the increasing concentration of resources at the high energy frontier threatens to leave a hole in this essential area of the Standard Model.

We have initiated a programme to evaluate the best opportunities in this area and have identified possible signals worthy of further study [1, 2, 3]. In this letter we note that  $e^+e^-$  annihilation in the vicinity of charm threshold, 4 - 4.5 GeV in  $E_{c.m.}$ , offers special opportunities: the lowest hybrid charmonia “ $H_c$ ” are predicted to exist at 4.1 - 4.2 GeV [4, 5], just above charm threshold where the conventional  $c\bar{c}$  are rather well understood and where there is a well known experimental anomaly which may be due to excitation of these hybrid states [6]. As a result of our recent investigations into the dynamics of the quark - flux-tube system [1, 2] we present new arguments that support this suggestion and propose further experimental tests. In particular we shall argue that  $\psi(4040)$  and  $\psi(4160)$  are strong mixtures of  $\psi(3S)$  and hybrid charmonium.

One of us (PRP) has shown [2] that, within the conventional charmonium picture, the hadronic width and branching ratios of  $\psi(4040)$  and the narrowness of the  $\psi(4415)$  constrain the  $\psi(4040)$  to be the  $3S$  (as opposed to  $2D$ ) and the  $\psi(4415)$  to be  $4S$  (or possibly  $5S$ ) states; this independently supports the spectroscopic assignments of many potential models. The  $\psi(4160)$  hadronic width is consistent with it being any of  $3S, 4S$  or  $2D$  [2] and hence, by elimination, one is tempted to deduce the  $2D$  or possibly the  $4S$  [7] assignment for it. However,  $\Gamma^{ee}(4160) \sim \Gamma^{ee}(4040)$  which is inconsistent with any of these pictures. The simplest explanation is that these states are roughly 50 : 50 mixtures of  $\psi(3S)$  and an “inert” state that is essentially decoupled from  $e^+e^-$  as originally suggested by Ono [6].

The unresolved problem is what “inert” state the  $\psi(3S)$  is mixing with: dynamics within standard spectroscopy appear unable to generate such a mixing consistently. Mixing between  $\psi(3S) - \psi(2D)$  via a tensor force appears to be inadequate given the known limited mixing for the  $2S - 1D$  states  $\psi(3685; 3770)$  [6, 8, 9]. Strong mixing between  $c\bar{c}$  and  $D\bar{D}$  coupled channels is incompatible with the observed narrow widths (of order tens of MeV) of the  $\psi(4040), \psi(4160)$  [6]. The maximum theoretical mixing of  $\psi(3S) - \psi(2D)$  generated in the literature, to the best of our knowledge, is some 10% [6, 8] in amplitude.

However, a mixing of 50% can arise naturally if there is mass degeneracy between two “primitive” states. The  $\psi(3S)$  and  $\psi(2D)$  are not expected to be degenerate enough for large mixing.<sup>2</sup> However, a new candidate for an “inert” state degenerate with  $\psi(3S)$  has recently emerged.

Numerical solutions of flux-tube charmonium spectroscopy predict that the lightest hybrid charmonium states are in the 4.1 - 4.2 GeV region [4], while QCD inspired potential models with long range linear behaviour uniformly predict that  $\psi(3S)$  is also in the narrow range 4.10 to 4.12 GeV [8, 10, 11]. This near degeneracy is also supported by lattice studies of  $c\bar{c}$  which predict that  $\psi(3S)$  and  $H_c$  are within 30 MeV of each other [5] and, again, in the 4.1 - 4.2 GeV region. Consequently it is probable that, within their widths, there will be

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<sup>2</sup>A *coulomb + linear* potential gives [22] a 2D-3S mass difference of 79 - 87 MeV for string tension  $b = 0.18 \pm 0.02 \text{ GeV}^2$ , a charm quark mass  $m_c = 1.5 \pm 0.3 \text{ GeV}$  and  $\alpha_S = 0.4$ . The ranges of  $b$  and  $m_c$  are the maximal ones consistent with spectroscopy. The only way of reducing the 2D-3S mass difference significantly appears to be by reducing  $\alpha_S$ ; nonetheless it is still  $\sim 30 \text{ MeV}$  even in the extreme limit  $\alpha_S \rightarrow 0$ .

mass degeneracy of  $\psi(3S)$  and  $H_c$ , leading to strong mixings and splitting of the eigenvalues. If such a degeneracy occurs one immediately expects that the physical eigenstates will tend to be

$$\psi_{\pm} \simeq \frac{1}{\sqrt{2}}(\psi(3S) \pm H_c) \quad (1)$$

which we shall identify as  $\psi_- \equiv \psi(4040)$  and  $\psi_+ \equiv \psi(4160)$ .

The  $H_c$  component in eq. 1 will be “inert” for the following reason. A characteristic feature of hybrid mesons is the prediction that their decays to ground state mesons are suppressed and that the dominant coupling is to excited states [1, 12, 13, 14], in particular  $DD^{**}$  for which the threshold is  $\sim 4.3$  GeV. This pathway will hence be closed for  $\psi(4040)$  and  $\psi(4160)$  and consequently their hadronic decays will be driven by the  $\psi(3S)$  component. Thus the  $H_c$  component immediately satisfies the “inert” criterion for the additional piece and is partly responsible for the relatively narrow widths of these states.

Both the dominant production in  $e^+e^-$  annihilation and the prominent hadron decays will be driven by the  $\psi(3S)$  component, leading to intimate relationships between the properties of the two eigenstates. First, this naturally explains their leptonic widths

$$\Gamma^{e^+e^-}(\psi_+) \simeq \Gamma^{e^+e^-}(\psi_-) \simeq \frac{1}{2}\Gamma^{e^+e^-}(\psi(3S))$$

which is consistent with the fact that potential models uniformly predict a value for  $\Gamma^{ee}(3S)$  that is essentially a factor of two larger than the data in both cases.

We now consider the implications for hadronic decays with particular reference to our recent dynamical analyses [1, 2, 4].

The remarkable branching ratios (with phase space removed)

$$\psi(4040) \rightarrow DD : DD^* : D^*D^* \simeq 1 : 20 : 640 \quad (2)$$

differ considerably from those expected for a simple S-wave charmonium (1 : 4 : 7) and D-wave (1 : 1 : 4) [15, 16, 17]. Within a charmonium picture the favoured interpretation has been that these ratios arise as a consequence of nodes in the radial wave function of  $\psi(3S)$  which suppress  $DD$  and  $DD^*$  due to the coincidence of  $p(4040 \rightarrow DD, DD^*)$  lying near the Fourier transformed node (Figure 1) [2, 18]. This hypothesis is quantitatively consistent with the model description of other  $c\bar{c}$  dynamics [2] and we shall therefore adopt this as our point of departure. We note a useful kinematical coincidence.

$$\frac{p(4160 \rightarrow DD^*)}{p(4040 \rightarrow DD)} = \frac{0.75}{0.77} \quad (3)$$

$$\frac{p(4160 \rightarrow D^*D^*)}{p(4040 \rightarrow DD^*)} = \frac{0.54}{0.57} \quad (4)$$

If the decay amplitudes are insensitive to these small changes in momenta then one would expect immediate correlations along the following lines. If the  $DD$  and  $DD^*$  channels of  $\psi(4040)$  are suppressed due to the node structure of the  $3S$  wave function, and if the hadronic decays of both resonances  $\psi(4040)$  and  $\psi(4160)$  are due to a common  $\psi(3S)$  component, then one infers that there will be a corresponding suppression of  $DD^*$  and  $D^*D^*$  for the  $\psi(4160)$ . After taking due account of the spin weightings, one expects

$$\Gamma(4160 \rightarrow DD^*) \simeq 4\Gamma(4040 \rightarrow DD)$$

Table 1: Width ratios in the flux-tube model [21] in regions A, B and C of parameter space. The various width ratios are calculated at each “dot” in Figure 2 and the mean  $R$  and standard deviation  $\sigma$  are computed for each region. The “dots” in Figure 2 fit the experimental ratios in eq. 2 up to the  $1\sigma$  deviations given in ref. [19]. This translates into  $\sigma/R \sim 40\%$ . Ratios indicated with  $\sigma/R$  significantly smaller than this should hence be understood to contain parameter independent information, and are hence more model independent. No assumption about total widths are made. The first ratio would still be valid for  $\psi(4040)$  as pure 3S. The decays  $\psi_- \rightarrow D_s D_s$  in Region B and  $\psi_+ \rightarrow D_s^* D_s$  in Region C are at a node and hence very sensitive to parameters, indicated by an asterisk. The conventions and parameters are those of ref. [2].

Width Ratio	Region A		Region B		Region C	
	$R$	$\sigma/R$	$R$	$\sigma/R$	$R$	$\sigma/R$
$\psi_- \rightarrow D^* D^* / \psi_- \rightarrow D_s D_s$	4.0	20%	*	*	54	42%
$\psi_+ \rightarrow D^* D^* / \psi_+ \rightarrow D_s D_s$	36	24%	13%	29	29%	
$\psi_+ \rightarrow D^* D^* / \psi_+ \rightarrow D_s^* D_s$	1.4	30%	8.8	45%	*	*
$\psi_+ \rightarrow DD / \psi_+ \rightarrow D^* D$	3.3	37%	.25	58%	3.4	48%
$\psi_+ \rightarrow D^* D^* / \psi_+ \rightarrow DD$	4.1	37%	10	30%	2.9	8%
$\psi_+ \rightarrow D^* D / \psi_- \rightarrow DD$	2.3	8%	5.6	8%	2.1	14%
$\psi_+ \rightarrow D^* D^* / \psi_- \rightarrow D^* D$	2.4	3%	1.2	5%	1.8	5%
$\psi_+ \rightarrow D_s^* D_s / \psi_- \rightarrow D_s D_s$	3.9	3%	*	*	*	*
$\psi_+ \rightarrow D^* D^* / \psi_- \rightarrow D^* D^*$	1.3	18%	.69	26%	1.1	23%
$\psi_- \rightarrow D^* D / \psi_+ \rightarrow D_s D_s$	15	22%	5.0	9%	16	24%

$$\Gamma(4160 \rightarrow D^* D^*) \simeq \frac{7}{4} \Gamma(4040 \rightarrow DD^*) \quad (5)$$

from which one can deduce the naïve expectation  $\Gamma(4160 \rightarrow D^* D) : \Gamma(4160 \rightarrow D^* D^*) = 1 : 3.5_{-1.3}^{+5.3}$  using the experimental ratios in ref. [19]. Detailed calculations following the parameters, conventions and methods of ref. [2] show modifications to these numbers indicated in Table 1.

The positions of the nodes in momentum space depend on the spatial spread of the wave functions which are summarised by a parameter  $\beta$  - see Figure 1. Depending on the magnitude of  $\beta$  one can find different bands of solutions in  $\beta$  space that fit the branching ratios of  $\psi(4040) \rightarrow DD : DD^* : D^* D^*$  (see Figure 2). Historically the solutions<sup>3</sup> of refs. [18, 23, 24] appear to be those of band “A” where the outgoing momenta  $p_{D^* D}$  and  $p_{DD}$  are both in the vicinity of the first node, thereby suppressing the  $DD$  and  $DD^*$  channels. A similar situation ensues for band “C” where both momenta are in the vicinity of the second node. Solution A gives

$$\sigma(e^+ e^- \rightarrow DD : DD^* : D^* D^* : D_s D_s : D_s D_s^*) \sim 3 : 1 : 14 : 0.4 : 10$$

<sup>3</sup> $\beta_A = \beta = 0.44$  GeV [18] was fitted from experiment.  $\beta_A = 0.485$  GeV,  $\beta = 0.39$  GeV [23] and  $\beta_A = 0.46$  GeV,  $\beta = 0.42$  GeV [24] were deduced from numerical wave functions consistent with spectroscopy. We obtain with a *coulomb + linear* potential that  $\beta_A = 0.41 - 0.48$  GeV for string tension  $b = 0.18 \pm 0.02$  GeV<sup>2</sup>, charm quark mass  $m_c = 1.5 \pm 0.3$  GeV and  $\alpha_S = 0.4 \pm 0.1$ . Restricting to  $b = 0.18$  GeV<sup>2</sup>,  $\alpha_S = 0.4$  and  $m_c = 1.5 - 1.8$  GeV preferred by potential models [8], we prefer  $\beta_A = 0.45 - 0.48$  GeV.  $(\beta_A, \beta)$  is defined in Figure 2.

Solution C gives qualitatively similar results to solution A (except for the  $D_s D_s^*$  mode)

$$\sigma(e^+e^- \rightarrow DD : DD^* : D^*D^* : D_s D_s : D_s D_s^*) \sim 3 : 1 : 10 : 0.3 : 0$$

In solution B,  $p_{DD}$  is near the second node and hence dramatically suppressed but  $p_{DD^*}$  is between the first and second nodes with a similar magnitude, but opposite sign, to that in solution A. When the magnitude of  $p$  is varied by the amounts in eqs. 3 - 4 the effect on solution B is to move  $DD^*$  into the region of a local maximum and to shift the  $DD$  away from the node (and hence to increase the amplitude) such that the effects of the radial nodes are washed out; as a result the rates  $DD : DD^* : D^*D^*$  tend to restore towards their “naive” spin counting value of 1:4:7. The actual ratios depend upon the parameters chosen within the band (see Table 1) the most probable results being for solution B

$$\sigma(e^+e^- \rightarrow DD : DD^* : D^*D^* : D_s D_s : D_s D_s^*) \sim 1 : 4 : 10 : 2 : 1$$

We shall consider the consequences of each of these “mathematical” solutions, even though the parameters of solution A are physically preferred.

The production of  $D_s$  and  $D_s^*$  from the  $\psi(4040)$  and  $\psi(4160)$  provide further consistency checks. The solution A which gave a dramatic set of branching ratios for the  $D$  system implies an effective “spin counting” result for the  $D_s$  states arising from the common  $3S$  component of the  $\psi(4040)$  and  $\psi(4160)$ . This is because

$$p(\psi(4160) \rightarrow D_s D_s^*) = 0.41 \text{ GeV}$$

$$p(\psi(4040) \rightarrow D_s D_s) = 0.45 \text{ GeV}$$

and hence their momenta are far from the nodes of the ( $3S$ ) component wave function. On the other hand,

$$p(\psi(4160) \rightarrow D_s D_s) = 0.67 \text{ GeV}$$

is coincident with the node: so in this scenario one has the result that, due to the common  $3S$  component, the spin counting is realised best between the two states

$$\Gamma(\psi(4160) \rightarrow D_s D_s^*) \sim 4\Gamma(\psi(4040) \rightarrow D_s D_s)$$

while

$$\Gamma(\psi(4160) \rightarrow D_s D_s) \sim 0.$$

This characteristic set of results is very different from those in solution B where proximity of the node suppresses  $\Gamma(\psi(4040) \rightarrow D_s D_s) \sim 0$ .

Thus in summary the three patterns that arise in this  $\psi(3S) - H_c$  mixing picture of the  $\psi(4040)$  and  $\psi(4160)$  are :

Solution A (physically realistic)

$$\psi(4040) \rightarrow DD < D_s D_s < D^* D < D^* D^*$$

$$\psi(4160) \rightarrow D_s D_s \lesssim D^* D < DD \lesssim D_s D_s^* \approx D^* D^* \quad (6)$$

Solution B

$$\psi(4040) \rightarrow D_s D_s < DD < D^* D < D^* D^*$$

$$\psi(4160) \rightarrow DD \approx D_s^* D_s \approx D_s D_s < D^* D < D^* D^* \quad (7)$$



### Solution C

$$\psi(4040) \rightarrow D_s D_s < DD < D^* D < D^* D^*$$

$$\psi(4160) \rightarrow D_s^* D_s < D_s D_s \lesssim D^* D < DD < D^* D^* \quad (8)$$

with quantitative measures given in table 1.

Independent evidence for the  $\psi(4040)$  and  $\psi(4160)$  consisting of  $\psi(3S)$  mixed with an inert state arises within this model from a study of their total widths. Figure 2 shows that the regions in parameter space consistent with the experimental widths are the same independent of the pair creation amplitude, implying a common dynamics.

If the tests in eqs. 6 - 8 are successful they will confirm the common presence of the  $3S$  component in the wave functions and that the additional component is inert to hadronic decays thereby implying a new dynamics beyond that in conventional charmonium. Proving that this is due to hybrid states would then require further signals in other partial waves since the above mixing will ironically have made the hybrid dynamics effectively invisible in the  $1^{--}$  channel. For example, vector hybrids would not then be a significant source of enhanced  $\psi(3685)$  at CDF [20]. However, if the exotic  $1^{-+}$  state lies below the  $\psi(4040)$  or  $\psi(4160)$ , there is the possibility of a direct spin flip M1 radiative transition of the hybrid component  $H_c \rightarrow \gamma 1^{-+}$  exposing the exotic hybrid charmonium unambiguously. The matrix element for  $H_c \rightarrow \gamma 1^{-+}$  is related to that of  $\psi \rightarrow \eta_c \gamma$  in the limit where the photon momentum tends to zero. The M1 transition involves  $c\bar{c}$  spin flip from  $S = 1$  to  $S = 0$ , the only additional feature involving the coupling of spin and the orbital angular momentum associated with the excited flux-tube for the  $H_c = 1^{-+}$  state. To the extent that the flux-tube excitation is transversely polarised [1, 21] it is effectively  $L_z = \pm 1$ . Hence,

$$M = \langle \psi_{\pm} | \vec{\mu} | H_c(1^{-+}) \rangle = \cos\theta \langle 11, 10 | 11 \rangle \mu$$

where  $\mu$  is the strength of the M1 transition  $\psi \rightarrow \eta_c \gamma$  and  $\cos\theta \sim \frac{1}{\sqrt{2}}$  is the  $\langle \psi_{\pm} | H_c \rangle$  mixing angle. Thus the relative rates scale as

$$\Gamma(\psi_{\pm} \rightarrow \gamma H_c(1^{-+})) \sim 0.3 \left( \frac{p}{118 \text{ MeV}} \right)^3 \text{ keV}$$

implying a branching ratio of  $O(10^{-5})$ . The subsequent transition

$$\psi_{\pm} \rightarrow H_c(1^{-+}) + \gamma \rightarrow \psi(3095)\gamma\gamma$$

may provide the  $\psi(3095)$  as a tag, though a dedicated search at a high intensity Tau Charm Factory may still be required to isolate this signal.

There may be analogous signals in the  $\Upsilon$  spectrum. There is a mass shift of the  $\Upsilon(4S)$  and the candidate  $\Upsilon(5S)(10580)$  that appears qualitatively similar to those of  $\psi(4040)$  and  $\psi(4160)$  which can conceivably be explained by an increase of 80 MeV in the  $\Upsilon(5S) - \Upsilon(4S)$  mass separation due to coupled channel effects [9]. Furthermore, there is a small discrepancy between leptonic widths and theory but not at any significant level. Any mixing with an ‘‘inert’’ hybrid here would require further states [6] to be seen, albeit at a low level, since the highest mass state  $\Upsilon(11020)$  is the sixth in the tower and is consistent with being  $6S$ , hence militating against strong mixings with additional degrees of freedom. Given that there will soon be extensive studies in the  $4S$  peak at B-factories it may be interesting to see if there are any anomalous radiative decays to  $1^{-+}$   $H_b$  state arising from a possible  $H_b$  component mixed into the  $4S$  peak. However, we consider the charmonium states  $\psi(4040), \psi(4160)$

currently to be the most likely examples of hybrid mixing in the heavy quark sector and we urge a dedicated study of the  $\psi(4160)$  in particular to test this hypothesis.

We wish to thank T. Barnes, J.M. Richard and W. Toki for comments.

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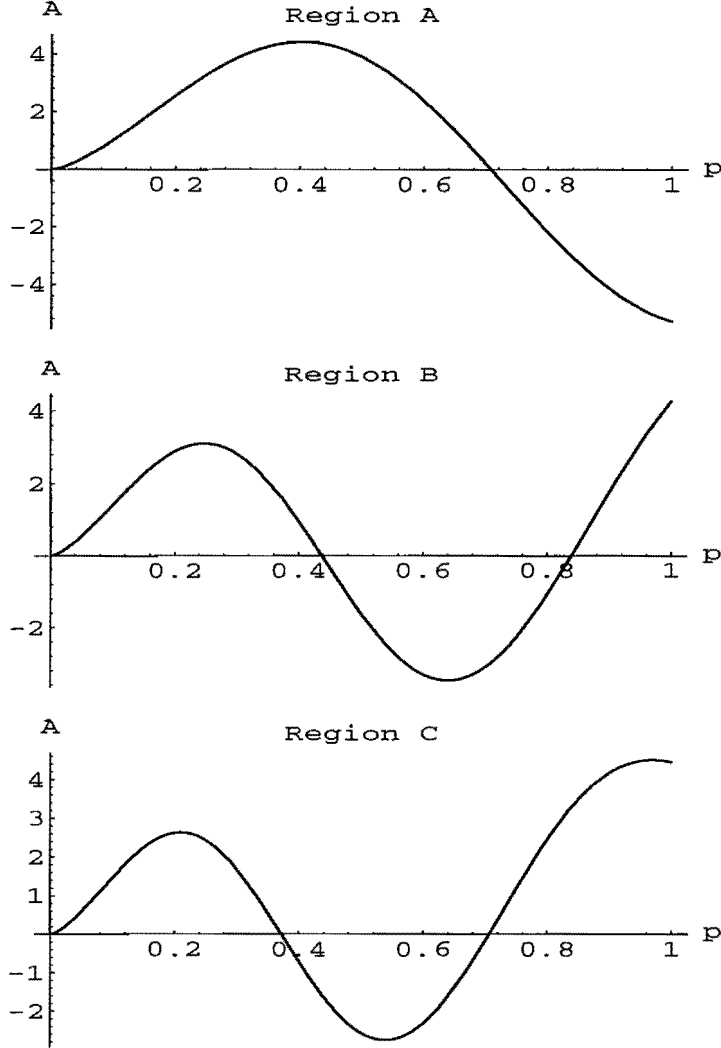


Figure 1: The  $3S \rightarrow 1S + 1S$  amplitude  $A \equiv \pm\sqrt{\tilde{\Gamma}(c\bar{c} \rightarrow c\bar{u} + \bar{c}u)}$  in  $MeV^{1/2}$  as a function of the CM momentum  $p$  in GeV. The physical  $\Gamma = SF \tilde{\Gamma} \tilde{M}_B \tilde{M}_C / \tilde{M}_A \simeq SF \tilde{\Gamma}$ , where the spin-averaged masses  $\tilde{M}$  are defined in ref. [2], and the spin-flavour factor  $SF$  is 1 ( $DD$ ), 4 ( $D^*D$ ), 7 ( $D^*D^*$ ),  $\frac{1}{2}$  ( $D_s D_s$ ) or 2 ( $D_s^* D_s$ ). We display  $A$  at typical  $(\beta_A, \beta)$  (in GeV) : (0.475, 0.455) (Region A), (0.290, 0.265) (Region B) and (0.245, 0.215) (Region C). The amplitude is slightly modified for  $\tilde{\Gamma}(c\bar{c} \rightarrow c\bar{s} + \bar{c}s)$  due to the  $s$ -quark mass differing from that of the  $u$ -quark [2].

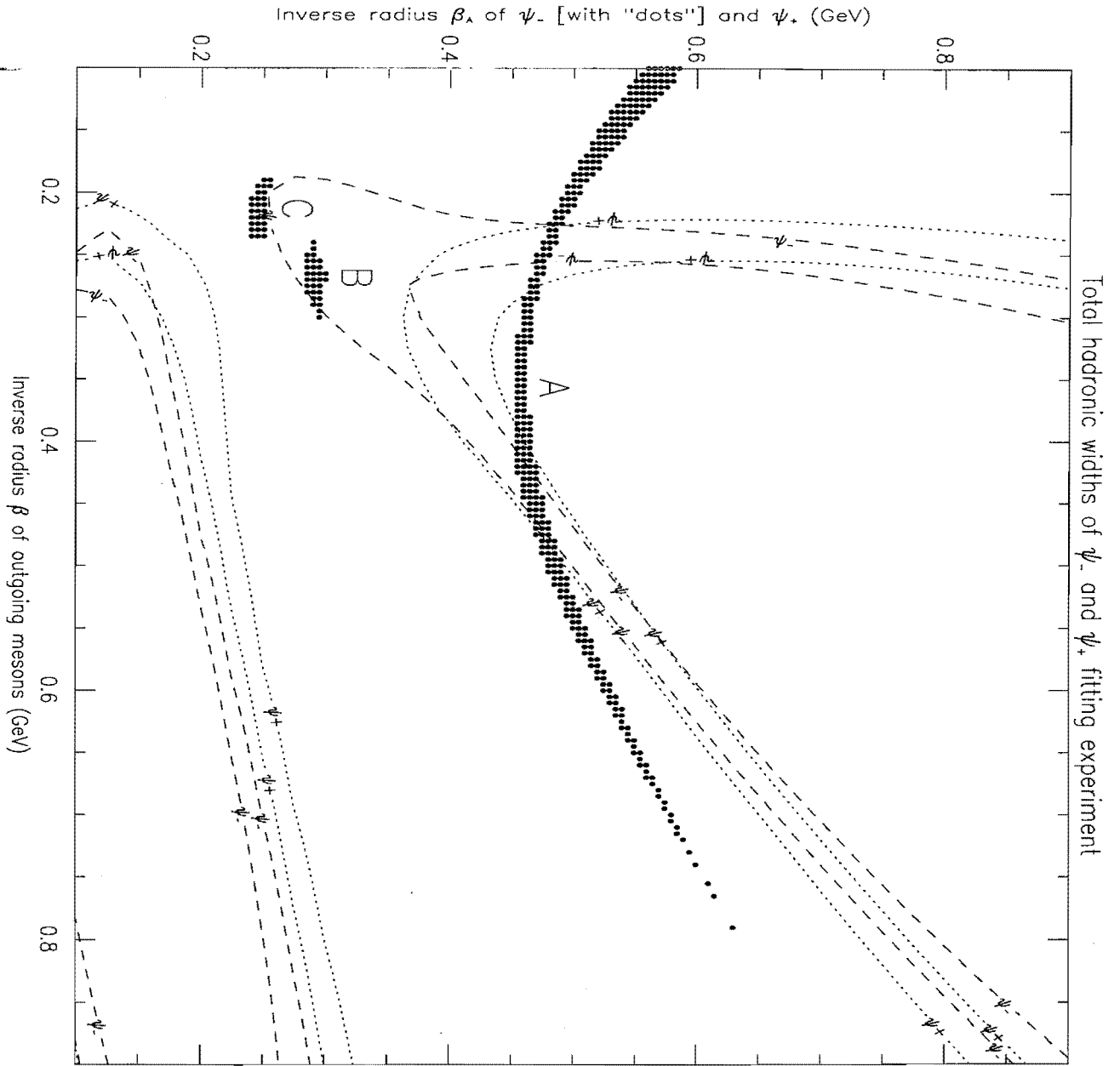


Figure 2: Total hadronic widths of  $\psi_- \equiv \psi(4040)$  and  $\psi_+ \equiv \psi(4160)$  fitting experiment. The contours enclose narrow regions with experimentally acceptable [19] total widths, indicating  $1\sigma$  variations from the mean experimental width. The regions where the  $\psi_-$  decay ratios fit experiment are indicated by the “dots”. We denote the regions by A, B and C.

