



**Technical Report**  
RAL-TR-95-069

# **Renormalization Group Evolution of R-parity-violating Yukawa Couplings**

**V Barger M S Berger R J N Phillips and T Wöhrmann**

December 1995

**© Council for the Central Laboratory of the Research Councils 1995**

Enquiries about copyright, reproduction and requests for additional copies of this report should be addressed to:

The Central Laboratory for the Research Councils  
Library and Information Services  
Rutherford Appleton Laboratory  
Chilton  
Didcot  
Oxfordshire  
OX11 0QX  
Tel: 01235 445384 Fax: 01235 446403  
E-mail [library@rl.ac.uk](mailto:library@rl.ac.uk)

**ISSN 1358-6254**

Neither the Council nor the Laboratory accept any responsibility for loss or damage arising from the use of information contained in any of their reports or in any communication about their tests or investigations.

**Rutherford Appleton Laboratory**

**MADPH-95-910**

**RAL-TR-95-069**

**IUHET-323**

November 1995

## **Renormalization group evolution of R-parity-violating Yukawa couplings**

V. Barger<sup>(a)</sup>, M.S. Berger<sup>(b)</sup>, R.J.N. Phillips<sup>(c)</sup>, and T. Wöhrmann<sup>(a)</sup>

*(a) Physics Department, University of Wisconsin, Madison, WI 53706, USA*

*(b) Physics Department, Indiana University, Bloomington, IN 47405, USA*

*(c) Rutherford Appleton Laboratory, Chilton, Didcot, Oxon OX11 0QX, UK*

### **Abstract**

We study the evolution of  $R$ -parity-violating (RPV) couplings in the minimum supersymmetric standard model, between the electroweak and grand unification scales, assuming a family hierarchy for these coupling strengths. Particular attention is given to solutions where both the  $R$ -conserving and  $R$ -violating top quark Yukawa couplings simultaneously approach infrared fixed points; these we analyse both algebraically and with numerical solutions of the evolution equations at one-loop level. We identify constraints on these couplings at the GUT scale, arising from lower limits on the top quark mass. We show that fixed points offer a new source of bounds on RPV couplings at the electroweak scale. We derive evolution equations for the CKM matrix, and show that RPV couplings affect the scaling of the unitarity triangle. The fixed-point behaviour is compatible with all present experimental constraints. However, fixed-point values of RPV top-quark couplings would require the corresponding sleptons or squarks to have mass  $\gtrsim m_t$  to suppress strong new top decays to sparticles.

## I. Introduction

Supersymmetry is a very attractive extension of the Standard Model (SM), with low-energy implications that are being actively pursued, both theoretically and experimentally [1, 2]. In the minimal supersymmetric extension of the standard model (MSSM), with minimum new particle content, a discrete symmetry ( $R$ -parity) is assumed to forbid rapid proton decay. In terms of baryon number  $B$ , lepton number  $L$  and spin  $S$ , the  $R$ -parity of a particle is  $R \equiv (-1)^{3B+L+2S}$ , with value  $R = +1$  for particles and  $R = -1$  for sparticles. An important consequence of  $R$ -conservation is that the lightest sparticle is stable and is thus a candidate for cold dark matter. However, since  $R$ -conservation is not theoretically motivated by any known principle, the possibility of  $R$ -nonconservation deserves equally serious consideration. In addition to the Yukawa superpotential in the MSSM

$$\mathcal{W} = (U)_{ab} H_2 Q_L^a \bar{U}_R^b + (D)_{ab} H_1 Q_L^a \bar{D}_R^b + (E)_{ab} H_1 L_L^a \bar{E}_R^b, \quad (1)$$

there are two classes of  $R$ -violating couplings in the MSSM superpotential, allowed by supersymmetry and renormalizability [3]. The superpotential terms for the first class violate lepton number  $L$ ,

$$\mathcal{W} = \lambda_{abc} L_L^a L_L^b \bar{E}_R^c + \lambda'_{abc} L_L^a Q_L^b \bar{D}_R^c, \quad (2)$$

while those of the second class violate baryon number  $B$ ,

$$\mathcal{W} = \lambda''_{abc} \bar{D}_R^a \bar{D}_R^b \bar{U}_R^c. \quad (3)$$

Here  $L, Q, \bar{E}, \bar{D}, \bar{U}$  stand for the doublet lepton, doublet quark, singlet antilepton, singlet  $d$ -type antiquark, singlet  $u$ -type antiquark superfields, respectively, and  $a, b, c$  are generation indices. The  $(U)_{ab}$ ,  $(D)_{ab}$  and  $(E)_{ab}$  in Eq. (1) are the Yukawa coupling matrices. In our notation, the superfields above are the weak interaction eigenstates, which might be expected as the natural choice at the grand unified scale, rather than the mass eigenstates. The Yukawa couplings  $\lambda_{abc}$  and  $\lambda''_{abc}$  are antisymmetric in their first two indices because of superfield antisymmetry. These superpotential terms lead to the interaction lagrangians

$$\mathcal{L} = \lambda_{abc} \{ \tilde{\nu}_{aL} \bar{e}_{cR} e_{bL} + \bar{e}_{bL} \bar{e}_{cR} \nu_{aL} + (\bar{e}_{cR})^* (\tilde{\nu}_{aL})^c e_{bL} - (a \leftrightarrow b) \} + h.c. \quad (4)$$

for the  $\lambda$ -terms, whereas the  $\lambda'$ -terms yield

$$\mathcal{L} = \lambda'_{abc} \{ \tilde{\nu}_{aL} \bar{d}_{cR} d_{bL} + \bar{d}_{bL} \bar{d}_{cR} \nu_{aL} + (\bar{d}_{cR})^* (\tilde{\nu}_{aL})^c d_{bL} \}$$

$$-\bar{e}_{aL}\bar{d}_{cR}u_{bL} - \tilde{u}_{bL}\bar{d}_{cR}e_{aL} - (\bar{d}_{cR})^*(\bar{e}_{aL})^c u_{bL} \} + h.c. , \quad (5)$$

with corresponding terms for each of these generations. In the case of a  $B$ -violating superpotential the lagrangian reads

$$\mathcal{L} = \lambda''_{abc} \{ u_c^c d_a^c \tilde{d}_b^* + u_c^c \tilde{d}_a^* d_b^c + \tilde{u}_c^* d_a^c d_b^c \} + h.c. \quad (6)$$

To escape the proton-lifetime constraints, it is sufficient that only one of these classes be absent or very highly suppressed. Phenomenological studies of the consequences of  $R$ -parity violation (RPV) have placed constraints on the various couplings  $\lambda_{abc}, \lambda'_{abc}, \lambda''_{abc}$  [4, 5, 6, 7, 8], but considerable latitude remains for RPV.

Studies of the renormalization group evolution equations (RGE), relating couplings at the electroweak scale to their values at the grand unification (GUT) scale, have led to new insights and constraints on the observable low-energy parameters in the  $R$ -conserving scenario. It therefore seems worthwhile to see what can be learned from similar studies of RPV scenarios. An initial study of this type addressed the evolution of  $\lambda''_{133}$  and  $\lambda''_{233}$  couplings [8]. This was subsequently extended to all the baryon-violating couplings  $\lambda''_{ijk}$  [9]. In the present work we undertake a somewhat more general study of the RGE for RPV interactions, paying particular attention to solutions for which both the  $R$ -conserving and  $R$ -violating top quark Yukawa couplings simultaneously approach infrared fixed points. Such fixed-point behaviour requires a coupling  $\lambda, \lambda'$ , or  $\lambda''$  to be of order unity at the electroweak scale. After our study was completed, a related work on RGE for RPV couplings appeared [10], which however has a different focus and is largely complementary to the present paper.

In the context of grand unified theories one is led to consider the possible unification of RPV parameters. If for example the RPV interactions arose from an  $SU(5)$ -invariant term, then in fact the  $L$ -violating RPV couplings would be related to the  $B$ -violating ones [11] at the GUT scale. We could then no longer set one or the other arbitrarily to zero and the proton lifetime (which places very strong constraints on products of  $L$ -violating and  $B$ -violating RPV couplings, typically requiring products  $\lambda'\lambda''$  to be smaller than  $5 \times 10^{-17}$  [11]) would strongly constrain all types of RPV couplings. It can be argued that some products of  $B$ -violating and  $L$ -violating couplings, containing several high-generation indices, would not contribute directly to proton decay [12]; however, proton decay would still be induced at the one-loop level by flavor mixing [11], so in fact

all RPV couplings would have to be very small. In such scenarios the fixed-point solutions for RPV couplings would be excluded; our present studies therefore implicitly assume that this kind of RPV unification does not occur. Furthermore, since RPV unification is analogous to the popular hypothesis of  $\lambda_b = \lambda_\tau$  Yukawa unification, it would appear somewhat inconsistent (though not completely unthinkable) to assume one without the other. Accordingly, in our present work, we do not try to impose the additional constraint of  $\lambda_b = \lambda_\tau$  unification.

## II. Renormalization group equations and fixed points

For any trilinear term in the superpotential  $d_{abc}\Phi^a\Phi^b\Phi^c$  involving superfields  $\Phi^a, \Phi^b, \Phi^c$ , the evolution of the couplings  $d_{abc}$  with the scale  $\mu$  is given by the RGE

$$\mu \frac{\partial}{\partial \mu} d_{abc} = \gamma_a^e d_{ebc} + \gamma_b^e d_{aec} + \gamma_c^e d_{abe}, \quad (7)$$

where the  $\gamma_a^e$  are elements of the anomalous dimension matrix. Table I gives the anomalous dimensions for the superfields. The first column of the table gives the results for the MSSM in matrix form; here  $\mathbf{U}$ ,  $\mathbf{D}$  and  $\mathbf{E}$  are the matrices of Yukawa couplings to the up-quarks, down-quarks and charged leptons, respectively, and a unit matrix is understood in front of the terms involving  $SU(3)$ ,  $SU(2)$  and  $U(1)$  gauge couplings  $g_3$ ,  $g_2$  and  $g_1$  and the terms with traces. The second column of Table I gives the additions to the anomalous dimension matrix due to  $L$ -violating terms  $\lambda_{abc}$  and  $\lambda'_{abc}$ , while the third column gives the corresponding additions due to  $B$ -violating  $\lambda''_{abc}$  terms. In our notation, an RPV-coupling with upper indices is the complex conjugate of the same coupling with lower indices, e.g.  $\lambda^{abc} = \lambda_{abc}^*$ .

The evolution equations for the  $R$ -conserving Yukawa matrices  $U, D, E$  of Eq. (1) are obtained from Eq. (7) with the index  $c$  belonging to a Higgs field. The general forms of the RGE are

$$\mu \frac{\partial}{\partial \mu} (U)_{ab} = (U)_{ib} \gamma_{Q_a}^{Q_i} + (U)_{ai} \gamma_{U_b}^{\bar{U}_i} + (U)_{ab} \gamma_{H_2}^{H_2}, \quad (8)$$

$$\mu \frac{\partial}{\partial \mu} (D)_{ab} = (D)_{ib} \gamma_{Q_a}^{Q_i} + (D)_{ai} \gamma_{D_b}^{\bar{D}_i} + (D)_{ab} \gamma_{H_1}^{H_1}, \quad (9)$$

$$\mu \frac{\partial}{\partial \mu} (E)_{ab} = (E)_{ib} \gamma_{L_a}^{L_i} + (E)_{ai} \gamma_{E_b}^{\bar{E}_i} + (E)_{ab} \gamma_{H_1}^{H_1}. \quad (10)$$

When we solve Eqs. (8)–(10) for the general  $R$ -parity violating case, we get additional contributions from Hermitian matrices involving the RPV couplings that are analogous to

Table I:  $16\pi^2\gamma_{\phi_j}^{\phi_i}$  in the MSSM plus additional terms for lepton or baryon number violating couplings, where  $i$  and  $j$  are flavor indices.

$\phi_{i,j}$	<i>MSSM</i>	Lepton # Violation	Baryon # Violation
$L_{i,j}$	$EE^\dagger - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2$	$\lambda_{iab}\lambda^{jab} + 3\lambda'_{iab}\lambda'^{jab}$	—
$E_{i,j}$	$2E^\dagger E - \frac{6}{5}g_1^2$	$\lambda^{abi}\lambda_{abj}$	—
$D_{i,j}$	$2D^\dagger D - \frac{8}{3}g_3^2 - \frac{2}{15}g_1^2$	$2\lambda'^{abi}\lambda'_{abj}$	$2\lambda''^{iab}\lambda''_{jab}$
$U_{i,j}$	$2U^\dagger U - \frac{8}{3}g_3^2 - \frac{8}{15}g_1^2$	—	$\lambda''^{abi}\lambda''_{abj}$
$Q_{i,j}$	$UU^\dagger + DD^\dagger - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{1}{30}g_1^2$	$\lambda'_{aib}\lambda'^{ajb}$	—
$H_1$	$Tr(EE^\dagger) + 3Tr(DD^\dagger) - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2$	—	—
$H_2$	$3Tr(UU^\dagger) - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2$	—	—

combinations like  $D^\dagger D$  for the usual Yukawa matrices. For example the matrix equation for the Yukawa matrices  $U$  and  $D$  become

$$\frac{dU}{dt} = \frac{1}{16\pi^2} \left[ \left[ -\frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{13}{15}\alpha_1 + 3UU^\dagger + DD^\dagger + Tr[3UU^\dagger] + M'^{(Q)} \right] U + UM''^{(U)} \right], \quad (11)$$

$$\frac{dD}{dt} = \frac{1}{16\pi^2} \left[ \left[ -\frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{7}{15}\alpha_1 + 3DD^\dagger + UU^\dagger + Tr[3DD^\dagger + EE^\dagger] + M'^{(Q)} \right] D + 2DM''^{(D)} + 2DM'^{(D)} \right], \quad (12)$$

where  $M'_{ij}{}^{(Q)} \equiv \lambda'_{aib}\lambda'^{ajb}$ ,  $M'_{ij}{}^{(D)} \equiv \lambda'^{abi}\lambda'_{abj}$ ,  $M''_{ij}{}^{(U)} \equiv \lambda''^{abi}\lambda''_{abj}$  and  $M''_{ij}{}^{(D)} \equiv \lambda''^{iab}\lambda''_{jab}$  are the combinations of RPV couplings appearing in Table I. The variable is

$$t = \ln(\mu/M_G) \quad (13)$$

where  $\mu$  is the running mass scale and  $M_G$  is the GUT unification mass.

The gauge couplings are not affected by the presence of  $R$ -violating couplings at the one-loop level.

The third generation Yukawa couplings are dominant, so if we retain in the anomalous dimensions only the (3,3) elements  $\lambda_t, \lambda_b, \lambda_\tau$  in  $U, D, E$ , setting all other elements to zero, Eqs. (8)–(10) read

$$\mu \frac{\partial}{\partial \mu} \lambda_t = \lambda_t \left[ \gamma_{Q_3}^{Q_3} + \gamma_{\bar{U}_3}^{\bar{U}_3} + \gamma_{H_2}^{H_2} \right], \quad (14)$$

$$\mu \frac{\partial}{\partial \mu} \lambda_b = \lambda_b \left[ \gamma_{Q_3}^{Q_3} + \gamma_{D_3}^{\bar{D}_3} + \gamma_{H_1}^{H_1} \right], \quad (15)$$

$$\mu \frac{\partial}{\partial \mu} \lambda_\tau = \lambda_\tau \left[ \gamma_{L_3}^{L_3} + \gamma_{E_3}^{\bar{E}_3} + \gamma_{H_1}^{H_1} \right], \quad (16)$$

Since there are 36 independent RPV couplings  $\lambda_{abc}, \lambda'_{abc}$  in the  $L$ -violating sector (9 independent couplings  $\lambda''_{abc}$  in the  $B$ -violating sector) to be added to the three dominant  $R$ -conserving Higgs couplings  $\lambda_t, \lambda_b, \lambda_\tau$ , we would have to consider 39 (12) coupled non-linear evolution equations, in general. Some further radical simplifications in the RPV sector are clearly needed to make the system of equations tractable.

It is plausible that there may exist a generational hierarchy among the RPV couplings, analogous to that of the conventional Higgs couplings; indeed, the RPV couplings to higher generations evolve more strongly due to larger Higgs couplings in their RGE, and hence have the potential to take larger values than RPV couplings to lower generations. Thus we consider retaining only the couplings  $\lambda_{233}$  and  $\lambda'_{333}$ , or  $\lambda''_{233}$ , neglecting all others. This restriction is also motivated by the fact that the experimental upper limits are stronger for the couplings with lower indices.

To simplify the form of the RGE, we adopt the following notation:

$$Y_i = \frac{1}{4\pi} \lambda_i^2 \quad (i = t, b, \tau), \quad Y'' = \frac{1}{4\pi} \lambda''_{233}, \quad Y' = \frac{1}{4\pi} \lambda'_{333}, \quad Y = \frac{1}{4\pi} \lambda_{233}^2.$$

The one-loop RGE then take the following forms, where  $\alpha_i = \frac{1}{4\pi} g_i^2$ ,

$$\frac{d\alpha_i}{dt} = \frac{1}{2\pi} b_i \alpha_i^2, \quad b_i = \{33/5, 1, -3\} \quad (17)$$

$$\frac{dY_t}{dt} = \frac{1}{2\pi} Y_t \left( 6Y_t + Y_b + Y_\tau + 2Y'' - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{13}{15}\alpha_1 \right) \quad (18)$$

$$\frac{dY_b}{dt} = \frac{1}{2\pi} Y_b \left( Y_t + 6Y_b + Y_\tau + 3Y' + 2Y'' - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{7}{15}\alpha_1 \right) \quad (19)$$

$$\frac{dY_\tau}{dt} = \frac{1}{2\pi} Y_\tau \left( 3Y_b + 4Y_\tau + 3Y + 3Y' - 3\alpha_2 - \frac{9}{5}\alpha_1 \right) \quad (20)$$

$$\frac{dY}{dt} = \frac{1}{2\pi} Y \left( 3Y_\tau + 3Y + 3Y' - 3\alpha_2 - \frac{9}{5}\alpha_1 \right) \quad (21)$$

$$\frac{dY'}{dt} = \frac{1}{2\pi} Y' \left( Y_t + 3Y_b + Y_\tau + Y + 6Y' - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{7}{15}\alpha_1 \right) \quad (22)$$

$$\frac{dY''}{dt} = \frac{1}{2\pi} Y'' \left( 2Y_t + 2Y_b + 6Y'' - 8\alpha_3 - \frac{4}{5}\alpha_1 \right) \quad (23)$$

Here it is understood that one takes *either*  $Y = Y' = 0$  *or*  $Y'' = 0$ .

An extremely interesting possibility in the RGE is that  $Y_t$  is large at the GUT scale and consequently is driven toward a fixed point at the electroweak scale [13, 14]. In



particular, in the MSSM  $\lambda_t \rightarrow 1.1$  as  $\mu \rightarrow m_t$ ; since  $\lambda_t(m_t) = \sqrt{2}m_t(m_t)/(v \sin \beta)$ , this leads to the relation, for low  $\tan \beta$  [14]

$$m_t(\text{pole}) = (200 \text{ GeV}) \sin \beta, \quad (24)$$

where  $\tan \beta = v_2/v_1$  is the ratio of the Higgs vevs and  $m_t(\text{pole})$  is the mass at the  $t$ -propagator pole. It is interesting to examine the impact of RPV couplings on this fixed-point result [8].

## A. $\lambda_t$ fixed point in the MSSM

We first review the  $\lambda_t$  fixed-point behavior in the MSSM limit, where RPV couplings are neglected. Setting  $dY_t/dt \simeq 0$  at  $\mu \simeq m_t$  gives the fixed-point condition

$$6Y_t + Y_b = \frac{16}{3}\alpha_3 + 3\alpha_2 + \frac{13}{15}\alpha_1. \quad (25)$$

The  $\lambda_t$  and  $\lambda_b$  couplings at  $\mu = m_t$  are related to the running masses

$$\lambda_t(m_t) = \frac{\sqrt{2}m_t(m_t)}{v \sin \beta}, \quad \lambda_b(m_t) = \frac{\sqrt{2}m_b(m_b)}{\eta_b v \cos \beta}, \quad (26)$$

with  $v = (\sqrt{2}G_F)^{-1/2} = 246 \text{ GeV}$ . Here  $\eta_b$  gives the QCD/QED running of  $m_b(\mu)$  between  $\mu = m_b$  and  $\mu = m_t$ ;  $\eta_b \simeq 1.5$  for  $\alpha_s(m_t) \simeq 0.10$  [14]. Thus we can express  $\lambda_b(m_t)$  in terms of  $\lambda_t(m_t)$ ,  $\tan \beta$  and the known running masses:

$$\lambda_b(m_t) = \frac{m_b(m_b)}{m_t(m_t)} \frac{\tan \beta}{\eta_b} \lambda_t(m_t) \simeq 0.017 \tan \beta \lambda_t(m_t), \quad (27)$$

taking  $m_b(m_b) = 4.25 \text{ GeV}$ ,  $m_t(m_t) = 167 \text{ GeV}$ , and hence

$$Y_b(m_t) \simeq 3 \times 10^{-4} \tan^2 \beta Y_t(m_t). \quad (28)$$

For small or moderate values  $\tan \beta \lesssim 20$ , we obtain  $Y_b/(6Y_t) < 0.02$  so we can safely neglect the  $Y_b$  contribution. In this case, taking the approximate values

$$\alpha_3 = 1/10, \quad \alpha_2 = 1/30, \quad \alpha_1 = 1/58 \quad \text{at } \mu = m_t, \quad (29)$$

we find the numerical value

$$Y_t(m_t) = 0.108, \quad \lambda_t(m_t) = 1.16, \quad (30)$$

For large  $\tan \beta \sim m_t/m_b$ , we can express the  $\lambda_t$  fixed-point relation as

$$Y_t(m_t) = \frac{\lambda_t^2(m_t)}{4\pi} = \left( \frac{8}{9}\alpha_3 + \frac{1}{2}\alpha_2 + \frac{13}{90}\alpha_1 \right) / \left( 1 + 5 \times 10^{-5} \tan^2 \beta \right). \quad (31)$$

## B. $\lambda''$ , $\lambda_t$ simultaneous fixed points

Next we consider the  $B$ -violating scenario with  $Y = Y' = 0$  and  $Y''$  non-zero, investigating the possibility that fixed-point limits are approached for both  $Y_t$  and  $Y''$  couplings, as found numerically in Ref. [8] (note that these authors use a different definition of  $\lambda''_{abc}$ ). This requires  $dY_t/dt \simeq 0$  and  $dY''/dt \simeq 0$  at  $\mu \simeq m_t$ , giving the conditions

$$6Y_t + Y_b + 2Y'' - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{13}{15}\alpha_1 \simeq 0, \quad (32)$$

$$2Y_t + 2Y_b + 6Y'' - 8\alpha_3 - \frac{4}{5}\alpha_1 \simeq 0. \quad (33)$$

Taking linear combinations to solve for  $Y_t$  and  $Y''$  we obtain (with  $Y_b \ll Y_t$ )

$$Y_t \simeq \frac{1}{16} \left( 8\alpha_3 + 9\alpha_2 + \frac{9}{5}\alpha_1 \right) \simeq 0.071, \quad \lambda_t \simeq 0.94, \quad (34)$$

$$Y'' \simeq \frac{1}{16} \left( \frac{56}{3}\alpha_3 - 3\alpha_2 + \frac{23}{15}\alpha_1 \right) \simeq 0.112, \quad \lambda''_{233} \simeq 1.18, \quad (35)$$

showing a considerable downward displacement in  $\lambda_t$  due to  $\lambda''_{233}$ . Such a large value of  $\lambda''_{233}$  would imply substantial  $t \rightarrow b\bar{s}, s\bar{b}$  decay, if kinematically allowed.

If both  $\lambda_t$  and  $\lambda''_{233}$  fixed points are realized as above, then the predicted physical top quark mass is

$$m_t(\text{pole}) \simeq (150 \text{ GeV}) \sin \beta. \quad (36)$$

Even for moderate values of  $\tan \beta$  ( $\tan \beta > 5$ ) one has  $\sin \beta \simeq 1$  ( $\sin \beta > 0.98$ ). This prediction is at the lower end of the present data [15, 16]:

$$m_t = 176 \pm 8 \pm 10 \text{ GeV} \quad (CDF), \quad m_t = 199^{+10}_{-21} \pm 22 \text{ GeV} \quad (D0). \quad (37)$$

When the data become more precise, the fixed-point possibility for  $\lambda''_{233}$  could be excluded, if the measured central value of  $m_t$  is unchanged.

One can also consider the case of large  $\tan \beta$  where the coupling  $Y_b$  is non-negligible, and in fact may be near its own fixed point. In that case we add another equation,  $dY_b/dt \simeq 0$ , to those above. This gives

$$Y_t + 6Y_b + Y_\tau + 2Y'' - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{7}{15}\alpha_1 \simeq 0. \quad (38)$$

A new coupling  $Y_\tau$  enters here, but it can be related to  $Y_b$  since

$$\lambda_\tau(m_t) = \frac{\sqrt{2}m_\tau(m_t)}{\eta_\tau v \cos \beta}, \quad (39)$$

and hence

$$\lambda_\tau(m_t) = \frac{m_\tau(m_\tau) \eta_b}{m_b(m_b) \eta_\tau} \lambda_b(m_t) = 0.6 \lambda_b(m_t), \quad Y_\tau(m_t) = 0.4 Y_b(m_t), \quad (40)$$

by arguments similar to those above relating  $\lambda_b(m_t)$  to  $\lambda_t(m_t)$ . Then we have three simultaneous equations in three unknowns, that give the solutions

$$Y_t \simeq 0.067, \quad \lambda_t \simeq 0.92, \quad (41)$$

$$Y_b \simeq 0.061, \quad \lambda_b \simeq 0.88, \quad (42)$$

$$Y'' \simeq 0.092, \quad \lambda_{233}'' \simeq 1.08. \quad (43)$$

### C. $\lambda, \lambda', \lambda_t$ simultaneous fixed points

If instead fixed points should occur simultaneously for  $Y_t$  and  $Y'$ , the conditions at  $\mu \simeq m_t$ , found from  $dY_t/dt \simeq 0$  and  $dY'/dt \simeq 0$ , are

$$Y_t = \frac{1}{35} \left[ \frac{80}{3} \alpha_3 + 15 \alpha_2 + \frac{71}{15} \alpha_1 - 3Y_b + Y_\tau + Y \right], \quad (44)$$

$$Y' = \frac{1}{35} \left[ \frac{80}{3} \alpha_3 + 15 \alpha_2 + \frac{29}{15} \alpha_1 - 17Y_b - 6Y_\tau - 6Y \right]. \quad (45)$$

If  $Y$  is small and we also neglect  $Y_b$  and  $Y_\tau$  (e.g. assuming small  $\tan \beta$ ), then  $Y_t$  and  $Y'$  approach almost the same fixed-point value

$$\lambda_t(m_t) \simeq \lambda'_{333} \simeq 1.07. \quad (46)$$

Alternatively, if  $Y_b$  is large, all three couplings  $Y_t, Y_b$  and  $Y'$  can approach fixed points; the solution of the corresponding three equations gives

$$\lambda_t(m_t) \simeq 1.05, \quad \lambda_b \simeq \lambda'_{333} \simeq 0.86. \quad (47)$$

In both the above cases  $\lambda_t(m_t)$  is only slightly displaced below the MSSM value, while  $\lambda'_{333}$  has quite a large value. The latter would imply substantial  $t \rightarrow b\bar{\tau}, \bar{\tau}b$  decays, if kinematically allowed; the  $t \rightarrow b\bar{\tau}$  mode is more likely, since  $\bar{\tau}$  is usually expected to be lighter than  $\bar{b}$ , and we discuss its implications later.

If  $Y'$  is negligible,  $Y_t$  and  $Y$  can approach fixed points simultaneously; in this case the two conditions essentially decouple, giving the MSSM result for  $Y_t$ . If  $Y_b$  and  $Y_\tau$  are negligible, the solution is

$$\lambda_t(m_t) \simeq 1.16, \quad \lambda_{233} \simeq 0.74, \quad (48)$$

but if  $Y_b$  too is large and approaches its fixed point, the three corresponding conditions give

$$\lambda_t(m_t) \simeq 1.09, \quad \lambda_b \simeq 1.04, \quad \lambda_{233} \simeq 0.40. \quad (49)$$

## D. CKM evolution

The presence of non-zero RPV couplings can also change the evolution of CKM mixing angles. This has interesting implications for the prediction of fermion mixings at the electroweak scale from an ansatz for Yukawa matrices at the GUT scale. In a model such as the MSSM (or the SM) with no RPV terms, the evolution of the CKM angles at the one-loop level comes entirely from the Yukawa matrix terms in the anomalous dimension  $\gamma_{Q_j}^{Q_i}$ . The Yukawa matrices  $U$  and  $D$  can be diagonalized by bi-unitary transformations

$$U^{diag} = V_U^L U V_U^{R\dagger}, \quad (50)$$

$$D^{diag} = V_D^L D V_D^{R\dagger}. \quad (51)$$

The CKM matrix is then given by

$$V \equiv V_U^L V_D^{L\dagger}. \quad (52)$$

In the presence of RPV there are additional contributions to the anomalous dimensions and hence to the CKM RGE's. Consider for example the case in which only the  $\lambda''$  couplings are nonzero, for which there are new contributions  $M_{ij}''^{(U)}$  and  $M_{ij}''^{(D)}$  to the RGE's as defined following Eq. (12). The RPV contributions to the RGE's can be diagonalized by

$$M''^{(U),diag} = V_{(U)}^R M''^{(U)} V_{(U)}^{R\dagger} \equiv \{\lambda_u''^2, \lambda_c''^2, \lambda_t''^2\}, \quad (53)$$

$$M''^{(D),diag} = V_{(D)}^R M''^{(D)} V_{(D)}^{R\dagger} \equiv \{\lambda_d''^2, \lambda_s''^2, \lambda_b''^2\}, \quad (54)$$

for which new matrices

$$V^{(U)} \equiv V_U^R V_{(U)}^{R\dagger}, \quad (55)$$

$$V^{(D)} \equiv V_D^R V_{(D)}^{R\dagger}, \quad (56)$$

can be defined. We find the RGE's take the form

$$\begin{aligned} \frac{dV_{i\alpha}}{dt} = & \frac{1}{16\pi^2} \left[ \sum_{\beta, j \neq i} \frac{\lambda_i^2 + \lambda_j^2}{\lambda_i^2 - \lambda_j^2} \lambda_\beta^2 V_{i\beta} V_{j\beta}^* V_{j\alpha} + \sum_{j, \beta \neq \alpha} \frac{\lambda_\alpha^2 + \lambda_\beta^2}{\lambda_\alpha^2 - \lambda_\beta^2} \lambda_j^2 V_{j\beta}^* V_{j\alpha} V_{i\beta} \right. \\ & \left. + \sum_{k, j \neq i} \frac{\lambda_i \lambda_j}{\lambda_i^2 - \lambda_j^2} \lambda_k''^2 V_{ik}^{(U)} V_{jk}^{(U)*} V_{j\alpha} + \sum_{\gamma, \beta \neq \alpha} \frac{2\lambda_\alpha \lambda_\beta}{\lambda_\alpha^2 - \lambda_\beta^2} \lambda_\gamma''^2 V_{\gamma\beta}^{(D)*} V_{\gamma\alpha}^{(D)} V_{i\beta} \right]. \quad (57) \end{aligned}$$

where  $i, j, k = u, c, t$  and  $\alpha, \beta, \gamma = d, s, b$ . One observes that generally there is a contribution to the evolution of the CKM matrix from the RPV sector.

Assuming, as we do, that only the RPV couplings  $\lambda_{233}$ ,  $\lambda'_{333}$  or  $\lambda''_{233}$  are non-zero, the off-diagonal elements of the matrices defined in Eqs. (55) and (56) vanish. Then the one-loop RGEs for mixing angles and the  $CP$ -violation parameter  $J = \text{Im}(V_{ud}V_{cs}V_{us}^*V_{cd}^*)$  have the same forms as in the MSSM, namely [17]

$$\frac{dW}{dt} = \frac{W}{16\pi^2} (\lambda_t^2 + \lambda_b^2), \quad (58)$$

where  $W = |V_{ub}|^2, |V_{cb}|^2, |V_{td}|^2, |V_{ts}|^2$  or  $J$ . Nevertheless the evolution of CKM angles differs from the MSSM because the evolution of the Yukawa couplings on the right hand side is altered by the RPV couplings.

### III. Numerical RGE Studies

In the previous section, we identified the quasi-infrared fixed points that can be determined through the algebraic solutions to the RGE equations. The one-loop RGEs form a set of coupled first-order differential equations that must be solved numerically.

Figure 1 shows the fixed point behaviour of each of the three RPV couplings considered in this paper ( $\lambda''_{233}, \lambda'_{333}, \lambda_{233}$ ) along with the corresponding fixed point behaviour for  $\lambda_t$ , assuming that  $\tan\beta$  is small and hence  $\lambda_b$  and  $\lambda_\tau$  are negligible. It can be seen that for all  $\lambda \gtrsim 1$  at the GUT scale, the respective Yukawa coupling approaches its fixed point at the electroweak scale. These infrared fixed points provide the theoretical upper limits for the RPV-Yukawa couplings at the electroweak scale summarized in Table II. The numerical evolution of the fixed points approaches but does not exactly reproduce the approximate analytical values Eqs. (34), (35), (46) and (48).

We obtain additional restrictions on the RPV couplings from the experimental lower bound on  $m_t$  (that we take to be  $m_t > 150$  GeV [15, 16]). These additional limits are shown in Fig. 2; the dark shaded region is excluded in all types of models only by assuming this lower bound on the top mass.

One might hope that RPV interactions could help to explain the measured value of  $R_b = \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$ , which differs from the SM prediction by over three standard deviations. However, while their contributions can have either sign, the RPV couplings must be significantly above their fixed-point values to explain the full

Table II: Fixed points for the different Yukawa couplings  $\lambda$  in different models for i)  $\tan\beta \lesssim 30$  and ii)  $\tan\beta \sim m_t/m_b$ . In the case of large  $\tan\beta$ ,  $\lambda_b$  also reaches a fixed point.

	Model	$\lambda_t$	$\lambda_b$	$\lambda_{233}$	$\lambda'_{333}$	$\lambda''_{233}$
i)	MSSM	1.06	–	–	–	–
	Lepton # Violation ( $\lambda \gg \lambda'$ )	1.06	–	1.04	–	–
	Lepton # Violation ( $\lambda' \gg \lambda$ )	0.99	–	–	0.97	–
	Baryon # Violation	0.90	–	–	–	1.02
ii)	MSSM	1.00	0.92	–	–	–
	Lepton # Violation ( $\lambda \gg \lambda'$ )	0.99	0.98	1.04	–	–
	Lepton # Violation ( $\lambda' \gg \lambda$ )	0.96	0.81	–	0.80	–
	Baryon # Violation	0.87	0.85	–	–	0.92

discrepancy [5]. In the case of lepton RPV the bounds on the leptonic partial widths are always strong enough to prevent RPV couplings from taking such large values.

Next we address the question, whether RPV couplings will significantly change the relation between electroweak scale and GUT scale values of the off-diagonal terms of the CKM matrix. When the masses and mixings of the CKM matrix satisfy a hierarchy, these relations are given by

$$W(\mu) = W(\text{GUT})S(\mu),$$

where  $W$  is a CKM matrix element connecting the third generation to one of the lighter generations, and  $S$  is a scaling factor [17]. The other CKM elements do not change with scale to leading order in the hierarchy. The scaling factor  $S(\mu)$  is determined by integrating Eq. (58) together with the other RGEs. In Fig. 3 we show the dependence of the scaling factor  $S$  on the GUT-scale RPV couplings  $\lambda_{233}$ ,  $\lambda'_{333}$  and  $\lambda''_{233}$  respectively.

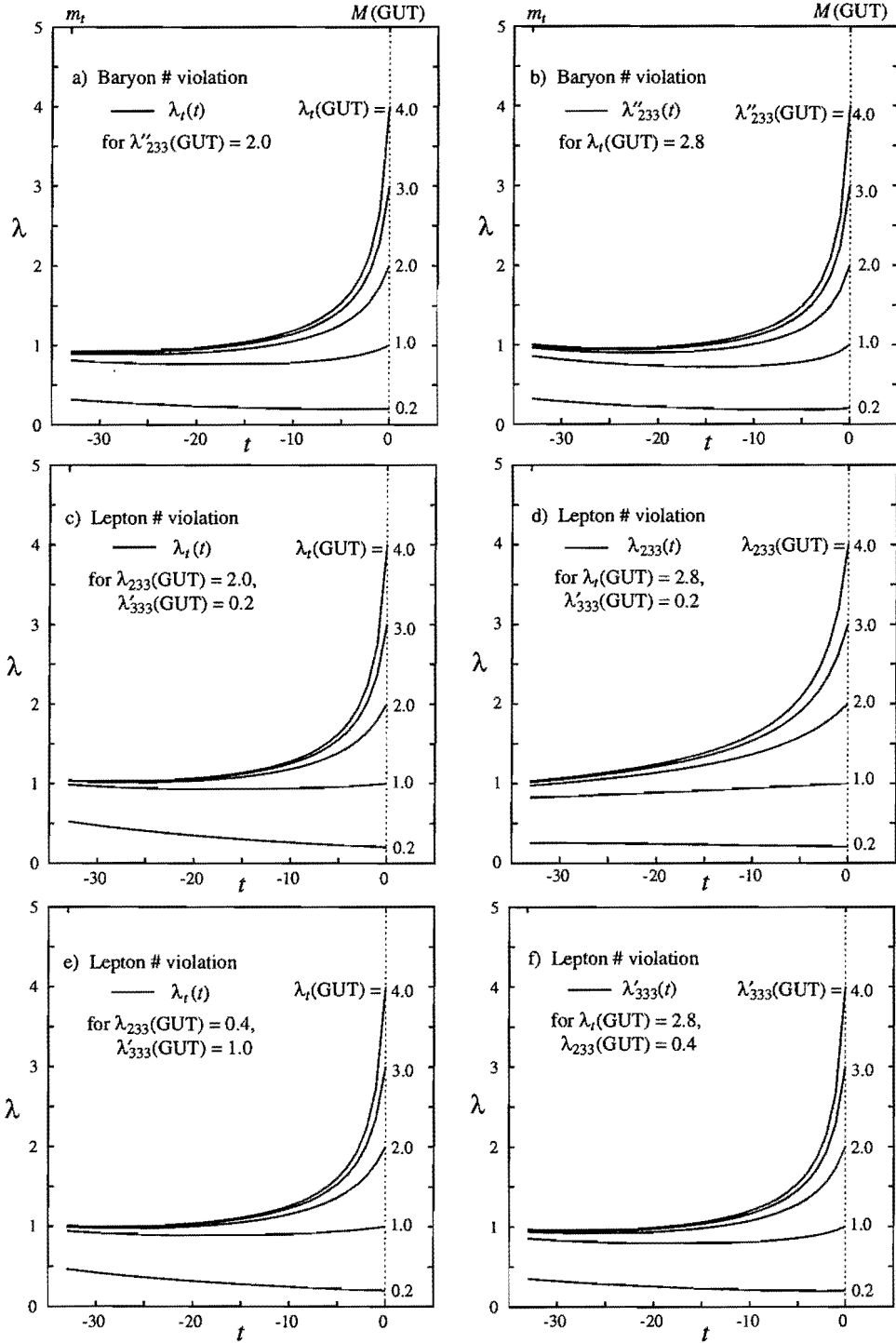


Fig. 1. Couplings  $\lambda$  as a function of the energy scale  $t$  for  $\lambda_t$  in (a) baryon number RPV, (c) lepton number RPV with  $\lambda_{233} \gg \lambda'_{333}$  and (e) lepton number RPV with  $\lambda'_{333} \gg \lambda_{233}$  for different starting points at the GUT scale ( $t = 0$ ). Panels (b), (d) and (f) show the same for  $\lambda'_{233}$ ,  $\lambda_{233}$  ( $\lambda_{233} \gg \lambda'_{333}$ ) and  $\lambda'_{333}$  ( $\lambda'_{333} \gg \lambda_{233}$ ) respectively. Here  $t \simeq -33$  represents the electroweak scale, where these couplings reach their fixed points.

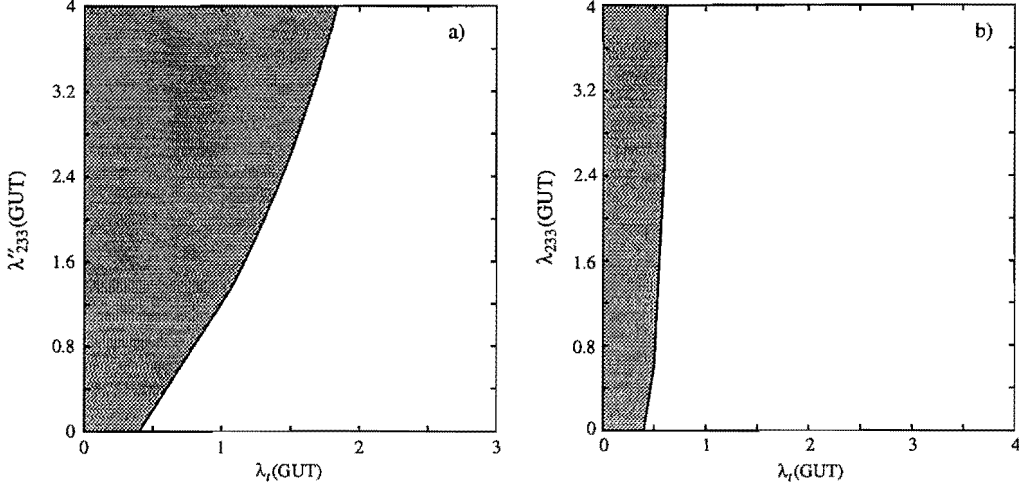


Fig. 2. Excluded regions in the (a)  $\lambda_t(\text{GUT}), \lambda''_{233}(\text{GUT})$  plane and (b)  $\lambda_t(\text{GUT}), \lambda_{233}(\text{GUT})$  ( $\lambda_{233}(\text{GUT}) = 2\lambda'_{333}(\text{GUT})$ ) plane obtained from  $m_t > 150$  GeV.

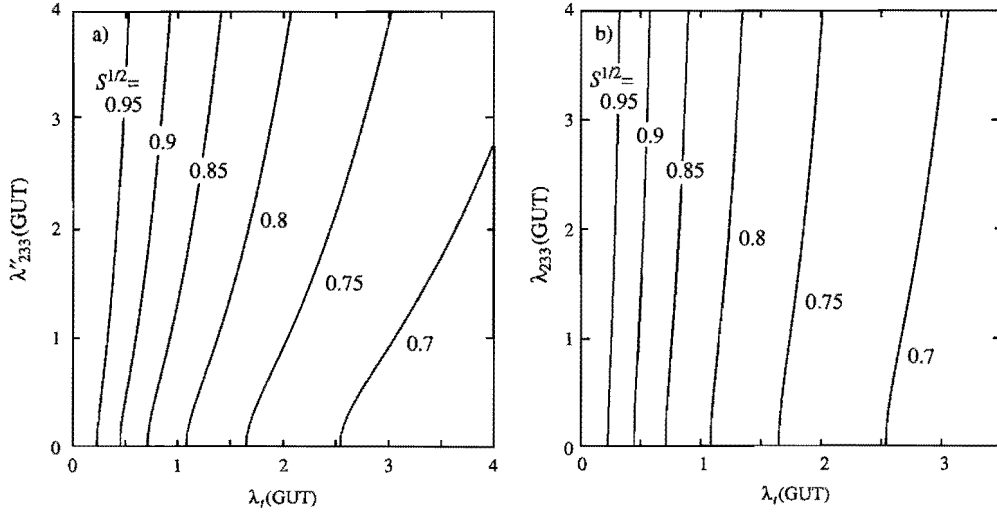


Fig. 3. Contours of constant  $S^{1/2}$  for different values of (a)  $\lambda''_{233}(\text{GUT})$  and  $\lambda_t(\text{GUT})$  (baryon number violation) and (b)  $\lambda_{233}(\text{GUT}) = 2\lambda'_{333}(\text{GUT})$  and  $\lambda_t(\text{GUT})$  (lepton number violation).

#### IV. RPV decays of the top quark

The RPV couplings  $\lambda''_{233}$  and  $\lambda'_{333}$  give rise to new decay modes of the top quark [18], if the necessary squark or slepton masses are small enough.

The  $L$ -violating coupling  $\lambda'_{333}$  leads to  $t_R \rightarrow b_R \bar{\bar{\tau}}_R, \bar{\bar{b}}_R \bar{\tau}_R$  decays, with partial widths [18]

$$\Gamma(t \rightarrow b \bar{\bar{\tau}}) = \frac{(\lambda'_{333})^2}{32\pi} m_t (1 - m_{\bar{\tau}}^2/m_t^2)^2, \quad (59)$$

$$\Gamma(t \rightarrow \bar{\bar{b}} \bar{\tau}) = \frac{(\lambda'_{333})^2}{32\pi} m_t (1 - m_{\bar{b}}^2/m_t^2)^2, \quad (60)$$

neglecting  $m_b$  and  $m_\tau$ . The former mode is more likely to be accessible, since sleptons



are expected to be lighter than squarks. Since the SM top decay has partial width

$$\Gamma(t \rightarrow bW) = \frac{G_F m_t^3 |V_{tb}|^2}{8\pi\sqrt{2}} (1 - M_W^2/m_t^2)^2 (1 + 2M_W^2/m_t^2), \quad (61)$$

the ratio of RPV to SM decays would be typically

$$\Gamma(t \rightarrow b\tilde{\tau}^+)/\Gamma(t \rightarrow bW^+) \simeq 0.70 (\lambda'_{333})^2 \quad (\text{for } m_{\tilde{\tau}} \simeq M_W). \quad (62)$$

It is natural to assume that  $\tilde{\tau}$  would decay mostly to  $\tau$  plus the lightest neutralino  $\chi_1^0$  (which is also probably the lightest sparticle), followed by the RPV decay  $\chi_1^0 \rightarrow b\bar{b}\nu_\tau(\bar{\nu}_\tau)$ , with a short lifetime [19]

$$\tau(\chi_1^0 \rightarrow b\bar{b}\nu_\tau, b\bar{b}\bar{\nu}_\tau) \sim 3 \times 10^{-21} \text{ sec } (m_{\tilde{\tau}}/m_\chi)^4 (100 \text{ GeV}/m_\chi)/(\lambda'_{333})^2, \quad (63)$$

giving altogether

$$t \rightarrow b\tilde{\tau}^+ \rightarrow b\tau\chi_1^0 \rightarrow b\bar{b}\bar{b}\tau^+\nu_\tau(\bar{\nu}_\tau). \quad (64)$$

This mode could in principle be identified experimentally, e.g. by exploiting the large number of potentially taggable  $b$ -jets and the presence of a tau. However, it would not be readily confused with the SM decay modes  $t \rightarrow bW^+ \rightarrow bq\bar{q}'$ ,  $b\ell\nu$ , ( $\ell = e, \mu$ ), that form the basis of the presently detected  $p\bar{p} \rightarrow t\bar{t}X$  signals in the  $(W \rightarrow \ell\nu) + 4jet$  and dilepton channels (neglecting leptons from  $\tau \rightarrow \ell\nu\nu$  that suffer from a small branching fraction and a soft spectrum). On the contrary, the RPV mode would deplete the SM signals by competition. With  $m_{\tilde{\tau}} \sim M_W$ , fixed-point values  $\lambda'_{333} \simeq 0.9$  (Fig.1) would suppress the SM signal rate by a factor  $(1 + 0.70(\lambda'_{333})^2)^{-2} \simeq 0.4$ , in contradiction to experiment where  $p\bar{p} \rightarrow t\bar{t}X \rightarrow b\bar{b}WWX$  signals tend if anything to exceed SM expectations [15, 16]. We conclude that either the fixed-point value is not approached or the  $\tilde{\tau}$  mass is higher and reduces the RPV effect (e.g.  $m_{\tilde{\tau}} = 150$  GeV with  $\lambda'_{333} = 0.9$  would suppress the SM signal rate by 0.88 instead). Note that our discussion hinges on the fact that the RPV decays of present interest would *not* contribute to SM top signals; it is quite different from the approach of Ref. [7], which considers RPV couplings that would give hard electrons or muons and contribute in conventional top searches.

Similarly, the  $B$ -violating coupling  $\lambda''_{233}$  leads to  $t_R \rightarrow \bar{b}_R\bar{\tilde{s}}_R, \bar{\tilde{b}}_R\bar{s}_R$  decays, with partial widths

$$\Gamma(t \rightarrow \bar{b}\bar{\tilde{s}}) = \Gamma(t \rightarrow \bar{\tilde{b}}\bar{s}) = \frac{(\lambda''_{233})^2}{32\pi} m_t (1 - m_{\tilde{q}}^2/m_t^2)^2, \quad (65)$$

neglecting  $m_b$  and  $m_s$ , and assuming a common squark mass  $m_{\tilde{q}} = m_{\tilde{s}} = m_{\tilde{q}}$ . If the squarks were no heavier than 150 GeV, say, the ratio of RPV to SM decays would be

$$\Gamma(t \rightarrow \bar{b}\bar{s}, \bar{\tilde{b}}\bar{\tilde{s}})/\Gamma(t \rightarrow bW^+) \simeq 0.16 (\lambda''_{233})^2 \quad (\text{for } m_{\tilde{q}} = 150 \text{ GeV}). \quad (66)$$

These RPV decays would plausibly be followed by  $\tilde{q} \rightarrow q\chi_1^0$  and  $\chi_1^0 \rightarrow cbs, \bar{c}\bar{b}\bar{s}$  (via the same  $\lambda''_{233}$  coupling with a short lifetime analogous to Eq.(63)), giving altogether

$$t \rightarrow (b\bar{s}, s\bar{b}) \rightarrow bs\chi_1^0 \rightarrow (cbbbs, \bar{c}\bar{b}\bar{b}\bar{s}). \quad (67)$$

This all-hadronic mode could in principle be identified experimentally, through the multiple  $b$ -jets plus the  $t \rightarrow 5$ -jet and  $\chi_1^0 \rightarrow 3$ -jet invariant mass constraints. However, it would not be readily mistaken for the SM hadronic mode  $t \rightarrow bW \rightarrow 3$ -jet, and would simply reduce all the SM top signal rates. If the coupling approached the fixed-point value  $\lambda''_{233} \simeq 1.0$ , while  $m_{\tilde{q}} \simeq 150$  GeV as assumed in Eq.(66), the SM top signals would be suppressed by a factor  $(1 + 0.16(\lambda''_{233})^2)^{-2} \simeq 0.75$ , which is strongly disfavored by the present data [15, 16] but perhaps not yet firmly excluded.

If indeed the  $s$ - and  $b$ -squarks were lighter than  $t$  to allow the  $B$ -violating modes above, it is quite likely that the  $R$ -conserving decay  $t \rightarrow \tilde{t}\chi_1^0$  would also be allowed, followed by  $\tilde{t} \rightarrow c\chi_1^0$  (via a loop) and  $B$ -violating decays for both neutralinos, with net effect

$$t \rightarrow \tilde{t}\chi_1^0 \rightarrow c\chi_1^0\chi_1^0 \rightarrow (cccbbbb, cccb\bar{c}\bar{b}\bar{b}, c\bar{c}\bar{c}\bar{b}\bar{b}\bar{b}\bar{b}). \quad (68)$$

This seven-quark mode would look quite unlike the usual SM modes and would further suppress the SM signal rates. Depending on details of the sparticle spectrum, however, other decays such as  $\tilde{t} \rightarrow bW\chi_1^0$  might take part too, leading to different final states; no general statement can be made except that they too would dilute the SM signals and therefore cannot be very important.

## V. Conclusions

The renormalization group evolution of the Standard Yukawa couplings can be affected by the presence of RPV couplings. In this paper we have done the following:

- We have identified the fixed points that occur in the RPV couplings, under the usual assumption that only  $B$ -violating or only  $L$ -violating RPV interactions exist.

- These fixed points provide process-independent upper bounds on RPV couplings at the electroweak scale; we confirm previously obtained bounds in the  $B$ -violating case and provide new results for the  $L$ -violating case [Fig.1].
- We have also addressed scenarios with large  $\tan\beta$  where  $\lambda_b$  too can reach a fixed point.
- The fixed point values are summarized in Table II. It is interesting that they are compatible with all present experimental constraints.
- However, fixed-point values of the  $L$ -violating coupling  $\lambda'_{333}$  or the  $B$ -violating coupling  $\lambda''_{233}$  would require the corresponding sparticles to have mass  $\gtrsim m_t$  to prevent unacceptably large fractions of top decay to sleptons or squarks.
- The fixed points lead to constraints, correlating the RPV couplings with the top quark Yukawa coupling at the GUT scale, from lower bounds on the top mass [Fig.2].
- We have derived evolution equations for the CKM matrix and examined the evolution of the CKM mixing angles in the presence of RPV couplings [Fig.3]. In the most general case, new CKM-like angles occur in the RPV coupling sector and influence the scaling of the CKM unitarity triangle.

## Acknowledgements

VB thanks Herbi Dreiner for a discussion and the Institute for Theoretical Physics at the University of California, Santa Barbara for hospitality during part of this work. RJNP thanks the University of Wisconsin for hospitality at the start of this study. This research was supported in part by the U.S. Department of Energy under Grant Nos. DE-FG02-95ER40896 and DE-FG02-91ER40661, in part by the National Science Foundation under Grant No. PHY94-07194, and in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation and support by NSF. TW is supported by the *Deutsche Forschungsgemeinschaft* (DFG).

## References

- [1] For an introduction to supersymmetry, see J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton University Press, 1983); P. Fayet and S. Ferrara, Phys. Rep. **32**, 249 (1977); P. West, *Introduction to Supersymmetry and Supergravity* (World Scientific, 1986); R.N. Mohapatra, *Unification and Supersymmetry* (Springer-Verlag, 1986).
- [2] For phenomenological reviews of SUSY, see H.P. Nilles, Phys. Rep. **110**, 1 (1984); G.G. Ross, *Grand Unified Theories* (Benjamin Cummings, 1985); R. Arnowitt, A. Chamseddine and P. Nath, *Applied  $N = 1$  Supergravity* (World Scientific, 1984); H. Haber and G. Kane, Phys. Rep. **117**, 75 (1985); X. Tata, lectures at TASI 1995 (to be published); J. Bagger, *ibid.*
- [3] C.S. Aulakh and R.N. Mohapatra, Phys. Lett. **B119**, 316 (1982); L.J. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984); F. Zwirner, Phys. Lett. **B132**, 103 (1983); S. Dawson, Nucl. Phys. **B261**, 297 (1985); R. Barbieri and A. Masiero, Nucl. Phys. **B267**, 679 (1986); S. Dimopoulos and L.J. Hall, Phys. Lett. **B207**, 210 (1987); L. Hall, Mod. Phys. Lett. **A5**, 467 (1990); K.S. Babu and R.N. Mohapatra, Phys. Rev. **D42**, 3778 (1990).
- [4] V. Barger, G.F. Giudice, T. Han, Phys. Rev. **D40**, 2987 (1989).
- [5] K. Enqvist, A. Masiero and A. Riotto, Nucl. Phys. **B373**, 95 (1992); G. Bhattacharyya, J. Ellis and K. Sridhar, Mod. Phys. Lett. **A10**, 1583 (1995); G. Bhattacharyya, D. Choudhury and K. Sridhar, Phys. Lett. **B355**, 193 (1995).
- [6] R. Mohapatra, Phys. Rev. **D34**, 3457 (1986); H. Dreiner and G.G. Ross, Nucl. Phys. **B365**, 597 (1991); R. Godbole, P. Roy and X. Tata, Nucl. Phys. **B401**, 67 (1993).
- [7] K. Agashe and M. Graesser, LBL-37823, UCB-PTH-95/33, hep-ph/9510439.
- [8] B. Brahmachari and P. Roy, Phys. Rev. **D50**, R39 (1994); Erratum **D51**, 3974 (1989); see also J. McCurry, Oxford thesis 1993 (unpublished).
- [9] J. L. Goity and M. Sher, Phys. Lett. **B346**, 69 (1995).
- [10] H. Dreiner and H. Pois, Zurich preprint ETH-TH/95-30, hep-ph/9511444.

- [11] A. Smirnov and F. Vissani, ICTP preprint IC/95/122, hep-ph/9506416.
- [12] C.E. Carlson, P. Roy and M. Sher, Phys. Rev. Lett. **B357**, 99 (1995).
- [13] N. Cabibbo, L. Maiani, G. Parisi, and R. Petronzio, Nucl. Phys. **B158**, 295 (1979); B. Pendleton and G.G. Ross, Phys. Lett. **98B**, 291 (1981); C.T. Hill, Phys. Rev. **D24**, 691 (1981); E.A. Paschos, Z. Phys. **C26**, 235 (1984); J.W. Halley, E.A. Paschos, and J. Usler, Phys. Lett. **155B**, 107 (1985); J. Bagger, S. Dimopoulos, and E. Masso, Nucl. Phys. **B253**, 397 (1985); W. Zimmerman, Commun. Math. Phys. **97**, 211 (1985); M. Tanimoto, T. Hayashi, R. Najima, and S. Wakaizumi, Prog. Theor. Phys **76**, 1098 (1986); K.S. Babu and E. Ma, Europhys. Lett. **3**, 437 (1987); M. Tanimoto, Y. Suetake, and K. Seuba, Phys. Rev. **D36**, 2119 (1987); C.H. Albright and M. Lindner, Phys. Lett. **B213**, 347 (1988); Z. Phys. **C44**, 673 (1989); J. Kubo, K. Sibold, and W. Zimmerman, Phys. Lett. **200**, 191 (1989); C.D. Froggatt, I.G. Knowles, and R.G. Moorhouse, Phys. Lett. **B249**, 273 (1990); **B298**, 356 (1993); E.M. Fieire, G. Lazarides, and Q. Shafi, Mod. Phys. Lett. **A5**, 2453 (1990); W.A. Bardeen, M. Carena, T.E. Clark, K. Sasaki, and C.E.M. Wagner, Nucl. Phys. **B369**, 33 (1992); M. Carena, S. Pokorski, and C.E.M. Wagner, Nucl. Phys. **B406**, 59 (1993); W. Bardeen, M. Carena, S. Pokorski, and C.E.M. Wagner, Phys. Lett. **B320**, 110 (1994); E.G. Floratos and G.K. Leontaris, Phys. Lett. **B336**, 194 (1994).
- [14] V. Barger, M.S. Berger, and P. Ohmann, Phys. Rev. **D47**, 1093 (1993); V. Barger, M.S. Berger, P. Ohmann, and R.J.N. Phillips, Phys. Lett. **B314**, 351 (1993).
- [15] CDF collaboration: F. Abe et al, Phys. Rev. Lett. **73**, 225 (1994), Phys. Rev. **D50**, 2966 (1994) and FERMILAB-PUB-94/411-E.
- [16] D0 collaboration: S. Abachi et al, Phys. Rev. Lett. **72**, 2138 (1994), FERMILAB-Pub-94/354-E.
- [17] V. Barger, M.S. Berger, and P. Ohmann, Phys. Rev. **D47**, 2038 (1993).
- [18] H. Dreiner and R.J.N. Phillips, Nucl. Phys. **B367**, 591 (1991).
- [19] S. Dawson, Nucl. Phys. **B261**, 297 (1985).

