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Collective Plasma Processes and the Solar Neutrino Problem

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Collective Plasma Processes and the Solar Neutrino Problem

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Abstract

This report contains a list of papers which were prepared during an investigation of the effects of collective processes in the dense plasma of the solar interior. These papers have a bearing on the solar neutrino problem outlined in the first paper. The effect of all the processes discussed in the report is an overall reduction in the solar opacity of approximately 9% . We have not included the collective effects in line emission.

Contents

Plasma Effects in the Solar Core and the Solar Neutrino Problem

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Abstract

In this paper we review recent corrections to the theory of photon transport in a dense hot plasma, (similar to conditions in the solar interior) and show that previous calculations overestimate the solar opacity. We also identify new effects

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which have never been taken into account. It is shown that these can significantly reduce the opacity and can therefore change the prediction of the solar neutrino flux from Standard Solar Models.

Recently many corrections to the radiative transport in the solar interior were reported[1] which, all together, could significantly reduce the solar opacity and alter the neutrino flux predictions. The parameters in the deep solar interior where neutrinos are produced show that the frequency range most important for radiation transport is that for which plasma collective effects dominate. Near the solar centre the differences between the plasma frequency $\omega_{pe} \simeq \sqrt{\frac{4\pi n_e e^2}{m_e}} \simeq 4.8 \times 10^{17} s^{-1}$ (for $n_e \simeq 5.7 \times 10^{25} cm^{-3}$) and the maximum frequency in the black-body spectrum $\omega_{max} \simeq \frac{3T}{\hbar} \simeq 2. \times 10^{18} s^{-1}$ (for $T \sim 1.5\ 10^7 K$) is small and the frequencies responsible for energy transfer are not much greater than ω_{pe}. Therefore corrections due to the refractive index have also to be taken into account. On the other hand collective effects change the cross-section both for scattering and bremsstrahlung, which give the largest contribution to the energy transport in the solar interior for $\lambda > \lambda_D$ (λ is the photon wavelength and λ_D is the Debye length) or for $\omega < \omega_{pe} \frac{c}{v_{Te}} \simeq 9 \times 10^{18} s^{-1}$ where $v_{Te} = (T/m_e)^{\frac{1}{2}}$ is the electron thermal velocity. This last expression shows that for the range of frequencies responsible for energy transport at the solar centre collective effects should be taken into account. Since temperature and density decrease with radial distance r from the centre, collective and relativistic corrections will of course reduce at larger distances from the centre. For example at $r = 0.2 R\odot$, $T \simeq 9.25 \times 10^6 K$ and

$n_e \simeq 6.0 \times 10^{24} cm^{-3}$, $\omega_{max} \simeq 3.7 \times 10^{18} sec^{-1}$ and $\omega_{pe}\frac{c}{v_{Te}} \simeq 5 \times 10^{18} sec^{-1}$ collective effects will be smaller but not negligible. The radiation flux and neutrino flux are created in the core and correspond approximately to the observed flux. The radial dependence of the effects should be investigated numerically, here we only give the results at the solar center.

Previous calculations[2] have taken into account some of the collective effects in scattering, relativistic and degeneracy effects have also been incorporated[3] but many other corrections described below have not been taken into account. By reconsidering all these effects we find the decrease in the solar opacity can be as high as 9% in the central region of the sun (see table 1) and is possibly sufficient to account for both the measured neutrino flux and solar luminosity. The effects we have improved or considered for the first time are

a) Doppler and collisional broadening and shifting of the Raman resonance in photon scattering by electrons [4],

b) Raman scattering on thermal plasmons[5],

c) Relativistic corrections in the nonlinear response of the electron polarization cloud surrounding the ions and electrons which scatter the waves[6],

d) Collective effects in bremsstrahlung[7],

e) Relativistic effects in bremsstrahlung[8],

f) Stimulated scattering and frequency diffusion[9]

g) Density inhomogeneity effects[10],

h) Refractive index corrections[10],

i) Quantum corrections to scattering[11],

At the solar centre the reduction in opacity due to the above effects accounts for $\sim 2/3$ of the total opacity since line absorption by iron accounts for $\sim \frac{1}{3}$. To these effects one should add the correction due to partial electron degeneracy[12] which we found to give a 2% decrease in the opacity (we recalculated the effect of degeneracy as given by Rose[3] but removing the relativistic effects and the line-absorption which were also taken into account by Rose) and also the effect of ion-correlations on bremsstrahlung[13] which gives a 1.5% decrease, both calculated at the solar centre. We next give a short description of each effect contributing to the solar opacity calculation.

First of all an important point to note is that in the case when collective effects dominate the whole physical picture of the objects which scatter and emit the radiation changes[14,15]. Plasmas no longer can be considered as a collection of free particles but as a collection of particles shielded by opposite sign particles (electrons and ions). Although this shielding is produced statistically by the other fluctuating particles the objects which are involved in scattering and emission look more like neutral classical atoms than free particles. This statement is fundamental in plasma physics, since the particles which are shielding, at the same time are being shielded and are the particles taking part in emission and scattering. The picture of shielded particles arises from the fluctuation theory after averaging of the natural fluctuations of a large system of almost free particles. The result of this approach when the wavelength of the radiation is comparable or larger than the Debye radius is that the shielding electrons oscillate in the radiation field as well as the shielded electrons. The charge of the shielding cloud is equal and opposite to the charge that it is shielding. For the case of shielded ions only the electrons in the shielding cloud oscillate in the radiation field, the ions can be assumed to be infinitely massive. For the case of electrons shielding electrons the shielded electron contribution to the scattered radiation is out of phase with the radiation produced by the shielding electrons in the cloud and almost cancel each other. Thus with increasing wavelength ($\lambda > \lambda_D$) the cross-section of scattering on electrons rapidly decreases. For the shielded ion the radiation is only due to the shielding cloud of electrons oscillating in the radiation field. However the ions are responsible for energy and momentum exchange during the scattering process. Thus with increasing λ (for $\lambda > \lambda_D$) the scattering on electrons shielding the ions increases and they will dominate in the scattering process (scattering on electrons shielding ions will hereforth be referred to as ion scattering) and even the heavy ions or groups of heavy ions (C,N,O and other species) will contribute to the scattering appreciably. The scattering is determined by the collective parameter $\delta_e = \frac{\omega_{pe}^2}{2\omega^2}\frac{c^2}{v_{Te}^2}$. For $\delta_e \gg 1$ scattering is due mainly to the electrons shielding the ions while for $\delta_e \ll 1$ it is due to free electrons (Thomson scattering). For the solar interior one should estimate δ_e for the frequencies which give the maximum contribution to the solar opacity. Since the contribution of bremsstrahlung reduces significantly for frequencies lower than $\omega_{max} \sim \frac{3T}{\hbar}$, the main contribution is due to frequencies already on the exponentially decreasing part of the Planck distribution. We find that the result is thus sensitive to small changes in T for the existing standard solar models and for the expected values of T in the solar interior δ_e at the peak of the Planck distribution is of the order of one, ie both electrons and ions play almost equal roles in scattering. It is therefore important that relativistic corrections to electron scattering be extended to ion scattering as well,that is to scattering by electrons shielding the ions. Inverse bremsstrahlung (free-free transitions) also contributes as much as scattering to the opacity.

The effect of transition- bremsstrahlung (dynamical screening) has never been taken into account in calculating the solar opacity, only static screening has previously been used.[16] When dynamical screening is used[7], the shielding of ions in bremsstrahlung of incoming electrons is compensated by emission of the shielding cloud, making the collective effects in bremsstrahlung less than previously found[16]. The collective effects on inverse bremsstrahlung can in general be large but for the most important frequencies in transporting the radiation in the solar interior they are less than 1% (see table).

A decrease of 3% was previously found using static screening but it also included electron degeneracy effects: since these were independently found to give a 2% decrease we can estimate that $\sim 1\%$ is the decrease due to static screening in bremsstrahlung.

The relativistic effects in the transport of radiation in the solar interior is another problem that we have re-considered. When collective effects are important ($\delta_e \geq 1$) scattering and bremsstrahlung change substantially from the case when the collective effects are negligible ($\delta_e \ll 1$). On the other hand relativistic corrections enter differently in the transport equation and in the cross-sections. Since the oscillating polarization cloud of ions is produced by electrons the relativistic correction appears for scattering by ions as well, but is quite different from the relativistic correction due to scattering by electrons.[6] The relativistic correction for scattering on electrons includes[6]: a) correction to the matrix elements of the polarization cloud, b) corrections to the dynamics and distribution function of scattering electrons (known previously as relativistic corrections to Thomson scattering but in the case when collective effects are important these are altered since the change in the matrix element describing these corrections is multiplied by the total matrix element including the collective effects). The Doppler shift in the frequency of the scattered waves and the effect of collisions broaden the Raman resonance in scattering[4]: this effect is of the same order ie., $\left(\frac{v_{Te}}{c}\right)^2$ as the relativistic corrections and should consistently be included when relativistic corrections are taken into account. The table below gives the contribution of these effects to the opacity at the solar centre.

All relativistic corrections were calculated to order v_{Te}^2/c^2. Previously[3] the relativistic corrections were taken as a multiplicative factor in the cross-section,which is only correct for the case $\delta_e \ll 1$, when collective effects can be neglected.

There are also additional relativistic corrections appearing in the transport equation[9,10]. The first is related to the Doppler frequency shift during the scattering and results in additional terms proportional to the derivatives in frequency of the radiative flux in the transport equation, these are called the frequency diffusion terms[9]. Also stimulated scattering is of the same order as all other relativistic corrections[9] and cannot be neglected. Stimulated scattering and frequency diffusion were considered before but without collective effects[18]. Another effect which leads also to frequency diffusion is the density inhomogeneity in the solar interior. We have shown that for $\delta_e \sim 1$ this effect (which was not considered before) is of the order of other relativistic effects[10]. Also of the same order is the quantum effect due to recoil in scattering which, according to Klein-Nischina is of the order of $\frac{2\hbar\omega}{m_e c^2} \sim \frac{6v_{Te}^2}{c^2}$ for $\hbar\omega \simeq 3T$. We have generalized[11] the Klein-Nischina correction to take into account the collective effects and found the dependence of these corrections on δ_e. This correction always reduces the opacity and is doubled in the transport cross-section.

There also exists a direct Raman scattering of radiation on thermal plasmons, the strength of which is proportional to the energy in the plasmon fields. This effect increases the solar opacity, but for $k^{p\ell}_{max} \approx \frac{1}{2\lambda_D}$ (where $k^{p\ell}$ is the plasmon wave number) this increase is found to be negligible[5].

Other effects which we considered, but did not include because they are negligible compared to the effects in the table, are:

1. Energy transfer by plasmons (small due to the small group velocity, but note that the energy in plasmons is larger than the energy in radiation)
2. Convection of plasmons by a flux of particles taking part in heat transfer (larger than the energy transfer by plasmons)
3. Convection of plasmons by the flux of photons (small)
4. Collective effects in electron -electron collision bremsstrahlung (the collective effects change this bremsstrahlung substantially). This is of order v^4_{Te}/c^4.
5. Collective effects in ion-ion collision bremsstrahlung (collective effects increase substantially this type of emission but it is still small). Also of order v^4_{Te}/c^4.
6. Collective scattering through virtual transverse waves (of the order of v^4_{Te}/c^4)
7. Tail formation in the proton distribution due to thermonuclear α-particle relaxations. The resulting tail of energetic protons is small and the effect neglible.

Neglecting the terms with derivatives in frequency[9] in the frequency diffusion terms, it is well known that from the transport equation that it is possible to derive the equation of radiative transfer by introducing the Rosseland mean opacity κ_R:

$$L = -\frac{16\pi}{3}\int_o^{R\odot}\frac{1}{\rho}\left(\frac{1}{\kappa_R}+\frac{1}{\kappa_c}\right)r^2\frac{d}{dr}\left(\sigma T^4\right)dr \tag{1}$$

where L is the total luminosity, ρ the mass density, $R\odot$ the solar radius, σ the Stefan's constant, $T(r)$ the temperature and r the radial distance from the Sun's centre. In (1) κ_c is the conductive opacity and κ_R the Rosseland radiative opacity, which includes all the physics of radiative transport and is defined as[2]:

$$\frac{1}{\rho\kappa_R} = \int_{\omega_{pe}}^{\infty}\frac{\frac{\partial B_\omega}{\partial T}}{n_e\left(\sigma^{sc}+\sigma^{ff}+\sigma^{L}\right)}d\omega \Big/ \int_o^{\infty}\frac{\partial B_\omega}{\partial T}d\omega \tag{2}$$

where $B_\omega(T)$ is the Planck function, n_e the electron density and $\sigma^{sc}_{(\omega)}, \sigma^{ff}_{(\omega)}, \sigma^{L}_{(\omega)}$ are the cross-sections for scattering, inverse bremsstrahlung and line absorption respectively.

In the solar core $\kappa_R \ll \kappa_c$ and radiative transport dominates. It is well known that uncertainties in the value of the opacity can have significant effects on the predictions of the SSM[19,20], particularly in the value of the solar central temperature T_c.

The opacity currently used in SSM calculations has an uncertainty estimated at 5% maximum[19] and this is considered to be too small to seriously affect the SSM predictions of the neutrino flux.

Here we have calculated, with a relative error lower than 10^{-4}, the change in opacity due to each effect taken separately (to show the relative importance of the effects). The results, for values of the parameters corresponding to the Sun's centre where the effects are maximum, are given in Table 1 where

$$\delta\kappa^{(i)} = \kappa_R^{(i)} - \kappa_R^{(o)} \tag{3}$$

with $\kappa_R^{(o)}$ given by eq.(2) without corrections and $\kappa_R^{(i)}$ is the result of eq.(2) where the i-th correction is inserted. In Table 1 the effect of stimulated scattering and frequency diffusion has been calculated using perturbation theory[9]. Since we did not make any corrections to the line absorption σ^L (due to partially ionized iron in the solar centre) which we did not include it in the calculation of opacities. It is known that the effect of iron accounts for $\simeq 1/3$ of the total opacity[3] (at the solar centre) but it can be omitted to calculate the relative importance of the various effects. An estimate of the total change to opacity (including line absorption) can be found multiplying our results by $\frac{2}{3}$.

The sum of all the corrections in the Table (assuming validity of the perturbation expansion) shows the decrease of opacity to be expected from the combined effect of all corrections: a reduction of 7.3% of the total opacity. The correction from ion correlations[13] (−1.5%) should also be taken into account: the total decrease of opacity is then of order 8.8%, larger than the 5% currently assumed[19] and possibly large enough to change substantially the prediction of the SSM on the neutrino flux. The quantum corrections given in the table are preliminary and it is probably larger, also some recent findings point to a lesser roler of line absorption on Iron ions which will also increase the total reduction in opacity.

The role of stimulated scattering and frequency diffusion however changes the whole structure of the equation of radiative transfer: the concept of mean opacity can no longer be introduced[9] and the RHS of eq.(1) has to be substituted with the integral over all frequencies and radial distance of the solution of the differential equation for the radiative flux $F^o_\omega(r)$ given by

$$A^{(2)}_\omega \omega^2 \frac{\partial^2 F^o_\omega}{\partial \omega^2} + A^{(1)}_\omega \omega \frac{\partial F^o_\omega}{\partial \omega} + A^{(o)}_\omega F^o_\omega = \Lambda_\omega \frac{dT}{dr} \tag{4}$$

where the coefficients $A^{(2)}_\omega, A^{(1)}_\omega, A^{(o)}_\omega$ and Λ_ω are given in Ref.9 in terms of the cross-sections including collective effects. From the solution of eq.(4), which depends on the choice of appropriate boundary conditions, the total luminosity can be written as

$$L = \int_o^{R\odot} 4\pi r^2 \left(\int_o^\infty F^o_\omega(r) d\omega \right) dr \tag{5}$$

Eq.(5) is the new "equation of radiative transfer" replacing eq.(1) when stimulated scattering and frequency diffusion are taken into account[9]. When they are not taken into account the derivative terms in eq.(4) are absent and eq.(4), integrated over r, becomes eq.(1).

It should be noted in eq.(4) that the coefficients in front of the derivatives are small parameters. In the absence of collective effects these coefficients are independent of frequency. Therefore by changing the variable ω as is usually done in a mathematical analysis of equations with a small coefficient in the higher derivative terms the small parameter remains. In the collective case the coefficients are frequency dependent which means that a change in variable results in mathematical problems which will be the subject of further investigation.

In conclusion we have shown that the present theory of radiation transport in the solar interior may not be accurate enough for the prediction of the neutrino flux. We have outlined new effects to be incorporated in the SSM and the equation of radiative transfer has to be modified. The effects discussed here depend on both density and temperature, they have their maximum at the solar centre and decrease with radial distance. But it is important to notice that the inner core is also the region where the thermonuclear reactions occur, ie the region of neutrino production. But there are also other points that need revision such as the role of collective effects in the cross-section for line absorption σ^L and in the dynamic screening of thermonuclear reactions should also be reconsidered in more detail. The solar interior is a plasma and should be appropriately treated as such.

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Table 1

N	Name of the effect	$\delta\kappa/\kappa$ (in %)
1	Doppler and collisional broadening and shifting of the Raman resonance in photon scattering by the electrons[4]	-3.0
2	Relativistic corrections in the nonlinear response of the electron polarization cloud for scattering[6]	-0.2
3	Stimulated scattering and frequency diffusion[9]	-4.5
4	Collective effects in bremsstrahlung[7]	-0.2
5	Density inhomogeneity[10]	-0.1
6	Refractive index corrections[10]	+0.1
7	Quantum recoil correction in collective scattering[11]	-1.0
8	Degeneracy effect on scattering	-2.0
	Sum of the effects	-10.9
	Estimated change of total opacity $\simeq \frac{2}{3}(-18.4) \simeq$	-7.3

Adding the ion correlation which gives a correction -1.5% result in the total change in opacity of -8.8%

COLLECTIVE PLASMA EFFECTS IN SCATTERING OF RADIATION IN ASTROPHYSICAL PLASMAS

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Abstract

Collective plasma effects are considered in the scattering of radiation in astrophysical plasmas. They appear to be important over a broad range of frequencies $\omega_{pe} < \omega < \omega_{pe} c/v_{Te}$ (ω_{pe} is the plasma frequency, c is the light velocity and v_{Te} is the

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electron thermal velocity). The contribution to the radiative transport due to collective scattering in plasmas and a generalization of Kompaneets equation are found. Both the stimulated and spontaneous scattering and the contribution of scattering on electrons and ions are taken into account. In the transport cross-sections the scattering on ions starts to contribute for $\omega \leq 3\omega_{pe}c/v_{Te}$ while in the generalized Kompaneets equation the scattering on ions dominate for $\omega \leq \omega_{pe}(c/v_{Te})(m_e/m_i)^{1/4}$. It is shown that the contribution related to the change of frequency during the stimulated and spontaneous scattering, modifies the structure of the transport equation. We show that this new transport equation contains a derivative of the intensity with respect to the frequency and in general such an equation does not allow the use of the concept of opacity as normally defined. Application of the results of collective scattering of radiation in accretion discs, supernova remnants and solar radiation transport is discussed.

1 Introduction

Scattering of radiation has important applications in the interpretation of many astrophysical observations such as radiation from accretion discs, expanding supernova shells and photospheres of stars. Scattering of radiation also contributes to the opacity of matter in stellar interiors, where it can be one of the main factors in controlling radiation transport. In stellar atmospheres accretion discs and supernova remnants the scattering is considered to be Thomson scattering or Compton scattering, i.e. scattering on free electrons in vacuum (Pozdnyakov et al., 1983) and plasma collective effects were either completely neglected or not analysed in detail. The well known Sunyaev-Zeldovich effect of "comptonization" (Sunyaev, Zeldovich 1970) just describes scattering on free electrons ignoring plasma collective effects. Scattering on free electrons is generally referred to as Thompson scattering if the photon energy is much less than the electron rest energy ($\hbar\omega \ll m_e c^2$) and Compton scattering if the photon energy is comparable or greater than the electron rest energy ($\hbar\omega \geq m_e c^2$). Thomson scattering is a classical effect while Compton scattering is the quantum mechanical process. For photons with energies greater than $m_e c^2$ the effects of the order $\hbar\omega\sigma_T/m_e c^2$, are Compton effects, σ_T is the Thomson cross-section. But for photons with the energies much less than $m_e c^2$ the effects of the order of $(v_{Te}^2/c^2)\,\sigma_T$ (where v_{Te} is the electron velocity) are relativistic Thomson classical effects. The "comptonization" considered in (Pozdnyakov et al., 1983), (Syunaev and Zeldovich, 1970) is the relativistic Thomson effect. In astrophysics the term "comptonization" is often used for Thomson scattering for the case when the energy of photons is much less than m_ec^2 and can be of the order of the particle kinetic energy (or their temperature). In this limit the whole effect is classical and the widely used Kompaneets equation (Kompaneets, 1957) does not contain the Planck constant. To be precise we will use the word "comptonization" for Thomson relativistic scattering or for effects described by the Kompaneets equation in quotation marks.

Scattering in plasmas in general cannot be considered as being due to free electrons since collective effects dominate in many cases. Collective effects are important when the wavelength of the radiation becomes comparable or larger than the Debye length λ_D (for $T_e \approx T_i$ it is of the order of the electron Debye length $\lambda_{De} = \sqrt{T_e/4\pi n_e e^2}$). This condition is satisfied in a plasma for frequencies ω in the range

$$\omega_{pe} < \omega < \omega_{pe}\frac{c}{v_{Te}} \tag{1}$$

where ω_{pe} is the plasma frequency, $\omega_{pe} = \sqrt{4\pi n_e e^2/m_e}$, c is the speed of light and v_{Te} is the electron thermal speed.

The Sunyaev-Zeldovich effect in the form it is normally applied deals with the frequency range $\omega \geq 3\omega_{pe}c/v_{Te}$ in which case collective effects can be ignored. However, there are many astrophysical situations where the radiation falls in the frequency range defined by eq.(1). The condition that the wavelength be greater than the Debye wavelength for collective effects to be important, can be expressed as $\delta_e \geq 1$ where the "collective" parameter δ_e is defined as

$$\delta_e = \frac{\omega_{pe}^2}{\omega^2}\frac{c^2}{2v_{Te}^2} \tag{2}$$

The cross-section of scattering can be expressed as a functions of the collective parameter δ_e (see Appendix). In astrophysical plasmas the range of frequencies determined by equation (1) where collective effects are important is broad and can cover about three orders of magnitude in frequency. Note that the lower limit ω_{pe} is set by the radiation dispersion relation where for frequencies less than ω_{pe} the waves become evanescent and cannot propagate, i.e. this is the cut-off frequency for radiation propagation. For ω close to ω_{pe} the collective parameter is no longer defined by equation (2) but by equation (A8). The collective parameter given by Eq.(2) determines the scattering on electrons, while the scattering on ions depends on the collective ion parameter which contains the effective ion charge given by eq.(14). The collective ion parameter given by eq. (A5) depends on the parameter δ_e, the effective ion charge and on the ratio of electron to ion temperatures (in the case where they are not equal).

When we discuss ion scattering we are referring to the scattering by the cloud of shielding electrons. For the range defined by Eq.(1) where the collective effects become important the presence of shielding clouds both for electrons and ions is crucial. The contributions to scattering of the shielding cloud of electrons and shielded electrons almost cancel each other, while in the case of ions only the shielding electrons contribute to scattering ($m_i \gg m_e$). Through the ambipolar electric field the energy and momentum of electrons shielding the ions is transferred to the ions. During the scattering on the ions through the shielding electrons only the ion distribution function changes. The cross-section of scattering on ions in the strongly collective regime is of the order of the Thomson cross-section of scattering on free electrons (Ginzburg and Tsytovich, 1991).

One can often find in astrophysical literature the statement that although it is possible to separate the collective effects in scattering into correlations due to electrons and ions it is incorrect to interpret these as scattering on electrons and ions and that the photons are only scattered by electrons. The scattering in the collective regime is mainly by ions. This can be rigorously proved using the standard method of plasma fluctuation theory. This theory allows us not only to describe the scattering of photons by the plasma particles but also to describe the change of the plasma particle distribution function due to scattering. From the energy and momentum conservation law the energy and momentum change of photons is equal and opposite in sign to the energy and momentum change of the particles. Moreover it is possible to find whether the energy is transferred to electrons or ions in the scattering process. The result is that the correct interpretation is such that the scattering in the collective regime is occurring mainly on ions. The correct interpretation immediately leads to many important conclusions including, as we show, a substantial increase of the well known Eddington luminosity limit. The reason why the misunderstanding can appear in the physical interpretation of scattering is that in the very first articles dealing with collective effects in scattering the particle distribution was considered as fixed and did not change. In the plasma literature it is established that for the modes with wavelength larger than the Debye length (such as all plasma waves as well as electromagnetic waves) the scattering is mainly produced by ions. This is well known and contained in textbooks of plasma physics (Davidson, 1980, Tsytovich, 1977) but in textbooks and monographs on astrophysics one can still find the statements that the scattering is due mainly to electrons. The latter is indeed correct outside the range given by relation (1).

The aim of the present paper is to investigate scattering in the range of frequencies where collective effects are important, i.e. for $\delta_e \geq 1$.

There are usually two important cases considered in the absence of collective effects: 1) time evolution of isotropic radiation governed by the Kompaneets equation and; 2) stationary slightly anisotropic radiation governed by spatial gradients which is the case of radiative transport. Our aim is to generalize both cases to the collective regime.

In the solar interior where the temperature $T\,(T_e = T_i = T)$ is of order $1.5keV$ and density $n_e \sim 5 \times 10^{25} cm^{-3}$ (Bachal, 1989) the range of frequencies of importance to radiative transport corresponds to $\delta_e \simeq 1$, hence plasma collective effects must be considered in the determination of the radiative transport and therefore the solar opacity. In the active galactic nuclei the plasma frequency could be of the order of $\omega_{pe} \approx 5 \times 10^{12} s^{-1}$

and temperature of order $10eV$ and the range of frequencies where collective effects are important is $\omega \leq 10^{15} sec^{-1}$, i.e. the optical range or lower. In supernovae explosion in its early stage even "comptonization" of soft γ-rays can be governed by collective effects. In the solar corona the scattering of radio waves is always determined by collective effects. These estimates show that collective effects are important in scattering of radiation in astrophysical plasmas. The effects of collective scattering surprisingly have not attracted much attention in astrophysical literature. The aim of the present article is to address this problem and to point out new qualitative effects introduced by collective scattering.

Previously collective effects in scattering have been considered for solar transport in the work of Beorcker (1987) and Huebner (1986) but they only considered spontaneous scattering ignoring stimulated scattering and also the contributions of electrons and ions was not separated. In the present paper we will give a more detailed analysis separating the contributions of electrons and ions, including frequencies close to the plasma frequency as well as considering different temperatures of electrons and ions. The transport equation for radiation in the absence of collective effects was previously considered by Sampson, (1959) and are widely used in astrophysical applications. Although the initial equations of Sampson, (1959) are correct taking into account both processes of spontaneous and stimulated scattering, and the zero order approximation in v_{Te}^2/c^2 is also correct, subsequent approximations made lead to incorrect results in all terms of the expansion in the parameter v_{Te}^2/c^2. In particular the assumption that the solution of the equation of radiative transfer has a certain form omitting the derivative of the intensity with respect to the frequency is not correct. We show that this assumption is wrong and that the transport equation does contain a term that is a derivative of the intensity with respect to the frequency. The presence of such a term is almost obvious from the structure of the Kompaneets equation (Kompaneets, 1957) which is used as the equation to describe the Sunyaev-Zeldovich effect (Syunaev and Zeldovich, 1970). Thus the aim of this paper will be not only to calculate the scattering taking into account collective plasma effects in the cross-section but also to find the equation which in the limit of high frequencies reduces to a new correct transport equation. This will clearly demonstrate that the more general radiation transport equation due to stimulated scattering processes are not the same as the result given by Sampson, (1959) in the high frequency limit. The effects we are interested in are of the same order of magnitude as the effects leading to the Kompaneets equation, i.e. of order $\sigma_T v_{Te}^2/c^2$ Thus our aim is the investigation of the effects of the order of that taken into account in the Kompaneets equation taking into account the collective plasma effects both for isotropic radiation and for radiative transport. We will show that the ion contribution to the transport equation starts for $\omega < \omega_{pe} 3c/v_{Te}$ while the contribution of ions to the Kompaneets equation starts for lower frequencies $\omega < \omega_{pe} c/v_{Te}(m_e/m_i)^{1/4}$.

The paper is organized in the following way. We start with a general equation for photon number density including spontaneous scattering on both electrons and ions and stimulated scattering on electrons and ions. We then derive a transport equation containing terms with a derivative of the intensity with respect to the radiation frequency (Tsytovich et al., 1995a). We then calculated in an explicit form the contribution of the scattering on electrons and ions in the transport equation for an arbitrary value of the collective parameter δ_e, average ion charge and the ratio of electron to ion temperature. We then illustrate the dependence of the total and the transport cross-sections of scattering on the collective parameter, electron to ion temperature ratio and ion average charge. We find a new expression for the Eddington luminosity limit and investigate the influence of collisional and Doppler broadening on scattering in the solar interior and show that they can change the solar opacity by more than 3%(Tsytovich et al., 1995b). Then we calculate the terms with a derivative with respect to the frequency in the transport equation of photons. These terms appear from stimulated and spontaneous scattering in a way similar to that for the Kompaneets equation and one can show some analogy with "comptonization". These terms in the transport equation are nevertheless quite different from that in the Kompaneets equation and they take into account collective plasma effects. Although the relative order of the contribution of terms with a derivative of intensity with respect to the frequency is v_{Te}^2/c^2 the concept of opacity cannot be introduced in the usual way in the case where such terms are taken into account in the transport equation.

We then consider the case when the intensity of radiation exceeds substantially the thermal level and when stimulated scattering process dominate. This will allow us to identify the terms describing the stimulated scattering in a generalized Kompaneets equation in which the collective effects are taken into account. The Kompaneets equation is found by a correspondence principle for arbitrary values of the collective parameter δ_e. Then we give the generalized Kompaneets equation when both the spontaneous and stimulated scattering on both electrons and ions is taken into account and analyse the dependence of the collective integral in this new equation on the collective parameter. In the discussion we give the result of the change to the solar opacity and discuss the possibility of a new type of solution of the transport equation, as well as the applications of the results to different astrophysical observations. Appendix A contains an explicit expression for the transport cross-section including the usual cross-section of spontaneous scattering on electrons and ions for arbitrary values of δ_e taking into account the refractive index of photons (the latter is important when the frequency of radiation is close to the plasma frequency). In the literature one can only find the sum of the transport cross-sections for scattering on electrons and ions. The separation of the scattering effects by electrons and by ions could be of importance for other astrophysical applications. The previous results which give only the sum of these cross-sections leads to some misunderstanding since one can have the impression that the collective effects change only the scattering on electrons. Given in the Appendix are also the expressions used to calculate numerically the change in the solar opacity in the case when Doppler and collisional broadening of the scattering processes are taken into account.

2 General equations

In the following we use the results already well known in the plasma literature (Tsytovich, 1977, Ginzburg and Tsytovich, 1991) and partially the results of the papers by Tsytovich et al., (Tsytovich et al., 1994a,b,c). There exist two small parameters in the general equation for the change of photon distribution due to the scattering on thermal non-relativistic particles: 1) the ratio of the square of the mean thermal particle velocity to the square of the velocity of light and 2) the ratio of the photon energy $\hbar\omega$ to the electron rest mass energy $m_e c^2$. The first parameter is classical while the second is quantum. In the case we are interested in the problem of scattering for the photon energy of the order of the thermal particle energy and the relativistic corrections of the order of v_{Te}^2/c^2 are of the same order of magnitude since $\hbar\omega/m_e c^2 = (\hbar\omega/T)(v_{Te}^2/c^2)$. We will consider here the completely classical case when one neglects the effects related to the second small parameter but take into account the effects linear in the first small parameter. This is possible only in the limit $\hbar\omega \ll T$. For a range of applications it is desirable to consider arbitrary ratios of $\hbar\omega$ to T. But these effects should be considered separately. Indeed the generalization of the Klein-Nishina cross-sections taking into account the collective plasma effects is very complicated to describe. Fortunately the generalization of Kompaneets equation can be found by using a classical approach. The reason for this is that the Kompaneets equation has a classical limit or more precisely by expressing the photon occupation number through the intensity of radiation this equation is completely classical. The occupation number can be determined by the quantum processes and the Planck distribution satisfies the Kompaneets equation.

The general equation for photon density number $N_{\mathbf{k}}$ (photon distribution function describing the dependence of the number of photons in wave number $\mathbf{k}$)) can be written if the probability of scattering is known. For simplicity we will neglect in this section the refractive index corrections to the photon dispersion and will assume that the frequency of photons is given by $\omega = kc$ where c is the light velocity (in the Appendix we give also a more general result taking into account the fact that the refractive index differs from 1). The wave vector of the scattered wave we denote as $\mathbf{k}'$ and frequency we denote as $\omega' = k'c$. The probability of scattering on electrons and (ions) is denoted as $W^{(e),(i)}_{\mathbf{k},\mathbf{k}'}$

In the classical approach the transport equation for photons can be written in the form:

$$\begin{aligned}\frac{\partial N_{\mathbf{k}}}{\partial t} + \frac{\mathbf{k}}{k}c \cdot \frac{\partial N_{\mathbf{k}}}{\partial \mathbf{r}} &= -N_{\mathbf{k}} \int \left[W^{(e)}_{\mathbf{k},\mathbf{k}'} f^{(e)}(\mathbf{p}) + \sum_i W^{(i)}_{\mathbf{k},\mathbf{k}'} f^{(i)}(\mathbf{p}) \right] \frac{d^3p d^3k'}{(2\pi)^6} + \\ &+ \int N_{\mathbf{k}'} \left[W^{(e)}_{\mathbf{k},\mathbf{k}'} f^{(e)}(\mathbf{p}) + \sum_i W^{(i)}_{\mathbf{k},\mathbf{k}'} f^{(i)}(\mathbf{p}) \right] \frac{d^3p d^3k'}{(2\pi)^6} + \\ &+ N_{\mathbf{k}} \int N_{\mathbf{k}'} \hbar (\mathbf{k} - \mathbf{k}') \cdot \left[W^{(e)}_{\mathbf{k},\mathbf{k}'} \frac{\partial f^{(e)}(\mathbf{p})}{\partial \mathbf{p}} + \sum_i W^{(i)}_{\mathbf{k},\mathbf{k}'} \frac{\partial f^{(i)}(\mathbf{p})}{\partial \mathbf{p}} \right] \frac{d^3p d^3k'}{(2\pi)^6} \qquad (3)\end{aligned}$$

where $f^{(\alpha)}(\mathbf{p})$ is the distribution function for electrons ($\alpha = e$) or ions ($\alpha = i$). For steady state radiation transport problems the time derivative in the left hand side of this equation is zero and the photon occupation number is close to the thermal distribution with a small deviation described by the first Legendre polynomial with the angle along the direction of inhomogeniety. For the equation generalizing the Kompaneets equation the spatial inhomogeniety is absent and the left hand side of the equation contains only the time derivative, while on the right hand side the occupation number of photons are arbitrary and in general is a nonequilibrium function of the frequency of photons only, i.e. the photons are supposed to be distributed isotopically. We consider in detail both cases. The first two terms on the right hand side of eq.(3) represent spontaneous processes and the last term gives the contribution of stimulated scattering on electrons and ions. The scattering probabilities contain the relativistic corrections of order v_{Te}^2/c^2 (Tsytovich et al., 1995c) (no corrections of the order of v_{Te}/c are present). On the other hand the relative change of photon frequency during the scattering is at least first order in this parameter. Since all the first order terms vanish due to the symmetry of the particle distribution we can calculate all effects related to the frequency change by using the expression for the probability in which the relativistic corrections are neglected. We thus take into account only the Doppler shift of the frequency of scattered waves.

The probabilities are given by

$$W^{(e)}_{\mathbf{k},\mathbf{k}'} = \frac{(2\pi)^3 e^4}{2m_e^2 \omega\omega'}(1+x^2)\left|\frac{1+\sum_i \chi^{(i)}_{\mathbf{q},\Omega}}{\epsilon_{\mathbf{q},\Omega}}\right|^2 \delta(\Omega - \mathbf{q}\cdot\mathbf{v}) \tag{4}$$

$$W^{(i)}_{\mathbf{k},\mathbf{k}'} = \frac{(2\pi)^3 e^4}{2m_e^2 \omega\omega'}(1+x^2)\left|\frac{\chi^{(e)}_{\mathbf{q},\Omega}}{\epsilon_{\mathbf{q},\Omega}}\right|^2 \delta(\Omega - \mathbf{q}\cdot\mathbf{v}) \qquad (5)$$

where $\mathbf{q} = \mathbf{k} - \mathbf{k}', \Omega = \omega - \omega', \theta_{\mathbf{k},\mathbf{k}'}$ is the angle of the $\mathbf{k}$ and $\mathbf{k}'$ vectors, $x = \cos\theta_{\mathbf{k},\mathbf{k}'}$, $\chi^{(i)}_{\mathbf{k},\omega}$ are the ion susceptibilities, the $\chi^{(e)}_{\mathbf{k},\omega}$ is the electron susceptibility, $\epsilon_{\mathbf{k},\omega}$ the plasma dielectric function, the summation is over all ion species i.

These probabilities still contain the relativistic effect through the delta-function describing the conservation of energy in the process of scattering, but they are due to the dispersion of the photons ($\omega = kc$). Until the frequency is specified the probabilities do not contain the velocity of light. In fact the relativistic corrections described by the delta-function are due only to the Doppler effect of the scattering waves. The relativistic corrections to the probabilities (4),(5) where found in (Tsytovich et al., 1995c).

Our aim will be to describe the process related to the change of the photon frequency in the photon equation. In the case where we neglect the change in photon frequency and include only the spontaneous terms in the transport equation the latter can be reduced to the standard form widely used in astrophysical applications (Bachal, 1989)

$$c\frac{1}{3}\frac{\partial B^T_\omega}{\partial r} = -n_e \sigma^{(0)}_{eff}(\omega)\mathcal{F}_\omega \qquad (6)$$

where B^T_ω is the Planck distribution function of photons $\mathcal{F}_\omega$ is the energy flux at frequency ω (their relation with photon occupation number has the usual form, see also eq.(21)) and $\sigma_{eff}^{(0)}(\omega)$ is the effective transport cross-section. The latter should in the general case contain not only the scattering cross-section but as well the effective cross-sections of bremsstrahlung and line absorption. These should be added to the expressions we obtain from equation (3).

Integrating eq.(6) over frequencies and defining the Rosseland opacity as

$$\frac{1}{\rho\kappa_R^{(0)}} = \int_{\omega_{pe}}^{\infty} \frac{1}{n_e \sigma_{eff}^{(0)}(\omega)} \frac{\partial B_\omega^T}{\partial T} d\omega \Bigg/ \int_0^{\infty} \frac{\partial B_\omega^T}{\partial T} d\omega \tag{7}$$

where ρ is the mass density, and the superscript$^{(0)}$ in the expression for opacity correspond to the superscript$^{(0)}$ in the cross-section. In the case where the transport equation found from the general equation (3) has the same form as eq.(6) but with a different value of the cross section $\sigma(\omega)$ we will use the notation for the opacity without the superscript$^{(0)}$. The usual expression for the transport equation is found for the total energy flux $\mathcal{F} = \int \mathcal{F}_\omega d\omega$ and is given by

$$\mathcal{F} = -\left(\int_0^{\infty} \frac{\partial B_\omega^T}{\partial T} d\omega\right) \frac{1}{3\rho\kappa_R^{(0)}} \frac{dT}{dr} \tag{8}$$

It is useful to introduce the value

$$z \equiv \frac{\hbar\omega}{T} \tag{9}$$

and consider the dependence of the cross-section on z. Taking into account that

$$\frac{\partial B_\omega^T}{\partial T} \propto \frac{z e^z}{(e^z-1)^2}$$

we can write the relative change of the opacity in the form

$$\frac{\delta \kappa_R}{\kappa_R^{(0)}} \equiv \frac{\kappa_R - \kappa_R^{(0)}}{\kappa_R^{(0)}} = \left(\int_{\omega_{pe}}^{\infty} \frac{z^4 e^z}{\sigma_{eff}^{(0)}(z)(e^z-1)^2} \Big/ \int_{\omega_{pe}}^{\infty} \frac{z^4 e^z}{\sigma(z)(e^z-1)^2} \right) - 1 \qquad (10)$$

The collective parameter δ_e can also be expressed through z as

$$\delta_e = \frac{c^2}{v_{Te}^2} \frac{z_0^2}{2z^2}; \qquad z_0 = \frac{\hbar \omega_{pe}}{T} \qquad (11)$$

In the case where we take into account the change of the frequency during the photon scattering the transport equation can no longer be written in the form of eq.(6). We can easily show that the transport equation will contain derivatives of the intensity over the frequency. Since the stimulated scattering is already first order in the parameter v_{Te}/c (according to (3) it contains $(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v} = \omega - \omega'$) it will lead to a term linear in the derivative of the number of photons with respect to the frequency. The spontaneous terms will lead to the second derivative of the intensity with respect to the frequency. Noting that the other terms only give a contribution of the order of v_{Te}^2/c^2 to the usual form of the transport equation we can see that the transport equation should have the form

$$c\frac{1}{3}\frac{\partial B_\omega^T}{\partial r} = -n_e\left\{\left(\sigma^{(0)}_{eff}(z)+\frac{v_{Te}^2}{c^2}\delta\sigma(z)\right)\mathcal{F}_\omega + \right.$$
$$\left. +\frac{v_{Te}^2}{c^2}\left[(\sigma^{st}(z)+\sigma_1^{sp}(z))z^4\frac{\partial}{\partial z}\frac{\mathcal{F}_\omega}{z^3}+\sigma_2^{sp}(z)z^5\frac{\partial^2}{\partial z^2}\frac{\mathcal{F}_\omega}{z^3}\right]\right\} \qquad (12)$$

Eq.(12) contains the derivative $\partial/\partial z$ of the number of photons taking part in the transport of radiation (which is proportional to $\mathcal{F}_\omega/\omega^3$), the superscript st denotes the stimulated scattering and the superscript ^{sp} denotes the spontaneous scattering. Equation (12) serves as a definition of $\sigma^{st}(z)$, $\sigma_1^{sp}(z)$, $\sigma_2^{sp}(z)$ and $\delta\sigma(z)$ and it cannot be converted to the familiar form given by eq.(6) therefore the concept of opacity is in general not applicable.

Our aim will be to prove that the transport equation has indeed the form of eq.(12) and to find explicit expressions for the cross-sections of scattering including the collective effects in the change of frequency of photons in the scattering process.

3 Collective effects in the transport equation for photons

We start with a consideration of spontaneous scattering leaving in the transport equation only the first terms of eq.(3) and integrating over the thermal velocity components of the particle distribution perpendicular to the vector $\mathbf{k}-\mathbf{k}'=\mathbf{q}$;

$$\frac{\mathbf{k}}{k}c\cdot\frac{\partial N_{\mathbf{k}}}{\partial \mathbf{r}} = -\frac{3}{8}n_e c\sigma_T\int_{-\infty}^{+\infty}\frac{dy}{\sqrt{\pi}}e^{-y^2}\int_{-1}^{1}dx\int d\omega'\,(N_{\mathbf{k}}-N_{\mathbf{k}'})\times$$
$$\times(1+x^2)\frac{\omega'}{\omega}\delta(\Omega-\mathbf{q}\cdot\mathbf{v})\left[\frac{1}{|F_e|^2}+<Z>\frac{1}{|F_i|^2}\right] \tag{13}$$

The first term in the square brackets describes the scattering on electrons while the second term in the square brackets describes the scattering on ions and

$$y=\frac{\mathbf{q}\cdot\mathbf{v}}{\sqrt{2}qv_{Te}};x=cos\theta_{\mathbf{k},\mathbf{k}'};<Z>=\frac{\sum_i Z_i^2 n_i}{\sum_i Z_i n_i} \tag{14}$$

n_i is the density of the ions of type i ,$< Z >$ is the effective ion charge, y is the dimensionless component of the velocity parallel to $\mathbf{q}$, $cos\theta_{\mathbf{k},\mathbf{k}'}$ is the angle of scattering. We have introduced the Thomson cross-section σ_T given by

$$\sigma_T=\frac{8}{3}\pi\frac{e^4}{m_e^2c^4} \tag{15}$$

and have used the relation

$$\int\frac{d^3k'}{\omega'}=2\pi\int_{-1}^{1}dx\int_0^{\infty}dk'\frac{k'^2}{\omega'^2}=\frac{2\pi}{c^3}\int_{-1}^{1}dx\int_0^{\infty}d\omega'\omega'$$

The form factors F_e and F_i for electrons and ions depend on the collective parameter δ_e defined by equation (2); in the first approximation they have the form

$$F_e \approx 1 + \frac{\delta_e}{1-x} W(y)$$

$$F_i \approx \frac{(1-x)}{\delta_e} \left(1 + \frac{\delta_e}{1-x} + < Z > \frac{\delta_e W(y)}{(1-x)} \frac{T_e}{T_i} \right) \tag{16}$$

and

$$W(y) = 1 - 2ye^{-y^2} \int_0^y e^{t^2} dt + i\sqrt{\pi} e^{-y^2}$$

Where we have taken into account the temperature of electrons, T_e which is not equal to the temperature of ions T_i.

Let us now consider the case when the distribution of photons is close to a thermal distribution with a deviation from it described by the first Legendre polynomial containing an angle between the photon wave number and the temperature inhomogeniety direction. The latter we denote by the unit vector $\mathbf{n}$. We use the natural expansion for $N_{\mathbf{k}}$ (see also Tsytovich et al., 1994, 1995a).

$$N_{\mathbf{k}} = N_\omega^T + \cos\theta_{\mathbf{k},\mathbf{n}} \delta N_\omega \tag{17}$$

where N_ω^T is the thermal photon distribution given by $N^T = (e^z - 1)^{-1}$ and δN_ω is the deviation from the thermal distribution responsible for the flux of radiation.

After substitution of eq.(17) in eq.(13) the l.h.s. of the transport equation becomes

$$c\cos\theta_{\mathbf{k},\mathbf{n}}\frac{\partial N_\omega^{(0)}}{\partial r} \tag{18}$$

and the linearized r.h.s. of the transport equation (the thermal distribution N_ω^T gives exactly zero in the r.h.s.) will contain $\cos\theta_{\mathbf{k},\mathbf{n}}$ and $\cos\theta_{\mathbf{k}',\mathbf{n}}$. Since the angular dependence of the probability entering the transport equation is only through $\cos\theta_{\mathbf{k},\mathbf{k}'}$ we can average $\cos\theta_{\mathbf{k}',\mathbf{n}}$ over the angles perpendicular to the $\mathbf{k},\mathbf{k}'$ plane such that

$$< \cos\theta_{\mathbf{k}',\mathbf{n}} >= \cos\theta_{\mathbf{k},\mathbf{n}}\cos\theta_{\mathbf{k},\mathbf{k}'} = x\cos\theta_{\mathbf{k},\mathbf{n}} \tag{19}$$

The transport equation (12) will have a form coinciding with the standard form (7) in which only the scattering processes are taken into account such that

$$\frac{\partial N^T(z)}{\partial r} = -n_e\left(\sigma_e^{tr}(z) + \sigma_i^{tr}(z)\right)\delta N_\omega \tag{20}$$

The relation between the Planck function B_ω^T and the energy flux $\mathcal{F}_\omega$ is given by the usual relations

$$B_\omega^T = \frac{1}{\pi^2}\hbar\frac{\omega^3}{c^3}N_\omega^T; \qquad \mathcal{F}_\omega = \frac{\hbar\omega^3}{3\pi c^4}\delta N_\omega \tag{21}$$

The transport cross-sections for scattering on electrons and ions can be found explicitly by using the general dispersion relation relating the real and imaginary part of the plasma dielectric function and are given in Appendix A. They depend on three parameters $< Z >$, δ_e and T_e/T_i. Also the total (not the transport) cross-sections are given in the Appendix both for electrons and ions.

For the parameters of the solar interior $n_e = 5.4 \times 10^{25} cm^{-3}, T_e = T_i = 1.5 keV, v_{Te} = 1.53 \times 10^9 cm/s, z_o = \hbar\omega_{pe}/T = 0.21$ and $< Z >= 1.53$, we plot in Fig.1 the dependence of the transport cross-sections of scattering on electrons and ions (in units of Thomson cross-section) as a function of the frequency normalized to the plasma frequency. It should be noted that even for frequencies larger than $\omega_{pe}c/v_{Te} \approx 20$ the cross-section of scattering on electrons is still about 15% less than the Thomson cross-section (at the $\omega_{pe}c/v_{Te} \approx 20$ it is 25% less). The collective effects begin to become unimportant for $\omega > 3\omega_{pe}c/v_{Te}$. The scattering on ions is seen to dominate at frequencies less than $10\omega_{pe}$. The total transport cross-section decreases continuously with decreasing frequency. The rapid decrease of the ion cross-section and total cross-section of scattering close to the plasma frequency is due to the refractive index effect, the waves become evanescent below the plasma frequency. Shown in Fig.1 is also the curve proportional to $z^4e^z/(e^z-1)^2$ which as we have shown enters in the definition of the Rosseland opacity. Only the frequencies about the maximum of this curve contribute significantly to the opacity (in Fig.1 we have plotted $f(z) = (1/5)z^4e^z/(e^z - 1)^2$) for demonstration reasons to have all curves on the same figure). This curve shows that in the solar interior the collective effects in scattering are always dominant.

In the results given in the expressions for the transport cross-section the integration over the frequencies ω' take into account the integration through the Raman resonance ($\epsilon_{\Omega,\mathbf{q}} \approx 0$) where the frequency difference in scattering coincides with the plasma frequency. Due to this resonance the Doppler and collisional broadening are not taken into account in the analytic expressions used to obtain Fig.1, these can change the cross-sections and hence the value of the opacity. In the Appendix we give the expressions for the form-factor of scattering on electrons which takes into account both the Doppler and collisional broadening of the Raman resonance. We use these expressions in the numerical calculation of the change of solar opacity introduced by these broadening effects. The collective effects as described above are taken as a zero approximation together with the bremsstrahlung cross-sections. The solar parameters were taken to be the same as those in Fig.1.

The result obtained for the % change in opacity due to the broadening effects is

$$\frac{\delta\kappa_R}{\kappa_R} \approx -3\% \tag{22}$$

This correction to the solar opacity is significant for the estimation of the central solar temperature from the solar standard model and will therefore have an effect on the high energy neutrino flux. In eq.(22) the line absorption is not taken into account in the κ_R.

In Fig.2 and Fig.3 we show the dependence of the transport and total cross-sections of scattering as a function of the collective parameter δ_e and the electron to ion temperature ratio. From Fig.3 one can see that the total cross-section of scattering decreases as the temperature ratio T_e/T_i increases. A substantial decrease in the total cross-section of scattering occurs when the temperature ratio T_e/T_i increases from 1 to 5 and further increases of the temperature ratio does not decrease the cross-section of scattering substantially as can be seen from Fig.3. Nevertheless the cross-section of scattering can decrease by a factor of 10.

This effect will be very important in lowering the Eddington limiting luminosity. One can expect that for high luminosity the electrons are quickly heated by radiation and thus can have a temperature much larger than the ions. This result shows that the collective scattering can increase the luminosity above the Eddington limit but only in the window where the collective effects dominate. This seems to be a very important consequence of the influence of collective effects in quasars and stellar objects.

Concerning the solar interior it is rather difficult to imagine the electron temperatures higher than the ion temperatures because of the high rate of collisions which equalizes the temperatures. Nevertheless it should be mentioned that the rate at which the electrons absorb radiation in the solar interior is 5 times larger than the rate at which they transfer their energy to ions, which in principle can lead to a difference of electron and ion temperatures. In the presence of instabilities the electron temperatures can also differ from the ion temperature. For completeness we give in Fig.4 for the parameters in the solar interior already given above the results of calculations for the change in the solar opacity as a function of the temperature ratio. We can see that the maximum decrease of solar opacity can be no more than 4%.

Shown in Fig.5 is the dependence of the total cross-section of scattering on the average ion charge. During the evolution of stars the average ion charge increases and thus the radiative pressure increases. This effect should also be important in astrophysical applications.

4 Collective effects in frequency diffusion during the radiative transport

We now calculate the role of the frequency change during the transport of radiation. This effect can be considered as "comptonization" during transport processes. But since in this article we restrict ourselves to the classical limit $\hbar\omega \ll T$ we will be able to calculate the effect of "comptonization" only in this limit.

We start with the electron contribution to stimulated scattering in the transport equation, which is now written as

$$\begin{aligned}\frac{\mathbf{k}}{k}c \cdot \frac{\partial N_{\mathbf{k}}}{\partial \mathbf{r}} = -\frac{3}{8} n_e c \sigma_T \int_{-\infty}^{+\infty} \frac{dy}{\sqrt{\pi}} e^{-y^2} \int_{-1}^{1} dx \int \frac{d\omega'}{|\epsilon_{\mathbf{q},\Omega}|^2} N_{\mathbf{k}} N_{\mathbf{k}'} \times \\ \times (1+x^2) \frac{\hbar \mathbf{q} \cdot \mathbf{v}}{m_e v_{Te}^2} \frac{\omega'}{\omega} \delta(\Omega - \mathbf{q} \cdot \mathbf{v}) \qquad (23)\end{aligned}$$

Using the expansion in the frequency difference given in the Appendix we find the value of the stimulated scattering cross-section entering in equation (12) (here $z \ll 1$)

$$\sigma^{st}(z) = 3\sigma_T \int_{-\infty}^{+\infty} \frac{dy}{\sqrt{\pi}} e^{-y^2} y^2 \int_{-1}^{1} dx \frac{x(1-x)(1+x^2)}{|F_e(x,y)|^2} \tag{24}$$

A similar expansion in the frequency difference in the spontaneous terms gives

$$\sigma_1^{sp}(z) = 3\sigma_T \int y^2 \int_{-1}^{1} dx \frac{1+x^2}{|F_e(x,y)|^2} \left\{ x(1-x) + \delta_e x \mathrm{Re}\left(F_e(x,y) W(y)\right) |F_e(x,y)|^2 \right\} \tag{25}$$

and

$$\sigma_2^{sp}(z) = \frac{1}{4} \sigma^{st}(z) \tag{26}$$

Collecting these expressions we can write the following transport equation which takes into account the "comptonization" in the classical limit;

$$\frac{1}{3} \frac{\partial B_\omega^T}{\partial r} = n_e \left[\sigma(z) \mathcal{F}_\omega + \frac{v_{Te}^2}{c^2} \frac{\partial}{\partial z} I^{tr}(\delta_e)\, z^8 \frac{\partial}{\partial z} \frac{\mathcal{F}_\omega}{z^3} \right] \tag{27}$$

The whole effect of "comptonization" during the radiative transfer described by (27) can be expressed through a single integral $I^{tr}(\delta_e)$

$$I^{tr}(\delta_e) = \frac{3}{4} \int_{-\infty}^{+\infty} \frac{dy}{\sqrt{\pi}} e^{-y^2} y^2 \int_{-1}^{1} dx \frac{x(1-x)^3(1+x^2)}{|1 - x + \delta_e W(y)|^2} \tag{28}$$

In (27) $\sigma(z) = \sigma^{(0)}(z) + \delta\sigma(z)$ (see (12)), thus $\delta\sigma(z)$ is related only to relativistic corrections to the probability of scattering and is given by the expression found in (Tsytovich et al., 1995a) and takes into account three effects: 1) relativistic corrections to Thomson scattering, 2) relativistic corrections to the electron shielding cloud, 3) relativistic corrections to the electron distribution function.

The transport "comptonization" integral $I^{tr}(\delta_e)$ describes the dependence of the "comptonization" in the transport on the collective parameter δ_e. As we will show later this collective integral differs from that entering in the generalized Kompaneets equation. In the limit of high frequencies when the collective effects are unimportant the transport integral $I^{tr}(\delta_e)$ is equal to $-4/5$ while in the strongly collective case it is equal to

$$I^{tr}(\delta_e) = -\frac{102}{35\delta_e^2}\int_{-\infty}^{+\infty}\frac{dy}{\sqrt{\pi}}\frac{e^{-y^2}y^2}{|W(y)|^2} \approx -\frac{306}{35\delta_e^2} \tag{29}$$

The numerical value of the integral in eq.(29) is found in Appendix A.

In the limit $\delta_e \to 0$ the scattering can be considered as scattering on free electrons and our analytical (exact) results are better used in astrophysical applications in the high frequency limit but not the results of Sampson (1959) which are partially computational and in which terms with derivatives of the intensity with respect to the frequency are neglected. The presence of such terms change qualitively the transport equation giving rise to important changes to the spectra of radiation. We not only improve the Sampson results but give a more general result valid also for the frequency range when the collective effects are important.

5 Generalization of Kompaneets equation for collective plasma effects

To derive a generalization of Kompaneets equation by taking into account the collective plasma effects we start with a consideration of stimulated scattering. We consider here the case of isotropic intense superthermal radiation where stimulated scattering processes dominate spontaneous scattering. It will be possible to include also the process of spontaneous scattering and find the generalized Kompaneets equation for any intensity of radiation including those close to the thermal radiation.

Neglecting the spontaneous processes and spatial variation we can write the starting equation in the form

$$\begin{aligned}\frac{\partial N_\omega}{\partial t} &= -\frac{3}{8} n_e c \sigma_T \int_{-\infty}^{+\infty} \frac{dy}{\sqrt{\pi}} e^{-y^2} \int_{-1}^{1} dx \int \frac{d\omega'}{|\epsilon_{\mathbf{q},\Omega}|^2} \\ &\quad \frac{\hbar \mathbf{q}\cdot\mathbf{v}}{m_e v_{Te}^2}(1+x^2) N_{\omega'} N_\omega \frac{\omega'}{\omega} \delta(\Omega - \mathbf{q}\cdot\mathbf{v})\end{aligned} \tag{30}$$

An expansion in the parameter v_{Te}/c gives

$$\frac{\partial N_\omega}{\partial t} = \frac{3\hbar}{4 m_e c} \sigma_T n_e \frac{1}{\omega^2} \frac{\partial}{\partial \omega} \left[\omega^4 N_\omega^2 \int_{-\infty}^{+\infty} \frac{dy}{\sqrt{\pi}} e^{-y^2} y^2 \int_{-1}^{1} dx \frac{(1+x^2)(1-x)}{|F_e(x,y)|^2} \right] \tag{31}$$

By introducing the dimensional parameter ℓ, corresponding to the photon path length.

$$\ell \equiv ct\sigma_T \frac{v_{Te}^2}{c^2} n_e \tag{32}$$

we get the generalized Kompaneets equation, which in the limit $\delta_e = 0$ reduces to the usual Kompaneets equation (Kompaneets, 1957)

$$\frac{\partial N_z}{\partial \ell} = \frac{1}{z_2}\frac{\partial}{\partial z}\left[I_e^k(\delta_e) z^4 N_z^2\right] \tag{33}$$

where

$$I_e^k(\delta_e) = \frac{3}{4}\int_{-1}^{1} dx(1+x^2)(1-x)^3 \int_{-\infty}^{+\infty} \frac{y^2 dy e^{-y^2}}{\sqrt{\pi}} \frac{1}{|1-x+\delta_e W(y)|^2} \tag{34}$$

The dimensionless optical depth $\tau\,(=\sigma_T n_e ct)$ is related to ℓ by the relation $\ell = (v_{Te}^2/c^2)\,\tau$.

In the derivation of equation (33) we have taken into account that δ_e is a function of z and that $\partial \ln \delta_e(z)/\partial \ln z = -2$.

In the limit $\delta_e \to 0$ we have $I(\delta_e) \to 1$ and equation (33) reduces to the term in the Kompaneets equation describing stimulated scattering.

Although we calculate here only the terms describing stimulated scattering assuming that the intensity of radiation exceeds substantially the thermal level, the results can be easily generalized for the case when the intensity is of any value including that close to the thermal level. We need only to take into account the processes of spontaneous scattering. They can be calculated by the same procedure from the general expression for the probability of scattering. But there is no need to perform such a calculation since we can find these terms by the correspondence principle. Namely we put the requirements that the thermal distribution satisfies the final equation and that it is converted to the Kompaneets equation in the limit $\delta_e \rightarrow 0$, and then get

$$\frac{\partial N_z}{\partial l} = \frac{1}{z^2}\frac{\partial}{\partial z}\left[I_e^k(\delta_e) z^4 \left(N_z^2 + N_z + z\frac{\partial}{\partial z}N_z\right)\right] \tag{35}$$

In the limit $\delta_e \gg 1$ where the collective effects dominate the integral entering in the generalized Kompaneets equation is practically equal to 3 (see the Appendix for the numerical value of the integral):

$$I_e^k(\delta_e) \approx \frac{66}{5}\frac{1}{\delta_e^2} \tag{36}$$

Given in Fig.6 are the results of numerical computation of the integral $I^k(\delta_e)$ as a function of δ_e, Fig.7 contains a comparison of the value of the integral $I^k(\delta_e)$ with its asymptotic curve for $\delta_e \gg 1$ given by expression (36). As can be seen from these curves the asymptotic is reached only for $\delta_e \approx 60$. Shown in Fig.8 is the change in $I^k(\delta_e)$ for the range $1 < \delta_e < 10$. The numerical calculations of the integrals for the coefficients (see Appendix) leads to the following expression for this integral in the case $\delta_e \ll 1$

$$I^k(\delta_e) = 1 - 0.06969 \times \delta_e^2 - 0.01104 \times \delta_e^2 \ln \delta_e \tag{37}$$

and Fig.9 shows this dependence.

From the probability for scattering on ions given by eq.(5), we can can find the contribution of ions in equation (35). One should then substitute $I_e^k + I_i^k$ for I_e^k in eq.(35) where

$$I_i^k(\delta_e) = \frac{3}{4} \int_{-1}^{1} dx (1 + x^2)(1 - x) \sum_i \frac{Z_i^2 n_i m_e}{m_i \sum_j Z_j n_j} \times$$
$$\int_{-\infty}^{+\infty} \frac{dy}{\sqrt{\pi}} e^{-y^2} y^2 \frac{\delta_e^2}{|1 - x + \delta_e + < Z > \delta_e W(y) \frac{T_e}{T_i}|^2} \tag{38}$$

where Z_i, n_i and m_i are the charge, density and the mass of the ions of type i respectively.

In the limit $\delta_e \ll 1$ the contribution of ions is relatively small of order $\delta_e^2 m_e / m_i$. For $\delta_e \gg 1$ one can find the range of frequencies where the ions dominate by comparing expression (36) with the following expression for I_i

$$I_i^k(\delta_e) \approx \frac{3}{(1+ < Z >)^2} \sum_i \frac{Z_i n_i m_e}{m_i} \frac{1}{\sum_i n_i Z_i} \tag{39}$$

6 Discussion of the results

We have considered the two cases of scattering of most interest in astrophysical applications:

1) when the collective scattering and change in photon frequency contribute to radiative transport changing the structure of the transport equation,

2) when the collective stimulated scattering and change in photon frequency modifies the isotropic part of the radiation leading to collective effects in "comptonization" described by a new generalized Kompaneets equation.

It appears that both equations describe diffusion in frequency in a similar but quite different manner. The point which needs to be discussed is the role of ions in radiative transport and also in the generalized Kompaneets equation. The cross-section of scattering on ions increases with increasing δ_e and in the limit $\delta_e \gg 1$ the cross-section of scattering on ions approaches the value of the Thompson cross-section on free electrons. Nevertheless stimulated scattering on ions and the change in photon frequency in the process of scattering on ions contains an additional parameter m_e/m_i. On the other hand the stimulated scattering on electrons decreases with the second power of δ_e and thus as a fourth power of frequency. Thus for frequencies of the order of

$$\omega \approx \omega_{pe} \frac{c}{v_{Te}} \left(\frac{1}{1+ < Z >} \right)^{\frac{1}{2}} \left(\frac{m_e}{m_i} \right)^{\frac{1}{4}} \tag{40}$$

stimulated scattering on ions and the change of photon frequency due to the process of scattering on ions becomes important. A similar substitution as that leading to eq.(38) should be made in the transport integral $I^{tr}(\delta_e)$ for the case when ions start to dominate in the "comptonization" effects in the transport of radiation. The difference between I^{tr} and I^k is in the expression under the angular integral.

For the case of radiative transport in the solar interior the value of the frequency given by eq.(40) is too low to contribute appreciably to the solar opacity. Indeed the maximum of the function $z^4 e^z/(e^z - 1)^2$ entering in the expression for the solar opacity is $z \approx 3.83$, while the value of z which corresponds to the frequency given by eq.(38) is 0.1, thus scattering and frequency diffusion on ions cannot be appreciable for radiation transport in the solar interior.

In radio sources where c/v_{Te} could be of the order of 10^3 the range of frequencies in which stimulated scattering on ions will be important can be substantially broad from ω_{pe} up to $\omega_{pe}(m_e/m_i)^{1/4}(c/v_{Te})$.

The results of the present paper can be applied to many astrophysical problems in which "comptonization" was considered previously but neglected without good reason the plasma collective effects. For large differences of frequencies of the observed radiation and the maximum frequency in the range defined by eq.(1) the previous result will not be changed. But the results for scattering of the radio waves in the solar corona for example should be completely collective this fact has not been realized in the present literature. Also scattering and "comptonization" in accretion disks should be dominated by collective effects for a range of frequencies. All the applications depend on whether or not the frequency is in the range defined by eq.(1).

Concerning the result of the change of solar opacity due to the broadening of the Raman resonance we should emphasise that the percentage change is given by the value in which collective effects were already taken into account in the zero approximation. Thus these results give an additional decrease of the opacity as related to the value which already takes into account collective scattering. In our calculations in the zero approximation only collective scattering and bremsstrahlung were taken into account but not the line absorption for which the absorption on iron ions is known to be the most important. Thus the figure given plays the role of illustration indicating that the effect is really large. In the case where the absorption on iron ions contributes about one third to the total absorption we should take our estimate to be a factor two thirds less which gives about a 5% decrease of opacity which is still an appreciable value. But there are also other contributions discussed in references (Tsytovich et al., 1994, 1995a, 1995b). The reason why we do not include the contribution of line absorption of iron is that up to the present time this absorption was calculated without taking into account the collective effects. But as we have shown the range of frequencies in the solar interior corresponds to values when collective effects are very important. We therefore conclude that one should find the collective expressions for line absorption, without this expression the results will still be approximate. Our aim was only to show that the contribution of broadening of the Raman resonance is indeed large.

The last point we want to discuss is the validity of the perturbation theory in the transport equation we obtained earlier which has a small parameter in the highest derivative w.r.t. the frequency. Obviously the mathematics exclude the use of a perturbation theory in this case. But the point is that the structure of the transport equation obtained is such that by any change of variables the small parameter in the highest derivative can not be eliminated. This means that the usual mathematical statements can not be applied to this equation. Our preliminary investigation of the solutions of the transport equation obtained without using the perturbation approach shows that there exists some interesting peculiarities in these solutions and the corrections can not be of the order of v_{Te}^2/c^2 but of the order of v_{Te}/c. This result can be obtained from an exact solution of the transport equation in which the collective effects are ignored. An open question exists whether these peculiarities will survive in the case where the collective effects are taken into account.

The most interesting result is the possibility of exceeding the Eddington limit which is very probable if there exists in astrophysical quasi-stellar sources the possibility of the electron temperature exceeding the ion temperature. This condition is very often found in laboratory plasmas. A real application can be made after analysis of the possible difference of electron and ion temperatures in sources of cosmic radiation.

7 Aknowlegments

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8 Appendix

In the Appendix we will give in the zero approximation (when the Doppler and, collisional broadening of Raman resonance and the stimulated scattering are neglected) separate expressions for the cross-sections of scattering on electrons and ions. We neglect the effects of the order of m_e^2/m_i^2 but keep the collective effects. The probability of scattering on ions differs from (4) by substitution of $\chi^{(e)}_{\mathbf{q},\Omega}$ for $1 + \sum_i \chi^{(i)}_{\mathbf{q},\Omega}$. We denote differential cross-sections of scattering by $\sigma_{e,i}(\theta)$, the total cross-sections by $\sigma_{e,i}$ and the transport cross-sections by $\sigma^{tr}_{e,i}$ where

$$\sigma_{e,i} = \int \sigma_{e,i}(\theta)\sin\theta d\theta \tag{A1}$$

$$\sigma_{e,i}^{tr} = \int \sigma_{e,i}(\theta)(1-\cos\theta)d\theta \tag{A2}$$

We find

$$\sigma_e = \sigma_T\sqrt{1-\frac{z_0^2}{z^2}}\left\{1+\frac{3}{8}\delta_e\left[2+2\delta_e+(2+2\delta_e+\delta_e^2)\ln\frac{\delta_e}{2+\delta_e}\right]\right\} \tag{A3}$$

$$\begin{aligned}\sigma_i = \sigma_T\frac{3}{8}\delta_e\frac{T_i}{T_e}\sqrt{1-\frac{z_0^2}{z^2}}\Bigg\{(2+2\delta_e+\delta_e^2)\ln\frac{2+\delta_e}{\delta_e}-2-2\delta_e+ \\ +2+2\delta_i-(2+2\delta_i+\delta_i^2)\ln\frac{2+\delta_i}{\delta_i}\Bigg\}\end{aligned} \tag{A4}$$

where

$$\delta_i = \left(1+<Z>\frac{T_e}{T_i}\right)\delta_e \tag{A5}$$

and

$$\begin{aligned}\sigma_e^{tr} = \sigma_T\sqrt{1-\frac{z_0^2}{z^2}}\{1-\delta_e+ \\ +\frac{3}{8}\delta_e^2\left[(2+2\delta_e+\delta_e^2)\ln\frac{2+\delta_e}{\delta_e}-2-2\delta_e\right]\Bigg\}\end{aligned} \tag{A6}$$

$$\sigma_i^{tr} = \sigma_T\sqrt{1-\frac{z_0^2}{z^2}}\frac{3}{8}\frac{T_i}{T_e}\times$$

$$\times\left\{\left[\delta_e^2\left[-(2+2\delta_e+\delta_e^2)\ln\frac{2+\delta_e}{\delta_e}+2+2\delta_e\right]+\right.\right.$$
$$\left.\left.+\delta_i\delta_e\left[(2+2\delta_i=\delta_i^2)\ln\frac{2+\delta_i}{\delta_i}-2-2\delta_i\right]\right]\right\} \tag{A7}$$

The expressions given are valid also for the case where the frequency of the photon is close to the plasma frequency. This is the reason the factor $\sqrt{1-z_0^2/z^2}$ appears in front of the expressions given. The δ_e in these expressions is different from eq.(2) being defined as

$$\delta_e=\frac{c^2}{2v_{Te}^2}\frac{\omega_{pe}^2}{\omega^2-\omega_{pe}^2} \tag{A8}$$

and approaches expression (2) only in the limit $\omega \gg \omega_{pe}$. In the literature (see for example (Boercker 1987, Rose, 1993) one can find only the sum of the transport cross-section of scattering on electrons and ions in the limit $\omega \gg \omega_{pe}$ and $T_e = T_i$. For arbitrary values of the ratio ω/ω_{pe} but still $T_e = T_i$ this sum is equal to

$$\sigma_e^{tr}+\sigma_i^{tr}=\sigma_T\sqrt{1-\frac{\omega_{pe}^2}{\omega^2}}$$
$$\left\{1-\frac{3}{8}\delta_e\left[\delta_i(2+2\delta_i+\delta_i^2)\ln\frac{\delta_i}{2+\delta_i}+2\delta_i+2\delta_i^2+\frac{8}{3}\right]\right\} \tag{A9}$$

To calculate the effects of Doppler shifts in the scattering, we use the δ-function in the expression for the probability and find

$$\Omega\equiv\omega-\omega'=\mathbf{q}\cdot\mathbf{v}=\sqrt{2}qv_{Te}y \tag{A10}$$

where

$$q = |\mathbf{k} - \mathbf{k}'| = \frac{\omega}{c}\left[1 + \left(\frac{\omega'}{\omega}\right)^2 - 2\frac{\omega'}{\omega}x\right]^{\frac{1}{2}}$$

Solving this equation to second order in (v_{Te}/c) we obtain

$$\frac{\omega'}{\omega} = 1 - 2\frac{v_{Te}}{c}y\sqrt{1-x} + 2\left(\frac{v_{Te}}{c}\right)^2 y^2(1-x) \qquad (A11)$$

Then we can use

$$\frac{\hbar\mathbf{q}\cdot\mathbf{v}}{m_c v_{Te}^2}\frac{\omega'}{\omega}\delta(\Omega - \mathbf{q}\cdot\mathbf{v}) =$$

$$\frac{\hbar\omega}{T}2\frac{v_{Te}}{c}y\sqrt{1-x}\left(1 - 4\frac{v_{Te}}{c}y\sqrt{1-x}\right)\delta[g(\omega) - \omega'] \qquad (A12)$$

where

$$g(\omega) = \omega\left[1 - 2\frac{v_{Te}}{c}y\sqrt{1-x} + 2\left(\frac{v_{Te}}{c}\right)^2 y^2(1-x)\right] \qquad (A13)$$

In the transport equation the integration over ω' should be performed using the δ-function, and we define the function $F(x,y)$ as

$$F(x,y) = \epsilon_{\mathbf{q},\Omega}|_{\omega'=g(\omega)} \qquad (A14)$$

The explicit calculation of $F(x,y)$ can be done to second order in v_{Te}/c. Neglecting the ion contribution and taking into account the corrections due to electron-ion collisions and relativistic effects we find (the calculation of the collisional term describing the collisional broadening of Raman resonance can be found in ref.(Tsytovich et al., 1995a)

$$F(x,y) = 1 + C(x,y) + \frac{\delta_e}{1-x} \times$$
$$\left\{ W(y) \left[1 + 2\frac{v_{Te}}{c} y\sqrt{1-x} - 2\left(\frac{v_{Te}}{c}\right)^2 y^2(2-x) \right] + \left(\frac{v_{Te}}{c}\right)^2 \tilde{W}(y) \right\} \qquad (A15)$$

where

$$W(y) = 1 - 2ye^{-y^2} \int_0^y e^{t^2} dt + i\sqrt{\pi} y e^{-y^2} \qquad (A16)$$

is the plasma function and

$$\tilde{W}(y) = \frac{21}{8}(W(y) - 1) - 3y^2 W(y) - \frac{3}{y^4} W(y) - \frac{3}{4} y^2 \qquad (A17)$$

$$C(x,y) = \mathrm{i}\frac{< Z > \ln\Lambda}{24\pi^{\frac{3}{2}} n_e v_{Te}^3} \omega_{pe}^3 y \left(\frac{\delta_e}{1-x}\right)^{\frac{3}{2}} \left\{ \left[Ei(y^2) - \mathrm{i}\pi \right] e^{-y^2} - \frac{1}{y^2} \right\} \qquad (A18)$$

where $\ln\Lambda$ is the Coulomb logarithm, the mean ion charge is

$$< Z > = \frac{\sum_i Z_i^2 n_i}{\sum_i Z_i n_i} \qquad (A19)$$

and $Ei(y^2)$ is the integral exponent function defined by

$$Ei(y) = y^2 \exp(-y^2) \int_{-\infty}^{y^2} \frac{\exp(t)}{t} dt \qquad (A20)$$

The term $C(x, y)$ describes the binary collisions.

The integrals entering in the asymptotic expressions can be calculated numerically to yield

$$\int_{-\infty}^{+\infty} \frac{dy}{\sqrt{\pi}} e^{-y^2} \frac{y^2}{|W(y)|^2} = 3.00001 \qquad (A21)$$

$$\begin{gathered} \frac{3}{4} \int_{-\infty}^{+\infty} \frac{dy}{\sqrt{\pi}} e^{-y^2} \times \\ \left[2 \frac{\mathrm{Re}W(y)}{(\mathrm{Im}W(y))((\mathrm{Re}W(y))^2 - 3(\mathrm{Im}W(y))^2} \left(\frac{\pi}{2} - \arctan \left(\frac{ReW(y)}{\mathrm{Im}W(y)} \right) \right) \right. \\ \left. + \ln \left(\frac{(\mathrm{Re}W(y))^2 + (\mathrm{Im}W(y))^2}{4} \right) (3(\mathrm{Re}W(y))^2 - (\mathrm{Im}W(y))^2) \right] = \\ = 0.006969 \qquad (A22) \end{gathered}$$

$$\frac{3}{2} \int_{-\infty}^{+\infty} \frac{dy}{\sqrt{\pi}} e^{-y^2} \left(3(\mathrm{Re}W(y))^2 - (\mathrm{Im}W(y))^2 \right) = 0.0011038 \qquad (A23)$$

9 Figure captions

Fig.1. Dependence of the cross-sections of scattering of photons on electrons and ions in the solar interior on the photon frequency. Plasma electron density $n_e = 5.4 \times 10^{25} cm^{-3}, T_e = T_i = 1.5keV, v_{Te} = 1.53 \times 10^9 cm/s, z_0 = \hbar\omega_{pe}/T = 0.21, < Z >= 1.53$. The solid line-cross-section of scattering on electrons, the dotted line- the cross-sections of the sum of scattering on ions (abundance of elements is taken from a standard solar model (Bachal, 1989) the dashed line -the sum of scattering on electrons and ions, the dashed-dotted line -the 1/5 of form factor $z^4 \exp(z)/(\exp(z) - 1)^2$ which enters in the expression for opacity.

Fig.2. Dependence of cross-sections of scattering on the collective parameter δ_e. The figure gives both the transport cross-section and the usual cross-sections - solid and dotted lines (lower) for electrons, dashed and dashed-dotted lines for ions, solid and dotted lines (upper) for sum of electrons and ions. $< Z >= 1.53, T_e = T_i$

Fig.3. Dependence of the total transport cross-section of scattering on the collective parameter for different ratio of electron to ion temperature. The cross-sections decrease continuously with increase of $\tau = T_e/T_i$- the solid line (upper) $\tau = 1$, dotted line (upper) for $\tau = 2$, dashed line for $\tau = 3$, dashed-dotted line for $\tau = 4$, solid line (lower) for $\tau = 5$, dotted line (lower) for $\tau = 6$.

Fig.4. Dependence of solar opacity on the temperature ratio T_e/T_i for the parameters

given in Fig.1. The line absorption is not taken into account.

Fig.5. Dependence of the total transport cross-section of scattering on electrons and ions on the collective parameter δ_e for different average charges of ions -solid curve $< Z >= 1$, dotted curve $< Z >= 2$, dashed curve $< Z >= 4$.

Fig.6. Integral of the collective "comptonization" I^k for scattering on electrons as a function of the collective parameter δ_e.

Fig.7. Comparison of the dependence of the "comptonization" integral I^k for large value of collective parameter δ_e with asymtotic formula (36). Solid line the results of numerical calculation of the exact value of I^k. The dotted line the asymtotic dependence according to the formula (36).

Fig.8. Dependence of collective "comptonization" integral I^k on the collective parameter δ_e for the value of collective parameter of the order of 1.

Fig.9. The dependence of the collective "comptonization" integral on the parameter δ_e for the value of $\delta_e \ll 1$ when the collective effects are small.

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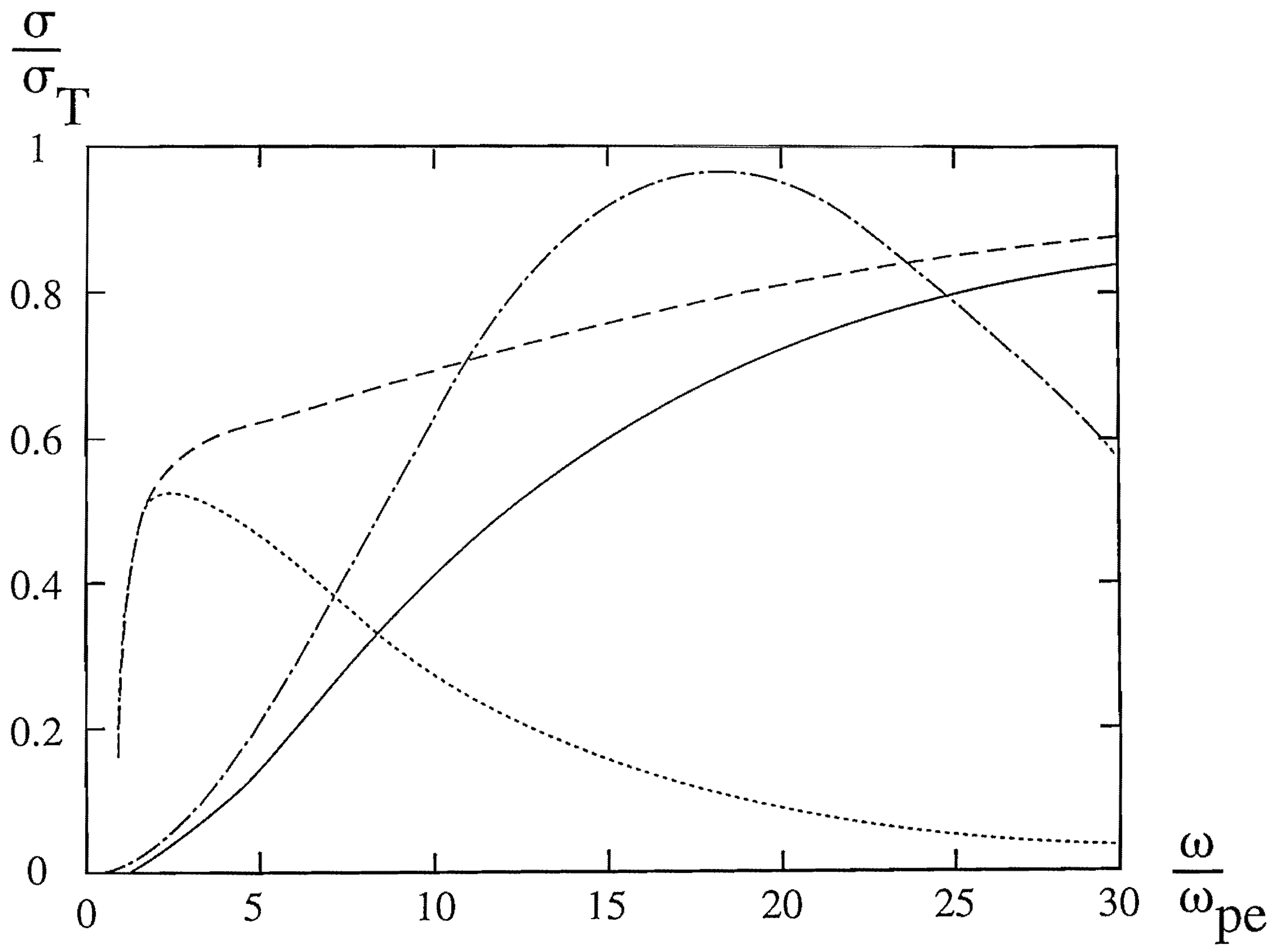
$\frac{\sigma}{\sigma_T}$
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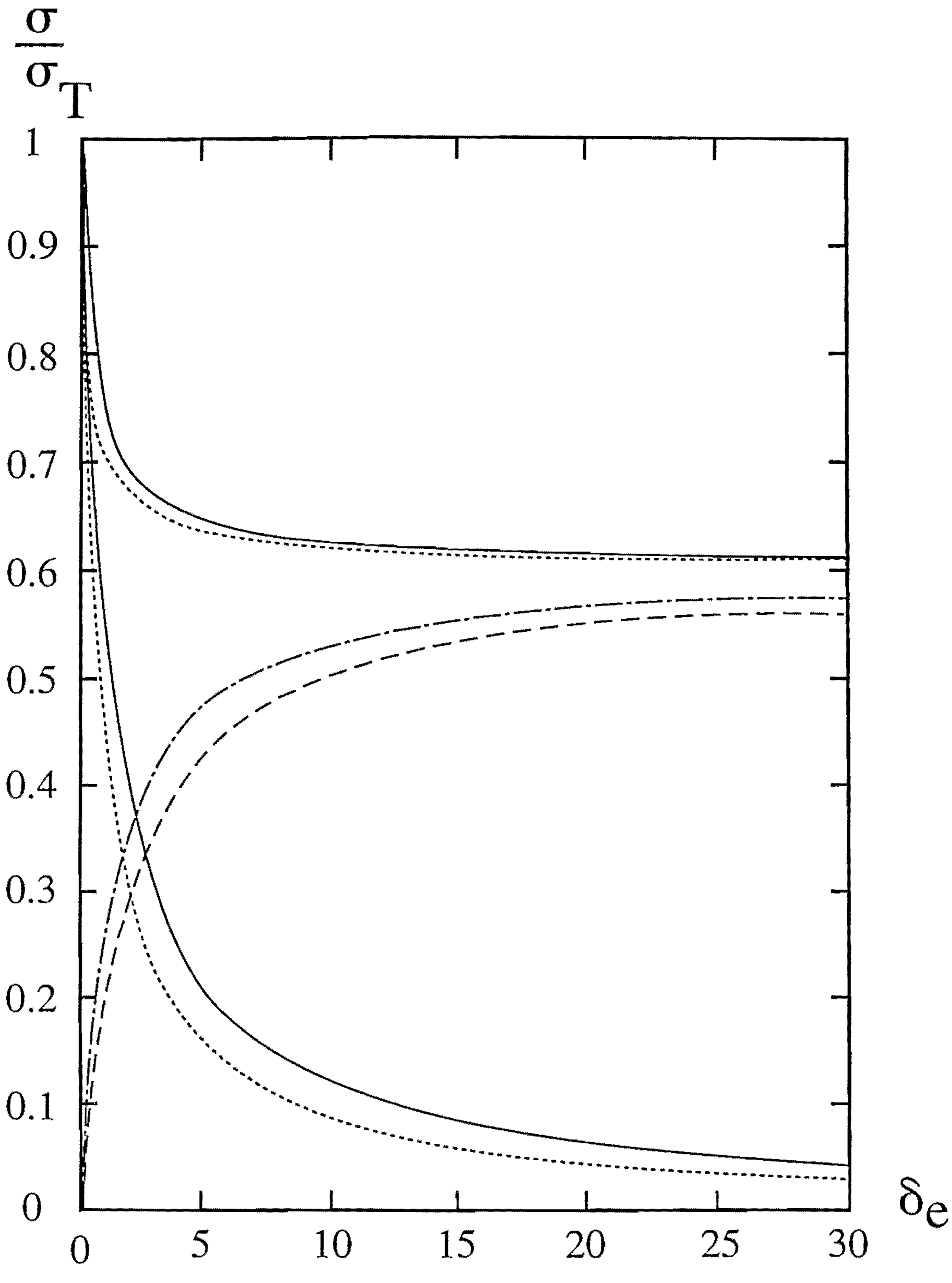
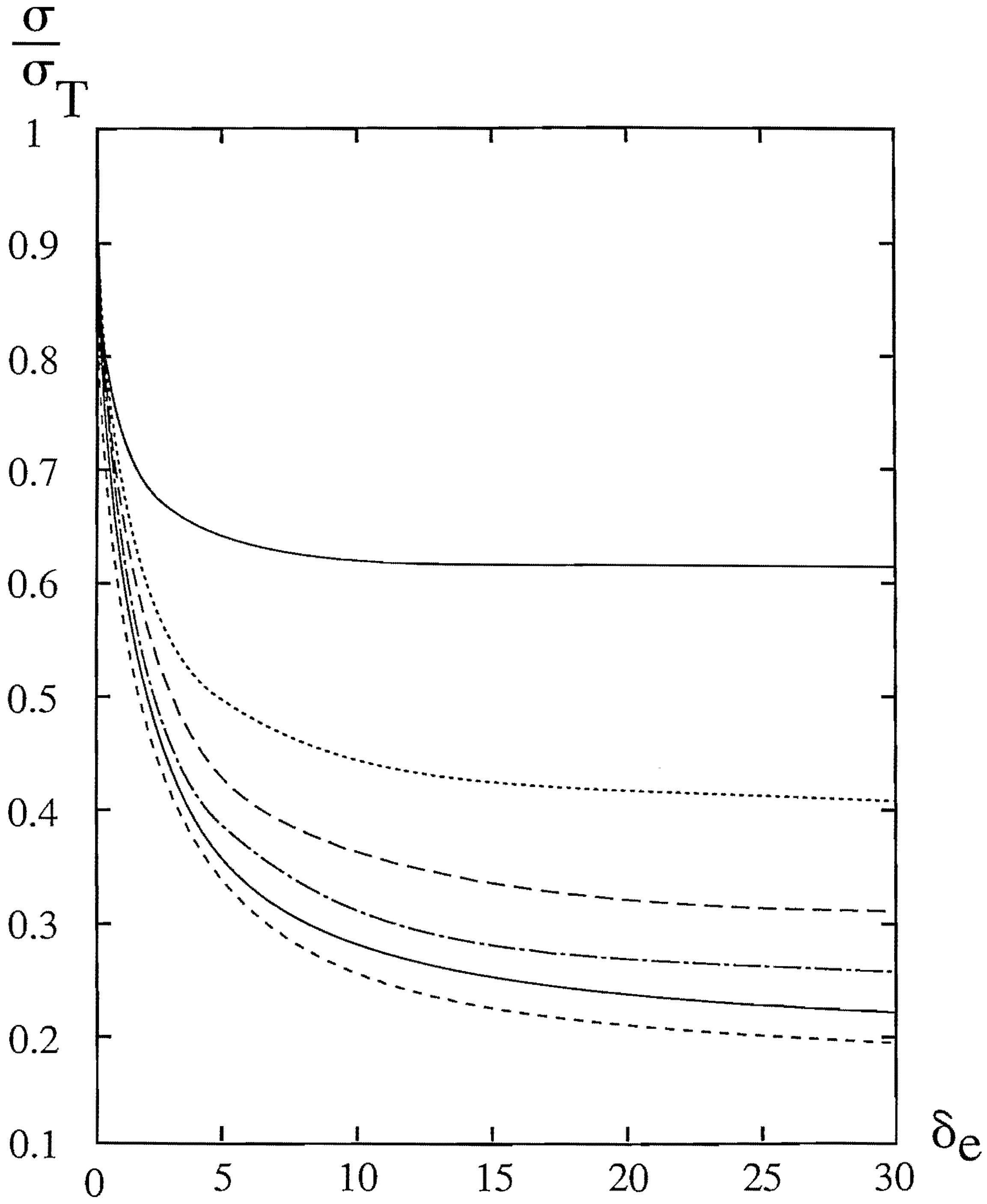
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
$\frac{\omega}{\omega_{pe}}$

Fig 1



$\frac{\sigma}{\sigma_T}$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30
δ_e

Fig 2


$\frac{\sigma}{\sigma_T}$
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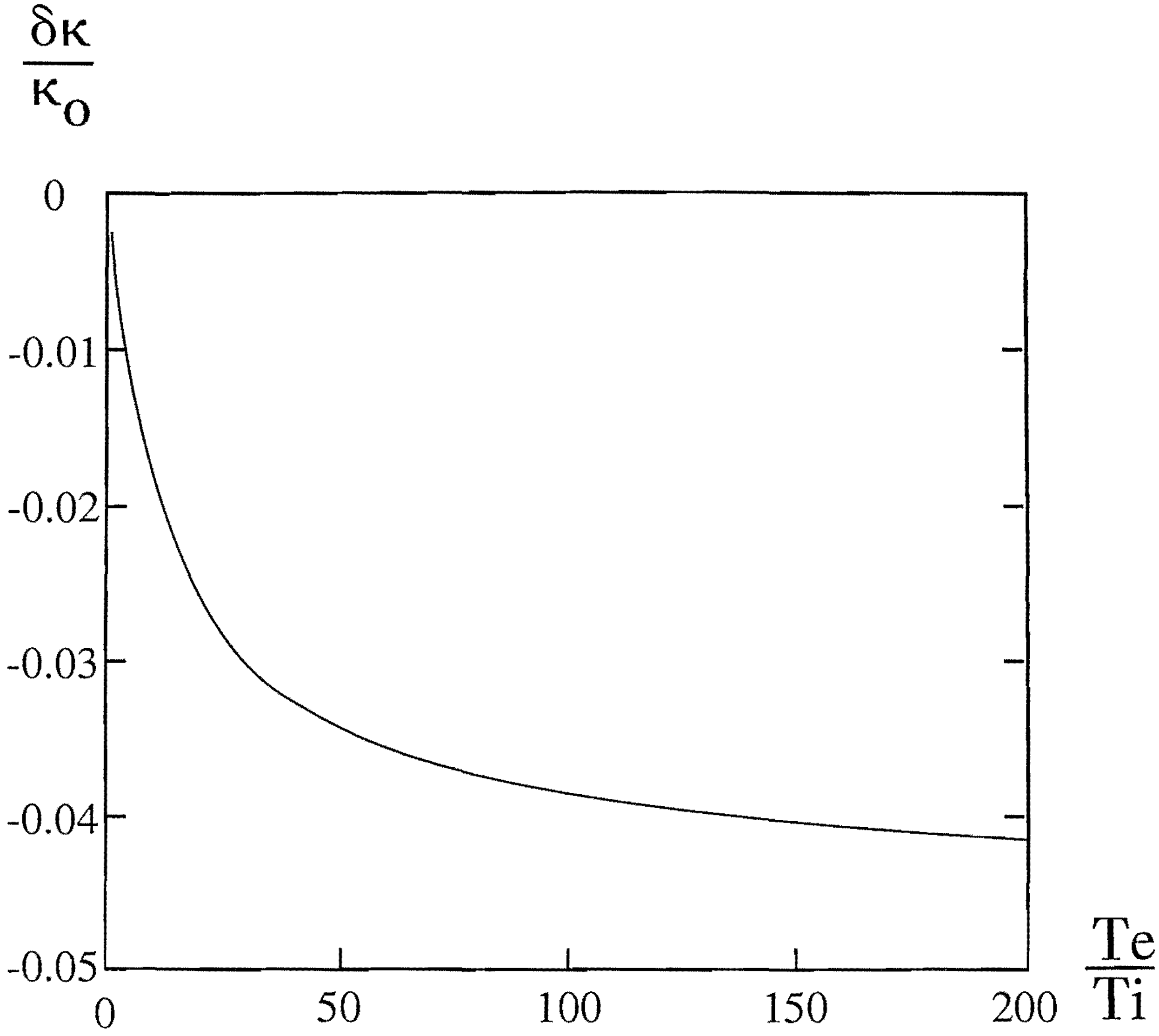
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
5
10
15
20
25
30
δ_e

Fig 3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-0.02
-0.03
-0.04
-0.05
0
50
100
150
200
$\frac{Te}{Ti}$

Fig 4

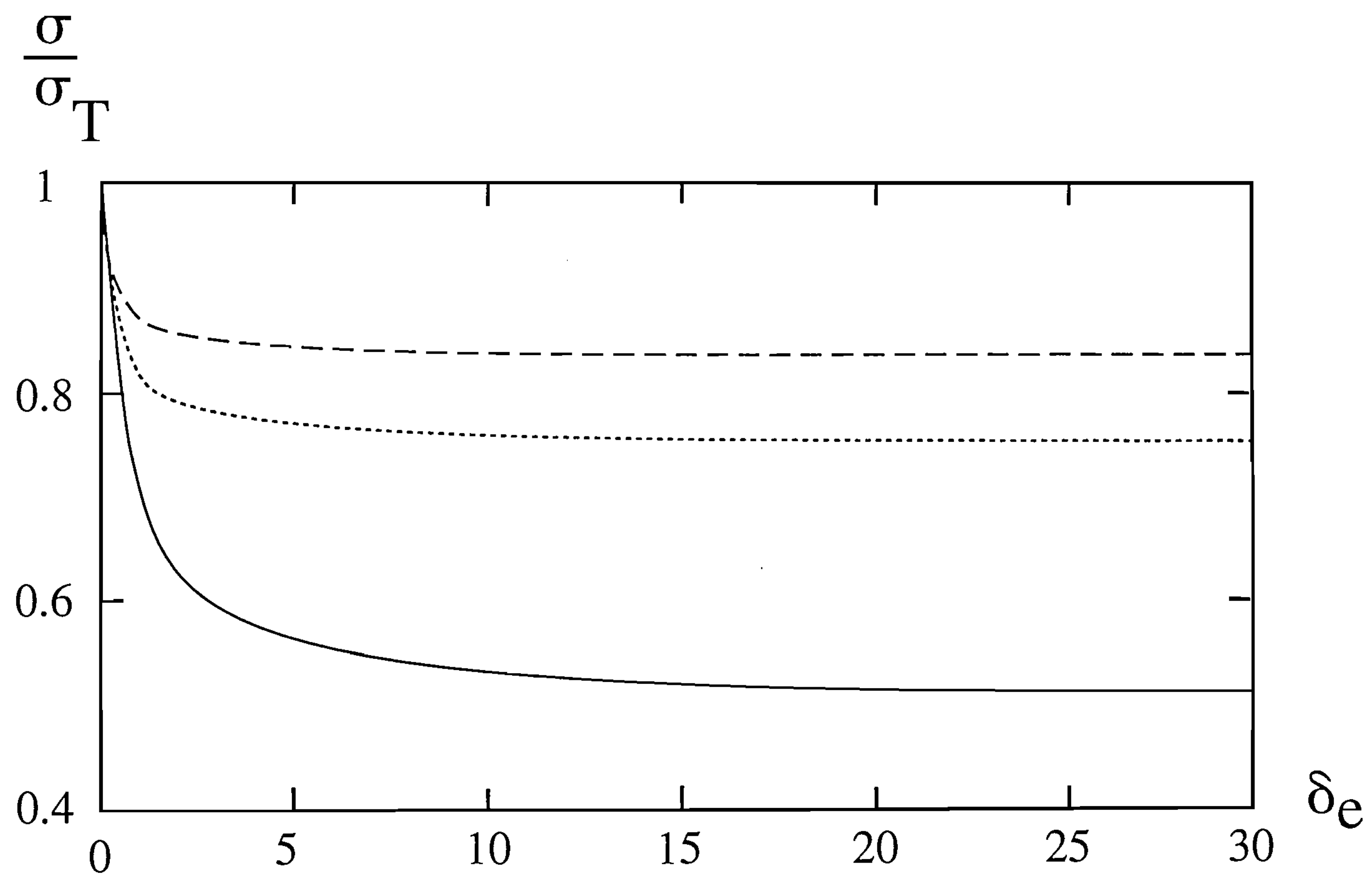

Fig 5

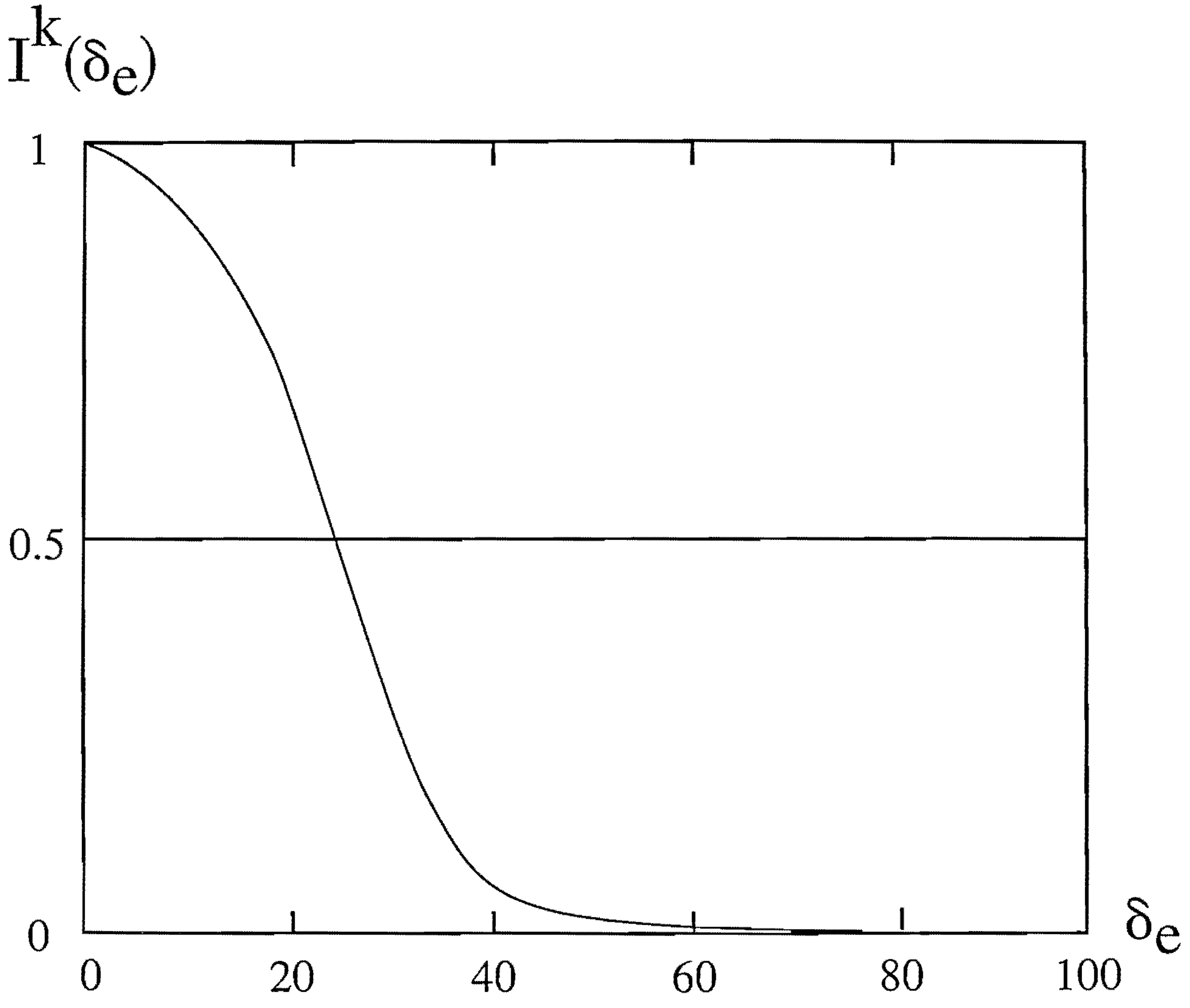

Fig 6

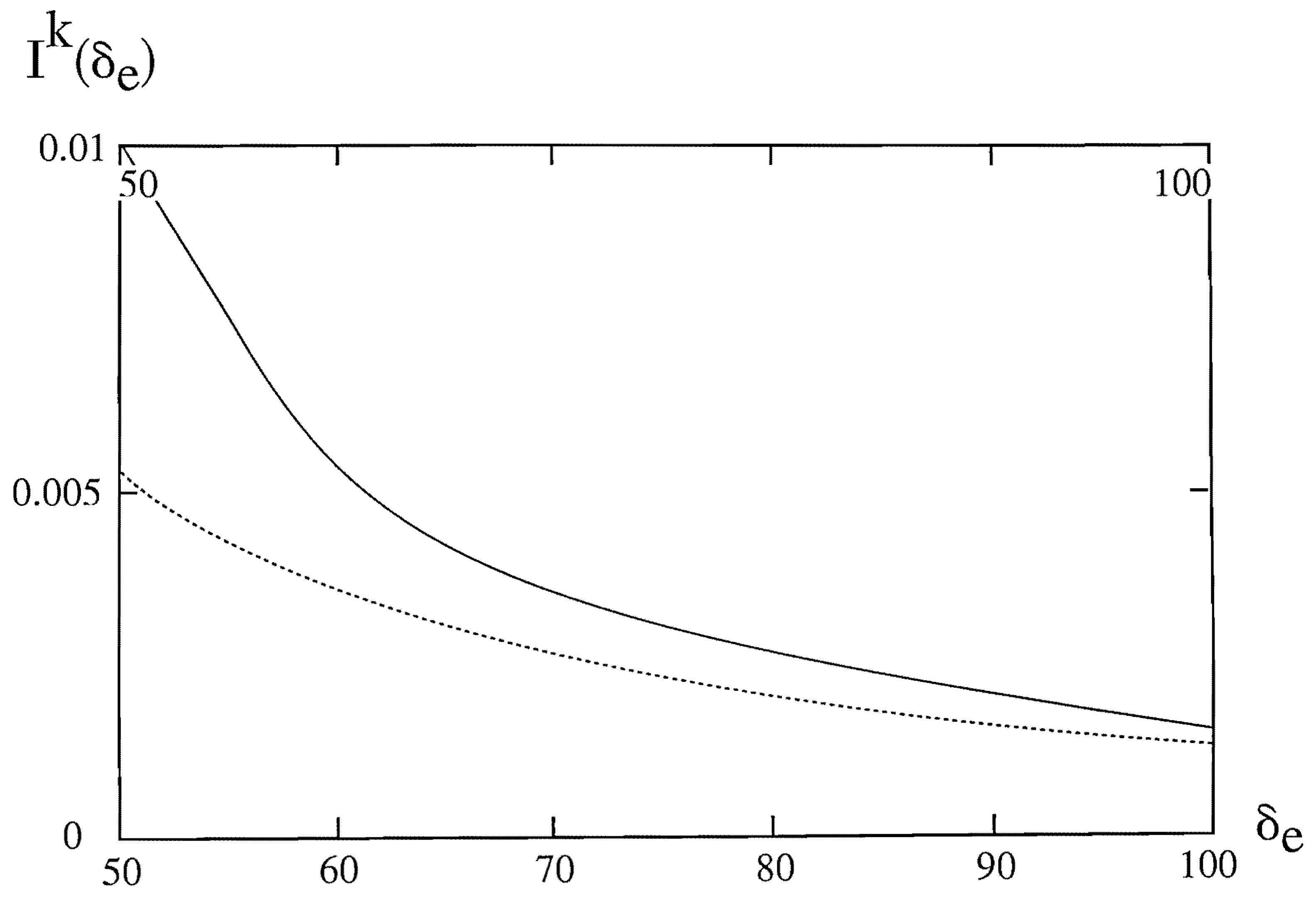

Fig 7

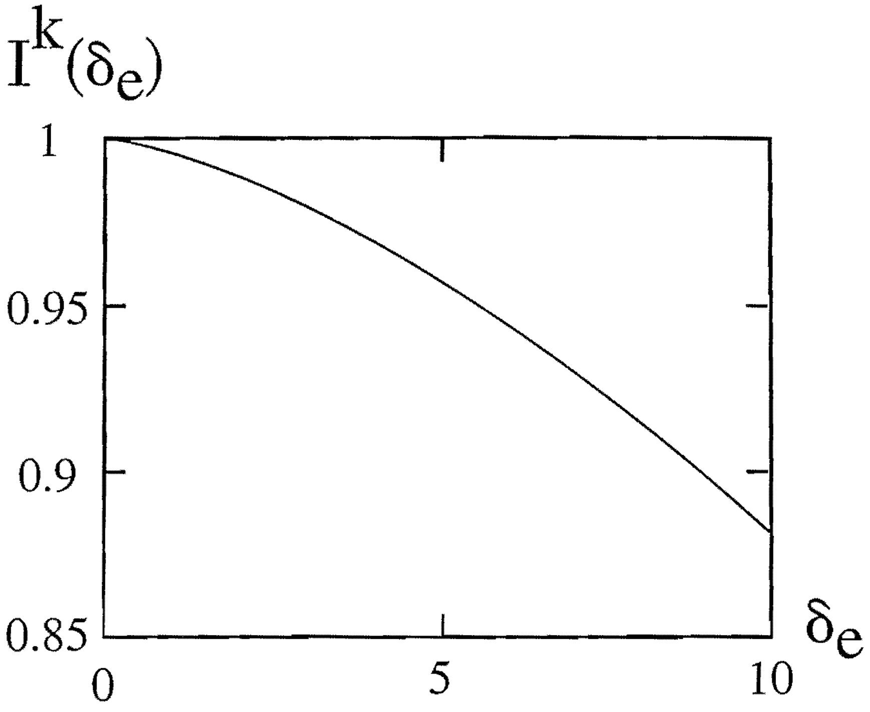

Fig 8

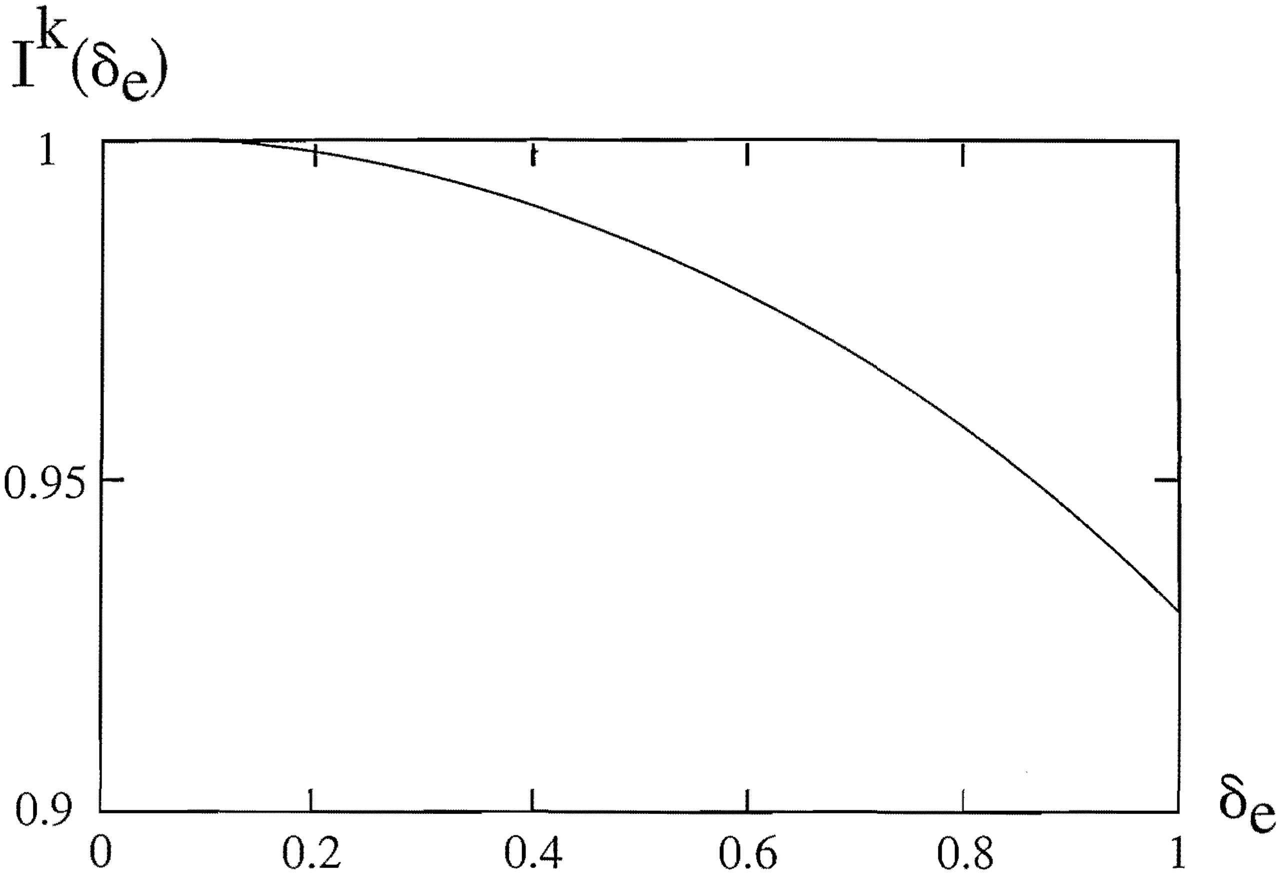

Fig 9

" THE EQUATION OF RADIATIVE TRANSFER IN THE SOLAR INTERIOR "

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ABSTRACT

We re-examine the theory of collective scattering, bremsstrahlung and the transport of radiation in the solar interior, including the effects of transition-bremsstrahlung, stimulated scattering, relativistic corrections and frequency diffusion during radiative transport, which transforms the transport equation into a differential equation (in frequency) for the radiative flux. When this is taken into account the transport equation can no longer be converted into the equation of "radiative transfer" where the total flux of radiation integrated over all frequencies is related to the temperature gradient via the Rosseland mean opacity. The concept of radiative opacity can only be introduced if the frequency diffusion is ignored or treated as a perturbation. In this case the change in the radiative opacity due to these effects is discussed in view of its importance for the neutrino production rate in the solar interior and compared with previous calculations.

Subiect headings : plasmas-opacities

1.INTRODUCTION

In the Standard Solar Model (SSM) the opacity due to scattering and inverse bremsstrahlung (free-free transitions) of radiation in the dense hot plasma of the Solar interior is an important parameter in determining the radiative transfer and total luminosity L_T which is given by:

$$L_T = -\frac{16\pi}{3}\int_0^{R_\odot}\frac{1}{\rho}\left(\frac{1}{\kappa_R}+\frac{1}{\kappa_c}\right)r^2\frac{d}{dr}(\sigma T^4)dr \tag{1}$$

where ρ is the mass density, σ Stefan's constant, T the temperatureand κ_R, κ_c are the radiative and conductive opacity respectively.

SSM results are very sensitive to the opacity [Bahcall 1989, Thurck-Chieze et al.1993] and in the central region of the Sun the radiative opacity dominates with the main contribution to κ_R arising from scattering and inverse-bremsstrahlung and k_R is determined by an integral over all frequencies of radiation which can propagate in the Solar interior (see eq. 34).

It is well known that collective effects in a plasma can modify these processes when the wavelength of radiation λ is of the order or larger than the plasma Debye length λ_D:for the most important frequencies in the solar central region it is $\lambda/\lambda_D > 1$ and collective effects should be taken into account.

Previous calculations have considered collective effects on scattering but neglected stimulated scattering [Boercker 1987, Huebner 1986] or considered relativistic corrections and stimulated scattering but neglected collective effects [Sampson 1959, Buchler and Yueh 1976, Iglesias and Rogers 1991].

In this work we re-examine the corrections to the opacity due to collective and relativistic effects, including stimulated scattering and frequency diffusion [Tsytovich et al.1995a], collective effects incorporated into relativistic fluctuation scattering [Tsytovich et al.1995b,c], and the effects of density inhomogeneity and refractive index. The effect of density inhomogeneity is proportional to ω_{pe}^2/ω^2 where

$$\omega_{pe}^2 = \frac{4\pi e^2 n_e}{m_e}$$

is the plasma frequency and ω the radiation frequency; in the solar centre $\omega_{pe} \simeq 4,8 \times 10^{17} s^{-1}$ and $\omega \simeq 10^{18} s^{-1}$ (peak of the black body spectrum) and the effect is therefore of the same order as the relativistic corrections $\sim v_{Te}^2/c^2$ where $v_{Te} =$

$(T/m_e)^{\frac{1}{2}}$ is the electron thermal velocity and c the speed of light ($c \simeq 20 \div 30 v_{Te}$ in the solar interior).

It should also be stressed that the stimulated effects should always be taken into account for a thick medium (as the solar centre) giving corrections of order v_{Te}^2/c^2 for thermal radiation. It is also imperative to include the stimulated terms in the transport equation as only then an equilibrium will be achieved and the stimulated scattering also describes the change in frequency due to scattering (frequency diffusion).

Let us first review the physical interpretation of collective scattering in plasmas as given by Gailitis and Tsytovich (1965).

When the wavelength of the radiation is comparable to the Debye radius the electrons in the shielding clouds oscillate in the radiation field as well as the shielded electrons. Every shielded electron is also responsible for shielding [Rosenbluth and Rostoker 1962]. The charge of the shielding cloud is equal and opposite to the charge that it is shielding (only the shielding electrons oscillate in the field, the ions can be assumed to be infinitely massive for high frequency radiation). This part of scattering is denoted as "scattering on electrons". The shielded electron contribution to the scattered radiation is out of phase with the radiation produced by the shielding electrons in the cloud and almost cancel each other, whereas for shielded ions the radiation is only due to the shielding cloud of electrons (if the ions are assumed infinitely massive). The ions however are responsible for energy and momentum exchange during the scattering process: the coupling (via the ambipolar field)of the shielding electron cloud to the shielded ion allows the transfer of momentum and energy. This part of scattering is denoted as "scattering on ions". Even although in the collective regime the electrons are responsible for the scattered radiation it is the ions which control the energy and momentum exchange during the process : this can clearly be seen in eq.(5) where the probability of scattering on ions $W^i_{\underline{k},\underline{k}'}$ is multiplied on the ion distribution function. But clearly the relativistic corrections enter not only in the scattering on electrons but also in the scattering on ions (via the shielding electrons): these corrections have only recently been included in the total scattering cross-section [Tsytovich et al.1995c].

When stimulated scattering and all relativistic corrections are ignored the total transport scattering cross-section due to electrons and all ions can be written as [Boercker 1987,Tsytovich et al.1995c]:

$$\sigma^{tr}_{(\omega)} = \sigma^{tr}_e + \sigma^{tr}_{ions} =$$

$$\sigma_T \left\{ 1 - \frac{3}{8}\delta_e \left[\delta(2 + 2\delta + \delta^2) \ln \frac{\delta}{2+\delta} + 2\delta + 2\delta^2 + \frac{8}{3} \right] \right\} \qquad (2)$$

Here σ_T is the Thomson cross-section :

$$\sigma_T = \frac{8}{3}\pi \frac{e^4}{m_e^2 c^4}$$

the "collective parameter" δ_e is defined as :

$$\delta_e = \frac{\omega_{pe}^2}{2\omega^2 \varepsilon(\omega)} \frac{c^2}{v_{Te}^2} \simeq \frac{1}{2} \left(\frac{\lambda}{\lambda_D} \right)^2 \qquad (3)$$

where $\varepsilon(\omega)$ is the refractive index (neglecting thermal effects):

$$\varepsilon(\omega) = 1 - \frac{\omega_{pe}^2}{\omega^2}$$

and ω_{pe} the local plasma frequency with $n_e(r)$ a function of distance r from the Sun's centre.

In (3) we made use of the dispersion relation for electromagnetic radiation in plasmas :

$$\omega^2 = \omega_{pe}^2 + c^2 k^2$$

The parameter δ in (2) is defined as :

$$\delta = \delta_e(1 + \langle Z \rangle) \qquad (4)$$

where

$$\langle Z \rangle = \frac{\Sigma_i n_i Z_i^2}{\Sigma_i n_i Z_i} = \frac{\Sigma_i n_i Z_i^2}{n_e} \qquad (5)$$

is the "mean ion charge" in the plasma with n_e, n_i being the electron and type $-i$ ion number densities, Z_i the ion charge and the second equality in (5) takes into account charge neutrality.

In the solar interior the electrons are partially degenerate and corrections to eq.(2) due to electron degeneracy have been found by Huebner (1986) and Boercker (1987): their effect on the opacity has been found by Iglesias and Rogers (1991)and by Rose (1993) to give a decrease of 2% at the solar centre.This effect should be added to the other effects discussed in this paper (see Table I). Here we consider the relativistic corrections to the transport cross-section to second order in v_{Te}/c, including stimulated scattering, in a new "effective" cross-section for scattering which reduces,for $\frac{v_{Te}}{c} = 0$ to the zero-order expression (2).

The other process contributing to the opacity in the central region of the Sun is inverse-bremsstrahlung (free-free transitions) and we consider collective and relativistic corrections to the corresponding cross-section. Collective corrections to bremsstrahlung were taken into account by Iglesias and Rogers (1991) but they used static screening (which overestimates the effect) and did not include transition bremsstrahlung which was considered by Tsytovich et al.(1995d);relativistic corrections to bremsstrahlung were also introduced by Tsytovich et al.(1995e). It should also be noted that Dagdaviren and Koonin (1987) introduced a correction due to ion correlations which should also be added to the other effects (see Table I). We do not consider here atomic processes (line absorption) which should always be added in calculating the opacity [Huebner 1986]. Notice however that in the solar inner region, where the collective and relativistic corrections to scattering and bremsstrahlung are most important,it is known that their contribution to the opacity is $\simeq \frac{1}{3}$ and should be taken into account using opacity codes with bound-bound and bound-free cross-sections incorporated (see e.g. Rose (1993)). Of course these cross-sections also need corrections due to collective effects and this is the subject of future work. There is also a quantum correction for scattering on electrons due to the recoil effect [Tsytovich et al.1995f], which is of order

$$\frac{\hbar\omega}{m_e c^2} = \frac{\hbar\omega}{T}\left(\frac{v_{Te}}{c}\right)^2 \ll 1$$

but the numerical coefficient seems to be large enough to make this correction of the same order of the other quantum effects (-2% in the opacity).

The recoil correction should therefore also be added to the transport cross-section.

The most straight-forward way to introduce all the corrections to the radiative opacity is to start with the transport equation for radiation.

This is given in Section 2, where the scattering and inverse-bremsstrahlung cross-sections are also introduced.

In Section 3 the changes in the cross-section due to collective and relativistic effects are calculated and their effect on the radiative opacity is discussed and compared with previous calculations. In Section 4 the effect of frequency diffusion is used to derive a new form of the transport equation, a differential equation in frequency for the radiative flux. Its effect on the opacity is found using perturbation theory.

2. THE TRANSPORT EQUATION

The basic equation for photon-transport is most conveniently written in terms of the photon occupation number $N_{\underline{k}}$ (where $\hbar \underline{k}$ is the photon momentum).

Neglecting atomic processes (to be included separately) and in the classical limit $\hbar k \ll p$ where p is the particle momentum, i.e. for $\hbar\omega \ll T(c/v_{Te})$, which is well satisfied in the solar interior also for photon energies $\hbar\omega \simeq T$ since $v_{Te}/c \simeq 1/20$, the transport equation for the steady-state case

$$\frac{\partial N_{\underline{k}}}{\partial t} = 0$$

is given by [Ginzburg and Tsytovich 1990]:

$$\begin{aligned} \underline{v}_g \cdot \frac{\partial N_{\underline{k}}}{\partial \underline{r}} - \frac{\partial \omega}{\partial \underline{r}} \cdot \frac{\partial N_{\underline{k}}}{\partial \underline{k}} &= -\Sigma_\alpha \int W^\alpha_{\underline{k},\underline{k}'} f^\alpha(\underline{p}_\alpha)(N_{\underline{k}} - N_{\underline{k}'}) \frac{d^3p_\alpha d^3k'}{(2\pi)^6} + \\ &+ N_{\underline{k}} \int W^e_{\underline{k},\underline{k}'} \hbar(\underline{k} - \underline{k}') \cdot \frac{\partial f^e(\underline{p})}{\partial \underline{p}} N_{\underline{k}'} \frac{d^3p d^3k'}{(2\pi)^6} + 2\gamma_{\underline{k}} N_{\underline{k}} + \\ &+ \Sigma_{\alpha,\beta} \int W^b_{\alpha,\beta}(\underline{k},\underline{q}) f^\alpha(\underline{p}_\alpha) f^\beta(\underline{p}_\beta) \frac{d^3p_\alpha d^3p_\beta d^3q}{(2\pi)^9} \end{aligned} \tag{6}$$

where

$$\begin{aligned} \gamma_{\underline{k}} = \frac{1}{2}\Sigma_{\alpha,\beta} \int W^b_{\alpha,\beta}(\underline{k},\underline{q}) \Big[\Big(f^\beta(\underline{p}_\beta) - f^\beta(\underline{p}_\beta - \hbar q) \Big) f^\alpha(\underline{p}_\alpha) + \\ + \Big(f^\alpha(\underline{p}_\alpha) - f^\alpha(\underline{p}_\alpha - \hbar(\underline{k} - \underline{q}) \Big) f^\beta(\underline{p}_\beta) \Big] \frac{d^3p_\alpha d^3p_\beta d^3q}{(2\pi)^9} \end{aligned} \tag{7}$$

Here the second term in the LHS takes into account the density inhomogeneity (in the approximation of geometrical optics) and $\underline{v}_g$ is the group velocity of electromagnetic radiation:

$$\underline{v}_g = \frac{\partial \omega}{\partial \underline{k}} = c\sqrt{\varepsilon(\omega)}\frac{\underline{k}}{k} \tag{8}$$

In the RHS sum is over all plasma particles and the first term represents spontaneous scattering ($\underline{k} \to \underline{k}'$) on electrons and ions, the second term stimulated scattering on electrons (we neglect the stimulated scattering on ions [Tsytovich et al. 1995a]) and the last two terms account for spontaneous emission and induced emission and absorption (inverse bremsstrahlung) due to collisions of particles α and β in the plasma.

Here $W^e_{\underline{k},\underline{k}'}$ and $W^i_{\underline{k},\underline{k}'}$ are the scattering probabilities (on electrons and type-i ion respectively) and $W^b_{\alpha\beta}(\underline{k},\underline{q})$ is the probability of emission of a photon $(\hbar\underline{k},\hbar\omega)$ with momentum transfer $\hbar\underline{q}$ in the collision of particles α and β in the plasma; $f^\alpha(\underline{p}_\alpha)$ is the distribution function of particles of type α (electrons or ions) with momentum $\underline{p}_\alpha$ and is normalized to:

$$n_\alpha = \int f^\alpha(\underline{p}_\alpha)\frac{d^3p_\alpha}{(2\pi)^3} \tag{9}$$

where n_α is the number density of $\alpha - particles$ in the plasma.

The photon occupation number $N_{\underline{k}}$ is related to the radiation intensity per unit solid angle and frequency interval $I_{\underline{k}}$ by [Chiu 1968,Cox and Giuli 1968]:

$$I_{\underline{k}} = \frac{1}{2\pi^2}\varepsilon(\omega)\frac{\hbar\omega^3}{c^2}N_{\underline{k}} \tag{10}$$

In thermodynamic equilibrium the radiation field is isotropic and :

$$N_{\underline{k}} = N^{(0)}_\omega = \left(exp\left(\frac{\hbar\omega}{T}\right)-1\right)^{-1};I_{\underline{k}} = B_\omega = \varepsilon(\omega)B^{(0)}_\omega = \frac{\varepsilon(\omega)}{2\pi^2}\frac{\hbar\omega^3}{c^2}N^{(0)}_\omega \tag{11}$$

and the RHS of the transport equation (6) is exactly zero. The "generalized" planck function $B_\omega(T)$ given in eq. (11) includes the refractive index $\varepsilon(\omega)$ to account for dispersion in the medium [Cox and Giuli 1968].

In the solar interior (particularly in the deep interior where scattering and bremsstrahlung dominate the opacity) the solution to the transport equation in conditions near local thermodynamic equilibrium but in the presence of a photon flux is usually given in terms of the "diffusion approximation" [see e.g Chiu 1968]:

$$N_{\underline{k}} = N^{(0)}_\omega + \cos\theta_{\underline{k},\underline{n}}\delta N_\omega \tag{12}$$

where $\theta_{\underline{k},\underline{n}}$ is the angle between the wave-vector $\underline{k}$ and the radial direction $\underline{n} = \underline{r}/r$ and δN_ω is the small ($\delta N_\omega \ll N^{(0)}_\omega$) isotropic deviation related to the presence of a radiative flux:

$$F_\omega = \int I_{\underline{k}}\cos\theta_{\underline{k},\underline{n}}d\Omega_{\underline{k}} = \frac{1}{2\pi^2}\varepsilon(\omega)\frac{\hbar\omega^3}{c^2}\delta N_\omega\int\cos^2\theta_{\underline{k},\underline{n}}d\Omega_{\underline{k}} \tag{13}$$

where $d\Omega_{\underline{k}} = 2\pi\sin\theta_{\underline{k},\underline{n}}d\theta_{\underline{k},\underline{n}}$. Then:

$$L_\omega \equiv 4\pi r^2F_\omega = \frac{8}{3}\frac{\hbar\omega^3}{c^2}\varepsilon(\omega)\delta N_\omega r^2 \tag{14}$$

where we have also introduced the "shell luminosity" L_ω whose integral over all frequencies and radial distance is the total luminosity of eq. (1). It should be stressed that we use the diffusion approximation in the most general form,i.e. δN_ω in eq.(12) is an unknown quantity. This is important in the following when the introduction of frequency diffusion will give a differential equation for the (unknown) flux and differs from previous treatments by Sampson (1959) and Buchler and Yueh (1976) where a definite assumption is made for the flux of radiation [cfr.eq.(6) of Buchler and Yueh with our eq.(12)]. The transport equation (6) is then linearized using eq.(12); in the LHS we use the relations:

$$\frac{\partial N_\omega^{(0)}}{\partial \underline{k}} = \underline{v}_g \frac{\partial N_\omega^{(0)}}{\partial \omega}; \frac{\partial \omega}{\partial \underline{r}} = \frac{\omega_{pe}^2}{2\omega} \frac{1}{n_e} \frac{\partial n_e}{\partial \underline{r}} \tag{15}$$

and the transport equation becomes:

$$\cos\theta_{\underline{k},\underline{n}} c\sqrt{\varepsilon(\omega)} \left[\frac{\partial N_\omega^{(0)}}{\partial r} - \delta_e \frac{v_{Te}^2}{c^2} \frac{1}{n_e} \frac{\partial n_e}{\partial r} \omega \frac{\partial N_\omega^{(0)}}{\partial \omega} \right] = \Im_{\underline{k}}(\delta N_\omega) \tag{16}$$

where $\Im_{\underline{k}}(\delta N_\omega)$ represents the linearized RHS of eq. (6).

After substitution of eq.(12) for $N_{\underline{k}'}, \Im_{\underline{k}}(\delta N_\omega)$ contains $\cos\theta_{\underline{k}',\underline{n}}$ in integrals where the integration over the angle of $\underline{k}'$ should be performed; since the scattering probabilities only depend on $\underline{k} - \underline{k}'$, i.e. on the angle $\theta_{\underline{k},\underline{k}'}$, it is possible to substitute, in the integrals, the average value

$$< \cos\theta_{\underline{k}',\underline{n}} > = < \cos\theta_{\underline{k},\underline{n}} \cos\theta_{\underline{k},\underline{k}'} > +$$

$$+ < \sin\theta_{\underline{k},\underline{n}} \sin\theta_{\underline{k},\underline{k}'} \cos(\phi - \phi') > = \cos\theta_{\underline{k},\underline{n}} \cos\theta_{\underline{k},\underline{k}'}$$

(this is also discussed in more details in Chiu (1968)and Cox and Giuli (1968)). Using this result the (linearized) form of the RHS of eq.(6) takes the form (set $x = \cos\theta_{\underline{k},\underline{k}'}$):

$$\Im_{\underline{k}}(\delta N_\omega) = \cos\theta_{\underline{k},\underline{n}} \Im_\omega(\delta N_\omega) \tag{17}$$

where

$$\Im_\omega(\delta N_\omega) = -\Sigma_\alpha \int W_{\underline{k},\underline{k}'}^\alpha f^\alpha(\underline{p}_\alpha)(\delta N_\omega - x\delta N_{\omega'}) \frac{d^3 p_\alpha d^3 k'}{(2\pi)^6} +$$

$$+ \int W_{\underline{k},\underline{k}'}^e \hbar(\underline{k} - \underline{k}') \cdot \frac{\partial f^e(\underline{p})}{\partial \underline{p}} (N_{\omega'}^{(0)} \delta N_\omega + N_\omega^{(0)} x \delta N_{\omega'}) \frac{d^3 p d^3 k'}{(2\pi)^6} + 2\gamma_k \delta N_\omega \tag{18}$$

where the first term is spontaneous scattering, the second term is induced scattering (on electrons) and the last term is the inverse-bremsstrahlung. The absorption coefficient γ_k has been calculated, including the collective effects of transition-bremsstrahlung, by Tsytovich et al.(1995d) and is given in the Appendix.

Defining a bremsstrahlung "cross-section" $\sigma^{ff}(\omega)$ (free-free transitions) as

$$2\gamma_k \equiv -n_e c\sigma^{ff}(\omega) \tag{19}$$

we separate the contribution of the collective effects setting

$$\sigma^{ff}(\omega) = \sigma_0^{ff}(\omega) - \delta\sigma_c^{ff}(\omega) \tag{20}$$

where $\sigma_0^{ff}(\omega)$ is the cross-section without collective effects.

It is well known that collective effects including dinamical screening and transition bremsstrahlung can substantially decrease the bremsstrahlung cross-section in plasmas [Ginzburg and Tsytovich 1990] but their contribution to the solar opacity was not taken into account by Iglesias and Rogers (1991).

Also quantum and relativistic corrections have to be taken into account. The quantum corrections have been incorporated into $\sigma_0^{ff}(\omega)$ (see eg.Rose 1993); denoting the relativistic correction (to the non-collective term $\sigma_0^{ff}(\omega)$) as $\delta\sigma_R^{ff}(\omega)$ we finally write the free-free cross section in the form :

$$\sigma^{ff}(\omega) = \sigma_0^{ff}(\omega) - \delta\sigma_c^{ff}(\omega) - \delta\sigma_R^{ff}(\omega) \tag{21}$$

where $\sigma_0^{ff}(\omega)$ includes the quantum corrections [Rose 1993], $\delta\sigma_c^{ff}$ is due to collective effects and $\delta\sigma_R^{ff}$ is due to (non-collective) relativistic corrections (Tsytovich et al.1995e).They are both given in the Appendix.

Both these corrections have never been taken into account in the solar opacity. In the scattering terms of eq. (18) the general expression for the probabilities (fully relativistic and including collective effects [Ginzburg and Tsytovich 1990]) can be expanded in (v_{Te}/c)in the form [Tsytovich et al.1995c]:

$$W^{(\alpha)}_{\underline{k},\underline{k}'} = (W^{(\alpha)(0)}_{\underline{k},\underline{k}'} + \delta W^{(\alpha)}_{\underline{k},\underline{k}'})\delta(\omega - \omega' - \underline{q}\cdot\underline{v}^{\alpha}) \tag{22}$$

where $\delta W^{(\alpha)}_{\underline{k},\underline{k}'}$ is the relativistic correction to the collective scattering probability(it is of $0(v_{Te}^2/c^2)$) and the zero order (non-relativistic but with collective effects) probabilities for scattering on electrons and ions are given by:

$$W^{(e)(0)}_{\underline{k},\underline{k}'} = \frac{(2\pi)^3 e^4}{2m_e^2\omega^2}(1+x^2)\frac{1}{|\epsilon^{(0)}_{\underline{q}\Omega}|^2} \tag{23}$$

$$W^{(i)(0)}_{\underline{k},\underline{k}'} = \frac{(2\pi)^3 Z_i^2 e^4}{2m_e^2\omega^2}(1+x^2)\left|\frac{\chi^{(e)}_{\underline{q},\Omega}}{\epsilon^{(0)}_{\underline{q}\Omega}}\right|^2 \tag{24}$$

where m_e is the electron mass, Z_i the ion charge, $\chi^{(e)}$ is the electron susceptibility and $\epsilon^{(0)}_{\underline{q}\Omega}$ the plasma dielectric function without relativistic corrections:

$$\epsilon^{(0)}_{\underline{q}\Omega} = 1 + \chi^{(e)}_{\underline{q},\Omega} + \sum_i \chi^{(i)}_{\underline{q},\Omega} \tag{25}$$

where $\chi^{(i)}$ are the ion susceptibilities.

In (23) and (24) we have defined the quantities:

$$\underline{q} = \underline{k} - \underline{k}', \Omega = \omega - \omega' \tag{26}$$

Neglecting the frequency Doppler shift ($\omega' = \omega$) these are given by :

$$q = \sqrt{2}\frac{\omega}{c}(1-x)^{\frac{1}{2}}; \Omega = 0 \tag{27}$$

In the next order the Doppler shift can be included, using the conservation law for scattering on electrons (the $\delta - function$ in eq.(22))and Ω and q can be expanded in v_{Te}/c : to second order it is [Tsytovich et al.1995b]:

$$q = \frac{\omega}{c}\left(1 + \frac{\omega'^2}{\omega^2} - 2\frac{\omega'}{\omega}x\right)^{\frac{1}{2}} \tag{28}$$

with the Doppler shifted frequency given by :

$$\omega' = \omega\left[1 - 2\frac{v_{Te}}{c}(1-x)^{\frac{1}{2}}y + 2\frac{v_{Te}^2}{c^2}(1-x)y^2\right] \tag{29}$$

where

$$y = \frac{\underline{q}\cdot\underline{v}^e}{\sqrt{2}qv_{Te}} \tag{30}$$

is the normalized component of the electron velocity parallel to $\underline{q}$.

The non-relativistic case (27) is recovered from (28,29) for $v_{Te}/c \rightarrow 0$.

Ions are always considered to be non relativistic.

The expressions for the relativistic corrections $\delta W^{(\alpha)}_{\underline{k},\underline{k}'}$ can be found in Tsytovich et al. (1995c) and shall not be reported here.

Let us consider first the transport equation without relativistic corrections and assuming Maxwellian distributions $f_M^{(\alpha)}(\underline{v})$ for the plasma particles. In this case the induced scattering term should also be neglected since:

$$(\underline{k}-\underline{k}')\cdot\frac{\partial f_M^{(e)}(\underline{v})}{\partial \underline{p}} \propto (\underline{k}-\underline{k}')\cdot\underline{v} = \Omega \propto \frac{v_{Te}}{c}$$

and therefore the induced term is of the order of the relativistic corrections. Notice that for the same reason the induced scattering should consistently be included when relativistic corrections are taken into account.

Making use of the $\delta(\omega-\omega')$ in the zero-order probabilities (eq.27) and defining the "transport" cross section as:

$$\sigma_0^{tr}(\omega)=\frac{1}{n_e c}\Sigma_\alpha \int W^{\alpha(0)}_{\underline{k},\underline{k}'} f_M^\alpha(\underline{v}_\alpha)(1-x)d^3v_\alpha\frac{d^3k'}{(2\pi)^3}\delta(\omega-\omega') \tag{31}$$

which, performing the integrals, give the explicit result as in eq.(2) (see eg. Tsytovich et al 1995c) from eq.s (16) and (18) we have the zero-order transport equation:

$$\frac{\partial N_\omega^{(0)}}{\partial r} = -n_e\sigma_0^{tr}(\omega)\delta N_\omega - n_e\sigma_0^{ff}(\omega)\delta N_\omega \tag{32}$$

where we have also neglected the collective and relativistic corrections to inverse-bremsstrahlung. From eq. (11) and (14) this can be written as:

$$\frac{\partial B_\omega^{(0)}}{\partial T}\frac{dT}{dr} = -n_e(\sigma_0^{tr}(\omega)+\sigma_0^{ff}(\omega))\frac{3F_\omega^{(0)}}{4\pi} \tag{33}$$

where we have introduced the temperature gradient from:

$$\frac{\partial B_\omega^{(0)}}{\partial r} = \frac{\partial B_\omega^{(0)}}{\partial T}\frac{dT}{dr}$$

and $F_\omega^{(0)} = F_\omega/\varepsilon(\omega)$.

Defining the Rosseland mean opacity as :

$$\frac{1}{\rho\kappa_R^0} = \int_0^\infty \frac{\frac{\partial B_\omega^{(0)}}{\partial T}}{n_e(\sigma_0^{tr}+\sigma_0^{ff}+\sigma^L)}d\omega \bigg/ \int_0^\infty \frac{\partial B_\omega^{(0)}}{\partial T}d\omega \tag{34}$$

where ρ is the mass density, the transport equation (33), integrated over frequency and radial distance takes on the usual form (see eq.(1)). Notice that in the definition of κ_R^0 the cross section for line-absorption σ^L has also been included, as it should be added in the transport equation as stated before. The corrections to eqs.(33,34) will be discussed in the next section.

3. CORRECTIONS TO THE OPACITY

Returning to the general transport equation the following corrections have to be taken into account:

1)refractive index corrections:

In the zero order result (34) we have also neglected the refractive index $\sqrt{\varepsilon(\omega)}$ in the numerator:its(small)effect is to increase the opacity.As already mentioned, for frequencies of order ω_{pe} the effect of the refractive index should be taken into account: notice that $\varepsilon(\omega)$ is present both in the "generalized" Planck function $B(\omega)$ (eq. 11) and in the flux $F(\omega)$ (eq. 14) and it therefore cancels in (33) but the effect of $\sqrt{\varepsilon(\omega)}$ (from the group velocity) should be taken into account,as well as the effect in the collective parameter δ_e. In the Solar interior where $\delta_e \approx 1$ (near the peak of the Planck distribution),and it is (see eq. 3):

$$\sqrt{\varepsilon(\omega)} \approx 1 - \frac{v_{Te}^2}{c^2}$$

so that the "refractive index correction" is of the same order of the relativistic corrections and should consistently be included when these are considered.

2) relativistic correction to the electron Maxwellian distribution function [Tsytovich et al.1995b]:

$$f_{\underline{p}}^{(e)} = \frac{1}{D} f_M^{(e)}(\underline{v}) exp(-\frac{3v^4}{8v_{Te}^2c^2}); f_{\underline{p}}^{(i)} = f_M^{(i)}(\underline{v}) \qquad (35)$$

where the normalization constant D is given in the Appendix.

3) broadening of the Raman resonance occurring for $\varepsilon^{(0)}_{\underline{q},\Omega} \simeq 0$ at $\Omega \simeq \omega_{pe}$: relativistic and collisional corrections change the dielectric function to [Tsytovich et al.1995b]:

$$\varepsilon_{\underline{q},\Omega} = \varepsilon^{(0)}_{\underline{q},\Omega} + \delta\varepsilon_{\underline{q},\Omega} \qquad (36)$$

removing the resonance at $\Omega \simeq \omega_{pe}$ (see eqn.(A3) in the Appendix. This effect reduces the scattering cross-section and therefore the opacity.

4) relativistic corrections to collective scattering [Tsytovich et al. 1995c] taking into account the total scattering probability. If only Thomson scattering is considered (i.e. scattering from individual particles, no collective effects), the scattering probability is given by eq. (23) for $\varepsilon^{(0)}_{\underline{q},\Omega} = 1$ and the result by eq. (2)

for $\delta_e = 0$, that is $\sigma_0^{tr} = \sigma_T$. The relativistic corrections to Thomson scattering have been found by Buchler and Yueh (1976) but the relativistic corrections to collective scattering (including the interference between Thomson scattering from an individual particle and scattering from its shielding cloud in the plasma) have been found in Tsytovich et al.(1995c) and should be included.

5) diffusion in frequency [Tsytovich et al.1995b]: using the frequency Doppler shift (eq.29) we can expand $N_{\omega'}^{(0)}$ and $\delta N_{\omega'}$ in the integrals of eq.(18).

To second order in v_{Te}/c it is :

$$\delta N_{\omega'} = \delta N_\omega - \Omega \frac{\partial \delta N_\omega}{\partial \omega} + \frac{1}{2}\Omega^2 \frac{\partial^2 \delta N_\omega}{\partial \omega^2} \tag{37}$$

and similar expansion for $N_{\omega'}^{(0)}$. The zero order terms give the transport cross-section (from the first term of eq. (18)) but the presence of the other terms changes the transport equation into a differential equation in frequency. The effect on the opacity is found here using perturbation theory (see Section 4).

6) stimulated scattering: this is given by the second term in eqn.(18) and it contributes zero-order terms (from the zero-order terms in the expansion (37)) which are of the same order of the relativistic corrections to spontaneous scattering and diffusion terms from the derivatives in the expansion (37).Both have been taken into account by Tsytovich et al.(1995b). The effects on the opacity of stimulated scattering and frequency diffusion have been also found by Buchler and Yueh (1976) neglecting collective effects. Their general theory however (valid at all temperatures),which removes two invalid assumptions made earlier by Sampson (1959),has "serious numerical problems" at low temperature (solar temperature).For this particular case, $\frac{v_{Te}}{c} \ll 1$, the work of Tsytovich et al.(1995b,c) is therefore more consistent and more general (it includes collective effects). Returning to the general form of the transport equation (eq.s (16)- (18)), writing the density gradient in the form:

$$\frac{1}{n_e}\frac{\partial n_e}{\partial r} = f(P,T)\frac{1}{T}\frac{dT}{dr} \tag{38}$$

where $f(P,T) = \frac{dlogn_e}{dlogT}$ can be derived from the known thermodyna mic relations in the solar interior [see e.g.Cox and Giuli 1968] or numerically from tables of solar models, and using the property

$$\frac{\omega^4}{T}\frac{\partial}{\partial \omega}\left(\frac{B_\omega^{(0)}}{\omega^3}\right) = -\frac{\partial B_\omega^{(0)}}{\partial T}$$

the transport equation (16) can be written as:

$$\left[1+\delta_e \frac{v_{Te}^2}{c^2} f(P,T)\right] \sqrt{\varepsilon(\omega)} \frac{\partial B_\omega^{(0)}}{\partial T} \frac{dT}{dr} = \frac{\hbar\omega^3}{2\pi^2 c^3} \Im_\omega(\delta N_\omega) \tag{39}$$

where the term in square brackets gives the correction due to the density inhomogeneity.

The RHS, inserting the expansion for $N_{\omega'}^{(0)}$ and $\delta N_{\omega'}$ in eq. (18) can be written in the form:

$$\Im_\omega(\delta N_\omega) = -n_e c[\sigma_{(\omega)}^{tr} - \sigma_{(\omega)}^{st} + \sigma_{(\omega)}^{ff}]\delta N_{(\omega)} - n_e c(\hat{\sigma}^{tr} - \hat{\sigma}^{st})\delta N_\omega \tag{40}$$

where the transport and stimulated cross-sections (from the zero-order terms in the expansions) are defined as:

$$\sigma_{(\omega)}^{tr} = \frac{1}{n_e c} \Sigma_\alpha \int W_{\underline{k},\underline{k}'}^\alpha f_{\underline{p}_\alpha}^\alpha (1-x) \frac{d^3 p_\alpha d^3 k'}{(2\pi)^6} \tag{41}$$

$$\sigma_{(\omega)}^{st} = \frac{1}{n_e c} \int W_{\underline{k},\underline{k}'}^e \hbar(\underline{k}-\underline{k}') \cdot \frac{\partial f_M^e}{\partial \underline{p}} \left[(1+x)N_\omega^{(0)} + (\omega'-\omega)\frac{\partial N_\omega^{(0)}}{\partial \omega}\right] \frac{d^3 v d^3 k'}{(2\pi)^3} \tag{42}$$

Their explicit expressions are given in the Appendix.In the last term $\hat{\sigma}^{tr}, \hat{\sigma}^{st}$ are operators, from the derivatives in the expansions, given by:

$$\hat{\sigma}_{(\omega)}^{tr} = -\frac{1}{n_e c} \int W_{\underline{k},\underline{k}'}^e f_M^e(v) x \frac{d^3 v d^3 k'}{(2\pi)^3} \hat{O}_\omega \tag{43}$$

where :

$$\hat{O}_\omega = (\omega'-\omega)\frac{\partial}{\partial \omega} + \frac{1}{2}(\omega'-\omega)^2 \frac{\partial^2}{\partial \omega^2}$$

$$\hat{\sigma}_{(\omega)}^{st} = \frac{N_\omega^{(0)}}{n_e c} \int W_{\underline{k},\underline{k}'}^e \hbar(\underline{k}-\underline{k}') \cdot \frac{\partial f_M^e(v)}{\partial \underline{p}} x(\omega'-\omega) \frac{d^3 v d^3 k'}{(2\pi)^3} \frac{\partial}{\partial \omega} \tag{44}$$

and the expansion (29) should be used for $\omega - \omega'$. Notice that except in (41), where the relativistic corrections to the electron distribution function are taken into account, in eq.s (42) to (44) we have used the Maxwellian distribution since all these terms are already of $0(v_{Te}/c)$ (from $\omega' - \omega$ or $\underline{k} - \underline{k}'$).

If stimulated scattering and frequency diffusion terms are ignored ($\sigma^{st} = \hat{\sigma}^{tr} = \hat{\sigma}^{st} = 0$) the transport equation, from (39) and (40), becomes:

$$\left[1+\delta_e \frac{v_{Te}^2}{c^2} f(P,T)\right] \sqrt{\varepsilon(\omega)} \frac{\partial B_\omega^{(0)}}{\partial T} \frac{dT}{dr} = -n_e[\sigma_\omega^{tr} + \sigma_\omega^{ff}] \frac{3}{4\pi} F_\omega^{(0)} \tag{45}$$

which,integrating over all frequencies, can be written in the usual form (eq. 1) defining the opacity from eq. (45) as:

$$\frac{1}{\rho\kappa_R} = \int_{\omega_{pe}}^{\infty} \frac{\left[1+\delta_e \frac{v_{Te}^2}{c^2} f(P,T)\right] \sqrt{\varepsilon(\omega)}\frac{\partial B_\omega^{(0)}}{\partial T}}{n_e[\sigma^{tr}+\sigma^{ff}+\sigma^{L}]} d\omega \,/ \int_0^\infty \frac{\partial B_\omega^{(0)}}{\partial T} d\omega \qquad (46)$$

Equations (45,46) contain all the corrections from points 1-4 above and the corrections to inverse bremsstrahlung. The expression for $\sigma^{ff}_{(\omega)}$ is given by eq. (21); the results for $\sigma^{tr}_{(\omega)}$ are reported in the Appendix and should be compared with the zero-order expressions (31,2) Recalling that all corrections are "small" ($|\delta\sigma_i| \ll \sigma_0$) it is then possible to estimate the effect of each correction using perturbation theory in eq. (46) in the form of an expansion in the small quantities $\frac{\delta\sigma_i}{\sigma_0}$ and calculate each effect separately,that is :

$$\kappa_R = \kappa_R^0 - \sum_i \delta\kappa_R^{(i)} \qquad (47)$$

where the sum includes all the corrections discussed previously .

These corrections have been calculated numerically at the Sun's centre and the results are given in Table I (Tsytovich, Bingham, de Angelis, Forlani and Occorsio 1995). Adding the corrections due to electron degeneracy in which the relativistic correction non-correctly treated by (Rose 1993) are excluded but both the degeneracy effects is scattering and bremsstrahlung are taken into account according to methods used in (Rose 1993) and adding also effect due to ion correlations (Dagdaviren and Koonin 1987) which are approximately -1.5% the total decrease of solar opacity is of order 9%, much larger than the uncertainty in the opacity values (5%) suggested by (Thurck-Chieze et al. 1993). From the results in the Table it is clear that some corrections can be safely neglected but others are important and should be taken into account. Stimulated scattering and frequency diffusion effects also contribute about 3% in the given value of 9% [Tsytovich et al.1995b] and in the next section we shall consider the full equation (40) including stimulated scattering and frequency diffusion.

4. EFFECT OF STIMULATED SCATTERING AND FREQUENCY DIFFUSION

From eq.s (39) and (40) we have, introducing the energy flux F^0_ω, the general form of the transport equation :

$$\left[1+\delta_e\frac{v^2_{Te}}{c^2}f(P,T)\right]\sqrt{\varepsilon(\omega)}\frac{\partial B^{(0)}_\omega}{\partial T}\frac{dT}{dr}=$$

$$=-\frac{3}{4\pi}n_e\left\{(\sigma^{tr}-\sigma^{st}+\sigma^{ff}+\sigma^L)F^0_\omega+\omega^3(\hat{\sigma}^{tr}-\hat{\sigma}^{st})\frac{F^0_\omega}{\omega^3}\right\} \tag{48}$$

where we have explicitly included the cross-section from atomic processes $\sigma^L(\omega)$. The operators $\hat{\sigma}^{tr}$ and $\hat{\sigma}^{st}$ are defined in eq.s (43) and (44) and are relativistic corrections since both $\omega'-\omega$ and $\underline{k}'-\underline{k}$ are of $0(v_{Te}/c)$.

They have been calculated by Tsytovich et al.(1995a).

Using their expressions and setting:

$$\wedge_\omega=\frac{4\pi}{3}\left[1+\delta_e\frac{v^2_{Te}}{c^2}f(P,T)\right]\sqrt{\varepsilon(\omega)}\frac{\partial B^0_\omega}{\partial T} \tag{49}$$

$$\sigma_{eff}(\omega)=\sigma^{tr}_{(\omega)}-\sigma^{st}_{(\omega)}+\sigma^{ff}_{(\omega)}+\sigma^{L}_{(\omega)} \tag{50}$$

the transport equation (48) takes the form:

$$A^{(2)}_\omega\omega^2\frac{\partial^2\frac{F^0_\omega}{\omega^3}}{\partial\omega^2}+A^{(1)}_\omega\omega\frac{\partial\frac{F^0_\omega}{\omega^3}}{\partial\omega}+A^{(0)}_\omega F^0_\omega=-\wedge_\omega\frac{dT}{dr} \tag{51}$$

with the coefficients given by :

$$A^{(0)}_\omega=n_e\sigma_{eff}(\omega) \tag{52}$$

$$A^{(1)}_\omega=n_e\omega^3\sigma_T\frac{3}{4}\frac{v_{Te}}{c}\int_{-\infty}^{+\infty}dyy\frac{e^{-y^2}}{\sqrt{\pi}}\int_{-1}^{+1}\frac{dx}{|F(x,y)|^2}(1+x^2)x\sqrt{(1-x)}\cdot$$

$$\cdot\left[1-2\frac{v_{Te}}{c}y\sqrt{(1-x)}\right](2+\frac{z}{e^z-1}) \tag{53}$$

$$A_\omega^{(2)} = -n_e\omega^3\sigma_T\frac{3}{4}\frac{v_{Te}^2}{c^2}\int_{-\infty}^{+\infty} dy y^2\frac{e^{-y^2}}{\sqrt{\pi}}\int_{-1}^{+1} dx\frac{(1+x^2)(1-x)x}{|F(x,y)|^2} \qquad (54)$$

where $z = \frac{\hbar\omega}{T}$ and the function $F(x,y)$ is given in the Appendix.

Eq. (51) is the new “transport equation” and it should be used instead of eq. (1) in Solar Models. It has to be solved numerically with appropriate “boundary conditions” and then integrated over frequency and radial distance to find the total flux or luminosity.

It is clear that in this context a “mean opacity” cannot be formally defined. We have calculated the effect of the new terms on the opacity using perturbation theory (Tsytovich et al.1995b):the flux is assumed equal to a zero-order term (as given by eq.33) and a "small" additive part δF_ω due to the effect of the new terms.Eq.(51) is then linearized and solved for δF_ω : the solution,integrated over frequency,determines the change of opacity with respect to the zero-order value of eq.(34). The result is given in Table (see ref. Tsytovich, Bingham, de Angelis, Forlani and Occorsio (1995)).

5. CONCLUSION

We have shown that only when the diffusion in frequency is neglected the equation of transport for photons in the solar interior can be written in the traditional form of the equation of radiative transfer (eq. 1) with the radiative opacity, defined in eq. 46, including relativistic and collective corrections to scattering and bremsstrahlung as well as corrections due to refractive index and density inhomogeneity. An estimate of the effect of these corrections on the opacity is about 9% given in Table I. The effect of stimulated scattering and frequency diffusion however changes the transport equation into a differential equation in frequency (eq. 51) which has to be solved numerically,with appropriate boundary conditions.

It is not possible to define a radiative opacity and the transport equation no longer takes the familiar form of "radiative transfer" (eq.(1)). An estimate of the effect is reported in Table I when eq.(51) is solved using perturbation theory. The effect on the Solar model, in particular on the temperature in the region of neutrino production, can only be found from SSM calculations with the present transport equation (51) replacing the equation of radiative transfer (1).

APPENDIX

THE SCATTERING CROSS-SECTIONS.

The transport cross section, defined in eq. (41), has been calculated by Tsytovich et al.(1995a,b,c). Separating scattering on electrons and scattering on ions we write:

$$\sigma^{tr}(\omega) = \sigma_e^{tr}(\omega) + \sigma_{ions}^{tr}(\omega) \tag{A1}$$

Scattering on electrons is given by:

$$\sigma_e^{tr}(\omega) = \frac{3}{4}\sigma_T \int_{-\infty}^{+\infty} \frac{e^{-y^2}}{\sqrt{\pi}} dy \int_{-1}^{+1} \frac{dx}{|F(x,y)|^2}\cdot$$

$$\cdot\left\{\frac{1}{2}(1+x^2)(1-x)H(x,y) - \frac{v_{Te}^2}{c^2}[f(x,y)ReF(x,y) - \delta_e G(x,y)]\right\} \tag{A2}$$

where $ReF(x,y)$ is the real part of the dielectric function :

$$F(x,y) = \varepsilon_{q\Omega}|_{\omega'=\omega'(\omega)} = C(x,y) + 1 +$$

$$\frac{\delta_e}{1-x}W(y)\left[1 + 2\frac{v_{Te}}{c}y\sqrt{1-x} - 2\frac{v_{Te}^2}{c^2}y^2(2-x)\right] \tag{A3}$$

where $W(y)$ is the plasma function taking into account the relativistic corrections to the electron distribution function (given by the R-factor defined in A7):

$$W(y) = 1 - y\int_{-\infty}^{+\infty} \frac{e^{-t^2}}{\sqrt{\pi}} R(t)\frac{dt}{y-t} + i\sqrt{\pi}ye^{-y^2}R(y) \tag{A4}$$

and the principal value of the integral should be taken. Without relativistic corrections ($R = 1$) it reduces to the well-known zero- order form :

$$W^0(y) = 1 - 2ye^{-y^2}\int_0^y e^{t^2}dt + i\sqrt{\pi}ye^{-y^2}$$

The first term in A3 is the contribution of electron-ion collisions and has been given by Tsytovich et al.(1995a) as :

$$C(x,y) = i\frac{\langle Z\rangle \ln\Lambda\omega_{pe}^3}{24\pi^{\frac{3}{2}}n_e v_{Te}^3}y\frac{\delta_e^{\frac{3}{2}}}{(1-x)^{\frac{3}{2}}}\left\{[E_i(y^2) - i\pi]e^{-y^2} - \frac{1}{y^2}\right\} \tag{A5}$$

where $\ln\Lambda$ is the Coulomb logarithm for electron-ion collisions and $E_i(y^2)$ is the error function. For the conditions appropriate to the solar interior the Coulomb logarithm in eq.(A5)is given by :

$$\ln\Lambda = \ln(\frac{2T}{\hbar\omega_{pe}})$$

In (A3) the dielectric function, calculated for $\omega' = \omega'(\omega)$ as given by the expansion (29) is corrected for relativistic and collisional effects (Tsytovich et al.1995b,c) which eliminate the Raman resonance occurring for :

$$\varepsilon_{q\omega}^{(0)} = 1 + \frac{\delta_e}{1-x}W^0(y) \simeq 0$$

Of the three terms in the curly brackets in (A2) the fist comes from the relativistic correction to the electron distribution function (Tsytovich et al.1995b,c) and

$$H(x,y) = \left[1 - 3\frac{v_{Te}}{c}y\sqrt{1-x} + 2\frac{v_{Te}^2}{c^2}y^2(3-2x)\right] R(y) \tag{A6}$$

$$R(y) = \frac{1}{D}e^{-\frac{3}{2}\frac{v_{Te}^2}{c^2}y^4} \int_0^{\propto} xe^{-x^2[1+\frac{3}{2}\frac{v_{Te}^2}{c^2}(x^2+2y^2)]}dx \tag{A7}$$

$$D = \int_{-\infty}^{+\infty} e^{-y^2}e^{-\frac{3}{2}\frac{v_{Te}^2}{c^2}y^4}\frac{dy}{\sqrt{\pi}} \int_0^{\propto} xe^{-x^2[1+\frac{3}{2}\frac{v_{Te}^2}{c^2}(x^2+2y^2)]}dx \tag{A8}$$

The second term is due to the relativistic correction to the probability of Thomson scattering (single electron) [Tsytovich et al.1995c] and

$$f(x,y) = \frac{1}{\sqrt{1-x}}[3 - x + x^2 - x^3 + y^2(1 + x + x^2 - x^3)]$$

The last term is due to the relativistic corrections to the probability of collective scattering [Tsytovich et al.1995c] and

$$G(x,y) = 1+x+x^2-x^3+(\frac{9}{2}+2x+\frac{9}{2}x^2)W^0(y)-2(1+x+x^2-x^3)y^2W^0(y) \tag{A9}$$

For scattering on ions we have from eq. (41):

$$\sigma_{ions}^{tr}(\omega) = \sigma_{ions}^{tr(0)} + \delta\sigma_{ions} \tag{A10}$$

where the zero-order (non-relativistic) cross-section is given by the use of the zero-order probability (eq. 24) in eq. (41) and gives the known result

$$\sigma^{tr(0)}_{ion} = \frac{3}{8}\sigma_T \delta_e [\delta_e g_0(\delta_e) - \delta g_0(\delta)] \tag{A11}$$

where

$$g_0(Z) = 2 + 2Z - (2 + 2Z + Z^2)\ln\frac{2+Z}{Z} \tag{A12}$$

The relativistic correction, due to collective scattering on the (relativistic) electrons in the shielding cloud, is given by Tsytovich et al.(1995c) as :

$$\delta\sigma_{ions} = \frac{3}{4}\sigma_T \left(\frac{v_{Te}}{c}\right)^2 \delta_e [g_1(\delta_e) - g_1(\delta)] \tag{A13}$$

where

$$g_1(Z) = \frac{28}{3} + \frac{35}{3}Z + 11Z^2 + 2Z^3 + (2 - Z + \frac{7}{2}Z^2 + \frac{3}{2}Z^3 + Z^4)\ln\frac{2+Z}{Z} \tag{A14}$$

Notice that the sum of (A11) and the zero-order from (A2) (without relativistic and collisional corrections) gives the zero-order transport cross-section (eq.2).

The cross-section for stimulated scattering $\sigma^{st}(\omega)$, defined in eq. 42, has been calculated by Tsytovich et al.(1995b) in the form:

$$\frac{\sigma^{st}(\omega)}{\sigma_T} =$$

$$-\frac{3}{4}\frac{v_{Te}}{c}\frac{z}{e^z-1}\int_{-\infty}^{+\infty} dy y \frac{e^{-y^2}}{\sqrt{\pi}}\int_{-1}^{+1} dx \frac{(1+x^2)(1+x)\sqrt{1-x}}{|F(x,y)|^2} + \frac{3}{2}\left(\frac{v_{Te}}{c}\right)^2 \frac{z}{e^z-1}\cdot$$

$$\cdot\int_{-\infty}^{+\infty} dy y^2 \frac{e^{-y^2}}{\sqrt{\pi}}\int_{-1}^{+1}(1+x^2)(1-x)\left[2(1+x) - \frac{ze^z}{e^z-1}\right]\frac{dx}{|F(x,y)|^2} \tag{A15}$$

where $z = \hbar\omega/T$.

THE FREE-FREE CROSS SECTION

The absorption coefficient $2\gamma_k$ (eq.19) has been calculated by Tsytovich et al (1995d),including collective effects,in the form :

$$2\gamma_k = \frac{8\sqrt{\pi}}{\sqrt{2}}\sigma_T \frac{e^2 < Z > n_e^2 c^4}{v_{Te}\hbar\omega^3}(1 - e^z)\mathcal{F}(\omega) \tag{A16}$$

where $z = \frac{\hbar\omega}{T}$ and :

$$\mathcal{F}(\omega) = \int_{\frac{\sqrt{z}}{2}}^{\propto} \frac{dx}{x} e^{-(x+\frac{z}{4x})^2} \left(1 + \frac{1 + 2\frac{\omega_{pe}^2}{\omega^2} x^2}{\left[1 + 2\frac{\omega_{pe}^2}{\omega^2}(1+ < Z >)x^2\right] |1 + 2\frac{\omega_{pe}^2}{\omega^2} x^2 W(x)|^2} \right) \tag{A17}$$

where $W(x)$ is the plasma function, the collective effects are all included in the integral $\mathcal{F}(\omega)$,which differs from previous results for the presence of terms due to transition bremsstrahlung. When collective effects are neglected the function $\mathcal{F}(\omega)$ becomes :

$$\mathcal{F}_o(\omega) = 2 \int_{\frac{\sqrt{z}}{2}}^{\infty} \frac{dx}{x} e^{-(x+\frac{z}{4x})^2} = \int_{\sqrt{\frac{2\hbar\omega}{m_e}}}^{\infty} \frac{e^{-\frac{v^2}{2v_{Te}^2}}}{v_{Te}^2} v \ln \frac{v + v'}{v - v'} dv \tag{A18}$$

where

$$v' = \sqrt{v^2 - \frac{2\hbar\omega}{m_e}}$$

and the second equality can easily be proved integrating by parts.Eq.(A16) with $\mathcal{F}_o(\omega)$ is the known classical non-collective result for electron-ion bremsstrahlung and Maxwellian electrons. The collective correction to inverse-bremsstrahlung defined in eq.(21) is then given by :

$$\delta\sigma_c^{ff}(\omega) = \frac{8\sqrt{\pi}}{\sqrt{2}} \sigma_T \frac{e^2 < Z > n_e c^3}{v_{Te}\hbar\omega^3} (e^z - 1)[\mathcal{F}_0(\omega) - \mathcal{F}(\omega)] \tag{A19}$$

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TABLE CAPTION

Change of opacity at the solar centre (parameters from Bahcall 1989) due to the effects listed in the first column. Here $\delta\kappa = \kappa_R - \kappa_R^0$ as given by eqs.(34,46), both calculated without σ_L (first column). The second column, ($\simeq \frac{2}{3}$ of the value in the first column) is an estimate of the change of total opacity if line absorption is included. The last column reports previous results from Iglesias and Rogers (1991): relativistic corrections, without collective effects and "static" collective effects on inverse bremsstrahlung (which overestimate the change of opacity) corrections, due to electron degeneracy (Rose 1993, based on Boercker 1987) where the relativistic and quantum recoil correction were treated without collective effect, and ion correlations (Dagdeviren and Koonin 1987) for the change of total opacity at the solar centre. The effect of quantum recoil in scattering is reported from Tsytovich et al., (1995f). (These corrections calculated with collective effects taken into account).

The electron degeneracy effect was calculated by us using the results of (Rose 1993) but excluding the relativistic and quantum recoil corrections which in (Rose 1993) were taken into account together with degeneracy corrections, but without the collective effects. This is performed to find contribution of the degeneracy effect only. The difference between the first and the third column number in the electron degeneracy effect is that in (Rose 1993) the line absorption is taken into account but not in our calculations in the whole first column. Thus to look on the relativistic non-collective corrections in (Rose 1993) one should compare the second and the third column. The 2/3 estimate for line absorption is not exact and varies from one SSM to another SSM, it could be as low as $\frac{1}{4}$. But for any existing SSM the recalculation of our correction to that which takes into account the line absorption in κ_R^0 is rather simple.

The degeneracy effect is small and other corrections calculated in first and second columns with the collective effects taken into account can be summed with the degeneracy correction.

The last line of the Table contains in the first two columns results of our calculations which do not take into account the line absorption (first column) and with line absorption taken into account (second column). The total number in the second column for the total change of opacity is obtained by the sum of the total number of the second column and the ion correlation effect off order -1.5% (the later takes into account the line absorption but for particular SSM).

TABLE I:Change of opacity $\delta\kappa/\kappa_R^0$ (in %)at the solar centre

Broadening of Raman resonance	−3.0	−2.0
Relativistic collective scattering	−0.2	−0.1
Stimulated scattering & frequency diffusion	−4.5	−3.0
Collective bremsstrahlung	−0.2	−0.1
Density inhomogeneity	−0.1	−0.07
Refractive index	+0.1	+0.07
Quantum recoil in scattering	−1.0	−0.7
Electron degeneracy	−2.0	−1.3
Ion correlations	−−	−1.5
Total	−10.9	−8.8

BROADENING OF THE RAMAN RESONANCE IN PHOTON SCATTERING IN PLASMAS

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ABSTRACT

The broadening of the Raman resonance, due to the Doppler effect and particle collisions, is considered for the scattering of radiation in plasmas. The result is used to calculate the transport cross-section for the plasma in the Solar interior and it is shown that the broadening effect can reduce the solar opacity. This result should therefore be taken into account in the Standard Solar Model, since a reduced opacity can change the predicted core temperature and hence the predicted flux of solar neutrinos.

1 INTRODUCTION

Scattering of radiation in plasmas is an important topic in the interpretation of many laboratory and astrophysical observations such as radiation from accreting disks, expanding supernova shells and photospheres of stars. Scattering of radiation also contributes to the opacity of matter in stellar interiors. In this paper we reconsider the scattering of radiation in plasmas taking into account the broadening of the Raman resonance due to the Doppler frequency shift and particle collisions. The general equation for the photon occupation number $N_{\underline{k}}$ can be written in terms of the scattering probabilities. Let $W^{\alpha}_{\underline{p},\underline{k},\underline{k}'}$ be the probability per unit time that an incident photon $(\omega,\underline{k})$ be scattered in a plasma consisting of electrons (e) and ions(i) by a particle of type $\alpha(e,i)$ with momentum $\underline{p}$ into a photon $(\omega',\underline{k}')$; then the kinetic equation for scattering in the classical limit is [1,2]

$$\frac{dN_{\underline{k}}}{dt} = -\sum_{\alpha}\int W^{(\alpha)}_{\underline{p},\underline{k},\underline{k}'}(N_{\underline{k}} - N_{\underline{k}'})f^{\alpha}(\underline{p})\frac{d\underline{k}'}{(2\pi)^3}d\underline{p}+$$

$$\sum_{\alpha}\int N_{\underline{k}}N_{\underline{k}'}W^{(\alpha)}_{\underline{p},\underline{k},\underline{k}'}\hbar\underline{q}\cdot\frac{\partial f^{\alpha}(\underline{p})}{\partial\underline{p}}\frac{d\underline{k}'}{(2\pi)^3}d\underline{p} \tag{1}$$

where the summation is over the electrons ($\alpha = e$) and all ion species ($\alpha = i$), f^{α} is the distribution function of particles of type α,$\underline{q} = \underline{k} - \underline{k}'$ and the operator on the left hand side is

$$\frac{dN_{\underline{k}}}{dt} = \frac{\partial N_{\underline{k}}}{\partial t} + \frac{\partial\omega}{\partial\underline{k}}\cdot\frac{\partial N_{\underline{k}}}{\partial\underline{r}} \tag{2}$$

The first term in the kinetic equation is spontaneous scattering, the second term represents stimulated scattering. In the non-relativistic approximation and in the high frequency limit the probability of scattering on electrons and ions are [2]

$$W^{(e)}_{\underline{p},\underline{k},\underline{k}'} = \frac{e^4}{2m_e^2\omega\omega'}(1 + cos^2\theta_{\underline{k},\underline{k}'})\frac{|1 +_i \chi^{i}_{q,\Omega}|^2}{\epsilon_{\underline{q}\Omega}}\delta(\Omega - \underline{q}\cdot\underline{v}) \tag{3}$$

$$W^{(i)}_{\underline{p},\underline{k},\underline{k}'} = Z_i^2\frac{e^4}{2m_e^2\omega\omega'}(1 + cos^2\theta_{\underline{k},\underline{k}'})\left|\frac{\chi^{(e)}_{\underline{q}\Omega}}{\epsilon_{\underline{q}\Omega}}\right|^2\delta(\Omega - \underline{q}\cdot\underline{v}) \tag{4}$$

where $\Omega = \omega - \omega'$, $\theta_{\underline{k},\underline{k}'}$ is the angle between the wave-vectors, Z_i is the ion charge and $\chi^{(e)}, \chi^{(i)}$ are the plasma electron and ion susceptibilities, such that the plasma permittivity is given by

$$\epsilon_{\underline{q}\Omega} = 1 + \chi^{(e)}_{\underline{q}\Omega} + \sum_{i}\chi^{(i)}_{\underline{q}\Omega} \tag{5}$$

where the summation is over all ion species.

The Raman resonance in photon scattering corresponds to

$$\epsilon_{\underline{q}\Omega} \simeq 0$$

which can happen for

$$\Omega \equiv \omega' - \omega \simeq \pm\omega_{\underline{q}}$$

where $\omega_{\underline{q}}$ is the eigen frequency of a plasma (Langmuir) mode at frequencies near the electron plasma frequency ω_{pe}

$$\omega' - \omega \simeq \pm\omega_{pe}, \qquad \omega_{pe} \simeq \omega_{\underline{q}} \simeq \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2}$$

.

The Raman resonance can occur in the electron scattering term where the Doppler shift due to electrons is $\omega - \omega' = (\underline{k} - \underline{k}') \cdot \underline{v}^e$; for the ions the Doppler shift $\omega - \omega' = (\underline{k} - \underline{k}') \cdot \underline{v}^i$ is usually small.

In the usual treatment of scattering, neglecting $\chi^{(i)}$ it is found in the first approximation (i.e.- neglecting the Doppler shift) that from the transport equation the scattering cross-section is proportional to

$$\int \frac{1}{\omega'} Im \left(\frac{1}{\epsilon^{(e)}_{\underline{q}\Omega}}\right) d\omega' \tag{6}$$

.

where $\epsilon^{(e)}_{\underline{q}\Omega} = 1 + \chi^{(e)}_{\underline{q}\Omega}$.

This result is well known [3] and we write it here only to show that the Raman resonance finally appears in the transport equation as $1/\epsilon^{(e)}_{\underline{q}\Omega}$ integrated over the scattered frequency ω'.

The usual integration of equation (6) (see e.g. ref [3]) is performed using the causality principle: the function $1/\epsilon^{(e)}_{\underline{q}\Omega}$ has no poles in the upper complex ω'-plane and thus the resonance in the integral appears as a principal value where the large positive and negative values of $1/\epsilon^{(e)}_{\underline{q}\Omega}$ close to the resonance compensate each other and the final result is proportional to $Re\left(1/\epsilon^{(e)}_{\underline{q},0}\right)$ which does not contain a resonance.

There are at least three processes that can produce a broadening of the resonance

i) the Doppler shift in the scattered frequencies;

ii) relativistic effects in the electron distribution;

iii) binary electron-ion collisions.

It is the aim of the present paper to consider these processes and find the correct expression for the probabilities in the transport equation.

In Section 2 the corrections (i) and (ii) are found. In Section 3 the corrections due to particle collisions are found. In Section 4 we test the effect of these corrections on the Rosseland opacity in the Solar interior.

Photon scattering is an important factor in determining the opacity in the solar interior [8]. Given that the solar neutrino flux in the ^8B channel is very sensitive to small changes to the opacity therefore even small, higher-order corrections to the present calculation of the transport cross-section for scattering in the solar interior can be important [5].

2. DOPPLER AND RELATIVISTIC CORRECTIONS TO THE RAMAN RESONANCE

The Doppler shift is given by the delta function in the expression for the scattering probability, eqs.(3) and (4). Introducing the dimensionless velocity parallel to $\underline{q}$ as

$$y = \frac{\underline{q} \cdot \underline{v}}{\sqrt{2} q v_{Te}} \tag{7}$$

where $v_{Te} = \sqrt{(\frac{T_e}{m_e})}$ is the electron thermal speed and T_e the electron temperature (energy units), the relation $\Omega = \underline{q} \cdot \underline{v}$ for $k = \frac{\omega}{c}, k' = \frac{\omega'}{c}$ can be written in the form

$$1 - \frac{\omega'}{\omega} = \sqrt{2}\frac{v_{Te}}{c} y \sqrt{[1 + (\frac{\omega'}{\omega})^2 - 2x\frac{\omega'}{\omega}]},$$

where $x = \cos\theta_{\underline{k},\underline{k}'}$. To second order in $\frac{v_{Te}}{c}$ this gives

$$\omega' = \omega - (\underline{k} - \underline{k}') \cdot \underline{v} \simeq \omega \left[1 - 2\frac{v_{Te}}{c}\mu^{1/2} y + 2\frac{v_{Te}^2}{c^2}\mu y^2\right] \tag{8}$$

$$q^2 = \left|\underline{k} - \underline{k}'\right|^2 = 2(\frac{\omega}{c})^2 \mu[1 - 4\frac{v_{Te}}{c} y \sqrt{(\mu)} + 4\left(\frac{v_{Te}}{c}\right)^2 y^2(1 + \mu)] \tag{9}$$

where we have introduced the notation $\mu = 1 - x$. From the definition of $\epsilon^{(e)}_{\underline{q}\Omega}$, after integration over the component of velocity perpendicular to $\underline{q}$, for a Maxwellian distribution function we have

$$\epsilon^{(e)}_{\underline{q}\Omega} = 1 + \left(\frac{\omega_{pe}}{q v_{Te}}\right)^2 W(y) \tag{10}$$

where the plasma function W is given by

$$W(z) = 1 - z \int \frac{e^{-y^2}}{z - y - i0} \frac{dy}{\sqrt{\pi}} = 1 - 2ze^{-z^2} \int_0^z e^{t^2} dt + i\sqrt{\pi} z e^{-z^2} \tag{11}$$

Expanding $\frac{1}{q^2}$ in eq.(10) we find (to second order in v_{Te}/c)

$$\epsilon^{(e)}_{\underline{q}\Omega} = 1 + \frac{\delta_e}{\mu} \left\{ W(y) \left[1 + 2\frac{v_{Te}}{c} y \mu^{1/2} - 2\frac{v_{Te}^2}{c^2} y^2 (1 + \mu)\right]\right\} \tag{12}$$

where δ_e is a “collective” parameter for scattering given by

$$\delta_e = \frac{\omega_{pe}^2}{2\omega^2}\frac{c^2}{v_{Te}^2} \tag{13}$$

Finally in the transport equation using the expansion (8) we can write

$$\int \frac{d^3k'}{\omega\omega'}\delta(\Omega - \underline{q}\cdot\underline{v}) = \frac{2\pi}{c^3}\int_{-1}^{1} dx \int d\omega' A(x,y)\delta(\omega' - \omega'(\omega))$$

where $\omega'(\omega)$ is given by eq.(8) and

$$A(x,y) = 1 - 3\left(\frac{v_{Te}}{c}\right)\mu^{1/2}y + 2\left(\frac{v_{Te}}{c}\right)^2 y^2(1+2\mu) \tag{14}$$

It should be stressed that although expression (12) appears in the denominator in the transport equation, no further expansion in (v_{Te}/c) can be done since there is a resonance (Raman resonance) when the denominator is zero in the first approximation. Equation (12) takes into account only the Doppler shift of the scattered frequency. For consistency it is also necessary to introduce the relativistic corrections to the electron distribution function in the expression for the dielectric function.

Taking into account the first relativistic correction, the energy of an electron is given by

$$\epsilon = \frac{1}{2}m_e v^2\left(1 + \frac{3}{4}\frac{v^2}{c^2}\right)$$

To this order the normalized electron distribution is

$$f^{(e)}(v) = C\frac{n_e}{(2\pi)^{3/2}v_{Te}^3}e^{-\frac{v^2}{2v_{Te}^2}}e^{-\frac{3}{8}\frac{v^4}{v_{Te}^2c^2}}$$

where C is the normalization constant (a function of $\frac{v_{Te}}{c}$) to be calculated from the condition

$$\int f^e(v)d^3v = n_e$$

Integrating over the velocities perpendicular to $\underline{q}$ and using again eq.(7) for the normalized parallel velocities we have

$$f_{||}^{(e)}(y) = \int f^{(e)}(v)d\underline{v}_{\perp} = \frac{n_e}{\sqrt{\pi}}e^{-y^2}R(y) \tag{15}$$

where

$$R(y) = Ce^{-\frac{3}{2}\frac{v_{Te}^2}{c^2}y^4}\int_0^{\infty} xe^{-x^2[1+\frac{3}{2}\frac{v_{Te}^2}{c^2}(x^2+2y^2)]}dx \tag{16}$$

The electron permittivity including the relativistic corrections is given by

$$\epsilon^{(e)}_{q,\Omega} = 1 - \frac{4\pi e^2}{q^2 T_e} \int \frac{(\underline{q}\cdot\underline{v}) f^{(e)}(\underline{v})}{\Omega - \underline{q}\cdot\underline{v}} d^3 v = 1 + \frac{\omega^2_{pe}}{q^2 v^2_{Te}} W_R(z) \tag{17}$$

where

$$z = \frac{\Omega}{\sqrt{2} q v_{Te}} \tag{18}$$

and

$$W_R(z) = 1 - z \int_{-\infty}^{+\infty} \frac{e^{-y^2}}{\sqrt{\pi}} R(y) \frac{dy}{z-y} + i\sqrt{\pi} z e^{-z^2} R(z) \tag{19}$$

and the principal value of the integral should be taken. Then in eq.(12) the function W has to be replaced by W_R and we have (notice that in the transport equation $z = y$ from the delta-function)

$$\epsilon^{(e)}_{q,\Omega} = 1 + \frac{\delta_e}{\mu} W_R(y)[1 + 2\frac{v_{Te}}{c} y\sqrt{\mu} - 2\frac{v^2_{Te}}{c^2} y^2 (1+\mu)] \tag{20}$$

which now includes also the relativistic correction to the distribution function.

3. CORRECTIONS DUE TO COLLISIONS

We consider in this section the corrections to the electron susceptibility $\chi^{(e)}_{k\omega}$ due to electron-ion collisions. Near resonance $\epsilon^{(e)}_{k\omega} = 1 + \chi^{(e)}_{k\omega} \simeq 0$ a change in $\delta\chi^{(e)}_{k\omega}$ due to collisions can be important, even if it is a small correction, as it determines the width and shape of the resonance.

It should be stressed that the collisional corrections are considered here for the longitudinal electron permittivity and they therefore describe the effect of inverse bremsstrahlung of longitudinal waves. The collisional corrections can thus be considered as the influence of inverse bremsstrahlung of longitudinal waves on the scattering of the transverse waves.

Since the direct (spontaneous) bremsstrahlung of longitudinal waves is much more effective than that of transverse waves (for non relativistic velocities) this cross-effect is important, but its consequences have really to be taken into account only inside the Raman resonance, when $\chi^{(e)}_{k\omega} \simeq -1$.

For longitudinal waves we shall derive the effect of the electron-ion collisions starting with the kinetic equation for the electron distribution function $f(\underline{r}, \underline{v}, t)$ including the Landau term to lowest order

$$\frac{\partial f}{\partial t} + (\underline{v} \cdot \underline{\nabla}) f - \frac{e}{m_e} \underline{E} \cdot \frac{\partial f}{\partial \underline{v}} = \sum_i \frac{2\pi ln \Lambda \, n_i Z_i^2 e^4}{m_e^2} \frac{\partial}{\partial v_l} \frac{1}{v} (\delta_{lj} - \frac{v_l v_j}{v^2}) \frac{\partial f}{\partial v_j} \qquad (21)$$

where the sum is over all ion species (of charge Z_i and density n_i) and $ln\Lambda$ is the Coulomb logarithm and the relative velocity has been taken as the electron velocity.

We solve eq.(21) using perturbation theory, i.e. we write $f(\underline{r}, \underline{v}, t)$ in the form

$$f(\underline{r}, \underline{v}, t) = F(v) + \delta f^{(0)} + \delta f^{(1)} \qquad (22)$$

where $F(\underline{v})$ is the initial (Maxwellian) distribution, $\delta f^{(0)}$ is the perturbation due to the field $\underline{E}$ and $\delta f^{(1)}$ is due to collisions.

The two perturbations satisfy the following equations

$$\frac{\partial \delta f^{(0)}}{\partial t} + (\underline{v} \cdot \underline{\nabla}) \delta f^{(0)} = \frac{e}{m_e} \underline{E} \cdot \frac{\partial F(v)}{\partial \underline{v}} = -\frac{e}{m_e} \frac{\underline{E} \cdot \underline{v}}{v_{Te}^2} F(v) \qquad (23)$$

$$\frac{\partial \delta f^{(1)}}{\partial t} + (\underline{v}\cdot\underline{\nabla})\delta f^{(1)} = \sum_i \frac{2\pi ln \wedge n_i Z_i^2 e^4}{m_e^2} \frac{\partial}{\partial v_l}\frac{1}{v}(\delta_{lj} - \frac{v_l v_j}{v^2})\frac{\partial \delta f^{(0)}}{\partial v_j} \tag{24}$$

The solution to eq.(23) in Fourier space is

$$\delta f^{(0)}_{k\omega} = -i\frac{e}{(\omega - \underline{k}\cdot\underline{v})}\frac{\underline{k}\cdot\underline{v}}{m_e k v_{Te}^2}F(v)E_k \tag{25}$$

where we have used $\underline{E}_k = \frac{\underline{k}}{k}E_k$ for the longitudinal field.
Notice that $\delta f^{(0)}_{k\omega}$ depends only on the angle

$$x = \frac{\underline{k}\cdot\underline{v}}{kv} \tag{26}$$

and on the absolute value of the velocity v and can be written as

$$\delta f^{(0)}_{k\omega} = -i\frac{e}{m_e k v_{Te}^2}F(v)\left(\frac{\omega}{\omega - \underline{k}\cdot\underline{v}} - 1\right)E_k \tag{27}$$

In equation (24) we can then transform the derivatives with respect to velocity components into derivatives with respect to x and the solution in Fourier space is then

$$\delta f^{(1)}_{k\omega} = \frac{i}{\omega - \underline{k}\cdot\underline{v}}\langle Z\rangle \frac{\omega_{pe}^2 e^2 ln\wedge}{2 m_e v^3}\frac{\partial(1-x^2)}{\partial x}\frac{\partial \delta f^{(0)}_{k\omega}}{\partial x} \tag{28}$$

or, using (27)

$$\delta f^{(1)}_{k\omega} = \langle Z\rangle \frac{e^3 \omega_{pe}^2 \omega ln\wedge}{m_e^2 v_{Te}^2 v^2}\frac{kv - \omega x}{(\omega - kvx)^4}F(v)E_k \tag{29}$$

where

$$\langle Z\rangle = \frac{\sum_i n_i Z_i^2}{n_e} = \frac{\sum_i n_i Z_i^2}{\sum_i n_i Z_i} \tag{30}$$

The change in the electron permittivity corresponding to the perturbation (29), due to collisions, is given by

$$\delta\epsilon^{(1)}_{k\omega} = \frac{4\pi e}{ikE_k}\int \delta f^{(1)}_{k\omega} d^3v = i\frac{\langle Z\rangle \omega_{pe}^6 ln\wedge z^4}{(2\pi)^{3/2} n_e \omega^3 v_{Te}^3}\int_0^\infty e^{-y^2}dy\int_{-1}^{1}\frac{zx - y}{(z - yx)^4}dx \tag{31}$$

where

$$z = \frac{\omega}{\sqrt{2}kv_{Te}} \; , \; y = \frac{v}{\sqrt{2}v_{Te}}$$

The last integral over the angle in equation (31) can be done to yield

$$\delta\epsilon_{k\omega}^{(1)} = \frac{2}{3}i\frac{\langle Z\rangle\omega_{pe}^{6} ln \wedge z^{4}}{(2\pi)^{3/2}n_e\omega^3 v_{Te}^3}\int_0^{\infty}\frac{ye^{-y^2}dy}{[(z+i0)^2-y^2]^2} \tag{32}$$

The integral in equation (32) is most conveniently expressed through the error-function

$$Ei(z^2) = \int_{-\infty}^{z^2}\frac{e^t}{t}dt \tag{33}$$

and

$$\int_0^{\infty}\frac{e^{-t}}{(z+i0)^2-t}dt = e^{-z^2}(Ei(z^2)-i\pi)$$

allowing the integral in equation (32) to become

$$2\int_0^{\infty}\frac{e^{-y^2}ydy}{[(z+i0)^2-y^2]^2} = -\frac{\partial}{\partial z^2}\int_0^{\infty}\frac{e^{-t}dt}{(z+i0)^2-t} = e^{-z^2}(Ei(z^2)-i\pi)-\frac{1}{z^2}$$

which results finally in the following expression for $\delta\epsilon_{k\omega}^{(1)}$

$$\delta\epsilon_{k\omega}^{(1)} = i\frac{\langle Z\rangle}{3(2\pi)^{3/2}}\left(\frac{\omega_{pe}}{\omega}\right)^3\frac{\omega_{pe}^3 ln \wedge z^4}{n_e v_{Te}^3}\left\{e^{-z^2}[Ei(z^2)-i\pi]-\frac{1}{z^2}\right\} \tag{34}$$

The probability of scattering depends on $\epsilon_{\underline{q},\Omega}$ and since we take $\Omega = \underline{q}\cdot\underline{v} = \sqrt{2}qv_{Te}y$ from the δ-function, therefore in equation (34) we have to take

$$z = \frac{\Omega}{\sqrt{2}qv_{Te}} = y \tag{35}$$

and to lowest order in q we take

$$\Omega = 2\omega\left(\frac{v_{Te}}{c}\right)\mu^{1/2}y \tag{36}$$

then

$$\delta\epsilon_{\underline{q}\Omega}^{(1)} = \frac{i\langle Z\rangle ln \wedge \delta_e^{3/2}}{24\pi^{3/2}\mu^{3/2}}\frac{\omega_{pe}^3}{n_e v_{Te}^3}y\left\{[Ei(y^2)-i\pi]e^{-y^2}-\frac{1}{y^2}\right\} \tag{37}$$

where δ_e is given by eq.(13).

Eq.(37) gives the change in the electron permittivity due to electron-ion collisions and has to be included in the r.h.s. of equation (12) to find the total corrections to the electron permittivity which can now be written in the final form

$$\epsilon_{q\Omega}^{(e)} = 1 + \frac{\delta_e}{\mu} W_R(y) \left[1 + \frac{2v_{Te}}{c} y \mu^{1/2} - 2\frac{v_{Te}^2}{c^2} y^2 (1+\mu) \right] +$$

$$+ i \frac{\langle Z \rangle ln \wedge \omega_{pe}^3}{24\pi^{3/2} n_e v_{Te}^3} y \frac{\delta_e^{3/2}}{\mu^{3/2}} \left\{ [Ei(y^2) - i\pi] e^{-y^2} - \frac{1}{y^2} \right\} \tag{38}$$

This last expression concludes the main aim of this work to find the plasma dielectric function including all the effects which broaden the Raman resonance in photon scattering in plasmas. It should be used in the scattering probabilities (eqs.3 and 4) in the transport equation (1) together with the result (13) which is a consequence of the Doppler frequency shift in photon scattering.

4. THE TRANSPORT EQUATION AND THE OPACITY IN THE SOLAR INTERIOR

In this Section we consider the effect of resonance broadening on the transport cross-section for photon scattering in the solar interior and calculate numerically the change in the opacity due to this effect. It should be noted that the absence of a resonance for scattering on ion sound waves is due to the fact that ion-sound waves are heavily damped for $T_i \simeq T_e$, which is assumed to be the case in the solar interior where there is a high collision rate. In the solar interior the scattering on electrons and ions give almost equal contributions and we include both in eq.(1). It is known (see e.g. Ref.[6]) that inverse bremsstrahlung dominates for low frequencies and that scattering at high frequencies ($\omega \gg \omega_{pe}$) really contributes to the opacity, which justifies our assumption of the high-frequency limit. Also notice that in the sun the peak of the black body spectrum occurs for $\omega_{peak} \sim (6 \div 7)\omega_{pe}$, thus the factor $(\omega/\omega_{pe})^2$ is large and satisfies this assumption.

The Raman resonance is allowed for $\omega > 2\omega_{pe}$ and is therefore possible for the most important frequencies. Also for $y \sim 6 \div 7$ the imaginary part related to Landau damping (see the function $W(y)$) is negligible so that in eq.(5) values very close to zero are possible and the Raman resonance is well pronounced. In the solar interior the electron-ion collision frequency is $\nu_{ei} \sim 10^{-1}\omega_{pe}$ and since the most important frequency domain, as we have seen, is $\omega \simeq (6 \div 7)\omega_{pe}$ then $\nu_{ei}/\omega \sim 1/60$ whereas $v_{Te}/c \sim 1/20$. We therefore expect the collisional corrections to be smaller than the Doppler corrections (at least in the central solar region): this is in fact confirmed by the numerical calculation. For the solution of eq.(1) in the solar interior the photon distribution is assumed close to a black-body distribution $N_\omega^{(o)}$, where

$$N_\omega^{(o)} = \{exp(\hbar\omega/T) - 1\}^{-1} \tag{39}$$

Assuming a small deviation δN_ω (responsible for the flux of radiation) which in the diffusion approximation can be written as [6]

$$N_{\underline{k}} = N_\omega^{(o)} + cos\theta_{\underline{k},\underline{n}}\delta N_\omega \qquad \delta N_\omega \ll N_\omega^{(o)} \tag{40}$$

where $\underline{n} = \underline{r}/r$ and $\theta_{\underline{k},\underline{n}}$ is the angle between the wave-vector $\underline{k}$ and the direction of the propagation of the flux $\underline{n}$.

It is important to stress that the classical forms of the transport equation and photon distribution (eq.s 1 and 39) can be used for $\hbar k \ll p$, where p is the particle momentum, i.e. for

$$\hbar\omega \ll T\frac{c}{v_{Te}} \tag{41}$$

This condition is therefore satisfied also for $\hbar\omega \sim T$ since $c/v_{Te} \geq 20$ in the solar interior.

The second term in (1) after substitution of (40) for $N_{\underline{k}'}$ contains the term $cos\theta_{\underline{k}',\underline{n}}$, since the $\underline{k}'$ integration also contains an integral over the angles of $\underline{k}'$ we can average $cos\theta_{\underline{k}',\underline{n}}$ in the plane perpendicular to $(\underline{k},\underline{k}')$ such that

$$\overline{cos\theta_{\underline{k}',\underline{n}}} = \overline{cos\theta_{\underline{k},\underline{n}}cos\theta_{\underline{k},\underline{k}'}} + \overline{sin\theta_{\underline{k},\underline{n}}\, sin\theta_{\underline{k},\underline{k}'}\, cos(\phi-\phi')} = cos\theta_{\underline{k},\underline{n}}cos\theta_{\underline{k},\underline{k}'}$$

Notice that this is possible since the probability $W^{(\alpha)}_{\underline{p},\underline{k},\underline{k}'}$ only depends on $\underline{q} = \underline{k} - \underline{k}'$ i.e. the angle $\theta_{\underline{k},\underline{k}'}$.

Equation (1) then becomes, assuming steady state and neglecting the stimulated term

$$\begin{aligned} cos\theta_{\underline{k},\underline{n}}c\frac{\partial N^{(o)}_{\omega}}{\partial r} &= -cos\theta_{\underline{k},\underline{n}}\delta N_{\omega}\sum_{\alpha}\int W^{(\alpha)}_{\underline{p},\underline{k},\underline{k}'}f^{\alpha}(\underline{v})d^3k'd\underline{v} + \\ &\quad cos\theta_{\underline{k},\underline{n}}\sum_{\alpha}\int W^{(\alpha)}_{\underline{p},\underline{k},\underline{k}'}\delta N_{\omega'}cos\theta_{\underline{k},\underline{k}'}\, f^{\alpha}(\underline{v})d^3k'd\underline{v} \quad (42) \longrightarrow \end{aligned}$$

To a first approximation we can take $\delta N_{\omega'} \approx \delta N_{\omega}$ in the last term of eq.(42) (notice that the additional terms neglected here lead to a diffusion of radiation in frequency which is the subject of a companion paper [9]. Introducing the black-body energy density B_{ω} and the energy flux $\mathcal{F}_{\omega}$, defined as

$$B_{\omega} = \frac{\hbar\omega^3}{2\pi^2c^2}N^{(o)}_{\omega}\ ;\ \mathcal{F}_{\omega} = \frac{2\hbar\omega^3}{3\pi c^2}\delta N_{\omega} \tag{43}$$

the transport equation takes the form

$$\frac{1}{3}\frac{\partial B_{\omega}}{\partial r} = -\frac{\mathcal{F}_{\omega}}{4\pi}n_e\sigma^{tr}(\omega) \tag{44}$$

where the transport cross-section of scattering is given by

$$\sigma^{tr}(\omega) = \frac{1}{cn_e}\sum_{\alpha}\int W^{(\alpha)}_{\underline{p},\underline{k},\underline{k}'}f^{\alpha}(\underline{v})d^3k'(1 - cos\theta_{\underline{k},\underline{k}'})d\underline{v} \tag{45}$$

where n_e is the electron number density.

If the Doppler shift is neglected then we have

$$\delta(\Omega - \underline{q} \cdot \underline{v}) \simeq \delta(\omega - \omega')$$

and the A factor from eq.(14) is equal to 1.

Consequently $\epsilon^{(e)}_{q\Omega}$ can be approximated as

$$\epsilon^{(e)}_{\underline{q},\omega\approx\omega'} = 1 + \frac{\omega^2_{pe}}{q^2 v^2_{Te}} W(y) \simeq 1 + \frac{\delta_e}{1 - cos\theta_{\underline{k},\underline{k}'}} W(y)$$

where in the second equality we have used to lowest order

$$q^2_{\omega\simeq\omega'} \simeq 2k^2(1 - cos\theta_{\underline{k},\underline{k}'}) = 2\,\frac{\omega^2}{c^2}(1 - cos\theta_{\underline{k},\underline{k}'}) \tag{46}$$

The zero-order transport cross-section $\sigma^{tr}_o(\omega)$ can then be found from (45) as the sum of two contributions

$$\sigma^{tr}_o(\omega) = \sigma^e(\omega) + \sigma^i(\omega)$$

$$\sigma^i(\omega) = \sum_{j=ions} \frac{n_j}{n_e} \sigma^j(\omega)$$

where $\sigma^e(\omega)$ is the cross-section of scattering on electrons and $\sigma^i(\omega)$ is the part due to scattering on ions, due to all ion species present with number density n_j.

The result is well known and can be written in the form (see e.g. Ref.[4])

$$\sigma^{tr}_o(\omega) = \sigma_T \left\{ 1 - \frac{3}{8}\delta_e \left[(2 + 2\delta + \delta^2)\delta\; ln\left(\frac{\delta}{2+\delta}\right) + 2\delta + 2\delta^2 + \frac{8}{3} \right] \right\} \tag{47}$$

where

$$\sigma_T = \frac{8}{3}\pi \left(\frac{e^2}{m_e c^2}\right)^2 \tag{48}$$

is the Thomson cross-section and

$$\delta = \delta_e(1 + \langle Z \rangle) \tag{49}$$

Eq.(47) takes into account the collective effects in scattering of radiation neglecting the broadening of the Raman resonance and the Doppler frequency shift.[1] Integrating over frequency the transport equation (44), including the effects of inverse bremsstrahlung and line-absorption which were not included in eq.(1), and defining $\mathcal{F}(r)$ as the integrated flux, we obtain the equation of radiative transfer in the form

$$\mathcal{F}(r) = -\left(\int_0^\infty \frac{\partial B_\omega}{\partial T} d\omega\right) \frac{4\pi}{3\rho\kappa} \frac{dT}{dr} \tag{50}$$

where we have used

$$\frac{\partial B_\omega}{\partial r} = \frac{\partial B_\omega}{\partial T} \frac{dT}{dr}$$

and defined the Rosseland opacity as

$$\rho\kappa = \frac{\int_0^\infty \frac{\partial B_\omega}{\partial T} d\omega}{\int_{\omega_{pe}}^\infty \frac{1}{n_e(\sigma^{tr}+\sigma^{br}+\sigma^L)} \frac{\partial B_\omega}{\partial T} d\omega} \tag{51}$$

where ρ is the mass density, σ^{br} and σ^L are the cross-sections for inverse bremsstrahlung (free-free transitions) and line absorption respectively. These have to be added to the scattering cross-section to account for all processes affecting photon propagation [6]. Since the total flux (integrated from zero to the solar radius) is constant (solar luminosity) small changes in κ can affect the temperature gradient in eq.(50) and lower values of κ can result in lower central temperatures and therefore in lower fluxes of neutrinos as suggested by the experimental results [11]. Denoting κ_o as the value of the opacity when σ_o^{tr} is used in the calculation, we have calculated κ using eq.(45) for σ^{tr} including the broadening corrections. Notice that also σ^{br} and σ^L have to be corrected for collective and relativistic effects this is done in separate papers[10,13]. In the present calculation our aim is only to find the effect due to broadening as given in this paper, we have therefore used the zero-order (single particle, non relativistic) inverse-bremsstrahlung cross-section and neglected line absorption. We have calculated κ for values of the parameters corresponding to the solar centre, as given in Ref.[8], and in this case line absorption is only due to iron (all other species being fully ionized) and is known to account for $\simeq \frac{1}{3}$ of the total opacity.

For the scattering on ions the broadening effects can be neglected and the ion contribution to the scattering cross-section is taken as

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[1] It was also previously used, including corrections due to electron degeneracy, calculate the opacity in the solar interior [7], but relativistic and Doppler corrections were incorporated in [7] incorrectly.

$$\frac{\sigma^{tr}_{ions}}{\sigma_T} = \frac{3}{8}\left\{\delta_e^2\left[(2+2\delta_e+\delta_e^2)ln\frac{\delta_e}{2+\delta_e}+2+2\delta_e\right]-\delta_e\delta\left[(2+2\delta+\delta^2)ln\frac{\delta}{2+\delta}+2+2\delta\right]\right\} \quad (52)$$

which is just the ion contribution to the cross-section (eq.47). The electron cross-section is found using (3), (13) and (38) in eq.(45) and can be written after integration over the perpendicular component of velocity using eq.(15) (since the probability does not depend on it) and over ω', as

$$\frac{\sigma^{tr}_{elec}}{\sigma_T} = \frac{3}{8}\int_{-\infty}^{+\infty} R(y)\frac{e^{-y^2}}{\sqrt{\pi}}dy\int_{-1}^{1} dx(1+x^2)(1-x)^4\frac{A(x,y)}{F_R^2(x,y,z)+F_I^2(x,y,z)} \quad (53)$$

where

$$z=\frac{\hbar\omega}{T}\ ,z_0=\frac{\hbar\omega_{pe}}{T}\ ,\delta_e(z)=\frac{z_0^2}{2z^2}\frac{m_ec^2}{T} \quad (54)$$

$$\begin{aligned}F_R(x,y,z) &= (1-x)^{\frac{3}{2}}+L\delta_e^{\frac{3}{2}}(z)y\pi e^{-y^2}+\\ &+\delta_e(z)(1-x)^{\frac{1}{2}}ReW_R(y)\left[1+2\left(\frac{T}{m_ec^2}\right)^{\frac{1}{2}}(1-x)^{\frac{1}{2}}y-2\frac{T}{m_ec^2}y^2(2-x)\right]\end{aligned} \quad (55)$$

$$\begin{aligned}F_I(x,y,z) &= L\delta_e^{\frac{3}{2}}(z)y\left[e^{-y^2}Ei(y^2)-\frac{1}{y^2}\right]+\\ &+\delta_e(z)(1-x)^{\frac{1}{2}}ImW_R(y)\left[1+2\left(\frac{T}{m_ec^2}\right)^{\frac{1}{2}}(1-x)^{\frac{1}{2}}y-2\frac{T}{m_ec^2}y^2(2-x)\right]\end{aligned} \quad (56)$$

where

$$L=\langle Z\rangle\frac{ln\Lambda}{24\pi^{\frac{3}{2}}}\frac{\omega_{pe}^3}{n_ev_{Te}^3} \quad (57)$$

In the solar interior we have used the Born approximation for the Coulomb logarithm

$$\ln\Lambda=\ln\frac{r_{max}}{r_{min}} \quad (58)$$

where

$$r_{max} = \frac{v_{Te}}{\omega_{pe}}; r_{min} = \frac{\hbar}{2m_e v_{Te}}$$

The continuous part of the opacity can finally be calculated as (see eq.(51))

$$\rho\kappa = \int_0^\infty \frac{z^4 e^z}{(e^z-1)^2} dz \left[\int_{z_0}^\infty \frac{z^4 e^z}{n_e[\sigma_{elec}^{tr}(z) + \sigma_{ion}^{tr}(z) + \sigma^{br}(z)](e^z-1)^2} dz \right]^{-1} \quad (59)$$

where the scattering cross-sections $\sigma_{ion}^{tr}(z)$ and $\sigma_{elec}^{tr}(z)$ are given by equations (52) and (53) and line-absorption has been neglected.

This provides a new expression for the opacity in the solar interior and includes effects of broadening of the Raman resonance due to frequency Doppler shift and collisions. This new result treats the ion and electron contributions to scattering separately and only corrects for the electron term. At the solar centre the numerical result gives a decrease of 3.0% with respect to κ_o (also calculated without line absorption). Of course the decrease of the total opacity will be smaller. It should be stressed that broadening of the Raman resonance is one of the effects which can modify the scattering cross-section and therefore the opacity. Other effects, which can be of the same order in the solar interior are considered in companion papers; these include the relativistic corrections to the scattering amplitudes[10], the contribution of induced scattering and frequency diffusion during radiative transport[9] and Raman scattering on plasmons[14]. Taken all together, including corrections to inverse-bremsstrahlung and line absorption, these corrections can decrease the solar opacity by a significant amount for the results of the Solar Model and the solar neutrino flux.

5.CONCLUSIONS

In the kinetic equation for photon scattering in plasmas we have considered the corrections to the dielectric function (broadening of the Raman resonance) due to the frequency Doppler shift, electron ion collisions and relativistic (electron) distribution function. The resulting form of the dielectric function is given by eq.(38). We have tested the importance of these corrections for the transport of radiation in the solar interior and found that they produce a reduction of 3.0% of the radiative opacity (scattering+inverse-bremsstrahlung) at the solar centre where the radiative opacity is known to contribute approximately $\frac{2}{3}$ of the total opacity, which includes absorption from partially ionized iron. This reduction is not negligible, given that the uncertainty in the opacity is currently assumed to be at most 5% in Solar Models [11]. However this is only one of a number of relativistic, quantum and collective corrections to radiation transport that can change the solar opacity and are taken into account in separate papers. Only the inclusion of all the effects in the equations of the solar model can provide an answer on the importance of these corrections for the solar neutrino problem since a decrease of the opacity could give lower temperatures in the core region, where thermonuclear reactions occur, and therefore lower neutrino fluxes possibly in agreement with experimental results.

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STIMULATED SCATTERING AND FREQUENCY DIFFUSION OF PHOTONS IN PLASMAS

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ABSTRACT

Stimulated scattering on electrons and frequency diffusion in the transport equation for photons in plasmas are considered including the collective plasma effects. The results are applied to the solar interior (where collective plasma effects are not negligible for a wide range of frequencies) to find the effect on the solar opacity.

1.INTRODUCTION

This is the third in a series of papers dealing with relativistic corrections to scattering of photons in plasmas and their effects on the solar opacity. In the first and third papers in the series [1],[2] the corrections to spontaneous scattering were found. Here we consider the induced scattering of photons by the plasma electrons and the effect of frequency diffusion taking into account the collective plasma effects.

The transport equation for the distribution function $N_{\underline{k}}$ of photons of wavenumber $\underline{k}$ and frequency $\omega_k = \omega(k)$ can be written, in the quasi-classical approximation, as [1]

$$\begin{aligned}\frac{dN_{\underline{k}}}{dt} = &- N_{\underline{k}} \int \left[W^{(e)}_{\underline{k},\underline{k}'} f^{(e)}(\underline{p}) + \sum_i W^{(i)}_{\underline{k},\underline{k}'} f^{(i)}(\underline{p}) \right] \frac{d^3p d^3k'}{(2\pi)^6} + \\ &+ \int N_{\underline{k}'} \left[W^{(e)}_{\underline{k},\underline{k}'} f^{(e)}(\underline{p}) + \sum_i W^{(i)}_{\underline{k},\underline{k}'} f^{(i)}(\underline{p}) \right] \frac{d^3p d^3k'}{(2\pi)^6} + \\ &+ N_{\underline{k}} \int N_{\underline{k}'} W^{(e)}_{\underline{k},\underline{k}'} \hbar(\underline{k} - \underline{k}') \cdot \frac{\partial f^{(e)}_{(\underline{p})}}{\partial \underline{p}} \frac{d^3p d^3k'}{(2\pi)^6} \end{aligned} \tag{1}$$

where

$$\frac{dN_{\underline{k}}}{dt} = \frac{\partial N_{\underline{k}}}{\partial t} + \underline{v}_g \cdot \frac{\partial N_{\underline{k}}}{\partial \underline{r}}$$

and

$$\underline{v}_g = c\sqrt{\epsilon(\omega)} \frac{\underline{k}}{k} \; , \qquad \epsilon(\omega) = \left(1 - \frac{\omega_{pe}^2}{\omega^2} \right)$$

is the group velocity, $\omega_{pe} = \sqrt{4\pi e^2 n_e / m_e}$ is the plasma frequency, $f^{(\alpha)}(\underline{p})$ is the electrons'($\alpha = e$) or ions' ($\alpha = i$) distribution function in the plasma and $W^{(\alpha)}_{\underline{k},\underline{k}'}$ is the probability of scattering on electrons or ions respectively.

The first two terms on the R.H.S. of (1) represent the spontaneous processes and the last term gives the contribution of stimulated scattering on electrons (we neglect stimulated scattering on ions which is smaller by the factor m_e/m_i).

In the present paper we evaluate the contributions of stimulated scattering on electrons (last term of eq.1) and of the frequency diffusion terms to the transport equation (Section 2). The effects on the Rosseland opacity in the solar interior are found in Section 3.

We shall consider the corrections up to second order in v_{Te}/c, where $v_{Te} = (T_e/m_e)^{\frac{1}{2}}$ is the electron thermal velocity.

The presence of $\underline{k}-\underline{k}'$ in the stimulated term makes this term already a 1^{st} order term in (v_{Te}/c) (from the Doppler shift due to the conservation law, expressed through the δ-function in the probability, see section 2). As a consequence we only need the zero order scattering probability and we can neglect the relativistic correction to $f^{(e)}(p)$ (which are also of order v_{Te}^2/c^2) and assume a Maxwellian distribution in the following. The relativistic corrections to the distribution function and to the scattering probabilities were considered in the first two papers [1,2].

2.THE TRANSPORT EQUATION

The zero order scattering probability on electrons can be written as [1]

$$W_o^{(e)} = \frac{(2\pi)^3 e^4}{2m_e^2\omega\omega'}(1+x^2)\left|\frac{1+\sum_i \chi_{q\Omega}^{(i)}}{\epsilon_{\underline{q}\Omega}}\right|^2 \delta(\Omega - \underline{q}\cdot\underline{v}) \tag{2}$$

where $\underline{q} = \underline{k} - \underline{k}', \Omega = \omega - \omega', \theta_{\underline{k},\underline{k}'}$ is the angle of the $\underline{k}$ and $\underline{k}'$ vectors, $x = \cos\theta_{\underline{k},\underline{k}'}$, $\chi_{\underline{k}\omega}^{(i)}$ are the ion susceptibilities, $\epsilon_{\underline{k}\omega}$ the plasma dielectric function.

In equation (1) we perform the integration over the component of the electron velocity perpendicular to $\underline{q}$, neglecting the ions and use the properties of the Maxwellian distribution then eq.(1) becomes

$$\frac{dN_{\underline{k}}}{dt} = n_e c\frac{3}{8}\sigma_T \int_{-\infty}^{+\infty}\frac{dy}{\sqrt{\pi}}e^{-y^2}\int_{-1}^{1} dx(1+x^2)\times$$

$$\times\int\frac{d\omega'}{|\epsilon_{\underline{q}\Omega}|^2}\left(N_{\underline{k}'} - N_{\underline{k}} - N_{\underline{k}}N_{\underline{k}'}\cdot\frac{\hbar \underline{q}\cdot\underline{v}}{m_e v_{Te}^2}\right)\frac{\omega'}{\omega}\sqrt{\epsilon(\omega')}\delta(\Omega - \underline{q}\cdot\underline{v}) \tag{3}$$

where

$$y = \frac{\underline{q}\cdot\underline{v}}{\sqrt{2}qv_{Te}} \tag{4}$$

is the dimensionless component of the velocity parallel to $\underline{q}$. We have introduced the Thomson cross-section σ_T given by

$$\sigma_T = \frac{8}{3}\pi\frac{e^4}{m_e^2 c^4}$$

and changed the k'-integration using the dispersion relation for electromagnetic waves

$$\int \frac{d^3k'}{\omega'} = 2\pi\int_{-1}^{1} dx \int_0^{\infty} dk'\frac{k'^2}{\omega'} = \frac{2\pi}{c^3}\int_{-1}^{1} dx \int d\omega'\omega'\sqrt{\epsilon(\omega')}$$

Let us consider, now, the usual expansion for $N_{\underline{k}}$ namely [1,2]

$$N_{\underline{k}} = N_\omega^{(0)} + \cos\theta_{\underline{k},\underline{n}}\delta N_\omega \tag{5}$$

where

$$N_\omega^{(0)} = \left\{exp\left(\frac{\hbar\omega}{T}\right) - 1\right\}^{-1} \tag{6}$$

is the thermal photon distribution, δN_ω the deviation responsible for the flux of radiation, $\underline{n}$ the unit vector in the direction of flux propagation and T the thermal energy.

After substitution of eq.(5) in eq.(3) we obtain the linearized transport equation (for steady-state) in the following form [the thermal distribution (6) gives exactly zero in the r.h.s. of eq.(1)]

$$\sqrt{\epsilon(\omega)}\frac{\partial N_\omega^{(0)}}{\partial r} = -n_e(\sigma^{tr}\delta N_\omega - \sigma^{st}\delta N_\omega) \tag{7}$$

where the first term was obtained in Paper I and is given by

$$\sigma^{tr}\delta N_\omega = -\frac{3}{8}\sigma_T\int_{-\infty}^{+\infty}\frac{dye^{-y^2}}{\sqrt{\pi}}\int_{-1}^{1}dx(1+x^2)\times$$

$$\int_0^{\infty}\frac{d\omega'}{|\epsilon_{\underline{q}\Omega}|^2}(x\delta N_{\omega'} - \delta N_\omega)\frac{\omega'}{\omega}\sqrt{\epsilon(\omega')}\delta(\Omega - \underline{q}\cdot\underline{v}) \tag{8}$$

and the second term (stimulated scattering) follows from eq.(3) and is given by

$$\sigma^{st}\delta N_\omega = -\frac{3}{8}\sigma_T\int_{-\infty}^{+\infty}\frac{dye^{-y^2}}{\sqrt{\pi}}\int_{-1}^{1}dx(1+x^2)\times$$

$$\int_0^{\infty}\frac{d\omega'}{|\epsilon_{\underline{q}\Omega}|^2}\left[N_{\omega'}^{(0)}\delta N_\omega + xN_\omega^{(0)}\delta N_{\omega'}\right]\frac{\hbar\underline{q}\cdot\underline{v}}{m_e v_{Te}^2}\frac{\omega'}{\omega}\sqrt{\epsilon(\omega')}\delta(\Omega - \underline{q}\cdot\underline{v}) \tag{9}$$

It is important to stress that in the zero-order (non relativistic) solution to these equations $\omega' = \omega$, since the Doppler shift (introduced by the δ -function [1]) is of order v_{Te}/c.

If the Doppler corrections are included expanding $N^{(0)}_{\omega'}$ and $\delta N_{\omega'}$ in powers of $\omega' - \omega$, σ^{tr} and σ^{st} become differential operators in frequency so that the transport equation contains diffusion terms in frequency.

From the δ-function in eq.(8,9) we have the Doppler shift

$$\Omega \equiv \omega - \omega' = \underline{q} \cdot \underline{v} = \sqrt{2} q v_{Te} y \tag{10}$$

where

$$q = |\underline{k} - \underline{k}'| = \frac{\omega}{c}\left[1 + \left(\frac{\omega'}{\omega}\right)^2 - 2\frac{\omega'}{\omega}x\right]^{\frac{1}{2}} \tag{11}$$

Solving this equation to second order in $\frac{v_{Te}}{c}$ we obtain

$$\frac{\omega'}{\omega} = 1 - 2\frac{v_{Te}}{c}y\sqrt{1-x} + 2\left(\frac{v_{Te}}{c}\right)^2 y^2(1-x) \tag{12}$$

Then in the integrand of eq.(8,9) we can use

$$\frac{\omega'}{\omega}\delta(\Omega - \underline{q}\cdot\underline{v}) = [1 - 3\frac{v_{Te}}{c}y\sqrt{1-x} + 2\frac{v_{Te}^2}{c^2}y^2(3-2x)]\delta(\omega'(\omega) - \omega'), \tag{13}$$

and

$$\frac{\hbar \underline{q}\cdot\underline{v}}{m_e v_{Te}^2}\frac{\omega'}{\omega}\sqrt{\epsilon(\omega')}\delta(\Omega - \underline{q}\cdot\underline{v}) =$$

$$+\frac{\hbar\omega}{T}2\frac{v_{Te}}{c}y\sqrt{1-x}\left(1 - 4\frac{v_{Te}}{c}y\sqrt{1-x}\right)\delta[\omega'(\omega) - \omega'] \tag{14}$$

where $\omega'(\omega)$ is given by eq.(12),

$$N^o_{\omega'} = N^o_\omega - 2\frac{v_{Te}}{c}y\sqrt{1-x}[1 - \frac{v_{Te}}{c}y\sqrt{1-x}]\omega\frac{\partial N^o_\omega}{\partial\omega} + 2\frac{v_{Te}^2}{c^2}y^2(1-x)\omega^2\frac{\partial^2 N^o_\omega}{\partial\omega^2} \tag{15}$$

and

$$\delta N_{\omega'} = \delta N_\omega + \left[-2\frac{v_{Te}}{c}y\sqrt{1-x} + 2\left(\frac{v_{Te}}{c}\right)^2 y^2(1-x)\right]\omega\frac{\partial\delta N_\omega}{\partial\omega}$$

$$+2\frac{v_{Te}^2}{c^2}y^2(1-x)\omega^2\frac{\partial^2\delta N_\omega}{\partial\omega^2} \tag{16}$$

Then, the substitution of (13-16) in (8,9) leads to the following expression for the cross-sections (performing the ω' integration using the delta-function)

$$\sigma^{tr}\delta N_\omega = (\sigma_o^{tr} + \hat{\sigma}^{tr})\delta N_\omega \tag{17}$$

where

$$\sigma_o^{tr} = \frac{3}{8}\sigma_T\sqrt{\epsilon(\omega)}\int_{-\infty}^{+\infty}\frac{dy}{\sqrt{\pi}}e^{-y^2}\int_{-1}^{+1}\frac{dx}{|F(x,y,z)|^2}(1+x^2)(1-x)$$
$$\left[1 - 3\frac{v_{Te}}{c}y\sqrt{1-x} + 2\frac{v_{Te}^2}{c^2}y^2(3-2x)\right] \tag{18}$$

with the operator given by

$$\hat{\sigma}^{tr} = \frac{3}{4}\sigma_T\int_{-\infty}^{+\infty}\frac{dy}{\sqrt{\pi}}e^{-y^2}\int_{-1}^{+1}\frac{dx}{|F(x,y,z)|^2}(1+x^2)x$$
$$\left\{\left[\frac{v_{Te}}{c}y\sqrt{1-x} - 4\frac{v_{Te}^2}{c^2}y^2(1-x)\right]\omega\frac{\partial}{\partial\omega} - \frac{v_{Te}^2}{c^2}y^2(1-x)\omega^2\frac{\partial^2}{\partial\omega^2}\right\} \tag{19}$$

and

$$\sigma^{st}\delta N_\omega = (\sigma_o^{st} + \hat{\sigma}^{st})\delta N_\omega \tag{20}$$

with

$$\sigma_o^{st} = -\frac{3}{4}\sigma_T\frac{v_{Te}}{c}\frac{z}{e^z-1}\int_{-\infty}^{+\infty}\frac{dy}{\sqrt{\pi}}ye^{-y^2}\int_{-1}^{1}dx\frac{(1+x^2)(1+x)\sqrt{(1-x)}}{|F(x,y,z)|^2}+ \tag{21}$$
$$+\frac{3}{2}\sigma_T\left(\frac{v_{Te}}{c}\right)^2\frac{z}{e^z-1}\int_{-\infty}^{+\infty}\frac{dy}{\sqrt{\pi}}y^2e^{-y^2}\int_{-1}^{1}\frac{dx}{|F(x,y,z)|^2}(1+x^2)(1-x)[2(1+x) - \frac{ze^z}{e^z-1}]$$

$$\hat{\sigma}^{st} = \frac{3}{2}\sigma_T(\frac{v_{Te}}{c})^2\frac{z}{e^z-1}\int_{-\infty}^{+\infty}\frac{dy}{\sqrt{\pi}}y^2e^{-y^2}\int_{-1}^{1}\frac{dx}{|F(x,y,z)|^2}(1+x^2)(1-x)x\omega\frac{\partial}{\partial\omega} \tag{22}$$

and we have introduced the notations

$$z = \frac{\hbar\omega}{T} \tag{23}$$

$$F(x,y,z) = \epsilon_{\underline{q}\Omega}|_{\omega'=\omega'(\omega)} \tag{24}$$

The explicit calculation of $F(x,y,z)$ has been done in reference 1.

Neglecting the ion contribution and taking into account the corrections due to collisions, Doppler shift and relativistic effects. These results conclude the aim of the present paper the zero order transport equation namely

$$\sqrt{\epsilon(\omega)}\frac{\partial N^o_\omega}{\partial r} = -n_e\sigma_o^{tr}\delta N_\omega + n_e\sigma^{st}\delta N_\omega \tag{25}$$

which can now be written as

$$\left[a^{(1)}_\omega\omega\frac{\partial}{\partial\omega} + a^{(2)}_\omega\omega^2\frac{\partial^2}{\partial\omega^2} + \sigma_o^{tr} - \sigma_o^{st}\right]\delta N_\omega = -\frac{\sqrt{\epsilon}}{n_e}\frac{\partial N^o_\omega}{\partial r} \tag{26}$$

where

$$a^{(1)}_\omega = \frac{3}{4}\sigma_T\frac{v_{Te}}{c}\int_{-\infty}^{+\infty}\frac{dy}{\sqrt{\pi}}ye^{-y^2}\times$$

$$\int_{-1}^{1}\frac{dx}{|F(x,y,z)|^2}(1+x^2)x\sqrt{1-x}[1-2\frac{v_{Te}}{c}y\sqrt{1-x}(2+\frac{z}{e^z-1})] \tag{27}$$

$$a^{(2)}_\omega = -\frac{3}{4}\sigma_T(\frac{v_{Te}}{c})^2\int_{-\infty}^{+\infty}\frac{dy}{\sqrt{\pi}}y^2e^{-y^2}\int_{-1}^{1}\frac{dx}{|F(x,y,z)|^2}(1+x^2)x(1-x). \tag{28}$$

3) THE OPACITY IN THE SOLAR INTERIOR

In the solar interior the transport equation is usually written in terms of the Planck function B_ω and the energy flux $\mathcal{F}_\omega$ defined as

$$B_\omega = \frac{\hbar}{2\pi^2}\frac{\omega^3}{c^2}N^o_\omega; \mathcal{F}_\omega = \frac{2}{3}\frac{\hbar\omega^3}{\pi c^2}\delta N_\omega \tag{29}$$

and takes the form

$$\frac{4\pi}{3}\sqrt{\epsilon(\omega)}\frac{\partial B_\omega}{\partial T}\frac{dT}{dr} = -n_e(\sigma^{eff} - \sigma_o^{st})\mathcal{F}_\omega - n_e\omega^3(a^{(1)}_\omega\omega\frac{\partial}{\partial\omega} + a^{(2)}_\omega\omega^2\frac{\partial^2}{\partial\omega^2})\frac{\mathcal{F}_\omega}{\omega^3} \tag{30}$$

where $\sigma^{eff}$is the total cross-section including inverse bremsstrahlung and line absorption.

Using perturbation theory we solved eq.(30) assuming

$$\mathcal{F}_\omega = \mathcal{F}^o_\omega + \delta\mathcal{F}_\omega$$

where

$$\mathcal{F}^o_\omega = -\frac{4\pi}{3n_e\sigma_{eff}}\frac{\partial B_\omega}{\partial T}\frac{dT}{dr} \tag{31}$$

is the zero order flux and the correction is due to the presence of stimulated scattering and frequency diffusion.

The solution for $\delta\mathcal{F}_\omega$, integrated over all frequencies, is used to calculate the correction to the opacity given by

$$\frac{\kappa - \kappa_o}{\kappa} = \frac{15}{4\pi^4}\rho\kappa_o \int_{\omega_{pe}}^{\infty} \frac{z^4 e^z}{(e^z - 1)^2 n_e \sigma_{eff}} G(z) dz \tag{32}$$

where ρ is the mass density,

$$G(z) = \frac{\sigma_o^{st}}{\sigma_{eff}} - \frac{1}{f(z)}(a_z^{(1)} z \frac{\partial}{\partial z} + a_z^{(2)} z^2 \frac{\partial^2}{\partial z^2}) \frac{f(z)}{\sigma_{eff}} \tag{33}$$

$$f(z) = \frac{z e^z}{(e^z - 1)^2}$$

and the zero order Rosseland mean opacity is given by

$$\frac{1}{\rho\kappa_o} = \frac{15}{4\pi^4} \int_{\omega_{pe}}^{\infty} \frac{z^4 e^z}{(e^z - 1)^2 n_e \sigma_{eff}} dz \tag{34}$$

We have calculated numerically the effect of the new terms using eq.(32) neglecting line-absorption and using the zero order cross-sections for scattering and inverse bremsstrahlung[1]. At the solar centre we found a reduction of the continuous opacity of 4.5%. The reduction of the total opacity of course is smaller, it is known that absorption from iron ions at the solar centre accounts for almost $\frac{1}{3}$ of the total opacity [3]. This result however is a part of the total correction to scattering of photons in the solar interior. Other corrections were found in the first two papers[1,2]. Taken all together these corrections amount to a decrease of opacity of order 12% when the ion correlations are not taken into account and of order of 14% when the ion correlations are taken into account, at the solar centre, which could be significant for the neutrino flux prediction from Standard Solar Models.

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RELATIVISTIC CORRECTIONS TO COLLECTIVE PHOTON SCATTERING IN PLASMAS

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ABSTRACT

Relativistic corrections to Thomson scattering of photons in plasmas are well known [1]. Here we consider the case when collective effects have also to be taken into account and find the relativistic corrections to collective scattering in plasmas. Collective effects are due to the shielding clouds and are important when the wavelength of the scattering radiation is of the same size or larger than the Debye wavelength. This is the case, for instance, in the solar interior where relativistic corrections to Thomson scattering of photons have recently been taken into account in the calculation of the solar opacity [2] with the result of a few percent decrease. As an application of our results we show that when the relativistic corrections to collective scattering are included, the decrease in the opacity is 0.2%.

1. INTRODUCTION

This is the second in a series of papers dealing with relativistic and collective corrections to scattering of photons in plasmas.

It is well known that in plasmas the scattering cross-section of photons on electrons and ions, taking into account the collective effects, can be expressed through the plasma dielectric permittivity $\epsilon_{\omega k}$ and to a first approximation the cross-section is proportional to $1/|\epsilon_{\omega k}|^2$. The corrections due to Doppler and collisional broadening of the Raman resonance at $\epsilon_{\omega k} \simeq 0$ and use of a relativistic distribution function for the electrons in the expression for $\epsilon_{\omega k}$ were considered in Ref [3], (Papaer I). The paper I in ref.[3] is the first paper in the series, and referred to in this paper as paper I.

The changes in the probability of collective scattering, which can no longer be expressed simply as $1/|\epsilon_{\omega k}|^2$ when relativistic effects are taken into account, are the subject of this paper.

The collective effects are important when the wavelength of the radiation becomes comparable or larger than the plasma Debye length and this condition can be expressed as $\delta_e \geq 1$, where the "collective" parameter δ_e is defined as

$$\delta_e = \frac{1}{2}\frac{\omega_{pe}^2}{\omega^2}\left(\frac{c}{v_{Te}}\right)^2 \tag{1}$$

where ω is the radiation frequency, $\omega_{pe} = \left(\frac{4\pi e^2 n_e}{m_e}\right)^{\frac{1}{2}}$ is the plasma frequency and $v_{Te} = (T_e/m_e)^{\frac{1}{2}}$ is the electron thermal speed (the temperature T_e is expressed in energy units).

The relativistic corrections to scattering have been previously taken into account in the form of a multiplicative factor in Thomson scattering [1,2], we will show that this is correct only when the collective effects are negligible.

Denoting by M^T and M^{coll} the amplitudes in the probabilities for Thomson scattering (by individual particles) and scattering by their shielding clouds respectively, the scattering cross-section is proportional to $|M^T + M^{coll}|^2$.

Let δM^T be the change in the amplitude of Thomson scattering due to relativistic corrections, the change in the cross-section will be proportional to $2Re\delta M^T(M^T + M^{coll})$. But there is also a relativistic correction to the matrix element δM^{coll} of collective scattering and the total change in the cross-section will be therefore proportional to $2Re\left[(\delta M^T + \delta M^{coll})(M^T + M^{coll})\right]$.

The problem is even more complicated in cases (like the solar interior) where both electrons and ions give almost equal contributions to scattering. For ions the Thomson scattering is negligible but, when collective effects are important, the scattering by ions through the electron screening clouds can not be neglected, and therefore relativistic corrections exist also for "scattering by the ions" (and depends on the electron thermal velocity). The relativistic corrections for scattering on electrons and with ions should be calculated separately. Scattering on ions, which is a typical collective effect, was not included in previous calculations[1]. The aim of the present paper is to give for the first time the calculations of all the relativistic corrections both for electrons and ions in the classical limit (i.e. neglecting quantum effects). In Section 2 the general expressions for the scattering probabilities are given. In Section 3 the relativistic corrections to the probabilities are found. In Section 4 the results are used to find the corrections to the transport cross-section in the solar interior and the change in the radiative opacity is calculated at the solar centre (where the corrections are largest). The effects due to the frequency Doppler shift, relativistic distribution function and broadening of the Raman resonance from paper I should be added to the effects calculated here to find the total change of cross-section due to collective relativistic effects. Additional relativistic effects appear in the equation for radiative transfer due to stimulated scattering and frequency diffusion (Ref.[3] Paper II). The present paper is the paper III in this series of papers. In calculation of solar opacity all relativistic collective effects considered in papers I, II, III should be taken into account.

2. THE PROBABILITY OF SCATTERING

The transport equation for photons can be written in terms of the scattering probabilities (see Paper I). We start with the expressions for the scattering probabilities including all the relativistic corrections. The probability that an incident wave $(\underline{k}, \omega^\sigma_k)$ is scattered by a particle of type α of momentum $\underline{p}$ into a wave $(\underline{k}', \omega^{\sigma'}_{k'})$, where σ, σ' denote the polarization of the wave, is given by [4].

$$W^\alpha_{\underline{p},\underline{k},\underline{k}'} = \sum_{\sigma\sigma'} \frac{4(2\pi)^3 |M^{(\alpha)}(\underline{k},\underline{k}')|^2}{\frac{\partial}{\partial\omega}\omega^2 \epsilon^\sigma_{\underline{k}\omega}\Big|_{\omega=\omega^\sigma_k} \frac{\partial}{\partial\omega'}\omega'^2 \epsilon^{\sigma'}_{\underline{k}'\omega'}\Big|_{\omega'=\omega^{\sigma'}_{k'}}} \delta(\omega^\sigma_k - \omega^{\sigma'}_{k'} - (\underline{k} - \underline{k}') \cdot \underline{v}^\alpha) \quad (2)$$

where the sum is over all polarizations, α is an electron or an ion of velocity $\underline{v}^\alpha$,

$$\epsilon^\sigma_{\underline{k}\omega} = \epsilon_{ij}(k) e^\sigma_{i\underline{k}} e^{\sigma'*}_{j\underline{k}'} \quad ; \; k \equiv (\underline{k}, \omega^\sigma_k)$$

where $\underline{e}^\sigma_{\underline{k}}$ and $\underline{e}^{\sigma'}_{k'}$ are the unit polarization vectors of the incident and scattered wave respectively and $\epsilon_{ij}(k)$ is the dielectric permittivity tensor of the plasma. For electromagnetic waves in the high frequency limit ($\omega \gg \omega_p$).

$$\epsilon_{ij}(k) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\epsilon(\omega); \qquad \epsilon^\sigma_{\underline{k}\omega} = \epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}; \qquad \omega_p^2 = \frac{4\pi n_e e^2}{m_e}$$

and we have

$$W^\alpha_{\underline{p}\underline{k},\underline{k}'} = \sum_{\sigma\sigma'} \frac{(2\pi)^3}{\omega^\sigma_k \omega^{\sigma'}_{k'}} \left|M^{(\alpha)}(\underline{k},\underline{k}')\right|^2 \delta(\omega^\sigma_k - \omega^{\sigma'}_{k'} - (\underline{k} - \underline{k}') \cdot \underline{v}^\alpha) \quad (3)$$

where the scattering amplitude $M^\alpha(\underline{k},\underline{k}')$ is given by

$$M^{(\alpha)}(\underline{k},\underline{k}') = M^{(\alpha)}_{ij}(\underline{k},\underline{k}') e^\sigma_{i\underline{k}} e^{\sigma'*}_{j\underline{k}'} \quad (4)$$

the matrix $M^{(\alpha)}_{ij}(\underline{k},\underline{k}')$ is the sum of two contributions

$$M^{(\alpha)}_{ij}(\underline{k},\underline{k}') = M^{(\alpha)T}_{ij} + M^{(\alpha)coll}_{ij} \quad (5)$$

where $M^{(\alpha)T}_{ij}$ describes Thomson scattering (by individual particles) (fully relativistic) and $M^{(\alpha)coll}_{ij}$ describes the collective effects.

These are given by [4]

$$M_{ij}^{(\alpha)T}(k,k') = i\frac{Z_\alpha^2 e^2 (1-(v^\alpha)^2/c^2)^{\frac{1}{2}}}{m_\alpha}\times \tag{6}$$

$$\times\left\{\delta_{ij} + \frac{v_i^\alpha k_j}{\omega - \underline{k}\cdot\underline{v}^\alpha} + \frac{v_j^\alpha k_i'}{\omega' - \underline{k}'\cdot\underline{v}^\alpha} + \frac{v_i^\alpha v_j^\alpha}{c^2}\frac{c^2\underline{k}\cdot\underline{k}' - \omega\omega'}{(\omega - \underline{k}\cdot\underline{v}^\alpha)(\omega' - \underline{k}'\cdot\underline{v}^\alpha)}\right\}$$

where $Z_\alpha e, m_\alpha$ are the charge ($Z_e = 1$) and mass of the particle α of velocity $\underline{v}^\alpha$ and we have used the notation $\omega = \omega_k^\sigma, \omega' = \omega_{k'}^{\sigma'}$ and $\omega' - \underline{k}'\cdot\underline{v}^\alpha = \omega - \underline{k}\cdot\underline{v}^\alpha$ from the δ function. The collective matrix is given by

$$M_{ij}^{(\alpha)coll} = S_{imj}(k,k')G_{ms}(k-k')2Z_\alpha e v_s^\alpha \omega' \tag{7}$$

where the Green function $G_{ms}(q)$ is given by (for $q = (\underline{k}-\underline{k}', \Omega), \Omega = \omega - \omega'$)

$$G_{ms}(q) = G_{ms}^l(q) + G_{ms}^t(q) \tag{8}$$

with

$$G_{ms}^l(q) = -i\frac{4\pi q_m q_s}{\Omega q^2 \epsilon_{\underline{q}\Omega}^l} \tag{9}$$

$$G_{ms}^t(q) = \left(\delta_{ms} - \frac{q_m q_s}{q^2}\right)\frac{4\pi i\Omega}{c^2 q^2 - \epsilon_{\underline{q}\Omega}^t} \tag{10}$$

where $\epsilon_{\underline{q}\Omega}^l, \epsilon_{\underline{q}\Omega}^t$ are the longitudinal and transverse part of the plasma permittivity and G_{ms}^l, G_{ms}^t describe scattering through the longitudinal and transverse plasma fluctuations respectively.

The scattering through the transverse virtual field (i.e. the contribution of G^t) is of the order

$$\frac{v_{Te}^2}{c^2}\frac{\omega_{pe}^2}{\omega^2} \sim \delta_e \frac{v_{Te}^4}{c^4}$$

and can therefore be neglected (except for the case $\delta_e \gg 1$) since we are interested in the effects to order v_{Te}^2/c^2 we shall take $G_{ms}(q) = G_{ms}^l(q)$ in the following.

Finally the non linear plasma response tensor S_{imj} in eq.(7) is given by [4]

$$S_{imj}(k,k') = -\frac{e^3}{2}\sum_{\alpha} Z_\alpha^3 \int \frac{v_i^\alpha}{\omega - \underline{k}\cdot\underline{v}^\alpha} \times \tag{11}$$

$$\times \left\{ \beta_{k'}^{js} \frac{\partial}{\partial p_s^\alpha} \frac{1}{\Omega - \underline{q}\cdot\underline{v}^\alpha} \beta_q^{mn} \frac{\partial}{\partial p_n^\alpha} + \beta_q^{ms} \frac{\partial}{\partial p_s^\alpha} \frac{1}{\omega' - \underline{k}'\cdot\underline{v}^\alpha} \beta_{k'}^{jn} \frac{\partial}{\partial p_n^\alpha} \right\} f^{(\alpha)}(\underline{p}_\alpha) \frac{d^3 p_\alpha}{(2\pi)^3}$$

where we have defined

$$\beta_k^{ij} = \delta_{ij}\left(1 - \frac{\underline{k}\cdot\underline{v}^\alpha}{\omega}\right) + \frac{k_j v_i^\alpha}{\omega} \tag{12}$$

$$\beta_q^{ij} = \delta_{ij}\left(1 - \frac{\underline{q}\cdot\underline{v}^\alpha}{\Omega}\right) + \frac{q_j v_i^\alpha}{\Omega} \tag{13}$$

and

$$n_\alpha = \int f^{(\alpha)}(\underline{p}_\alpha) \frac{d^3 p_\alpha}{(2\pi)^3}$$

is the density of particles α.

The non-linear response tensor equation (11) contains all the terms from the $\underline{v} \times \underline{B}$ term and is fully relativistic.

Finally, using eq.(9) in eq.(7) for $G_{ms}(q)$ and using the condition $\underline{q}\cdot\underline{v}^\alpha = \omega - \omega' = \Omega$ from the δ-function in (2) we can write a simplified expression for the collective scattering amplitude namely

$$M_{ij}^{(\alpha)coll}(k,k') = -8\pi i Z_\alpha e S_{imj}(k,k') \frac{q_m \omega'}{q^2 \epsilon_q^l} \tag{14}$$

In the following we shall also neglect thermal effects in the refractive index of electromagnetic waves and consider the high-frequency limit $\omega \gg \omega_{pe}$, i.e.

$$\omega_k^2 \simeq c^2 k^2 \tag{15}$$

3. RELATIVISTIC CORRECTIONS TO THE SCATTERING PROBABILITY

From eq's.(4) and (6) making use of the conservation law $\omega - \underline{k}\cdot\underline{v} = \omega' - \underline{k}'\cdot\underline{v}$ and expanding $(\omega - \underline{k}\cdot\underline{v})^{-1}$ for $\omega \gg \underline{k}\cdot\underline{v}$ we find the amplitude for Thomson scattering on electrons to be (neglecting terms higher than second order in velocity)

$$M^{(e)T}(k,k') = \frac{ie^2}{m_e}\left\{\underline{e}_k^{\sigma}\cdot\underline{e}_{k'}^{\sigma'*}\left(1-\frac{v^2}{2c^2}\right) + \Lambda^{(1)}_{\underline{k},\underline{k}'}(\underline{v}) + \Lambda^{(2)}_{\underline{k},\underline{k}'}(\underline{v})\right\} \qquad (16)$$

where

$$\Lambda^{(1)}_{\underline{k},\underline{k}'}(v) = \frac{(\underline{e}_k^{\sigma}\cdot\underline{v})(\underline{k}\cdot\underline{e}_{k'}^{\sigma'*})}{\omega} + \frac{(\underline{e}_{k'}^{\sigma'*}\cdot\underline{v})(\underline{k}'\cdot\underline{e}_k^{\sigma})}{\omega}$$

$$\Lambda^{(2)}_{\underline{k},\underline{k}'}(v) = \frac{\underline{k}\cdot\underline{v}}{\omega}\,\Lambda^{(1)}_{\underline{k},\underline{k}'}(\underline{v}) - \mu\frac{\underline{e}_k^{\sigma}\cdot\underline{v}\;\underline{e}_{k'}^{\sigma'*}\cdot\underline{v}}{c^2}$$

where $\mu = 1-\cos\theta, \theta$ is the angle between the $\underline{k}$ and $\underline{k}'$ vectors and we have used $k = k', \omega = \omega'$ in the second order terms, where the Doppler frequency shift, of order $\frac{v_{Te}}{c}$, can be neglected. Separating $\underline{v}$ in to parallel $(\underline{v}_{\parallel})$ and perpendicular $(\underline{v}_{\perp})$ components to the vector $\underline{q} = \underline{k} - \underline{k}'$, such that

$$\underline{v}_{\parallel} = \frac{\underline{q}\cdot\underline{v}}{q^2}\underline{q} \qquad ; \qquad \underline{v}_{\perp} = \underline{v} - \underline{v}_{\parallel}$$

making use of the polarization condition for transverse waves $\underline{e}_k^{\sigma}\cdot\underline{k} = \underline{e}_{k'}^{\sigma'}\cdot\underline{k}' = 0$, using the fact that, (since the probability does not depend on $\underline{v}_{\perp}$ the linear terms in $\underline{v}_{\perp}$) integrate to zero in the transport equation and can therefore be omitted and using the result

$$\Lambda^{(1)}_{\underline{k},\underline{k}'}(\underline{v}_{\parallel}) = 0 \qquad\qquad \Lambda^{(1)}_{\underline{k},\underline{k}'}(\underline{v}) = \Lambda^{(1)}_{\underline{k},\underline{k}'}(\underline{v}_{\perp}),$$

we separate the zero-order (non relativistic) term and the relativistic contribution and write (16) in the form

$$M^{(e)T}(k,k') = M_0^{(e)T} + \delta M^T \qquad (17)$$

where the zero order term is given by

$$M_0^{(e)T} = \frac{ie^2}{m_e}(\underline{e}_k^{\sigma}\cdot\underline{e}_{k'}^{\sigma'*}) \qquad (18)$$

and the the relativistic correction is given by

$$\delta M^T = \frac{ie^2}{m_e}\left[\left(\frac{v_{Te}}{c}\right)^2 y^2 \left(\frac{(\underline{e}_{\underline{k}}^{\sigma'*}\cdot\underline{k})(\underline{e}_{\underline{k}}^{\sigma}\cdot\underline{k}')}{k^2} - \underline{e}_{\underline{k}}^{\sigma}\cdot\underline{e}_{\underline{k}'}^{\sigma'*}\right) + \right.$$

$$\left. - \left(\frac{v_\perp}{c}\right)^2 \left(\mu \underline{e}_{\underline{k}}^{\sigma\perp}\cdot\underline{e}_{\underline{k}'}^{\sigma'\perp} + \frac{1}{2}\underline{e}_{\underline{k}}^{\sigma}\cdot\underline{e}_{\underline{k}'}^{\sigma'*}\right)\right] \tag{19}$$

and y as defined by equation (25).

For Thomson scattering on ions the relativistic correction is of order v_{Ti}^2/c^2 and can be neglected for $v_{Ti} \ll v_{Te}$. For the collective amplitude from eq.(14) we need the quantity

$$S_{ij} = S_{imj}q_m = -\frac{1}{2}e^3 \int \frac{v_i}{\omega - \underline{k}\cdot\underline{v}} \hat{O}_j f^{(e)}(p) \frac{d^3p}{(2\pi)^3} \tag{20}$$

where the operator $\hat{O}_j$ is given by

$$\hat{O}_j = \beta_{k'}^{js} \frac{\partial}{\partial p_s} \frac{1}{\Omega - \underline{q}\cdot\underline{v}} \underline{q}\cdot\frac{\partial}{\partial \underline{p}} + \underline{q}\cdot\frac{\partial}{\partial \underline{p}} \frac{1}{\omega' - \underline{k}'\cdot\underline{v}} \frac{\partial}{\partial p_j}$$

In eq.(20) we have considered only the electron response in eq.(11) (neglecting the ion non-linear current) and made use of the fact that assuming a Maxwellian distribution for the electrons the $\underline{v}\times\underline{B}$ term does not enter in front of $\frac{\partial f^{(e)}(p)}{\partial p_j}$ (last term in eq.(11)). Expanding for high frequency $\omega' \gg \underline{k}'\cdot\underline{v}, \omega \gg \underline{k}\cdot\underline{v}$ and integrating twice by parts using the relativistic expression for electrons

$$\frac{dv_i}{dp_j} = \frac{1}{m_e\gamma}\left(\delta_{ij} - \frac{v_iv_j}{c^2}\right) \simeq \frac{1}{m_e}\left(1 - \frac{v^2}{2c^2}\right)\left(\delta_{ij} - \frac{v_iv_j}{c^2}\right)$$

and multiplying for the polarization vectors according to eq.(4) we find [5].

$$S = e_{\underline{k}i}^{\sigma} e_{\underline{k}'j}^{\sigma'*} S_{ij} = -\frac{e^3}{2m_e^2\omega'} \int \frac{d^3p}{(2\pi)^3} f^{(e)}(p)\left(1 - \frac{v^2}{c^2}\right) A_{\underline{k},\underline{k}'}(\underline{v}) \frac{q^2 - \frac{\Omega}{c^2}\underline{q}\cdot\underline{v}}{(\Omega - \underline{q}\cdot\underline{v})^2} \tag{21}$$

where

$$A_{\underline{k},\underline{k}'}(\underline{v}) = \underline{e}_{\underline{k}}^{\sigma}\cdot\underline{e}_{\underline{k}'}^{\sigma'*} + \frac{(\underline{e}_{\underline{k}'}^{\sigma'*}\cdot\underline{k})(\underline{e}_{\underline{k}}^{\sigma}\cdot\underline{v})}{\omega} + \frac{(\underline{e}_{\underline{k}}^{\sigma}\cdot\underline{k}')(\underline{e}_{\underline{k}'}^{\sigma'*}\cdot\underline{v})}{\omega'} - \mu\frac{(\underline{e}_{\underline{k}}^{\sigma}\cdot\underline{v})(\underline{e}_{\underline{k}'}^{\sigma'*}\cdot\underline{v})}{c^2}$$

Separating the zero order and relativistic corrections in (21) it can be written as $S = S_o + \delta S$. Expressing the integrals in terms of the W-function

$$W(z) = 1 - z\int \frac{dt}{z - t}\frac{e^{-t^2}}{\sqrt{\pi}} = 1 - 2ze^{-z^2}\int_0^z e^{t^2}dt + i\sqrt{\pi}ze^{-z^2} \tag{22}$$

we find

$$S_o = \frac{e^3 n_e}{2 m_e^2 v_{Te}^2 \omega'} \underline{e}_{\underline{k}}^{\sigma} \cdot \underline{e}_{\underline{k}'}^{\sigma' *} W(y) \tag{23}$$

$$\delta S = \frac{e^3 n_e}{2 m_e^2 v_{Te}^2 \omega'} \frac{v_{Te}^2}{c^2} \left(\underline{e}_{\underline{k}}^{\sigma} \cdot \underline{e}_{\underline{k}'}^{\sigma' *} \left[(\frac{11}{2} - \mu - 2y^2) W(y) - 1 \right] + \right.$$

$$\left. - \frac{c^2}{\omega \omega'} (\underline{e}_{\underline{k}'}^{\sigma' *} \cdot \underline{k})(\underline{e}_{\underline{k}}^{\sigma} \cdot \underline{k}') \left[(1 - 2y^2) W(y) - 1 \right] \right) \tag{24}$$

where

$$y = \frac{\Omega}{\sqrt{2} v_{Te} q} = \frac{\underline{q} \cdot \underline{v}}{\sqrt{2} v_{Te} q} \tag{25}$$

and the last equality is valid when these expressions are used in the transport equation due to the presence of the delta-function in eq.(3).

From eq.'s (4) and (14) the amplitude for collective scattering can be written as

$$M_{coll}^{(\alpha)}(k, k') = - \frac{8 \pi i Z_\alpha e \omega'}{q^2 \epsilon_{\underline{q}\Omega}} (S_o + \delta S) \tag{26}$$

The zero-order (non relativistic) contribution to this expression is due to S_o. Separating again zero-order and relativistic corrections we write

$$M_{coll}^{(\alpha)}(k, k') = M_0^{(\alpha) coll} + \delta M_{coll}^{(\alpha)} \tag{27}$$

where

$$M_0^{(\alpha) coll} = - \frac{4 \pi i Z_\alpha e^4 n_e}{m_e^2 v_{Te}^2 q^2 \epsilon_{\underline{q}\Omega}} (\underline{e}_{\underline{k}}^{\sigma} \cdot \underline{e}_{\underline{k}'}^{\sigma' *}) W(y) \tag{28}$$

and

$$\delta M_{coll}^{(\alpha)} = - \frac{8 \pi i Z_\alpha e \omega'}{q^2 \epsilon_{\underline{q}\Omega}} \delta S \tag{29}$$

Equation (29) gives the relativistic correction to the scattering amplitude due to the oscillating electrons in the shielding cloud of an electron ($\alpha = e$) and of an ion ($\alpha = i$).

Let us calculate first the well known zero-order probabilities which were used in Paper I and will also be used in the next Section.

Introducing the electron susceptibility

$$\chi^{(e)}_{\underline{q}\Omega} = \frac{\omega_{pe}^2}{v_{Te}^2 q^2} W(y) \tag{30}$$

eq.(28) can be written as

$$M_0^{(\alpha)coll} = -\frac{iZ_\alpha e^2}{m_e} \frac{\chi^{(e)}_{\underline{q}\Omega}}{\epsilon_{\underline{q}\Omega}} \left(\underline{e}^{\sigma}_{\underline{k}} \cdot \underline{e}^{\sigma' *}_{\underline{k}'} \right) \tag{31}$$

where

$$\epsilon_{\underline{q}\Omega} = 1 + \chi^{(e)}_{\underline{q}\Omega} + \sum_i \chi^{(i)}_{\underline{q}\Omega} \tag{32}$$

and where $\chi^{(i)}_{\underline{q}\Omega}$ are the ion susceptibilities. In all corrections to the matrix elements we shall neglect the contribution of the ions and all other effects of order m_e/m_i.

From eq.'s (31) and (18) we have the zero-order total scattering amplitude (for scattering on electrons)

$$M_0^{(e)}(k,k') = M_0^{(e)T} + M_0^{(e)coll} = \frac{ie^2}{m_e} \frac{1}{\epsilon_{\underline{q}\Omega}} \left(\underline{e}^{\sigma}_{\underline{k}} \cdot \underline{e}^{\sigma' *}_{\underline{k}'} \right) \tag{33}$$

where we have neglected the ion susceptibilities.

Using an unpolarized incident wave, and averaging over the initial polarization

$$\sum_{\sigma\sigma'} |\underline{e}^{\sigma}_{\underline{k}} \cdot \underline{e}^{\sigma' *}_{\underline{k}'}|^2 = \frac{1}{2} \left(1 + cos^2 \theta_{\underline{k},\underline{k}'} \right) \tag{34}$$

we then find the zero-order probability for scattering on electrons from eq.(2) (both the relativistic corrections and Doppler shift corrections neglected)

$$W_o^{(e)} = \frac{e^4}{2m_e^2 \omega^2} \left(1 + cos^2 \theta_{\underline{k},\underline{k}'} \right) \frac{1}{|\epsilon_{\underline{q}\Omega}|^2} \delta(\omega - \omega') \tag{35}$$

For scattering on ions the Thomson scattering is negligible ($\sim m_e/m_i$) and

$$M_0^{(i)}(k,k') \simeq M_0^{(i)coll} = -\frac{ie^2}{m_e} \frac{\chi^{(e)}_{\underline{q}\Omega}}{\epsilon_{\underline{q}\Omega}} \left(\underline{e}^{\sigma}_{\underline{k}} \cdot \underline{e}^{\sigma' *}_{\underline{k}'} \right) \tag{36}$$

and the probability of scattering due to the ion contribution in the scattering form factor is given by

$$W_o^{(i)} = \frac{Z_i^2 e^4}{2m_e^2\omega^2}\left(1 + cos^2\theta_{\underline{k},\underline{k}'}\right)\left|\frac{\chi^{(e)}_{\underline{q}\Omega}}{\epsilon_{\underline{q}\Omega}}\right|^2 \delta(\omega - \omega') \tag{37}$$

Including the relativistic corrections the scattering amplitudes on electrons or ions are given by

$$M^{(e)} = M_0^{(e)} + \delta M^T + \delta M_{coll}^{(e)} \tag{38}$$

$$M^i = M_0^i + \delta M_{coll}^{(i)} \tag{39}$$

where $M_0^{(e)}$ and $M_0^{(i)}$ are given by eq.'s (33) and (36) and $\delta M_{coll}^{(\alpha)}$ is given by eq.(29).

Defining

$$\delta|M^{(\alpha)}|^2 = |M^{(\alpha)}|^2 - |M_0^{(\alpha)}|^2 \tag{40}$$

we have, for scattering on electrons

$$\delta|M^{(e)}|^2 = 2Re\left(M_0^{(e)} \cdot \delta M^{T*}\right) + 2Re\left(M_0^{(e)} \cdot \delta M_{coll}^{(e)*}\right) \tag{41}$$

Using eq.(33) for $M_0^{(e)}$, the first term in eq.(41) can be written as

$$2Re\left(M_0^{(e)} \cdot \delta M^{T*}\right) = 2Re\left[\frac{ie^2}{m_e}\frac{1}{\epsilon_{\underline{q}\Omega}}\left(\underline{e}_k^\sigma \cdot \underline{e}_{k'}^{\sigma'*}\right)\delta M^{T*}\right] \tag{42}$$

where δM^T is given by eq.(19) and the second term is

$$2Re\left\{M_0^{(e)} \cdot \delta M_{coll}^{(e)*}\right\} = -2Re\left\{\frac{8\pi e^3}{m_e}\frac{\underline{e}_k^\sigma \cdot \underline{e}_{k'}^{\sigma'*}}{q^2|\epsilon_{\underline{q}\Omega}|^2}\omega'\delta S^*\right\} \tag{43}$$

where δS is given by eq.(24). Collecting the two terms and summing over polarizations we finally find from eq.(41)

$$\sum_{\sigma\sigma'}\delta|M^{(e)}|^2 =$$

$$-\frac{e^4}{m_e^4}Re\left(\frac{1}{\epsilon_{\underline{q}\Omega}}\right)\left[\frac{v_{Te}^2}{c^2}y^2 f_1(x) + \frac{\underline{v}_\perp^2}{c^2}f_2(x)\right] + \frac{v_{Te}^2}{c^2}\frac{e^4}{m_e^2}\frac{\delta_e}{1-x}\frac{1}{|\epsilon_{\underline{q}\Omega}|^2}G(x,y) \tag{44}$$

where

$$f_1(x) = 1 + x + x^2 - x^3 \tag{45}$$

$$f_2(x) = \frac{1}{2}(3 - x + x^2 - x^3) \tag{46}$$

and

$$G(x,y) = 1+x+x^2-x^3+W(y)\left(\frac{9}{2}+2x+\frac{9}{2}x^2\right)-2y^2W(y)(1+x+x^2-x^3) \tag{47}$$

The scattering probability on electrons can finally be written as $W^{(e)}_{\underline{p},\underline{k},\underline{k}'} = W^{(e)}_o + \delta W^{(e)}$, where

$$\delta W^{(e)} = \sum_{\sigma\sigma'} \frac{(2\pi)^3}{\omega\omega'} \delta|M^{(e)}|^2 \delta(\omega - \omega' - \underline{q}\cdot\underline{v}) \tag{48}$$

and the sum over polarizations of $|\delta M^{(e)}|^2$ is given by eq.(44). The plasma permittivity $\epsilon_{\underline{q}\Omega}$ has been calculated in Paper I including Doppler, relativistic and collisional corrections.

Next we calculate the correction due to scattering on ions. For $v_{Ti} \ll v_{Te}$ we can ignore the corrections of order v_{Ti}/c (i.e. the Doppler corrections to Thomson scattering on ions) and only consider the corrections in the non-linear response of the electrons in the shielding clouds of ions.

For an ion of charge eZ_β from eq.(31) and eq.(29) we find $(\alpha = \beta)$

$$M_0^{(\beta)} = -\frac{ie^2 Z_\beta}{m_e} \frac{\chi^{(e)}_{\underline{q}\Omega}}{\epsilon_{\underline{q}\Omega}} \underline{e}^{\sigma}_{k} \cdot \underline{e}^{\sigma' *}_{k'} \tag{49}$$

$$\delta M^{(\beta)}_{coll} = -\frac{8\pi ie Z_\beta \omega'}{q^2 \epsilon_{\underline{q}\Omega}} \delta S \tag{50}$$

and

$$\delta|M^{(\beta)}|^2 = 2Re\{M_0^{(\beta)} \cdot \delta M^{(\beta)*}_{coll}\} \tag{51}$$

Notice that all the corrections in δS come from the electron shielding clouds and are functions of y (see (25) which we now denote as y_e). From the δ-function in the probability we have

$$y_e = \frac{\Omega}{\sqrt{2}v_{Te}q} = \frac{\underline{q}\cdot\underline{v}^{(\beta)}}{\sqrt{2}v_{Te}q} \simeq \frac{v_{Ti}}{v_{Te}} \tag{52}$$

For $v_{Ti} \ll v_{Te}$ we shall then assume $y_e \to 0, W(y_e) \to 1$ and take

$$\chi^{(e)}_{\underline{q}\Omega} \simeq \frac{\omega_{pe}^2}{v_{Te}^2 q^2} = \frac{\delta_e}{1-x}$$

$$\epsilon_{\underline{q}\Omega} = 1 + \frac{\delta_e}{1-x}\left[1 + \langle Z\rangle W(y_\beta)\right] \tag{53}$$

where we have defined $< Z >$ as

$$\langle Z\rangle = \frac{1}{n_e}\sum_\beta n_\beta Z_\beta^2 = \frac{\sum_\beta n_\beta Z_\beta^2}{\sum_\beta n_\beta Z_\beta} \tag{54}$$

and

$$y_\beta = \frac{\underline{q}\cdot\underline{v}^{(\beta)}}{\sqrt{2}qv_{Ti}}$$

From the results in the previous section, for $y_e \to 0$, we have

$$\sum_{\sigma\sigma'}\left(\underline{e}_k^\sigma \cdot \underline{e}_{k'}^{\sigma' *}\delta S^*\right) = \frac{e^3 n_e}{2m_e^2 c^2 \omega'}\frac{1}{2}\left(\frac{7}{2} + x + \frac{7}{2}x^2 + x^3\right)$$

Equation (51) gives

$$\sum_{\sigma\sigma'}\delta|M^{(\beta)}|^2 = \frac{v_{Te}^2}{c^2}\frac{\delta_e^2}{(1-x)^2}\frac{e^4 Z_\beta^2}{m_e^2}\frac{1}{|\epsilon_{\underline{q}\Omega}|^2}\left(\frac{7}{2} + x + \frac{7}{2}x^2 + x^3\right) \tag{55}$$

The scattering probability on ions can be written as $W^{(i)}_{\underline{p},\underline{k},\underline{k}'} = W_o^{(i)} + \delta W^{(i)}$ where

$$\delta W^{(i)} = \sum_{\sigma\sigma'\beta}\frac{(2\pi)^3}{\omega\omega'}\delta|M^{(\beta)}|^2\delta(\omega - \omega' - \underline{q}\cdot\underline{v}) \tag{56}$$

note that the sum over polarizations is given by eq.(55) and the plasma permittivity is given by eq.(53). Equations (48) and (56) are the main result of this paper and give the relativistic corrections to the zero order probabilities of scattering (given by eqs (35) and (37)). In the next Section we calculate their effect for the particular case of photon scattering in the solar interior.

4. THE TRANSPORT EQUATION AND THE TRANSPORT CROSS-SECTION

In the solar interior the steady state transport equation for the electromagnetic radiation can be written in the form (for $\omega \gg \omega_p$) [3]

$$c\frac{\underline{k}}{k}\cdot\frac{\partial N_{\underline{k}}}{\partial \underline{r}} = -N_{\underline{k}}\sum_{\alpha}\int W^{(\alpha)}_{\underline{p},\underline{k},\underline{k}'} f^{(\alpha}(\underline{p})\frac{d^3pd^3k'}{(2\pi)^6} + \\ +\sum_{\alpha}\int N_{\underline{k}'}W^{(\alpha)}_{\underline{p},\underline{k},\underline{k}'} f^{(\alpha)}(\underline{p})\frac{d^3pd^3k'}{(2\pi)^6} \qquad (57)$$

where the sum is over all plasma particles (electrons and all types of ions) with distribution function $f^{(\alpha)}(\underline{p})$, the scattering probability is given by eq.(3) and $N_{\underline{k}}$ is the distribution function (occupation number) of photons $(\underline{k},\omega_k)$ with group velocity $\underline{v}_g = c\frac{\underline{k}}{k}$ (in the limit of eq.(15)).

Only spontaneous scattering is taken into account in eq.(57) for the relativistic corrections. The contribution of the stimulated scattering has been considered separately (paper II). As shown in Paper I the solution to (57) can be found in the form of an expansion [3]

$$N_{\underline{k}} = N^{(o)}_{\omega} + cos\theta_{\underline{k},\underline{n}}\delta N_{\omega} \qquad (58)$$

where

$$N^{(o)}_{\omega} = [e^{h\omega/T} - 1]^{-1} \qquad (59)$$

is the equilibrium part (black-body radiation), δN_ω is the small $(\delta N_\omega \ll N^{(o)}_\omega)$ anisotropy part of the photon occupation number leading to the energy flux and $\theta_{\underline{k},\underline{n}}$ is the angle between $\underline{k}$ and the unit vector $\underline{n}$ in the direction of flux propagation $(\underline{r})$.

Using eq.(58) in eq.(59) and averaging, in the second term, over the angle $\theta_{\underline{k}',\underline{n}}$ as in ref.[3] such that

$$\overline{cos\theta_{\underline{k}',\underline{n}}} = cos\theta_{\underline{k},\underline{n}}cos\theta_{\underline{k},\underline{k}'}$$

the transport equation becomes

$$c\frac{\partial N^{(o)}_{\omega}}{\partial r} = -\delta N_{\omega}\sum_{\alpha}\int W^{(\alpha)}_{\underline{p},\underline{k},\underline{k}'} f^{(\alpha)}(\underline{p})\frac{d^3p}{(2\pi)^6}\frac{\omega'^2 d\omega'}{c^3}d\Omega_{\underline{k},\underline{k}'} + \\ +\sum_{\alpha}\int \delta N_{\omega'}W^{(\alpha)}_{\underline{p},\underline{k},\underline{k}'}cos\theta_{\underline{k},\underline{k}'} f^{(\alpha)}(\underline{p})\frac{d^3p}{(2\pi)^6}\frac{\omega'^2 d\omega'}{c^3}d\Omega_{\underline{k},\underline{k}'} \qquad (60)$$

where $d\Omega_{\underline{k},\underline{k}'} = 2\pi sin\theta_{\underline{k},\underline{k}'} d\theta_{\underline{k},\underline{k}'}$ is the solid angle of the $\underline{k}'$-vector and we have used (15) to transform d^3k' into $d\omega'$. Notice that this procedure is possible since the scattering probability depends only on the angle $\theta_{\underline{k},\underline{k}'}$.

From the δ-function in the scattering probability to lowest order $\omega' = \omega$ and eq.(60) leads to the well known expression for the transport equation in terms of the transport cross-section [3].

For scattering on electrons, when Doppler shift corrections are taken into account, the expansion of $\delta N_{\omega'}$ in the second term of eq.(60) will produce terms like $\frac{\partial N_\omega}{\partial \omega}, \frac{\partial^2 N_\omega}{\partial \omega^2}$ i.e. the terms describing diffusion in frequency during the transport of radiation. They have been taken into account and discussed in detail in a separate paper [6]. Here we shall consider the lowest order term $\delta N_{\omega'} = \delta N_\omega$ in the second term of eq.(60): this leads to the following transport cross-section for scattering (Paper I)

$$\sigma^{tr}(\omega) = \frac{1}{cn_e} \sum_{\alpha} \int W^{(\alpha)}_{\underline{p},\underline{k},\underline{k}'} f^{(\alpha)}(\underline{p}) (1 - cos\theta_{k,k'}) \frac{d^3p}{(2\pi)^6} \frac{\omega'^2 d\omega'}{c^3} d\Omega_{\underline{k},\underline{k}'} \tag{61}$$

and the transport equation (60) can then be written in the form

$$\frac{1}{3}\frac{\partial B_\omega}{\partial r} = -n_e \sigma^{tr}(\omega) \frac{\mathcal{F}_\omega}{4\pi} \tag{62}$$

where we have introduced the Planck function

$$B_\omega = \frac{\hbar\omega^3}{2\pi^2 c^2} N^o_\omega \tag{63}$$

and the energy flux

$$\mathcal{F}_\omega = \frac{2\hbar\omega^3}{3\pi c^2} \delta N_\omega \tag{64}$$

We shall return to the form of the transport equation after calculating the scattering amplitudes with the relativistic corrections and the corrections to the transport cross-section.

We stress that in the present work the distribution function $f^{(\alpha)}(\underline{p})$ in eq.(61) is taken to be a Maxwellian distribution in considering the relativistic corrections to the scattering probability. The correction containing the zero-order (non relativistic) probability and the relativistic correction to $f^{(\alpha)}(\underline{p})$ has already been found in paper I and, as discussed in the introduction, is one of the corrections to be added to the final result in a perturbative scheme to calculate the opacity. This has been defined and discussed in Paper I. Here we calculate the change in the radiative opacity due to the relativistic corrections to the scattering probabilities in the transport cross-section. From eq.(61) we write $\sigma^{tr}(\omega)$ in the form

$$\sigma^{tr}(\omega) = \sigma_0^{tr}(\omega) + \delta\sigma(\omega) \tag{65}$$

where the zero order term corresponds to the zero order probability (and has been calculated in Paper I) and $\delta\sigma(\omega)$ to the relativistic corrections found in this paper eq.s(48) and (56) and is

$$\delta\sigma(\omega) = \tag{66}$$

$$\frac{1}{c^4 n_e} \sum_{\alpha} \sum_{\sigma\sigma'} \left\{ \int \delta|M^{(\alpha)}|^2 f^{(\alpha)}(p)(1-x) \frac{d^3p}{(2\pi)^3} d\Omega_{\underline{k},\underline{k}'} \delta(\omega' - \omega) d\omega' \right\}$$

where $x = \cos\theta_{\underline{k},\underline{k}'}$. Using the results obtained in Section 3 we then find for scattering on electrons

$$\delta\sigma^{(e)} = -\frac{3}{4}\sigma_T \frac{v_{Te}^2}{c^2} \int_{-\infty}^{+\infty} \frac{dy}{\sqrt{\pi}} e^{-y^2} \int_{-1}^{+1} \frac{dx}{F_R^2 + F_I^2} \left[(1-x)^{\frac{5}{2}} F_R \left(y^2 f_1(x) + 2 f_2(x) \right) + \right.$$

$$\left. -\delta_e (1-x)^3 G(x,y) \right] \tag{67}$$

where f_1, f_2, G are given in eqs.(45 - 47), the functions F_R (x, y, ω) and $F_I(x, y, \omega)$ are the real and imaginary parts of the plasma permittivity (given explicitly in Paper I) and σ_T is the Thomson cross-section. For scattering on ions, recalling eqs.(53,55) we have

$$\delta\sigma^{(i)}(\omega) = \frac{3}{4}\sigma_T \left(\frac{v_{Te}}{c} \right)^2 < Z > \times$$

$$\times \int_{-1}^{+1} dx \int_{-\infty}^{+\infty} \frac{e^{-y^2}}{\sqrt{\pi}} dy \frac{\delta_e^2 (1-x)(\frac{7}{2} + x + \frac{7}{2}x^2 + x^3)}{|1 - x + \delta_e + < Z > \delta_e W(y)|^2} \tag{68}$$

Making use of the known result (from the fluctuation-dissipation theorem used also in deriving the zero order cross-sections)

$$\int_{-\infty}^{+\infty} \frac{e^{-y^2}}{\sqrt{\pi}} dy \frac{1}{|1 - x + \delta_e + < Z > \delta_e W(y)|^2} =$$

$$\frac{1}{(1 - x + \delta_e)[1 - x + (1 + < Z >)\delta_e]} \tag{69}$$

and performing the final angular integration we find the correction due to scattering on ions

$$\delta\sigma^{(i)}(\omega) = \frac{3}{4}\sigma_T \left(\frac{v_{Te}}{c}\right)^2 g(\delta_e, \delta) \tag{70}$$

where

$$\delta = \delta_e(1 + \langle Z \rangle) \tag{71}$$

and

$$g(\delta_e, \delta) = \delta_e(g_1(\delta_e) - g_1(\delta)) \tag{72}$$

with

$$g_1(z) = \left(2 - z + \frac{7}{2}z^2 + \frac{3}{2}z^3 + z^4\right) \ln \frac{2+z}{z} + \frac{28}{3} + \frac{35}{3}z + 11z^2 + 2z^3 \tag{73}$$

This concludes the calculation of the correction to the transport cross-section as given in eq.(66).

These relativistic corrections contribute to the overall changes in the opacity as described in Paper I. A numerical calculation at the solar centre (the parameters were taken from Ref.[6]) gives a decrease of radiative opacity of 0.2%. The decrease of the total opacity (including line absorption from iron ions) will therefore be even smaller, this is negligible compared to previous results [2] which did not include the collective effects.

5.CONCLUSIONS

For photon scattering in plasmas we have found the relativistic corrections (from the Lorentz force and particle dynamics) to the scattering probabilities entering the transport equation for radiation in plasmas. We have considered both corrections to scattering by individual electrons (Thomson scattering) and to scattering by electron clouds surrounding plasma electrons and ions (collective scattering). The results are summarized in equations (48) and (56). We have applied our results to the scattering of radiation in the solar interior, calculating the corrections to the transport cross-section and to the radiative opacity and found that the decrease of opacity $\sim 0.2\%$. The results can be applied also for relativistic correction in scattering of radiation in magnetically confined plasma for $\omega_{pe} \gg \omega_{ce} = \frac{eB}{m_e c}$.

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COLLECTIVE EFFECTS IN BREMSSTRAHLUNG IN DENSE PLASMAS

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ABSTRACT

The results of recent developments in the theory of fluctuations in a plasma shows that the previously used theory of bremsstrahlung is incomplete and the exact expressions for bremsstrahlung should include the transition bremsstrahlung. The collective effects in bremsstrahlung known previously as the Debye screening effect are changed to a qualitatively different structure, which removes the effect of the ion polarization in bremsstrahlung and introduces new effective polarization which depends on effective ion charge and electron velocity. The paper contains new explicit expressions for collective effects in bremsstrahlung replacing it by an effective polarization which depends both on average ion charge and electron velocity. The results may be relevant for application in dense plasma where the wavelength is larger than the Debye length. It is shown that for the problem of photon transport in the solar interior the correct collective corrections to the bremsstrahlung change the opacity only by $\sim -0.2\%$, less than was calculated previously when the collective effects in bremsstrahlung where estimated without taking the modern results of plasma fluctuation theory into account.

1. INTRODUCTION

The problem of collective effects in bremsstrahlung is rather old. The collective effects in bremsstrahlung were first considered in [1]. Independently a new effect called the transition bremsstrahlung was investigated for the first time in [2,3,4] it was not quite clear its relation with collective effects and the shielding effects considered in [1]. Later it was proved that to use the shielding effect in bremsstrahlung is wrong both from a physical point of view and from the point of view of a correct analytical description of the effect. These results are discussed in detail in the monograph [5]. Until the present time nevertheless in applications very often the old approach is used. For example in the recent paper [6] bremsstrahlung is treated with a so-called shielding factor due to electron-ion correlations. These results were applied to the problems of dense plasma in the solar interior [6].

We give a brief summary why the approach of the shielding factor is incorrect and what role is played by the new effect of transitional bremsstrahlung. The transitional bremsstrahlung from a physical point of view is a very simple effect described by coherent oscillation and wave emission of the ion screening cloud in the electron-ion collisions. This coherent oscillation almost cancels the effect of the screening of the ion field in the process of bremsstrahlung [5]. The last statement was illustrated in [5] by numerous examples and is true not only for bremsstrahlung in plasmas. The question then arises whether all collective effects in bremsstrahlung can be reduced to transition bremsstrahlung and what is the role of screening? The answer came from the recent development of the fluctuation theory of plasmas where it was realized that the proper description of bremsstrahlung can be obtained only with a nonlinear theory of fluctuations which was known (see [7]) but leads to a very cumbersome expressions. The general expressions are not so easy to use in practical applications and the physical meaning of the results obtained in fluctuation theory was not clear. Only recently a proof was obtained [8] for the general statement that the fluctuation theory gives exactly the sum of the matrix elements of the shielded usual bremsstrahlung and matrix element of the transition bremsstrahlung. Thus the fluctuation approach (which is only correct for such a system as a plasma) proves that there exists no other collective effects in bremsstrahlung but the transition bremsstrahlung if the screening effect is taken into account in the usual bremsstrahlung. The most important point is the possibility of cancelling parts of the matrix elements describing bremsstrahlung which is nothing but interference of two processes of emission and which leads to the cancelation of the effects of shielding by the transition bremsstrahlung. This means that all collective effects in bremsstrahlung should be reconsidered. The mathematical basis for this is given by the results of [4] in which explicit expressions for the matrix elements for both processes of bremsstrahlung were given and one can start to use them to derive explicit expressions which can be used in applications. This problem is the aim of the present paper. We will be able to give a simple and explicit expression for amplitudes of both processes and illustrate the effect of cancelation in amplitudes of bremsstrahlung. The results obtained are quite different from that used before for the bremsstrahlung in electron-electron and ion-ion collisions. In electron-ion collisions the collective effect appears to be determined by the dielectric function for electron velocity but not the ion velocity as it was previously. This leads to a stripping ie, not dressed, effect discussed in detail in [5]. The factor which determines the collective effects we obtain depends on the average ion charge and this reflects the new physical process of transition bremsstrahlung which also depends on the average ion charge. This paper contains for the first time a correct explicit expression for collective effects in bremsstrahlung which can be used in applications.

The collective effects are very important for transport of radiation in the solar interior where the maximum frequency of radiation (determined, for instance, by the Planck distribution in the solar interior) corresponds to a wavelength which is larger than the Debye length [12.13]. We have re-examine most of the possible collective plasma effects for different processes (alltogether about 10 processes) and find that previously existing calculations were not correct. The aim of this paper is not only to give a general expression for bremsstrahlung but also re-examine there role in the solar interior. Thus we re-examine here one of the collective effects among others which can be important in the solar interior. As a result we find that the collective effect in bremsstrahlung contributes less to the solar opacity as was previously thought, (this situation is not true for the other effects). Since from the point of view of plasma theory the previous investigation of collective effects in bremsstrahlung in the solar interior were not correct, the present paper presents the first proof that they are not large. On the other hand all the collective corrections to the opacity in the solar interior appears to be of the same sign, and in this case one needs to make a sum of all the corrections to have a realistic estimation of the total effect. In this case the calculated contribution of collective effects in bremsstrahlung to the value of the solar opacity should be counted among other contributions.

Obviously our general results will have other applications in plasmas, but we concentrate on the problem of photon transport in the solar interior where we find that the collective effects in bremsstrahlung are not as important as previously thought.

2. COLLECTIVE EFFECTS IN BREMSSTRAHLUNG

In a plasma each charge is screened by a cloud of equal and opposite charges distributed in a volume of linear dimension $\sim d$ (Debye length). The effect of the screening clouds (collective effects) change both the scattering cross-section and the cross-section of bremsstrahlung decreasing the scattering on electrons. The collective effect in scattering increases the scattering on ions (i.e. on the screening electron clouds of the ions) which becomes the dominating scattering process for $\lambda \gg d$ and is of order [9,10].

$$\sigma = \frac{< Z >}{1 + < Z >} \sigma_T \tag{1}$$

where

$$\sigma_T = \frac{8}{3} \pi \frac{e^4}{m_e^2 c^4}$$

is the Thomson cross-section and

$$< Z > = \frac{\sum_i n_i Z_i^2}{\sum_i n_i Z_i} \tag{2}$$

is the "mean charge" of the ions of charge Z_i and number density n_i.

We should expect similar qualitative changes also in bremsstrahlung since the physics is similar: not only is the central charge displaced in the field of another charge (the field of an incident wave in the case of scattering) but also its screening cloud [5]. If the field of a charge is decomposed into harmonics and considered in a sense as a superposition of (virtual) waves, the analogy of bremsstrahlung and scattering is obvious. In fact it is known that the probability of bremsstrahlung can be written as the product of the probability of particle collisions and the probability of scattering of the harmonics of the field (virtual waves) into electromagnetic radiation. It is clear therefore that we should expect the same qualitative changes appearing in bremsstrahlung.

It is known that identical particle collisions in vacuum give a negligible contribution since (in the dipole approximation) the rate of bremsstrahlung is proportional to $\left(\frac{e_\alpha}{m_\alpha} - \frac{e_\beta}{m_\beta}\right)$ and therefore vanishes for like particles.

But in a plasma for the ion-ion or electron-electron collisions the collective effects (the displacements of their screening clouds in the collision) can give an appreciable contribution, expecially for $\lambda \sim d$ when the dipole approximation is not valid (notice that the case $\lambda \sim d$ is of particular relevance in the sun's interior where the condition $\lambda \gg d$ is valid only for the lowest frequencies of the radiation spectrum, close to the plasma frequency).

In the following we shall therefore consider not only the emission due to electron-ion collisions but also due to ion-ion and electron-electron collisions.

In collisions all types of waves can be emitted, including longitudinal plasma waves [1].

In the present paper we start with a general approach using matrix elements of bremsstrahlung given in [4]. A big advantage in using the results of [4] is that they can lead to explicit expressions for collective effects in bremsstrahlung of electromagnetic waves which was previously not given in the literature in the form useful for applications.

Here we will consider the case of electromagnetic waves using an approach valid for any polarization of the emitted wave.

We shall make the following assumptions to simplify the general expression of ref.[4];

1) emission is treated as a classical process for the small impact parameters when the collective effects are important.

2) the velocities v_α, v_β of the colliding particles are non-relativistic;

3) the phase velocities of the waves are much larger than the particle velocity

$$\frac{\omega_k}{k} \gg v_\alpha, v_\beta \tag{3}$$

The first assumption is valid for

$$\hbar k \ll mv \tag{4a}$$

$$\hbar q \ll mv \tag{4b}$$

where q is the momentum transferred from one colliding particle to the other colliding particle during the process of bremsstrahlung (q is the momentum of a virtual wave).

From (4a) we have

$$\hbar\omega_k \ll mv^2 \left(\frac{\omega_k}{kv}\right) \tag{5}$$

For electromagnetic waves $\omega_k/k \sim c$ and for thermal particles equation (5) becomes

$$\hbar\omega_k \ll T\left(\frac{c}{v_T}\right)^{\frac{1}{2}} \tag{6}$$

where T is the thermal energy (temperature in energy units) and $v_T = (T/m)^{\frac{1}{2}}$ the thermal velocity.

Eq. (5), combined with eq. (3), means that the energy of an emitted photon can be of the order of the particle energy and condition (3) is still valid. For instance in the sun's interior where $\hbar\omega \sim T$ at the maximum of the Planck distribution,eq. (6) shows that condition (3) is still valid. Our assumption (1) is therefore really a consequence of the assumptions (2) and (3), but only for the emitted waves (i.e. (4a)). Condition (4b) is an important assumption. The results obtained here are expressed as an integral over all virtual momentum q and lead for electron-ion collisions to a logarithmic divergence which we will treat by introducing q_{max}. The lowest possible q is determined by the conservation law in the process of bremsstrahlung and for electron-ion collision is of the order of $\frac{\omega_k}{v}$, where v is the relative electron-ion velocity (practically the electron velocity since the ion velocity is usually much slower than the electron velocity). Thus our results for bremsstrahlung in electron-ion collisions will contain the logarithm

$$\ln\frac{q_{max}v}{\omega}$$

An important point for applications is that in the case $q_{max} \gg \frac{1}{d}$ (as it is for the solar interior) and for q of the order of q_{max} the collective effects in bremsstrahlung are important. Thus we can join our results with the known results for bremsstrahlung when the collective effects are neglected. This gives q_{max} of the order of $\frac{mv}{\hbar}$ (the quantum uncertainty principle gives an estimate even without any deep considerations). But in principle there is no problem to join the two results to find an exact value of the numerical coefficient under the logarithm i.e. the logarithm becomes

$$\ln\frac{mv^2}{\hbar\omega}$$

This problem can thus be solved in a simple way for electron-ion collisions.

For bremsstrahlung electron-electron or ion-ion collisions the problem is even simpler since the final integral with collective effects included converges in q and thus q_{max} does not play a significant role.

The second assumption ($v/c \ll 1$) makes all calculations much easier since in this case only the electrostatic virtual fields are important. The corresponding total probability of bremsstrahlung is given in reference [4] (p. 260), for an emitted wave of wave-number $\underline{k}$, frequency ω_k and unit polarization vector $\hat{e}^\sigma_{\underline{k}}$ (σ stands for the type of polarization) it is given by

$$W_{\alpha\beta}(\underline{k},\underline{q}) = 2(2\pi)^2 \frac{\left|\hat{e}^\sigma_{\underline{k}} \cdot (\underline{M}^\alpha + \underline{M}^\beta + \underline{M}^{\alpha\beta})\right|^2}{\hbar \left[\frac{\partial \omega^2 \epsilon^\sigma_{\omega,k}}{\partial \omega}\right]_{\omega=\omega^\sigma_k}} \delta(\omega^\sigma_k - \underline{k}\cdot\underline{v}_\alpha + \underline{q}\cdot(\underline{v}_\alpha - \underline{v}_\beta)) \quad (7)$$

where $\underline{q}$ is the wave-vector of the momentum transferred in the collision and

$$\epsilon^\sigma_{\omega,k} = \epsilon_{ij}(\omega,k) e^{\sigma *}_{\underline{k}i} e^\sigma_{\underline{k}j} \quad (8)$$

where $\epsilon_{ij}(\omega,k)$ is the plasma dielectric tensor. For electromagnetic waves

$$\omega^t_k = (c^2k^2 + \omega^2_{pe})^{\frac{1}{2}} \ ; \ \omega^2_{pe} = \frac{4\pi e^2 n_e}{m_e} \quad (9)$$

and, neglecting the thermal effects in the dielectric tensor we simply take

$$\epsilon^\sigma_{\omega,k} \equiv \epsilon(\omega) = 1 - \frac{\omega^2_{pe}}{\omega^2} \quad (10)$$

The vectors $\underline{M}^\alpha$ and $\underline{M}^\beta$ in eq. (3), given in reference [4] (see eq.s (5.60) and (5.61)), account for the oscillation (displacement of trajectory) of one charge (α or β) in the screened field of the other (β or α). The oscillations and emission of the screening clouds of particles α and β are taken into account in $\underline{M}^{\alpha\beta}$ (eq. (5.66) of reference [4]).

Here we shall derive expressions for these vectors using our assumptions (1-3). For $\underline{M}^\alpha$ from ref. [4] we write

$$M^\alpha_i = (2\pi)^3 e_\beta \wedge^\alpha_{ij} (\underline{q}) G_{jl}(q) v^\beta_l \quad (11)$$

where e_β is the charge of particle β with velocity $\underline{v}^\beta$, G_{jl} is the Green function of the field and we use here only its longitudinal part given by

$$G_{jl}(\underline{q}) = -i \frac{4\pi q_j q_l}{(\underline{q}\cdot\underline{v}_\beta) q^2 \epsilon_{\underline{q}\cdot\underline{v}_\beta, q}} \quad (12)$$

and then

$$G_{jl} v^l_\beta = -i \frac{4\pi q_j}{q^2 \epsilon_{\underline{q}\cdot\underline{v}_\beta, q}} \quad (13)$$

where $\epsilon_{\omega,k}$ is the longitudinal part of the dielectric permittivity. Using eq.(13) eq.(11) becomes

$$M_i^\alpha = -i\frac{(4\pi)(2\pi)^3 e_\beta}{q^2 \epsilon_{\underline{q}\cdot\underline{v}_\beta,q}} \Lambda_{ij}^\alpha(q_\alpha) q_j \tag{14}$$

The tensor $\Lambda_{ij}^\alpha(\underline{q})$ is given in reference [4] (eq. 4.143) and using the conservation law

$$\underline{q}\cdot\underline{v}_\alpha = \underline{q}\cdot\underline{v}_\beta - (\omega_k^\sigma - \underline{k}\cdot\underline{v}_\alpha) \tag{15}$$

from the δ-function in eq. (7), we find

$$\Lambda_{ij}^\alpha(\underline{q})q_j = \tag{16}$$

$$i\frac{e_\alpha^2\left(1-\frac{v_\alpha^2}{c^2}\right)^{1/2}}{(2\pi)^3 m_\alpha(\omega_k^\sigma - \underline{k}\cdot\underline{v}_\alpha)^2}\left[v_\alpha^i\left(\underline{k}\cdot\underline{q} - \frac{\omega_k^\sigma}{c^2}\underline{q}\cdot\underline{v}_\beta\right) + (\omega_k^\sigma - \underline{k}\cdot\underline{v}_\alpha)\left(q_i + \frac{v_\alpha^i}{c^2}\omega_k^\sigma\right)\right]$$

where e_α, v_α are the charge and velocity of particle α.

A further simplification comes from using our assumptions (2) and (3); from eq. (15) for $\omega_k^\sigma \gg \underline{k}\cdot\underline{v}_\alpha$ it follows that

$$q > \frac{\omega_k^\sigma}{|\underline{v}_\alpha - \underline{v}_\beta|} \geq \frac{\omega_k^\sigma}{v_\alpha}$$

i.e.

$$q \gg \frac{v_\alpha \omega_k^\sigma}{c^2} \simeq \frac{v_\alpha^2}{c^2}\frac{\omega_k^\sigma}{v_\alpha}$$

Using these simplifications in eq.(16) the second term in square brackets is $\omega_k^\sigma q_i$, the first term can be neglected since it is of order $kv_\alpha \ll \omega_k^\sigma$ and $v_\alpha v_\beta/c^2 \ll 1$: then we find from eq.(14)

$$\underline{M}^\alpha = \frac{4\pi e_\alpha^2 e_\beta}{\omega_k^\sigma m_\alpha q^2 \epsilon_{\underline{q}\cdot\underline{v}_\beta,q}}\underline{q} \tag{17}$$

This matrix element contains the screening in $\epsilon_{\underline{q}\cdot\underline{v}_\alpha,q}$ and is often used in calculations of collective effects in bremsstrahlung. We will see that due to interference effects the correct factor entering in the collective effect contains the electron, but not the ion velocity in the dielectric permittivity ϵ and thus describes the stripping of electron screening for high electron velocities.

With the same simplifications we also find for the other colliding particle

$$\underline{M}^{\beta} = \frac{4\pi e_\alpha e_\beta^2}{\omega_k^\sigma m_\beta |\underline{k} - \underline{q}|^2 \epsilon_{(\underline{k}-\underline{q})\cdot\underline{v}_\alpha, |\underline{k}-\underline{q}|}} (\underline{k} - \underline{q}) \tag{18}$$

In this case the α particle is light (electron) and the β particle is heavy (ion) and therefore this matrix element can be neglected. But this is not the case of ion-ion and electron-electron bremsstrahlung where the collective effects change drastically the emission power.

In the absence of the screening effects $\epsilon_{\omega k} \simeq 1$, $q \gg k$ we have

$$\underline{M}^{\alpha} + \underline{M}^{\beta} \sim e_\alpha e_\beta \left(\frac{e_\alpha}{m_\alpha} - \frac{e_\beta}{m_\beta} \right) \underline{q} \tag{19}$$

which reproduces the known result of negligible bremsstrahlung for $e_\alpha/m_\alpha \simeq e_\beta/m_\beta$.

For $\underline{M}^{\alpha\beta}$ (see eq. 5.66 and 6.43 in reference [4]) and using our assumptions we can substitute δ_{ij} for $\delta_{ij}(1 - \underline{k} \cdot \underline{v}/\omega) + k_i v_j/\omega$ in all factors in eq. (6.43) of ref [4] and ω for $\omega - \underline{k} \cdot \underline{v}$ in the denominator of the first factor, if we also assume that only the electrons contribute to the nonlinear current (i.e. ignore the ion contribution which is of the order of m_i^{-1}) we find after a lengthy calculation

$$\begin{aligned} \underline{M}^{\alpha\beta} = {} & \frac{4\pi e e_\alpha e_\beta \underline{q}}{\omega_k^\sigma m_e q^2 \epsilon_{\underline{q}\cdot v_\beta, q}} \left(\frac{\epsilon^{(e)}_{(\underline{k}-\underline{q})\cdot\underline{v}_\alpha, |\underline{k}-\underline{q}|} - 1}{\epsilon_{(\underline{k}-\underline{q})\cdot\underline{v}_\alpha, |\underline{k}-\underline{q}|}} \right) + \\ & + \frac{4\pi e e_\alpha e_\beta (\underline{k} - \underline{q})}{\omega_k^\sigma m_e |\underline{k} - \underline{q}|^2 \epsilon_{(\underline{k}-\underline{q})\cdot\underline{v}_\alpha, |\underline{k}-\underline{q}|}} \left(\frac{\epsilon^{(e)}_{\underline{q}\cdot\underline{v}_\beta, q} - 1}{\epsilon_{\underline{q}\cdot\underline{v}_\beta, q}} \right) \end{aligned} \tag{20}$$

where e is the electron charge, $\epsilon^{(e)}_{\omega,k} = 1 + \chi^{(e)}_{\omega,k}$ is the electron susceptibility and

$$\epsilon_{\omega,k} = 1 + \chi^{(e)}_{\omega,k} + \sum_i \chi^{(i)}_{\omega,k} \tag{21}$$

is the plasma susceptibility with $\chi^{(i)}_{\omega k}$ giving the contribution of type-i ion. The expression (20) is completely new and was not given previously in the literature. From (20) it can be seen that it gives the results very similar to (17) and (18) and can never be neglected.

A comparison of eq. (17) and (18) with eq. (20) shows that in general the contribution of the screening clouds $\underline{M}^{\alpha\beta}$ to bremsstrahlung is important since the expressions in brackets can be of the order of one.

Next we shall find expressions for the total $\underline{M}^{\alpha}+\underline{M}^{\beta}+\underline{M}^{\alpha\beta}$ entering eq. (7) for the probability, for the three possible collisions.

a) electron-electron collisions: ($\underline{v}$ and $\underline{v}'$ are the velocities of two colliding electrons).

$$\underline{M}^{(ee')}=\underline{M}^{e}+\underline{M}^{e'}+\underline{M}^{ee'}=-\frac{4\pi e^3\underline{q}}{m_e\omega_k^\sigma q^2\epsilon_{\underline{q}\cdot\underline{v}',q}}\frac{1+\sum_i\chi^{(i)}_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}{\epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}+$$
$$-\frac{4\pi e^3(\underline{k}-\underline{q})}{m_e\omega_k^\sigma|\underline{k}-\underline{q}|^2\epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}\frac{1+\sum_i\chi^{(i)}_{\underline{q}\cdot\underline{v}',q}}{\epsilon_{\underline{q}\cdot\underline{v}',q}}\qquad(22)$$

It should be noted that this expression is quite different from that used in the present literature. It can also be directly found from the fluctuation theory of plasmas (see [8,11]).

b) ion-ion collisions.

Due to the ion mass in the denominator we can neglect eq. (17) and (18) and only the contribution of the screening clouds eq. (20) is important ($\underline{v}$ and $\underline{v}'$ are the velocities of two colliding ions)

$$\underline{M}^{(ii')}=\underline{M}^{ii'}=\frac{4\pi e^3Z_iZ_i'\underline{q}}{m_c\omega_k^\sigma q^2\epsilon_{\underline{q}\cdot\underline{v}',q}}\frac{\chi^{(e)}_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}{\epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}+$$
$$+\frac{4\pi e^3Z_iZ_i'(\underline{k}-\underline{q})}{m_e\omega_k^\sigma|\underline{k}-\underline{q}|^2\epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}\frac{\chi^{(e)}_{\underline{q}\cdot\underline{v}',q}}{\epsilon_{\underline{q}\cdot\underline{v}',q}}\qquad(23)$$

c)electron-ion collisions

Let α be the electron and β the ion, then we can neglect eq. (18) (due to the ion mass in the denominator) and, denoting the electron velocity $\underline{v}$ and the ion velocity $\underline{v}'$, we find

$$\underline{M}^{(ei)}=\underline{M}^{e}+\underline{M}^{ei}=\frac{4\pi e^3Z_i\underline{q}}{m_e\omega_k^\sigma q^2\epsilon_{\underline{q}\cdot\underline{v}',q}}\frac{1+\sum_i\chi^{(i)}_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}{\epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}+$$
$$-\frac{4\pi e^3Z_i(\underline{k}-\underline{q})}{m_e\omega_k^\sigma|\underline{k}-\underline{q}|^2\epsilon_{\underline{q}\cdot\underline{v}',\underline{q}}}\frac{\chi^{(e)}_{\underline{q}\cdot\underline{v}',q}}{\epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}\qquad(24)$$

For longitudinal waves ($\hat{e}^\sigma_{\underline{k}} = \underline{k}/k, \omega^\sigma_k = \omega^e_k \simeq \omega_{pe}$) these expressions lead to the well known results of [8]. Here we shall consider the case of transverse waves ($\omega^\sigma_k = \omega^t_k, \hat{e}^\sigma_{\underline{k}} \cdot \underline{k} = 0$) and use them in eq. (7) to find the emission in the following section.

We conclude this section with some comments on equations (22-24).

First notice that these expressions satisfy the known relation between scattering and bremsstrahlung discussed in the introduction. In fact the scattering cross-section (taking into account the collective effects) is proportional to $|M_{scat}|^2$ where [9,10]

$$M^{(e)}_{scat} \simeq \frac{e^2}{m_e}(\hat{e}^{\sigma *}_{\underline{k}} \cdot e^{\sigma'}_{\underline{k}'}) \frac{1 + \sum_i \chi^{(i)}_{\omega-\omega',(\underline{k}-\underline{k}')\cdot\underline{v}}}{\epsilon_{\omega-\omega',(\underline{k}-\underline{k}')\cdot\underline{v}}}$$

for scattering on electrons, and

$$M^{(i)}_{scat} \simeq \frac{e^2}{m_e}(\hat{e}^{\sigma *}_{\underline{k}} \cdot e^{\sigma'}_{\underline{k}'}) \frac{\chi^{(e)}_{\omega-\omega',(\underline{k}-\underline{k}')\cdot\underline{v}}}{\epsilon_{\omega-\omega',(\underline{k}-\underline{k}')\cdot\underline{v}}}$$

for scattering on ions.

Thus the bremsstrahlung matrix elements can be expressed through the scattering matrix elements. From $M^{(ee')}$ in eq. (22) we can see that the two terms describe two effects. The first term gives the appearance of the harmonic $\underline{q}$ from the field of the electron with velocity $\underline{v}'$ and scattering on the other electron (with velocity $\underline{v}$). The second term gives the appearance of the harmonic $(\underline{k} - \underline{q})$ from the field of the electron with velocity $\underline{v}$ and scattering on the other electron (with velocity $\underline{v}'$).

The same can be seen in $M^{(ii')}$ (eq. 23) where either one of the ions scatters the harmonic appearing from the field of the other ion.

Notice that both $\underline{M}^{(ee')}$ and $\underline{M}^{(ii')}$ are invariant for $\underline{q} \leftrightarrow \underline{k} - \underline{q}$. The term $\underline{M}^{(ei)}$ (eq.24) describes both the scattering on an electron (first term) and the scattering on an ion (second term) of the harmonics in the field of an ion or an electron respectively.

Finally without collective effects only the electron-ion collision term survives (see eq. 19) with the first term in which the collective effects in scattering are also neglected ($\underline{q} \simeq \underline{k}$) and $v' \simeq 0$, i.e.

$$\underline{M}^{(ei)}_{non-collective} \sim \frac{4\pi e^3 Z_i \underline{q}}{m_e \omega_k q^2 \epsilon_{0,q}} \qquad (25)$$

The comparison of this expression with eq's (22-24) shows how important the changes can be in bremsstrahlung due to properly taking into account collective effects.

3.SPONTANEOUS AND STIMULATED EMISSION

We are interested in the total emission rate of electromagnetic radiation from a unit plasma volume particularly in the frequency domain

$$\omega_{pe} < \omega \leq \frac{T}{\hbar} \ll \frac{T}{\hbar}\left(\frac{c}{v_{Te}}\right)^{\frac{1}{2}} \tag{26}$$

which is the frequency range relevant to the solar interior.

The total intensity of emission is given by [3]

$$Q = Q_{ee} + \sum_i Q_{ei} + \sum_{i,i'} Q_{ii'} \tag{27}$$

where

$$Q_{\alpha\beta} = \int \hbar\omega_k W_{\alpha\beta}(\underline{k},\underline{q}) \frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\, f_\alpha(\underline{v}_\alpha) f_\beta(\underline{v}_\beta) d^3v_\alpha d^3v_\beta \tag{28}$$

and where $f_\alpha(\underline{v})$ is the distribution function of particles of type-α in the plasma, normalized to the number density n_α

$$n_\alpha = \int f_\alpha(\underline{v}_\alpha) d^3v_\alpha \tag{29}$$

For transverse waves we use the identity (in the following we use the notation $\omega_k = \omega$)

$$k^2 dk = \frac{\omega^2}{c^3}\sqrt{\epsilon(\omega)}d\omega \tag{30}$$

and defining the emission $Q_{\alpha\beta}^{(\omega)}$ in the frequency range $d\omega$ as

$$Q_{\alpha\beta} = \int Q_{\alpha\beta}(\omega) d\omega \tag{31}$$

we find from eq's (28) and (30) that

$$Q_{\alpha\beta}(\omega) = \frac{\hbar\omega^3}{c^3}\sqrt{\epsilon(\omega)} \int W_{\alpha\beta}(\underline{k},\underline{q}) \frac{d\Omega_{\underline{k}}}{(2\pi)^3} f_\alpha(\underline{v}_\alpha) f_\beta(\underline{v}_\beta) d^3v_\alpha d^3v_\beta \frac{d^3q}{(2\pi)^3} \tag{32}$$

where $d\Omega_k$ is the solid angle associated with $\underline{k}$.

Then from eq. (27) we get

$$Q(\omega) = Q_{ee}(\omega) + \sum_{i} Q_{ei}(\omega) + \sum_{i,i'} Q_{ii'}(\omega) \tag{33}$$

for the total intensity in the frequency interval $d\omega$.

The stimulated emission (or inverse bremsstrahlung) produces an absorption of waves with a damping rate (see ref. [3])

$$\gamma_{\underline{k}} = \gamma_{\underline{k}}^{ee} + \sum_{ei} \gamma_{\underline{k}}^{ei} + \sum_{ii'} \gamma_{\underline{k}}^{ii'} \tag{34}$$

where

$$\gamma_{\underline{k}}^{\alpha\beta} = \frac{1}{2} \int W_{\alpha\beta}(\underline{k}, \underline{q}) \left[\left(f(\underline{v}_\beta) - f(\underline{v}_\beta - \frac{\hbar \underline{q}}{m_\beta}) \right) f(\underline{v}_\alpha) + \right.$$

$$\left. \left(f(\underline{v}_\alpha) - f(\underline{v}_\alpha - \frac{\hbar(\underline{k} - \underline{q})}{m_\alpha}) \right) f(\underline{v}_\beta) \right] d^3 v_\alpha d^3 v_\beta \frac{d^3 q}{(2\pi)^3} \tag{35}$$

where $f(\underline{v}_\alpha)$ is the distribution function and the probability $W_{\alpha\beta}$ is given by eq.(7).

Energy conservation requires

$$\frac{p_\alpha^2}{2m_\alpha} + \frac{p_\beta^2}{2m_\beta} = \frac{\left[\underline{p}_\alpha - \hbar(\underline{k} - \underline{q})\right]^2}{2m_\alpha} + \frac{(\underline{p}_\beta - \hbar \underline{q})^2}{2m_\beta} + \hbar\omega_k$$

where $\underline{p}_\alpha = m_\alpha \underline{v}_\alpha$ is the particle momentum. For the electron-ion case, neglecting the small transfer of momentum to the ion, and for Maxwellian distributions the term in square brackets in the integral eq. (35) becomes

$$f(\underline{v}_e) f(\underline{v}_i) \left[1 - e^{\frac{\hbar\omega_k}{T}}\right]$$

and $\gamma_{\underline{k}}^{ei}$ can be written in the form

$$\gamma_{\underline{k}}^{ei} = \frac{1}{2}(1 - e^{\frac{\hbar\omega_k}{T}}) \int W_{ei}(\underline{k}, \underline{q}) f_e(\underline{v}_e) f_i(\underline{v}_i) d^3 v_e d^3 v_i \frac{d^3 q}{(2\pi)^3} \tag{35a}$$

or, comparing with (32) integrated over the solid angle, it is given by

$$\gamma_k^{ei} = \frac{1}{2} \frac{(1 - e^{\frac{\hbar\omega_k}{T}})}{(\hbar\omega_k/T)} \left[\frac{2\pi^2 c^3}{T\omega_k^2 \sqrt{\epsilon(\omega_k)}} Q_{ei}(\omega_k) \right] \tag{36}$$

where

$$\gamma_k^{ei} = \frac{1}{4\pi}\int \gamma_{\underline{k}}^{ei} d\Omega_{\underline{k}}$$

For the e-e and i-i cases the relation between $\gamma^{\alpha\beta}$ and $Q^{\alpha\beta}$ can only be found for $\hbar k \ll p$ and $\hbar q \ll p$ (which means $\hbar\omega_k \ll T$), in this limit we can expand

$$f(\underline{v}) - f(\underline{v} - \frac{\hbar \underline{k}}{m}) \simeq \frac{\hbar \underline{k}}{m} \cdot \frac{\partial f(\underline{v})}{\partial \underline{v}} \simeq -\frac{\hbar}{T}(\underline{k}\cdot\underline{v})f(\underline{v})$$

such that in the integrand of eq.(35) we find

$$\gamma_k^{\alpha\beta} = -\frac{2\pi^2 c^3}{2T\omega_k^2\sqrt{\epsilon(\omega_k)}} Q_{\alpha\beta}(\omega_k)$$

which only coincides with eq.(36) in the limit $\hbar\omega_k/T \ll 1$.

In the following we shall find that only the e-i bremsstrahlung is important and then use the more general relation given by eq.(36) to relate emission to absorption.

4. COLLECTIVE EFFECTS IN SPONTANEOUS AND STIMULATED BREMSSTRAHLUNG

We consider separately the three cases.

1) electron-ion bremsstrahlung.

Using eq.(24) in eq.(7) and then eq.(32) we get

$$Q_{ei}(\omega) =$$

$$= \frac{\sqrt{\epsilon(\omega)}e^6 Z_i^2}{m_e^2 c^3 \pi^2}\int f^{(e)}(\underline{v}) f^{(i)}(\underline{v}') d^3q\, d\Omega_k\, d^3v\, d^3v'\, \delta[\omega - \underline{q}\cdot\underline{v}' - (\underline{k}-\underline{q})\cdot\underline{v}] F_{k,q}(\underline{v},\underline{v}') \tag{37}$$

where

$$F_{k,q}(\underline{v},\underline{v}') = \frac{\frac{1}{2}|\underline{k}\times\underline{q}|^2}{k^2|\epsilon_{\underline{q}\cdot\underline{v}',q}\,\epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}|^2}\left|\frac{1+\sum_i \chi^{(i)}_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}{q^2} + \frac{\chi^{(e)}_{\underline{q}\cdot\underline{v}',q}}{|\underline{k}-\underline{q}|^2}\right|^2 \tag{38}$$

We now make the following simplifications; the ion susceptibility corresponding to the electron velocity ($\underline{v}$) is small and can be ignored with respect to 1; the electron susceptibility corresponding to the ion velocity ($\underline{v}'$) is practically the Debye screening, ie.,

$$\chi^{(e)}_{\underline{q}\cdot\underline{v}',q} \simeq \frac{\omega_{pe}^2}{q^2 v_{Te}^2}$$

We can also neglect the ion velocity v' in the δ-function.

Then we have, using Maxwellian distributions for $f^{(e)}(v), f^{i}(v')$ and integrating over the component of velocity perpendicular to $\underline{q}$ and to $(\underline{k}-\underline{q})$ respectively

$$Q_{ei}(\omega) =$$

$$= \frac{\sqrt{\epsilon(\omega)} e^6 Z_i^2 n_i n_e}{m_e^2 c^3 \pi^2} \int \frac{e^{-y^2}}{\sqrt{\pi}} \frac{e^{-y'^2}}{\sqrt{\pi}} d^3q\, d\Omega_{\underline{k}} F_{\underline{k}\underline{q}}(y,y') \delta(\omega - |\underline{k}-\underline{q}| v_{Te}\sqrt{2}y) dy dy' \tag{39}$$

where

$$F_{\underline{k}\underline{q}}(y,y') = \frac{\frac{1}{2}|\underline{k}\times\underline{q}|^2 \left|1 + \frac{\omega_{pe}^2}{|\underline{k}-\underline{q}|^2 v_{Te}^2}\right|^2}{q^4 k^2 \left|1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2} + \sum_i \frac{\omega_{pe}^2 W_i^2 n_i}{q^2 v_{Te}^2 n_e} W(y')\right|^2 \left|1 + \frac{\omega_{pe}^2}{|\underline{k}-\underline{q}|^2 v_{Te}^2} W(y)\right|^2}$$

where y and y' are the normalized velocity components parallel to $\underline{k}-\underline{q}$ and $\underline{q}$ respectively, i.e.

$$y = \frac{v_\parallel}{\sqrt{2}v_{Te}}, v_\parallel = \frac{(\underline{k}-\underline{q})\cdot\underline{v}}{|\underline{k}-\underline{q}|};\ y' = \frac{v'_\parallel}{\sqrt{2}v_{Ti}}, v'_\parallel = \frac{\underline{q}\cdot\underline{v}'}{q} \tag{40}$$

and $W(y)$ is the plasma function, from the susceptibilities can be written as

$$W(y) = 1 - 2ye^{-y^2}\int_0^y e^{t^2} dt + i\sqrt{\pi} y e^{-y^2} \tag{41}$$

From the δ-function in eq.(39) we see that for $k \sim q$

$$\omega = ck \sim v_{Te} q y$$

i.e. $y \sim c/v_{Te}$ and e^{-y^2} becomes negligible, we can then assume $k \ll q$ in the integral and simplify it to find

$$Q_{ei}(\omega) =$$

$$= \frac{16}{3} \frac{\sqrt{\epsilon(\omega)} e^6 Z_i^2 n_i n_e}{m_e^2 c^3} \int dq \int \frac{e^{-y^2}}{\sqrt{\pi}} \delta(\omega - q v_{Te} \sqrt{2} y) \frac{\left(1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2}\right)^2}{\left|1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2} W(y)\right|^2} dy I_q \tag{42}$$

where

$$I_q = \int \frac{e^{-y'^2}}{\sqrt{\pi}} \frac{dy'}{\left|1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2} + \sum_i \frac{\omega_{pe}^2 Z_i^2 n_i}{q^2 v_{Te}^2 n_e} W(y')\right|^2} \tag{43}$$

The y'-integral (the ion contribution) is well known from the theory of scattering and is given by

$$I_q = \frac{1}{\left(1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2}\right) \left[1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2} (1 + < Z >)\right]} \tag{44}$$

where $< Z >$ is given by eq.(2).Then from eq.(42) we have

$$\sum_i Q_{ei}(\omega) =$$

$$\frac{16}{3} \frac{\sqrt{\epsilon(\omega)} e^6 n_e^2 < Z >}{m_e^2 c^3} \int \left(1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2}\right) dq \int \frac{e^{-y^2}}{\sqrt{\pi}} \frac{\delta(\omega - q v_{Te} \sqrt{2} y)}{\left[1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2} (1 + < Z >)\right] \left|1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2} W(y)\right|^2} dy$$

Performing the y-integral using the δ-function and then introducing the new variable to transform the q-integral

$$x = \frac{\omega}{\sqrt{2} v_{Te} q} \tag{45}$$

we finally find

$$\sum_i Q_{ei}(\omega) =$$

$$= \sigma_T \frac{2}{\pi \sqrt{2\pi}} \left(\frac{c}{v_{Te}}\right) n_e^2 e^2 \sqrt{\epsilon(\omega)} < Z > \int_{x_{min}}^{\infty} \frac{dx}{x} \frac{e^{-x^2} \left(1 + 2 \frac{\omega_{pe}^2}{\omega^2} x^2\right)}{\left[1 + 2 \frac{\omega_{pe}^2}{\omega^2} (1 + < Z >) x^2\right] \left|1 + 2 \frac{\omega_{pe}^2}{\omega^2} x^2 W(x)\right|^2} \tag{46}$$

This result differs from previous ones on collective electron-ion bremsstrahlung (see e.g. Bekefi [1], Ichimaru [14]) for the contribution of the screening clouds of the colliding particles (transition bremsstrahlung) which is included here for the first time.

In the classical description for the probability the q-integral is taken between q_{min} and q_{max} (the Bethe-Heitler limit). In our case we point out that since for $q \sim q_{max}$ it is $\epsilon \sim 1$ (negligible collective effects) then we can take the known results from single particle emission to define q_{max} the same Bethe-Heitler value of the non-collective treatment and join the collective and single-particle results for some $q^* < q_{max}$.

Starting from the expression for the emitted power (eq.(37) of ref. [8]) but not performing the integration over the perpendicular component of the electron velocity (as done in ref.[8]) but only over the ion distribution and the angle between $\underline{v}$ and $\underline{q}$ using the δ-function, the result is

$$Q_{ei}(\omega) = \frac{16}{3} \frac{e^6 n_e^2 \langle Z \rangle}{m_e^2 c^3} \int d^3v \frac{e^{-\frac{v^2}{2v_{Te}^2}}}{(2\pi)^{3/2} v_{Te}^3} \frac{1}{v} \int_{q_{min}}^{q_{max}} \frac{dq}{q} \mathcal{H}(\omega, q) \tag{47}$$

where

$$\mathcal{H}(\omega, q) = \frac{1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2}}{\left[1 + (1 + \langle Z \rangle) \frac{\omega_{pe}^2}{q^2 v_{Te}^2}\right] |1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2} W(\frac{\omega}{\sqrt{2} q v_{Te}})|^2}, \tag{48}$$

$< Z >$ in the effective ion charge, and the limits of integration for the transferred momentum q can be found from the quantum conservation laws (v' is the electron velocity after the emission)

$$\underline{q} = \frac{m_e}{\hbar}(\underline{v} - \underline{v}') \; ; \; q = \frac{m_e}{\hbar} \sqrt{v^2 + v'^2 - 2vv' \cos\theta} \tag{49}$$

and

$$\frac{m_e}{2} v^2 = \frac{m_e}{2} v'^2 + \hbar\omega \; ; \; v' = \sqrt{v^2 - \frac{2\hbar\omega}{m_e}} \tag{50}$$

Then

$$q_{min} = \frac{m_e}{\hbar} \left(v - \sqrt{v^2 - \frac{2\hbar\omega}{m_e}} \right) \; ; \; q_{max} = \frac{m_e}{\hbar} \left(v + \sqrt{v^2 - \frac{2\hbar\omega}{m_e}} \right) \tag{51}$$

Notice that in most applications, including the solar interior we have the result

$$\frac{\omega_{pe}^2}{q_{max}^2 v_{Te}^2} \simeq \frac{\omega_{pe}^2 \hbar^2}{m_e^2 v_{Te}^2 v^2} \simeq \left(\frac{\hbar\omega_{pe}}{T} \right)^2 << 1 \tag{52}$$

and therefore near the upper limit ($q \simeq q_{max}$) $\mathcal{H}(q,\omega) \simeq 1$ (no collective effects).The corrections are therefore essential only for $\omega << m_e \frac{v^2}{\hbar}$ and this is the most important domain for the contribution to the opacity. Integrating by parts eq.(47), using $\mathcal{H}(\omega, q_{max}) = 1$ (see 52), introducing $y = \frac{v}{\sqrt{2}v_{Te}}$ and changing variable to

$$x = \frac{1}{2}[y + \sqrt{y^2 - z}] \tag{53}$$

the result is

$$Q_{ei} = \frac{16}{3}\sqrt{\frac{2}{\pi}} \frac{e^6 n_e^2 \langle Z\rangle \sqrt{\varepsilon(\omega)}}{m_e^2 c^3 v_{Te}} \mathcal{F}_{col}(\omega) \tag{54}$$

where $\mathcal{F}_{col}(\omega)$ is given by

$$\mathcal{F}_{col}(\omega) = \int_{\frac{\sqrt{z}}{2}}^{\infty} \frac{dx}{x} e^{-(x+\frac{z}{4x})^2} \left(1 + \frac{1 + 2\frac{\omega_{pe}^2}{\omega^2}x^2}{\left[1 + 2\frac{\omega_{pe}^2}{\omega^2}(1+ < Z >)x^2\right] |1 + 2\frac{\omega_{pe}^2}{\omega^2}x^2 W(x)|^2} \right) \tag{55}$$

where $z = \hbar\omega/T$

The absorption coefficient is given in terms of the emitted power (eq.36) and the result is

$$2\gamma_k = \frac{8\sqrt{\pi}}{\sqrt{2}} \sigma_T \frac{e^2 < Z > n_e^2 c^4}{v_{Te} \hbar \omega^3} (1 - e^z) \mathcal{F}_{col}(\omega) \tag{56}$$

When collective effects are neglected ($\mathcal{H} = 1$) the q-integral in (56) can be performed and the function $\mathcal{F}_{col}(\omega)$ of (54) becomes

$$\mathcal{F}_o(\omega) = 2\int_{\frac{\sqrt{z}}{2}}^{\infty} \frac{dx}{x} e^{-(x+\frac{z}{4x})^2} = \int_{\sqrt{\frac{2\hbar\omega}{m_e}}}^{\infty} \frac{e^{-\frac{v^2}{2v_{Te}^2}}}{v_{Te}^2} v \ln\frac{v+v'}{v-v'} dv \tag{57}$$

where v' is given by eq.(49) and the second equality can easily be proved by integrating by parts and using the substitution (52), eq.(54) with $\mathcal{F}_0(\omega)$ is the known non-collective result for electron-ion bremsstrahlung and Maxwellian electrons.

Equation (54) represents one of the main results of this paper and differs significantly from previous attempts such as the treatment of Ichimaru's [14] to include plasma collective effects in bremsstrahlung. In particular equation 9.132 of Ichimaru [14] does not contain $< Z >$, the effective Z value for the plasma, in the denominator. He also has $W(x) = 1$ for all values of x. The result obtained in [14] only includes the screening of the ion field for all electron velocities. This is the effect of permanent shielding irrespective of the electron velocity which is not correct for high electron velocity. The result obtained in the present paper shows that the screening effect is completelly cancelled by the effect of emission from the oscillating cloud and what is left is a new effect of redressing the screening cloud as soon as the electron velocities become larger than the electron mean thermal velocity. The oscillation of the electron cloud is also included in $\left[1 + 2\frac{\omega_{pe}^2}{\omega^2}\,(1+ < Z >)\,x^2\right]$ in the denominator. The result of [14] can be written by changeing $\mathcal{F}_{col}$ to $\mathcal{F}_{scr}$ where

$$\mathcal{F}_{scr} = \int_{\sqrt{z}/2}^{\infty} \frac{dx}{x} e^{-(x+\frac{z}{4x})^2} \left(1 + \frac{1}{1 + 2\frac{\omega_{pe}^2}{\omega^2} x^2}\right)$$

Fig.1 shows the difference between the result obtained from Ichimaru [14] and our result for $< Z >= 1, 4, 7$ and 10 (Figures 1a, 1b, 1c, 1d respectively) by plotting

$$f_{col} = \frac{\mathcal{F}_{col}(\omega)}{\mathcal{F}_0\omega)} \qquad f_{scr} = \frac{\mathcal{F}_{scr}(\omega)}{\mathcal{F}_0(\omega)}$$

as function of ω/ω_{pe}.

The solid curve is our case and the dotted line is obtained from equation 9.132 of Ichimaru. In all cases there is a significant difference for small values of $\frac{\omega}{\omega_{pe}}$, as ω/ω_{pe} increases both curves asymptotically approach 1. The main difference is that Ichimaru's result overestimates the value of radiation emitted and thus reduces the opacity more than our result.

2) electron-electron bremsstrahlung.

We now use eq.(22) in eq.(7) and then from eq.(32) we have

$$Q_{ee}(\omega) = \tag{58}$$

$$= \frac{\sqrt{\epsilon(\omega)}e^6}{\pi^2 m_e^2 c^3} \int f_e(\underline{v}) f_e(\underline{v}') d^3v d^3v' d^3q d\Omega_k \delta[\omega - \underline{q} \cdot \underline{v}' - (\underline{k} - \underline{q}) \cdot \underline{v}] F_{\underline{k},\underline{q}}(\underline{v}, \underline{v}')$$

where

$$F_{\underline{k},\underline{q}}(\underline{v}, \underline{v}') =$$

$$= \frac{\frac{1}{2}|\underline{k} \times \underline{q}|^2}{k^2 |\epsilon_{\underline{q}\cdot\underline{v}',q}\,\epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}|^2} \left| \frac{[1 + \sum_i \chi^{(i)}_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}]}{q^2} - \frac{[1 + \sum_i \chi^{(i)}_{\underline{q}\cdot\underline{v}',q}]}{|\underline{k} - \underline{q}|^2} \right|^2 \tag{59}$$

Again neglecting the ion susceptibilities (calculated for electron velocities) and taking

$$\epsilon_{\underline{x}\cdot\underline{v},x} \sim \epsilon^{(e)}_{\underline{x}\cdot\underline{v},x} = 1 + \frac{\omega_{pe}^2}{x^2 v_{Te}^2} W(y) \tag{60}$$

for $\underline{x} = \underline{q}$ or $\underline{x} = \underline{k} - \underline{q}$, the normalized parallel components of electron velocities being given again by eq.(40), we find with similar calculations

$$Q_{ee}(\omega) =$$

$$= \frac{\sqrt{\epsilon(\omega)} e^6 n_e^2}{\pi^2 m_e^2 c^3} \int \frac{e^{-y^2}}{\sqrt{\pi}} \int \frac{e^{-y'^2}}{\sqrt{\pi}} dy' \delta(\omega - q\sqrt{2} v_{Te} y' - |\underline{k} - \underline{q}|\sqrt{2} v_{Te} y) F_{\underline{k},\underline{q}}(y, y') d^3 q \tag{61}$$

with

$$F_{\underline{k},\underline{q}}(y, y') = \frac{\frac{1}{2}|\underline{k} \times \underline{q}|^2 |k^2 - 2(\underline{k} \cdot \underline{q})|^2}{k^2 q^4 |\underline{k} - \underline{q}|^4 \left| \left(1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2} W(y')\right) \left(1 + \frac{\omega_{pe}^2}{|\underline{k} - \underline{q}|^2 v_{Te}^2} W(y)\right) \right|^2} \tag{62}$$

Changing $\underline{q} \rightarrow \underline{q} + \frac{1}{2}\underline{k}$ in the q-integral it is then easy to see that (for $\omega \sim ck$) from the conservation law (δ-function) it is

$$k \sim \frac{v_{Te}}{c} q \ll q$$

and we can then neglect k with respect to q; the integral over angles can be done such that

$$\int \frac{(\underline{k} \cdot \underline{q})^2}{k^2} (\underline{k} \times \underline{q})^2 d\Omega_k = \frac{8}{15} \pi k^2 q^4$$

and we finally have

$$Q_{ee}(\omega) =$$

$$= \frac{64}{15} \frac{\sqrt{\epsilon(\omega)} e^6 n_e^2 k^2 v_{Te}}{m_e^2 c^3 \omega_{pe}} \int_0^\infty q^6 dq \int \frac{e^{-y^2}}{\sqrt{\pi}} dy \int \frac{e^{-y'^2}}{\sqrt{\pi}} dy' \frac{\delta[\omega - q\sqrt{2}\omega_{pe}(y + y')]}{|q^2 + W(y)|^2 |q^2 + W(y')|^2} \tag{63}$$

where we have introduced the new variable

$$q = \left(\frac{v_{Te}}{\omega_{pe}}\right) q$$

It can be seen from eq.(53) that compared to $Q_{ei}(\omega)$, $Q_{ee}(\omega)$ is of order $(v_{Te}/c)^2\omega_{pe}/\omega$ and can therefore be neglected.

3) ion-ion bremsstrahlung.

Using eq.(23) we find that the integrand of $Q_{ii}(\omega)$ contains the factor

$$\left| \frac{\chi^{(e)}_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}{q^2} - \frac{\chi^{(e)}_{\underline{q}\cdot\underline{v}',q}}{|\underline{k}-\underline{q}|^2} \right|^2$$

Using the same approximations as in previous calculations

$$\chi^{(e)}_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|} \simeq \frac{\omega_{pe}^2}{|\underline{k}-\underline{q}|^2 v_{Te}^2}; \qquad \chi^{(e)}_{\underline{q}\cdot\underline{v}',q} \simeq \frac{\omega_{pe}^2}{q^2 v_{Te}^2}$$

the result is zero for the ion-ion bremsstrahlung.

To next order in the susceptibilities, i.e.

$$\chi^{(e)}_{\omega,k} \simeq \frac{\omega_{pe}^2}{k^2 v_{Te}^2}\left(1 + i\sqrt{\pi}\frac{\omega}{\sqrt{2}k v_{Te}}\right)$$

we find

$$\left|\underline{M}^{(ii')}\right|^2 \propto \frac{|y - y'|^2}{v_{Te}^2}$$

where y, y' are defined in eq.(40). When integrated with the distribution functions of the ions to find Q_{ii} the result is $v_{Ti}^2/v_{Te}^2 = m_i/m_e$ smaller then Q_{ei}. The ion-ion bremsstrahlung can therefore be neglected.

5. Discussion of the results

We have succeeded for the first time to give a complete analytical description of collective effects in bremsstrahlung. It is shown that the collective effects in electron-ion collisions can not be described by some factor due to the screening effects as used previously. An important difference between ours and previous results is that we take into account the oscillation of the ion screening cloud. The results then become dependent on the effective ion charge $< Z >$. This dependence is absent in previous work. The partial cancelation in the matrix elements due to the oscillation of the ion cloud diminishes the actual contribution of collective effects in the total bremsstrahlung. The difference with previous results is also that the screening factor (see (48)) contains the electron thermal velocity in W. For all electrons with velocities larger than the average thermal velocity the screening becomes negligible (the stripping effect of [5]).

In the case of a Maxwellian distribution the contribution of the tail thermal electron appears without screening. But even for electrons with velocities much less than the mean thermal velocity the other factor $\left(1+ < Z > \frac{\omega_{pe}^2}{q^2 v_{Te}^2}\right)$ survives and describes the changes due to transition bremsstrahlung. The contribution of collective effects in total bremsstrahlung absorption becomes very important for the frequency close to the electron plasma frequencies. For example for parameters in the solar interior the reduction of the total bremsstrahlung coefficient is about 30%. But the effective frequencies responsible for energy transfer in the solar interior are $\frac{\hbar\omega}{T} \simeq 3.8$ and $\frac{\hbar\omega_{pe}}{T} \simeq 0.2$. The collective effects as can be seen from eq.(55) decrease with ω. We make an accurate numerical investigation of the possible change to the solar opacity due to collective bremsstrahlung corrections found in this paper. The numerics in general need precise calculations of complex integrals with high accuracy. Our results show that the change in the solar opacity due to collective bremsstrahlung is only -0.2% which is less than in previous calculations. The decrease of the total change in opacity is due to the correct treatment of collective effects including the effect of particle redressing (stripping of the shielding cloud). For a complete description of all collective effects in the solar interior that we are studying the result obtained is important as a first proof that the collective effects in bremsstrahlung in the solar interior are small. Other results concerning a substantial increase of ion-ion and electron-electron bremsstrahlung should also be included in further applications of the collective effects in bremsstrahlung. An important point is that the ion-ion bremsstrahlung is proportional to Z^4. For high Z plasmas this bremsstrahlung can exceed the usual one.

The main difference between our results and those of previous calculations which only includes screening of the ion field for all electron velocities that do not take into account the fact that screening decreases for energetic particles. In fact for electron velocities larger than the thermal velocity these electrons are not screened. Previous results calculated using static screening overestimate the effect of bremsstrahlung resulting in a smaller opacity. Our result which takes into account the velocity dependence on screening. There is a significant difference for small values of $\frac{\omega}{\omega_{pe}}$ as one would expect since collective effects are more important.

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Figure Captions

Figure 1. (a,b,c,d) represent the scattering including the full collective effects f_{col} solid line and including only the screening of the ion field, f_{scr} dashed line, for different effective values of $< Z >$ a)$< Z >= 1$, b) $< Z >= 4$, c) $< Z >= 7$, and d) $< Z >= 10$. The scattering is normalized to $\mathcal{F}_0(\omega)$ when collective effects are neglected. the result of Ichimaru's corresponds to the dashed curve ie for screening.

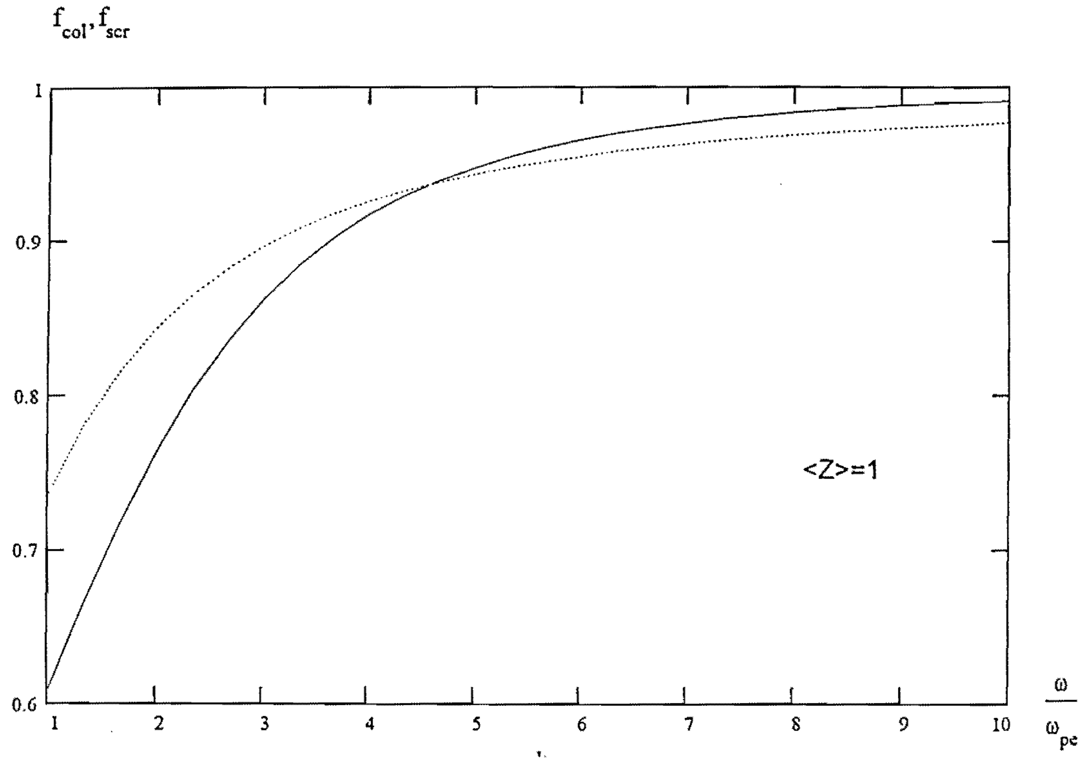

Fig 1a

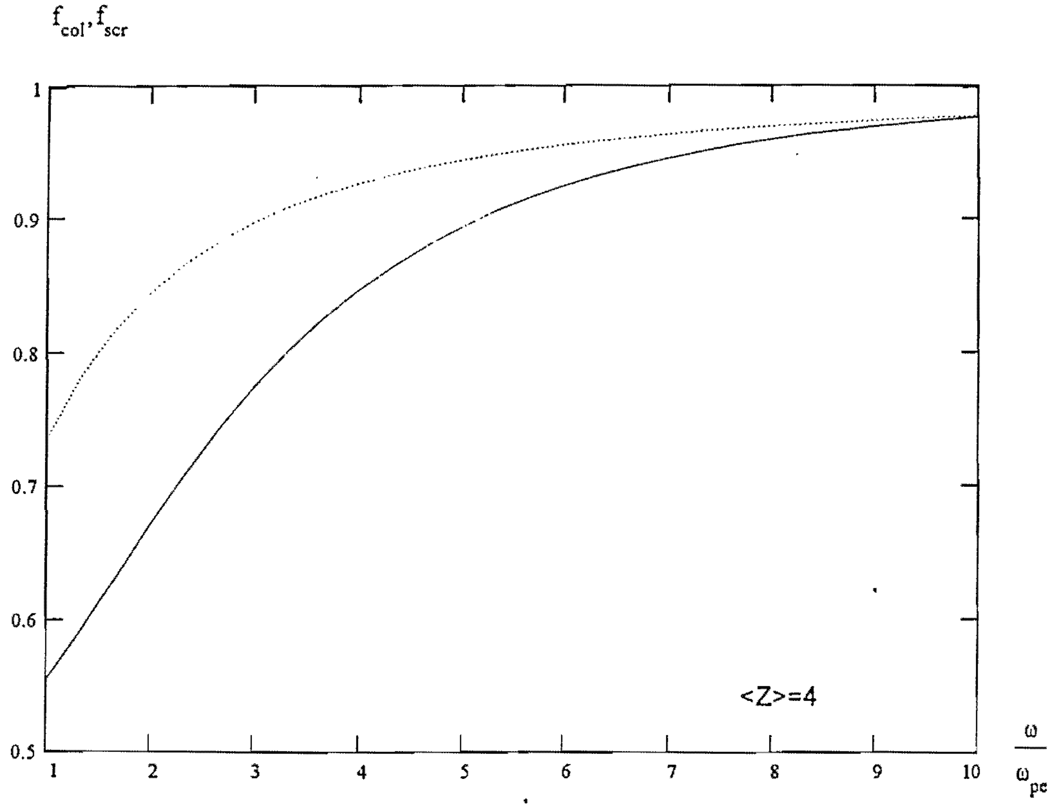

Fig 18

$1+\frac{\kappa}{3}$

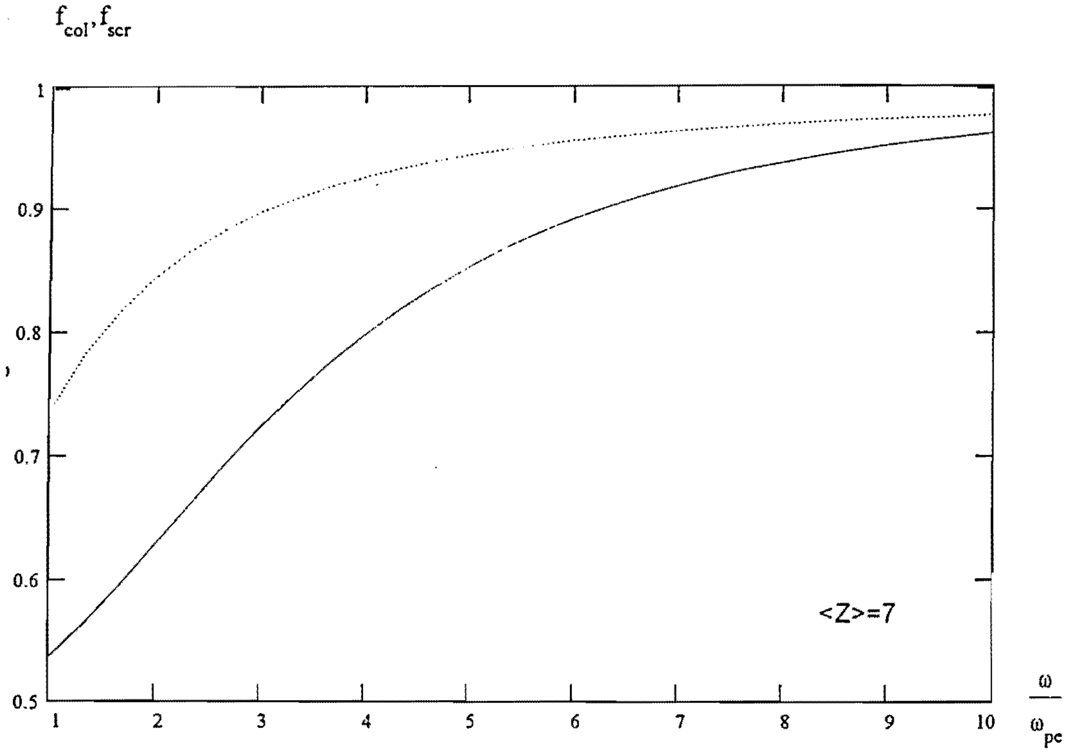

Fig 1c

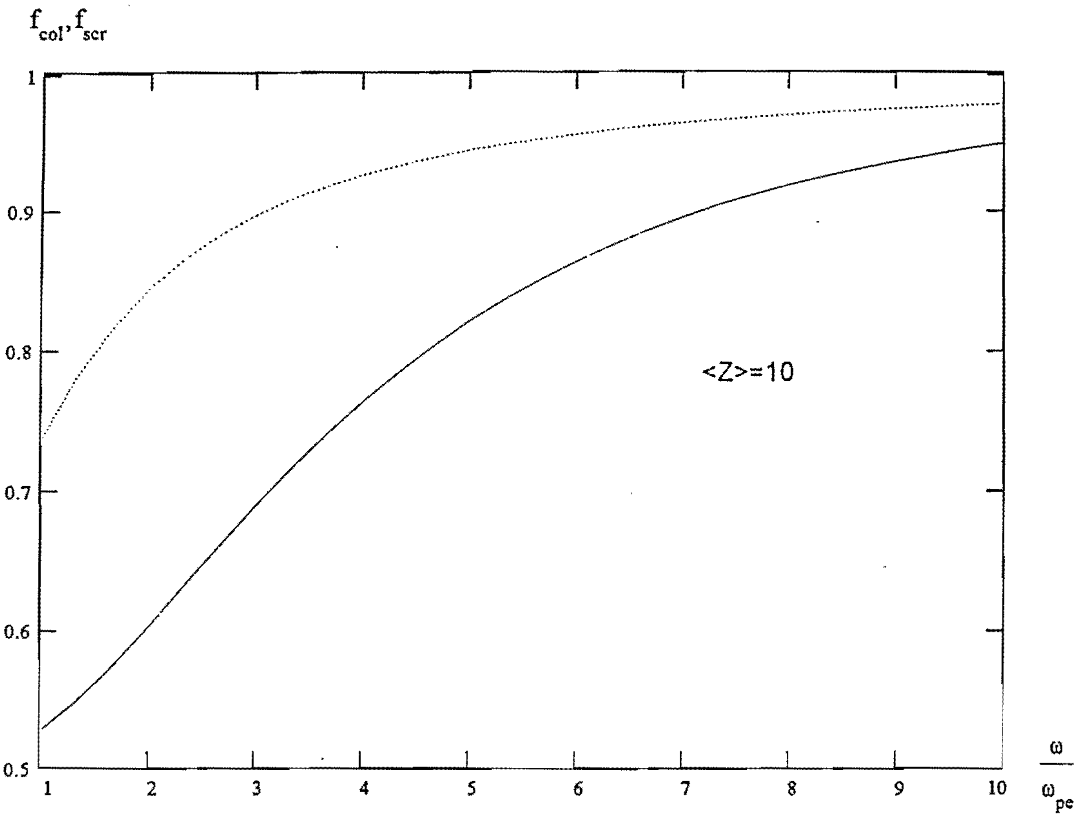

Fig 1d

RAMAN SCATTERING OF PHOTONS IN THE SOLAR INTERIOR

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ABSTRACT

We re-consider the transport equation in the solar interior and calculate the effects on the opacity of photon scattering near the Raman resonance. Both the spontaneous and stimulated scattering on thermal photons and thermal plasmons are taken into account and the changes in the opacity (with respect to previous calculations) are calculated numerically and found to be negligible with respect to relativistic corrections to photon scattering.

1. INTRODUCTION

It is well known that one of the possible "astrophysical solutions" of the solar neutrino problem is a lower opacity of the solar interior i.e. a more transparent Sun, that is a "cooler" Sun for a given luminosity. Given the high sensitivity of the B^8 and Be^7 neutrino fluxes to the solar temperature, even small changes in the opacity are important. One of the (small) effects which has never been considered in the solar interior is the Raman scattering of photons. In this paper we consider this effect and show that it is smaller than other corrections which have already been taken into account [1]. Raman scattering occurs when the "resonance" conditions:

$$\begin{cases} \omega_k = \omega_{k'} \pm \omega^{(l)}_{k-k'}; \qquad \omega^{l}_{k-k'} \simeq \omega_{pe} \\ \underline{k} = \underline{k}' \pm \underline{k}_p \end{cases} \tag{1}$$

are met in a plasma. Here $(\underline{k}, \omega_k)$ and $(\underline{k}', \omega_{k'})$ are wavenumbers and frequency of the incident and scattered transverse waves, $(\underline{k}_p, \omega_{pe})$ is the wavenumber and frequency of the plasma wave (ω_{pe} is the electron plasma frequency).

The resonance conditions require that the photon frequency ω be larger than $2\omega_{pe}$. When this is the case Raman scattering can be seen as a resonance in the usual scattering of electromagnetic waves in plasmas since the scattering probability is proportional to

$$\frac{Im\epsilon_{\omega-\omega',\underline{k}-\underline{k}'}}{|\epsilon_{\omega-\omega',\underline{k}-\underline{k}'}|^2} \tag{2}$$

where $\epsilon_{\omega,\underline{k}}$ is the plasma dielectric function and $\epsilon_{\omega_{pe},\underline{k}_p} \simeq 0$ for the plasma normal modes at $(\underline{k}_p, \omega_{pe})$. The total scattering cross-section, integrated over all wavenumbers $\underline{k}'$ (and therefore over all frequencies $\omega_{k'} = \omega'$) is proportional to [4]:

$$\int \frac{d\omega'}{\omega-\omega'} Im\left(\frac{1}{\epsilon_{\omega-\omega',\underline{k}-\underline{k}'}}\right) = -\pi\left[Re\left(\frac{1}{\epsilon_{0,\underline{k}-\underline{k}'}}\right) - 1\right] = -\pi\frac{\frac{\omega_{pe}^2}{q^2 v_{Te}^2}}{1+\frac{\omega_{pe}^2}{q^2 v_{Te}^2}} \tag{3}$$

where $q = |\underline{k}-\underline{k}'|, v_{Te} = (T/m_e)^{\frac{1}{2}}$is the electron thermal speed and the result is valid for $\omega_{pe} \ll \omega, \omega'$ when, to a first approximation, the Doppler shift can be neglected resulting in

$$\omega \simeq \omega'$$
$$q = \frac{\omega}{c}\sqrt{2}(1 - cos\theta_{\underline{k},\underline{k}'})^{\frac{1}{2}} \tag{4}$$

where $\theta_{\underline{k},\underline{k}'}$ is the angle between $\underline{k}$ and $\underline{k}'$ vectors. This result is well known and gives the expression for the transport cross-section in plasmas with the collective effects taken into account [4]. and it includes the resonant region (1) since the integration is over all frequencies .

The corrections to the next order, when the difference $\omega - \omega'$ is taken into account in the Raman resonance (i.e. eq.(4) is not valid) have also been found [2].

On the other hand inside the Raman resonance a more adequate description is to change eq.(2) into:

$$\delta(\epsilon_{\omega-\omega',\underline{k}-\underline{k}'}) = \pi \frac{\delta(\omega - \omega' + \omega_{pe}) + \delta(\omega - \omega' - \omega_{pe})}{\frac{\partial\epsilon}{\partial(\omega-\omega')}\Big|_{\omega-\omega'=\omega_{pe}}}$$

and then find (using the decay probability):

$$\int \frac{d\omega'}{\omega - \omega'} Im\left(\frac{1}{\epsilon_{\omega-\omega',\underline{k}-\underline{k}'}}\right) = -\pi \tag{5}$$

Clearly the results coincide in the limit

$$\frac{\omega_{pe}^2}{q^2 v_{Te}^2} = \frac{\lambda^2}{\lambda_D^2} \gg 1$$

where λ_D is the plasma Debye length and λ the photon wavelength. But at resonance $q = k_p$ and in a plasma with $k_p^2 v_{Te}^2 \ll \omega_{pe}^2$: the Raman scattering is indeed possible in the region where the plasma collective effects dominate namely for $\lambda \gg \lambda_D$, and (3) and (5) coincide.

Therefore the frequency domain important for Raman scattering is:

$$2\omega_{pe} < \omega \ll \omega_{pe}\frac{c}{v_{Te}} \tag{6}$$

In the solar interior $c \simeq 20 v_{Te}$ but the dominant frequency range (from the Planck spectrum) is $\omega \simeq (5-8)\omega_{pe}$ and the condition (6) is therefore satisfied in some range of frequencies.

The effects of Raman scattering seem therefore to be relevant to the solar interior [2]. Furthermore consider the energy density of photons and plasmons in the solar interior given by:

$$W^t = 2\int \hbar\omega_k N_k^t \frac{d^3k}{(2\pi)^3} = \frac{1}{\pi^2 c^3}\int_{\omega_{pe}}^{\infty} \frac{\hbar\omega^3\sqrt{\epsilon(\omega)}}{e^{\frac{\hbar\omega}{T}}-1}d\omega \tag{7}$$

$$W^l = \int \hbar\omega_{pe} N_k^l \frac{d^3k}{(2\pi)^3} = \int_0^{k_0} T\frac{k^2dk}{2\pi^2} \tag{8}$$

where N_k^t and N_k^l are the distributions (occupation numbers) of photons and plasmons respectively, $\epsilon(\omega) = (1-\omega_{pe}^2/\omega^2)$ and the last equalities follow for thermal black body distributions for photons and plasmons

$$N_k^{t(0)} = (e^{\frac{\hbar\omega}{T}} - 1)^{-1} \tag{9}$$

$$N_k^{l(0)} = \frac{T}{\hbar\omega_{pe}} \qquad (\hbar\omega_{pe} \ll T) \tag{10}$$

where T is the temperature (in energy units) and

$$k = \frac{\omega}{c}\sqrt{\epsilon(\omega)} \tag{11}$$

has been used for the photons in equation (7)

The upper limit of integration k_0 in equation (8) is some fraction of the Debye wave-number $k = 1/\lambda_D$ since for $k\lambda_D \geq 1$ plasmons are heavily damped and therefore the thermal distribution eq.(10) can only be valid for $k < k_0 = \frac{1}{n}k_D$ with $n \approx 3$. Since $k_0 \simeq 6T/\hbar c \simeq 2\frac{\omega_{max}}{c}$ the thermal energy in the plasmons is larger than the photon thermal energy for conditions appropriate to the solar interior ($\hbar\omega \simeq T$ from the Planck distribution).

Due to the low group velocity of plasmons the heat transport by photons is much more efficient: the plasmons are thermal to a good approximation but the photons have a flux component related to the temperature gradient in the Sun. But (given the higher energy of the plasmons) both thermal photons and plasmons contribute to the spontaneous and induced Raman scattering of the anisotropic part of the photon distribution, i.e. to the transport of photons. The contribution of thermal photons and plasmons to the induced Raman scattering of photons and their relative importance have never been taken into account to our knowledge.

It is also necessary to reconsider the spontaneous Raman scattering since the known result (see eq.(3)), which includes the Raman resonance, is valid under the conditions defined by eq.(4) i.e. neglecting the Doppler shift which is of order $\frac{\omega'-\omega}{\omega} \simeq \frac{v_{Te}}{c} \sim \frac{1}{20}$ outside the resonance but can be of order $\frac{\omega_{pe}}{\omega} \leq \frac{1}{2}$ (see eq.6) inside the resonance and cannot be neglected. In this paper we consider the transport equation for photons in the solar interior, including the induced processes, and find the corrections to the opacity due to Raman scattering, including the thermal plasmons.

2. TRANSPORT EQUATION FOR PHOTONS

We consider the contribution of spontaneous and induced Raman scattering to the transport equation for photons. In this case the probability of scattering can be written as the probability of decay of an electromagnetic wave $(\underline{k}, \omega_{\underline{k}'})$ into another electromagnetic wave $(\underline{k}', \omega_{\underline{k}}^{\prime})$ and a plasma wave $(\underline{k}^{l}, \omega^{l}_{\underline{k}-\underline{k}'})$ which is given by [3]:

$$W_{\underline{k},\underline{k}'} = \frac{\hbar e^2 \omega_{pe} |\underline{k} - \underline{k}'|^2}{16\pi m_e^2 \omega_{\underline{k}} \omega_{\underline{k}'}} (1 + cos^2\theta) \delta(\omega_{\underline{k}} - \omega_{\underline{k}'} - \omega^{l}_{\underline{k}-\underline{k}'}) \tag{12}$$

where θ is the angle between the vectors $\underline{k}$ and $\underline{k}'$.

The transport equation for photons, taking into account the Raman processes, can be written as [3]:

$$\underline{v}_g \cdot \frac{\partial N^t_{\underline{k}}}{\partial \underline{r}} = I^{sp}_{\underline{k}} + I^{t(st)}_{\underline{k}} + I^{l(st)}_{\underline{k}} \tag{13}$$

where

$$\underline{v}_g = c\sqrt{\epsilon(\omega)} \frac{\underline{k}}{k} \tag{14}$$

is the group velocity and on the rhs of eq.(13) we have separated the terms due to spontaneous scattering $(I^{sp}_{\underline{k}})$ and stimulated (induced) scattering on thermal photons $(I^{t(st)}_{\underline{k}})$ and on thermal plasmons $(I^{l(st)}_{\underline{k}})$. These are given by [3]:

$$I^{(sp)}_{\underline{k}} = -N^t_{\underline{k}} \int W_{\underline{k},\underline{k}'} d^3k' + \int N^t_{\underline{k}'} W_{\underline{k}',\underline{k}} d^3k' \tag{15}$$

$$I^{t(st)}_{\underline{k}} = N^t_{\underline{k}} \int N^t_{\underline{k}'} (W_{\underline{k},\underline{k}'} - W_{\underline{k}',\underline{k}}) d^3k' \tag{16}$$

$$I^{l(st)}_{\underline{k}} = -N^t_{\underline{k}} \int (N^l_{\underline{k}-\underline{k}'} W_{\underline{k},\underline{k}'} + N^l_{\underline{k}'-\underline{k}} W_{\underline{k}',\underline{k}}) d^3k' + $$

$$+ \int N^t_{\underline{k}'} (N^l_{\underline{k}-\underline{k}'} W_{\underline{k}',\underline{k}} + N^l_{\underline{k}'-\underline{k}} W_{\underline{k}',\underline{k}}) d^3k' \tag{17}$$

Only the spontaneous term $I^{sp}_{\underline{k}}$ has been previously considered. The first term on the rhs of eq.(15) gives the result of eq.(5) (or eq.(2) if the whole spectrum of scattered frequencies is taken into account) and is well known in the theory of collective scattering in plasmas [4]. The second term on the rhs of eq.(15) gives to lowest order the factor $(1 - cos\theta)$ (when combined with the first term) in the expression for the scattering cross-section and this is known as the "transport" cross-section [1].

Here we shall re-consider all three terms and evaluate their relative contribution.

The presence of $cos\theta_{\underline{k},\underline{n}}$ in the rhs of the transport equation ($\underline{n}$ being the unit vector in the direction of flux propagation), i.e.

$$\frac{\underline{k}}{k} \cdot \frac{\partial N^t_{\underline{k}}}{\partial \underline{r}} = cos\theta_{\underline{k},\underline{n}} \frac{\partial N^t_{\underline{k}}}{\partial r}$$

makes it possible to solve the transport equation by expanding $N^t_{\underline{k}}$ such that

$$N^t_{\underline{k}} = N^{t(o)}_k + \cos\theta_{\underline{k},\underline{n}} \delta N^t_k \tag{18}$$

where $N^{t(o)}_k$ is the thermal distribution of photons (eq.9), $\delta N^t_{\underline{k}} \ll N^{t(0)}_k$ is the small inhomogeneous part responsible for the flux of radiation.

For the plasmons we neglect, as discussed before, any deviations from the thermal distribution and take $N^l_k = N^{l(0)}_k$ in the transport equation, where the thermal plasmon distribution is given by eq.(10) for $k < k_0$. Recalling that the rhs of eq.(13) vanishes for a thermal distribution of photons and plasmons (note that this is true only when the stimulated processes are taken into account), substituting (18) in the rhs of eq.(13) only the deviations from thermal equilibrium survive and, to first order in δN^t_k, we have

$$\delta I^{sp}_{\underline{k}} = - cos\theta_{\underline{k}\underline{n}} \delta N^t_k \int W_{\underline{k},\underline{k}'} d^3k' + \int cos\theta_{\underline{k}'\underline{n}} \delta N^t_{k'} W_{\underline{k}',\underline{k}} d^3k' \tag{19}$$

$$\begin{aligned}\delta I^{t(st)}_{\underline{k}} =& cos\theta_{\underline{k}\underline{n}} \delta N^t_k \int N^{t(0)}_{k'} (W_{\underline{k},\underline{k}'} - W_{\underline{k}',\underline{k}}) d^3k' + \\ &+ N^{t(0)}_k \int cos\theta_{\underline{k}',\underline{n}} \delta N^t_{k'} (W_{\underline{k},\underline{k}'} - W_{\underline{k}',\underline{k}}) d^3k'\end{aligned} \tag{20}$$

$$\begin{aligned}\delta I^{l(st)}_{\underline{k}} =& \frac{T}{\hbar\omega_{pe}} \left\{ -cos\theta_{\underline{k},\underline{n}} \delta N^t_k \int (W_{\underline{k},\underline{k}'} + W_{\underline{k}',\underline{k}}) \theta(k_0 - q) d^3k' \right\} + \\ &+ \frac{T}{\hbar\omega_{pe}} \left\{ \int cos\theta_{\underline{k}',\underline{n}} \delta N^t_{k'} (W_{\underline{k},\underline{k}'} + W_{\underline{k}',\underline{k}}) \theta(k_0 - q) d^3k' \right\}\end{aligned} \tag{21}$$

where $q = |\underline{k} - \underline{k}'|$ in the θ function is given by eq.(4)

Since the decay probability $W_{\underline{k},\underline{k}'}$ depends only on the angle θ of $\underline{k}$ and $\underline{k}'$ we can average the $cos\theta_{\underline{k}',\underline{n}}$ in eq's (19-21) over the angles perpendicular to the $(\underline{k}, \underline{k}')$ plane such that:

$$\overline{cos\theta_{\underline{k}',\underline{n}}} = \overline{cos\theta cos\theta_{\underline{k},\underline{n}}} + \overline{sin\theta sin\theta_{\underline{k},\underline{n}} sin(\phi - \phi')} = \underline{cos\theta cos\theta_{\underline{k},\underline{n}}} \qquad (22)$$

Then eq's (19-21) are written in the form:

$$\delta I^{sp}_{\underline{k}} = cos\theta_{\underline{k},\underline{n}} \delta I^{sp}_{k} \qquad (23)$$

$$\delta I^{t(st)}_{\underline{k}} = cos\theta_{\underline{k},\underline{n}} \delta I^{t(st)}_{k} \qquad (24)$$

$$\delta I^{l(st)}_{\underline{k}} = cos\theta_{\underline{k},\underline{n}} \delta I^{l(st)}_{k} \qquad (25)$$

where each δI_k can be found from eq's (19-21). The $cos\theta_{\underline{k},\underline{n}}$ now can be cancelled from all terms in the transport equation which becomes

$$c\sqrt{\epsilon(\omega)}\frac{\partial N^{t(0)}_{k}}{\partial r} = \delta I^{sp}_{k} + \delta I^{t(st)}_{k} + \delta I^{l(st)}_{k} \qquad (26)$$

and all terms now depend only on $|\underline{k}|$ i.e. on $\omega = \omega_k$. We shall consider the contributions of the three terms separately in the following sections.

3. SPONTANEOUS RAMAN SCATTERING

From eq's (19), (22) and (23) we have:

$$\delta I^{sp}_{\underline{k}} = -\delta N^t_{\underline{k}} \int W_{\underline{k},\underline{k}'} d^3k' + \int cos\theta \delta N^t_{\underline{k}'} W_{\underline{k},\underline{k}'} d^3k' \tag{27}$$

This is a first order quantum correction (as can be seen from $\hbar$ in the expression for $W_{\underline{k},\underline{k}'}$) to the known zero-order term in Raman scattering which is given by

$$\int W^{sc}_{\underline{k},\underline{k}'} d^3k' = n_e c \sigma_T \frac{k^2 v^2_{Te}}{\omega^2_{pe}} \tag{28}$$

where n_e is the electron number density , σ_T the Thomson cross-section

$$\sigma_T = \frac{8}{3}\pi \frac{e^4}{m^2_e c^4} \tag{29}$$

and $W^{sc}_{\underline{k},\underline{k}'}$ is the probability of scattering in plasmas. The first term in eq.(27), using the decay probability (12) gives

$$\int W_{\underline{k},\underline{k}'} d^3k' = n_e c \sigma_T \frac{k^2 v^2_{Te}}{\omega^2_{pe}} \frac{\hbar \omega_{pe}}{T} \tag{30}$$

In the solar interior $\hbar\omega_{pe} \sim 0.2T$ so that the correction introduced by eq.(27) is small. Also notice that since $kv_{Te} \ll \omega_{pe}$ the contribution of Raman scattering ie eq.(28) is small as compared to the total scattering (over all frequencies) which is of the order of $n_e c \sigma_T$.

The correction due to the spontaneous term eq.(27) to the transport cross-section can therefore be neglected with respect to the zero order term in Raman scattering.

4.STIMULATED RAMAN SCATTERING ON THERMAL PHOTONS.

We show first of all that the stimulated scattering is a classical process and since $\hbar k \ll m v_{Te}$, then

$$\hbar\omega \ll T\left(\frac{c}{v_{Te}}\right) \tag{31}$$

which shows that we can use the decay probability eq.(12) for frequencies up to $\hbar\omega \sim T$.

From eq.'s(20),(22) and (24) we have:

$$\delta I_{\underline{k}}^{t(st)} = \delta N_k^t \int N_{k'}^{t(0)}(W_{\underline{k}',\underline{k}} - W_{\underline{k},\underline{k}'})d^3k' + N_k^{t(0)} \int cos\theta N_{k'}^t(W_{\underline{k}',\underline{k}} - W_{\underline{k},\underline{k}'})d^3k' \tag{32}$$

Using eq.(12) for the decay probability with

$$c^2|\underline{k} - \underline{k}'|^2 = \omega^2 + \omega'^2 - 2\omega\omega' cos\theta \tag{33}$$

and $\omega_{\underline{k}-\underline{k}'}^l \simeq \omega_{pe}$, we can perform the angular integrations in eq.(32) using $d^3k' = \frac{2\pi\omega'^2}{c^3} sin\theta d\theta$ to find

$$\begin{aligned}\delta I_\omega^{t(st)} =& \delta N_\omega^t \frac{\hbar e^2 \omega_{pe}}{3m_e^2 c^5 \omega} \int (\omega^2 + \omega'^2)\omega' N_{\omega'}^{t(0)} d\omega' [\delta(\omega' - \omega - \omega_{pe}) - \delta(\omega' - \omega + \omega_{pe})] + \\ & - N_\omega^{t(0)} \frac{4}{15} \frac{\hbar e^2 \omega_{pe}}{m_e^2 c^5} \int \omega'^2 \delta N_{\omega'}^t [\delta(\omega' - \omega - \omega_{pe}) - \delta(\omega' - \omega + \omega_{pe})]\end{aligned} \tag{34}$$

Expanding for $\omega_{pe} \ll \omega$ this gives

$$\delta I_\omega^{t(st)} = \frac{8}{3} \frac{\pi e^2 n_e}{m_e^3 c^5} \left[\delta N_\omega^t \frac{d(\hbar\omega^2 N_\omega^{t(0)})}{d\omega} - \frac{2}{5} N_\omega^{t(0)} \frac{d(\hbar\omega^2 \delta N_\omega^t)}{d\omega}\right] \tag{35}$$

Introducing the Thomson cross-section (eq.29) and using eq.(9) for the photon distribution eq.(35) can be written as

$$\frac{\delta I_\omega^{t(st)}}{n_e c \sigma_T} = \frac{T}{m_e c^2} \left[\delta N_\omega^t \frac{d}{d\omega}\left(\omega \frac{\hbar\omega/T}{e^{\hbar\omega/T} - 1}\right) - \frac{2}{5} \frac{\hbar\omega/T}{e^{\hbar\omega/T} - 1} \frac{1}{\omega} \frac{d}{d\omega}(\hbar\omega^2 \delta N_\omega^t)\right] \tag{36}$$

Introducing the energy flux L_ω at frequency ω which is defined as

$$L_\omega = \frac{2}{3\pi}\frac{\hbar\omega^3}{c^2}\delta N \tag{37}$$

we can write eq.(36) as

$$\frac{2\hbar\omega^3}{\pi 3c^2}\frac{\delta I_\omega^{t(st)}}{n_e c\sigma_T} = \frac{T}{m_e c^2}\left[L_\omega \frac{d}{d\omega}\left(\omega\frac{\hbar\omega/T}{e^{\hbar\omega/T}-1}\right) - \frac{2}{5}\frac{\hbar\omega/T}{e^{\hbar\omega/T}-1}\omega^2\frac{d}{d\omega}\left(\frac{1}{\omega}L_\omega\right)\right] \tag{38}$$

From the above expression we conclude that the stimulated Raman scattering on thermal photons: **a)** decreases the opacity since the first term on the rhs is larger than the second one; **b)** is of second order such that ie

$$\frac{T}{m_e c^2} = \left(\frac{v_{Te}}{c}\right)^2$$

and is a relativistic correction to collective scattering and therefore it cannot be neglected when relativistic corrections to scattering are considered.

5.STIMULATED RAMAN SCATTERING ON THERMAL PLASMONS

From eq's (21),(22) and (25) we have (setting $x = cos\theta_{\underline{k},\underline{k}'}$):

$$\delta I_\omega^{l(st)} = \tag{39}$$

$$-\frac{T}{\hbar\omega_{pe}}\left[\delta N_\omega^t \int [W_{\underline{k},\underline{k}'}\theta(k_0-q) + W_{\underline{k}',\underline{k}}\theta(k_0-q)]d^3k' - \int \delta N_{\omega'}^t cos\theta(W_{\underline{k},\underline{k}'} + W_{\underline{k}',\underline{k}})d^3k'\right]$$

Introducing the decay probability from eq.(12) we have, for $\omega_{\underline{k}-\underline{k}'}^l \simeq \omega_{pe}$:

$$\delta I_\omega^{l(st)} = -\frac{Te^2}{8\pi m_e^2\omega}\times$$

$$\times\left\{\delta N_\omega^t \int q^2(1+x^2)\theta(k_0-q)\frac{d^3k'}{\omega'} - \int \delta N_{\omega'}^t q^2(1+x^2)x\theta(k_0-q)\delta(\omega-\omega'-\omega_{pe})\frac{d^3k'}{\omega'}\right\}$$

where, for $\omega \simeq \omega'(\omega_{pe} \ll \omega), q = |\underline{k} - \underline{k}'|$ is given by eq.(4) and $k \simeq k' \simeq \frac{\omega}{c}$ so that the Riemann θ-function is:

$$\theta(k_0 - q) = \theta[k_0 - \sqrt{2}k'(1-x)^{\frac{1}{2}}]$$

Then, substituting $\delta N_{\omega'}^t \simeq \delta N_\omega^t$ in the second integral, we have:

$$\delta I_\omega^{l(st)} = -\delta N_\omega^t \frac{Te^2\omega}{2m_e^2c^3}\left\{\int_{-1}^{+1}(1-x)^2(1+x^2)dx \int_0^{k^*} k'dk'\right\}$$

where

$$k^* \equiv \frac{k_0}{\sqrt{2}(1-x)^{\frac{1}{2}}}$$

This gives, using $k_0 = \frac{1}{n}k_D(n \approx 3)$:

$$\frac{\delta I^{l(st)}}{n_e c\sigma_T} = -\frac{1}{2n^2}\delta N_\omega^t \tag{40}$$

Using again (37) we have the final form (to compare with eq.(38)):

$$\frac{2\pi\hbar\omega^3}{3\pi c^2}\frac{\delta I_\omega^{l(st)}}{n_e c\sigma_T} = -\frac{1}{2n^2}L_\omega \tag{41}$$

The sum of the two effects eq's (38 and 41) will next be used to find the change produced in the Rosseland opacity.

6.NUMERICAL ESTIMATES TO THE CHANGES IN THE SOLAR OPACITY

We calculate the correction to the Rosseland opacity due to stimulated Raman scattering (eq's (38) and (41)). The Rosseland opacity is defined as

$$\frac{1}{\rho\kappa_R} = \int_{\omega_{pe}}^{\infty} \frac{\sqrt{\epsilon(\omega)}\frac{\partial B_\omega}{\partial T}}{n_e\sigma_0(\omega)} d\omega \left(\int_0^{\infty} \frac{\partial B_\omega}{\partial T} d\omega \right)^{-1} \tag{42}$$

where $B_\omega(T)$ is the Planck function, and ρ is the mass density

$$B_\omega(T) = \frac{\hbar\omega^3}{2\pi^2c^2} N_\omega^{t(0)} \tag{43}$$

and σ_0 is the cross-section due to all absorption processes: scattering, inverse bremsstrahlung including the collective effects and lines [1].

The transport equation (26), multiplying both sides by $\hbar\omega^3/2\pi^2c^2$ and including the terms from spontaneous scattering and bremsstrahlung takes the form

$$\sqrt{\epsilon(\omega)}\frac{1}{3}\frac{\partial B_\omega}{\partial r} = -n_e\sigma_0(\omega)L_\omega + n_e\sigma_T\hat{\sigma}_{st}L_\omega \tag{44}$$

where $\hat{\sigma}_{st}$ is a differential operator given by (from the sum of eq's 38 and 41):

$$\hat{\sigma}_{st}L_\omega = \frac{T}{m_ec^2}\left\{ \left[\frac{z}{e^z-1} - \frac{z^2e^z}{(e^z-1)^2} \right] L_\omega - \frac{2}{5}\frac{z^3}{e^z-1}\frac{d}{dz}\left(\frac{1}{z}L_\omega\right) \right\} - \frac{L_\omega}{2n^2} \tag{45}$$

where

$$z = \frac{\hbar\omega}{T}, \qquad z_0 = \frac{\hbar\omega_{pe}}{T} \tag{46}$$

Neglecting the stimulated terms and denoting with $L_\omega^{(0)}$ the corresponding luminosity from the transport equation, which from eq.(44) is

$$L_\omega^{(0)} = -\frac{1}{3}\frac{dT}{dr}\frac{\sqrt{\epsilon(\omega)}\frac{\partial B_\omega}{\partial T}}{n_e\sigma_0(\omega)} \tag{47}$$

and, using the definition of the Rosseland opacity given by eq.(42), we find

$$L^{(0)} = \int_{\omega_{pe}}^{\infty} L_\omega^{(0)} d\omega = -\left(\int_0^{\infty} \frac{\partial B_\omega}{\partial T} d\omega \right) \frac{1}{3\kappa_R^{(0)}}\frac{dT}{dr} \tag{48}$$

which is the familiar form of the integrated transport equation without stimulated effects.

We shall include the stimulated terms on the rhs of eq.(44) using perturbation theory due to the small factor $(T/m_e c^2)$ The presence of these terms changes the luminosity which becomes $L_\omega = L_\omega^{(0)} + \delta L_\omega$, where, neglecting higher order terms δL_ω can be written as

$$\delta L_\omega = \frac{1}{\sigma_0(\omega)} \hat{\sigma}_{st} L_\omega^{(0)} \tag{49}$$

Then from eq's(45) and (49) we have, using eq.(47)

$$\delta L = \int_{\omega_{pe}}^{\infty} \delta L_\omega d\omega =$$

$$= - \left(\frac{T}{m_e c^2} \right) \frac{1}{3} \frac{dT}{dr} \frac{T}{\hbar} \int_{z_0}^{\infty} \left\{ \frac{\frac{\partial B_\omega}{\partial T}}{\sigma_0^2(z)} \left[\frac{z}{e^z - 1} - \frac{z^2 e^z}{(e^z - 1)^2} - \frac{m_e c^2}{T} \frac{1}{2n^2} \right] + \right.$$

$$\left. - \frac{2}{5} \frac{z^3}{e^z - 1} \frac{1}{\sigma_0(z)} \frac{d}{dz} \left(\frac{\frac{\partial B_\omega}{\partial T}}{z \sigma_0(z)} \right) \right\} dz \tag{50}$$

Introducing the change in the Rosseland opacity as $\kappa_R = \kappa_R^{(0)} - \delta\kappa$ we can write

$$L = L^{(0)} + \delta L = - \left(\int_0^\infty \frac{\partial B_\omega}{\partial T} d\omega \right) \frac{1}{3 \rho \kappa_R} \frac{dT}{dr} =$$

$$= - \left(\int_0^\infty \frac{\partial B}{\partial T} d\omega \right) \frac{1}{3 \rho \kappa_R^{(0)}} \frac{dT}{dr} \left(1 + \frac{\delta \kappa}{\kappa_R^{(0)}} \right) \tag{51}$$

and then from eq's (48) and (50) we find the change in the Rosseland opacity due to stimulated Raman scattering on thermal photons and thermal plasmons to be

$$\frac{\delta \kappa}{\kappa_0} = \left(\frac{T}{m_e c^2} \right) \frac{\kappa_R^{(0)}}{\int_0^\infty \frac{z^4 e^z}{(e^z - 1)^2} dz} \int_{z_0}^{\infty} \left\{ \frac{z^4 e^z}{\sigma_0^2(z)(e^z - 1)^2} \left[\frac{z}{e^z - 1} - \frac{z^2 e^z}{(e^z - 1)^2} - \frac{m_e c^2}{T} \frac{1}{2n^2} z_0^2 \right] + \right.$$

$$\left. - \frac{2}{5} \frac{z^3}{e^z - 1} \frac{1}{\sigma_0(z)} \frac{d}{dz} \left[\frac{z^3 e^z}{\sigma_0(z)(e^z - 1)^2} \right] \right\} dz \tag{52}$$

A numerical estimate for the parameter regime approximate to the Solar interior gives

$$\frac{\delta\kappa}{\kappa_0} \leq 0.001$$

This effect can be ignored when calculating the Solar opacity, since it is smaller by an order of magnitude in comparison with other effects on photon scattering such as relativistic corrections. We therefore conclude that Raman scattering can be neglected when calculating the Solar opacity.

Conclusions

In this paper we have calculated the effects of changes to the opacity in the solar interior due to photon scattering near the Raman resonance. We considered both the spontaneous and stimulated scattering on thermal plasmons and calculated changes to the solar opacity, which were found to be negligible with respect to relativistic corrections. The corrections found in this paper to the opacity are of order 0.1%, whereas, the relativistic corrections are more than an order of magnitude greater. We therefore conclude that spontaneous and stimulated scattering from plasmons at the Raman resonance are not important for photon transport in the centre of the sun. The solar neutrino problem cannot therefore be resolved by the effects considered in this paper.

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