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Authors: Haijun Zhang, Xiaojun Gu, Qin Yang, Wei Zhao, Feilong Jiang

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Knudsen Maximum Effect in Micro-Scale Gas Lubrication

Haijun Zhang¹ Xiaojun Gu² Qin Yang³ Wei Zhao¹ Feilong Jiang¹

1. College of Information Science and Engineering, Jiaxing University, Jiaxing 314001, China

2. Scientific Computing Department, STFC Daresbury Laboratory, Warrington WA4 4AD, UK

3. College of Data Science, Jiaxing University, Jiaxing 314001, China

ABSTRACT

Micro-scale gas lubrication has been studied for several decades, with extensive research on non-equilibrium flow effects like velocity slip and thermal creep. However, the Knudsen maximum effect in micro-scale gas lubrication has not been reported yet. To address this, we analyzed the load capacity characteristics of slider bearings with ultra-thin film gas lubrication equations derived from the linearized Boltzmann-BGK model equation (FK lubrication model) under the condition of constant bearing number. Our study reveals that there exists a maximum value of load capacity for slider bearings when the reference Knudsen number is about unity. This happens because the dimensionless mass flow rate of micro-scale gas flows has a minimum value when the reference Knudsen number approaches unity. Understanding the Knudsen maximum effect is crucial when designing micro-nano devices related to gas lubrication, as it implies that there exists an optimum clearance for maximum load capacity.

Keywords micro-scale gas lubrication, Knudsen maximum, maximum load capacity, reference Knudsen number

1 Introduction

In micro-scale gas lubrication under clearance conditions of submicron or less, the flow in the gas film cannot be regarded as a continuum flow since the molecular mean free path of gas is not negligible compared with the gas film thickness. Therefore, the micro-scale gas lubrication problems need to be solved using the kinetic theory of gases[1], the extended hydro-thermal-dynamic equations[2], the traditional Navier-Stokes equations with appropriate slip boundary conditions[3-9] and the direct simulation Monte Carlo method[10, 11]. Among these solution methods, the lubrication model derived from the kinetic theory of gases, i.e., the FK lubrication model, has been widely accepted as a more accurate model. Based on the FK lubrication model, the thermal-molecular gas film lubrication model for the heat-assisted magnetic recording head sliders was put forward and the effect of temperature on the flying characteristics of head sliders has been analyzed[12, 13].

In the kinetic theory of gases, the Knudsen number (Kn), defined as the ratio of the molecular mean free path to the characteristic length of gas flows[1], is one of the most critical parameters. For micro-scale gas lubrications, the characteristic length of gas flows is the

thickness of the gas film. The Knudsen minimum is an interesting phenomenon in the field of micro-scale gas flow and it was first observed in the experimental phenomena of the channel flow with varying channel width by Knudsen[14]. The Knudsen minimum for the pure gas flows has been investigated widely with both methods of experiment and numerical simulation[15-18]. It states that the dimensionless mass flow rate versus the Kn will exhibit a minimum value which occurs at a given value of Kn .

Since Burgdorfer first introduced the concept of the kinetic theory to the field of microscale gas lubrication[19], the micro-scale gas lubrication problem has been studied for more than sixty years and many slip-modified lubrication models considering some physical mechanisms and comparisons between the FK lubrication model and the slip-modified lubrication models has been given[20]. According to the FK lubrication model, the load capacity for the slider bearings would decrease when the reference Knudsen number increases for all the arbitrary Knudsen number ranges, i.e., the more rarefied the gas is, the less the load capacity of the slider bearings is.

However, our study reveals that it is not true when the reference Knudsen number is close to unity. Actually, the maximum load capacity for the slider bearings would be obtained when the reference Knudsen number is about unity. That is the innovation of our paper and also a new contribution in micro-scale gas lubrication. We call the maximum load capacity characteristics as Knudsen maximum effect.

In this paper, firstly, the pressure distribution and load capacities of micro-scale gas slider bearings were analyzed according to the FK lubrication model and the Knudsen maximum effect in micro-scale gas lubrication was investigated when the reference Knudsen number is about unity. Secondly, the Knudsen minimum effect of micro gas flows was taken into account and the inherent mechanism of the Knudsen maximum effect was put forward. Finally, the conclusion was given.

2 FK Lubrication Model

The typical example of micro-scale gas lubrication is the slider bearings employed in computer magnetic disk storage and a schematic diagram of the slider bearings is outlined in Fig.1. The upper wall is fixed and the lower wall has a length of l and moves to the right with the velocity U . The origin of the x -axis sits at the left end of the lower wall. The temperatures at both walls are assumed to be constant T . The gap between the two walls decreases along the x direction with a pitch angle of the slider, γ . It reaches a minimum thickness of the gas film, h_o , at the right end of the lower wall. The reference Knudsen number is defined as the ratio of the molecular mean free path, $\lambda_o = (\mu/p_a)\sqrt{\pi RT/2}$ to the minimum thickness of the gas film, i.e. $Kn_o = \lambda_o/h_o$. Here μ denotes the dynamical viscosity of the gas and p_a denotes the ambient gas pressure of the slider bearings.

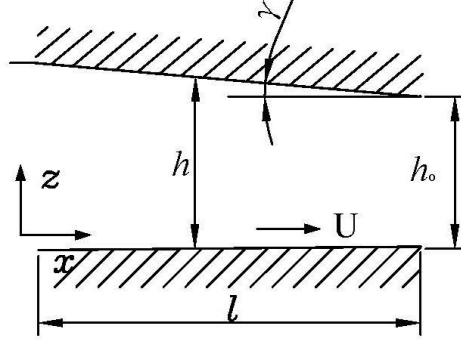


Fig. 1. Schematic diagram of a slider bearing.

Neglecting the complex derivation, the FK lubrication model of the dimensionless form is written as:

$$\frac{\partial}{\partial X} \left[\bar{Q}_p(D) P H^3 \frac{\partial P}{\partial X} \right] = \Lambda \frac{\partial P H}{\partial X}, \quad (1)$$

in which $\bar{Q}_p(D) = Q_p(D)/Q_{con}(D)$. P , H and X are the dimensionless pressure, thickness and length of the gas film and are defined as $P = p/p_a$, $H = h/h_o$ and $X = x/l$, respectively. The bearing number Λ is defined as $\Lambda = 6\mu U l / (p_a h_o^2)$. The dimensionless mass flow rate of the Poiseuille flow $Q_p(D)$ is given by[21]

$$Q_p(D) = \begin{cases} D/6 + 1.0162 + 1.0653/D - 2.1354/D^2, & D \geq 5, \\ 0.13852D + 1.25087 + 0.15653/D - 0.00969/D^2, & 0.15 \leq D < 5, \\ -2.22919D + 2.10673 + 0.01653/D - 0.0000694/D^2 & 0.01 \leq D < 0.15, \end{cases} \quad (2)$$

where D denotes inverse Knudsen number defined as $D = \sqrt{\pi}/(2Kn)$. $Kn = \lambda/h$ denotes the local Knudsen number of the micro-scale gas lubrication flow with a local mean free path $\lambda = (\mu/p)\sqrt{\pi RT/2}$. The continuum flow rate is calculated by $Q_{con}(D) = D/6$. Here it is necessary to note that D is related to the inverse Knudsen number $D_o = \sqrt{\pi}/(2Kn_o)$ by $D = D_o P H$.

3 Knudsen Maximum Effect

To clarify the Knudsen maximum effects in micro-scale gas lubrication, the FK lubrication model of the infinite-width slider bearings given in equation (1) has been solved numerically using the finite difference method[22, 23]. The calculating program was coded in Matlab software. The parameters of the infinite slider bearing were chosen from the previous literature[10]. The length of the slider bearing is $l = 5\mu m$. The pitch angle of the slider is fixed at $\gamma = 0.01$. The working lubrication media is argon gas with a dynamical viscosity $\mu = 2.08 \times 10^{-5} Pa \cdot s$. The ambient condition is set to be $T = 273K$ and $p_a = 101325 Pa$. The

gas constant for argon is $R = 208 \text{ J/kgK}$.

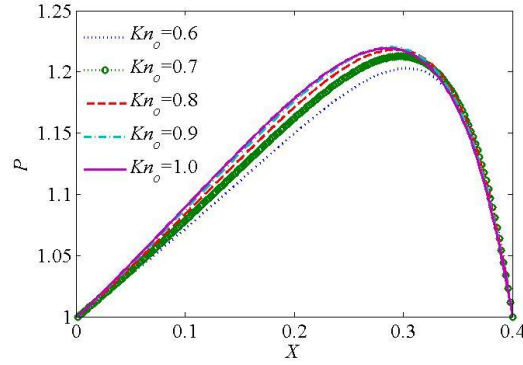


Fig. 2. Pressure distribution of the slider bearings with $\Lambda = 50$ and $Kn_o \leq 1$.

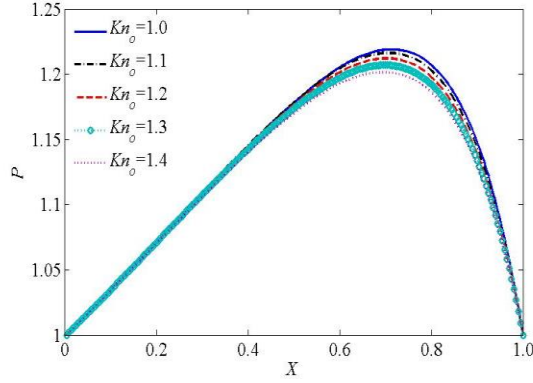
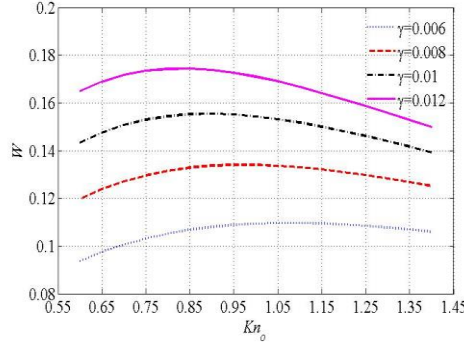


Fig. 3. Pressure distribution of the slider bearings with $\Lambda = 50$ and $Kn_o \geq 1$.

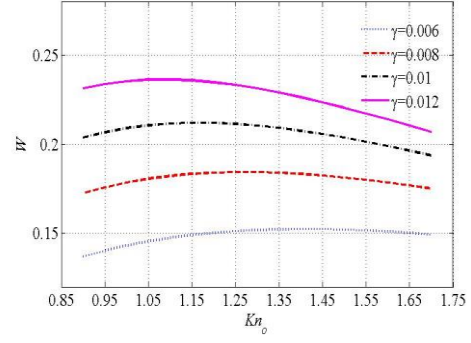
The pressure distributions of the infinite width slider bearing are presented in Fig. 2 for $Kn_o \leq 1$ and Fig. 3 for $Kn_o \geq 1$, respectively. The maximum value of the pressure profiles of the slider bearings increases with the reference Knudsen number increasing when $Kn_o \leq 1$, as shown in Fig. 2, which is contrary to the conventional justification in micro-scale gas lubrication. On the other hand, the maximum value of the pressure profiles of the slider bearings decreases with the reference Knudsen number increasing when $Kn_o \geq 1$, as indicated in Fig.3, which is following the conventional justification in micro-scale gas lubrication.

The load capacity of the infinite-width slider bearing W , is determined from the definite integral of its pressure distribution by

$$W = \frac{1}{p_a l} \int_0^l (p - p_a) dx = \int_0^1 (P - 1) dX \quad (3)$$



(a) $\Lambda = 50$



(b) $\Lambda = 80$

Fig. 4. Load capacities of the slider bearings under different pitch angles

The load capacities of the infinite-width slider bearings under different pitch angles for two fixed bearing numbers are plotted in Fig.4 (a) and (b), respectively. It can be seen that the load capacities of the slider bearings increase firstly, then decrease with as Kn_o increases, resulting in a maximum value when Kn_o is about unity. The larger the bearing number is, the larger the reference Knudsen number where the maximum value of load capacity occurs changes slightly.

4 Analysis and Discussion

From the above pressure distributions and load capacities of the slider bearings, we argue that the intrinsic physical mechanism of the Knudsen maximum would originate from the Knudsen minimum of micro-scale Poiseuille gas flow.

The Knudsen minimum of micro-scale gas Poiseuille flow can be simulated by solving the high-order moment equations[24] or the Navier-Stokes equation with the appropriate boundary conditions[25].

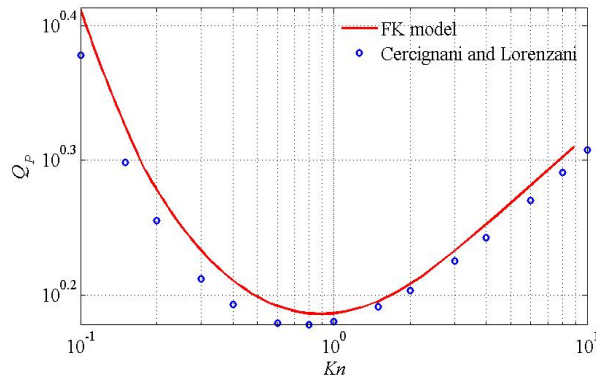


Fig. 5. The dimensionless mass flow rate of micro-scale gas Poiseuille flow

According to the equation (2), the dimensionless mass flow rate of micro-scale gas Poiseuille flow is calculated and shown in Fig. 5. To verify the existence of the Knudsen

minimum further, the mass flow rate from the Boltzmann equation by Cercignani and Lorenzani[26] is also plotted. It can be seen that the dimensionless mass flow rate would have a minimum value at the Knudsen number about unity.

The Knudsen minimum is the combination of two opposite effects from the rarefaction of the flow interacting in the centre and near the walls[27]. As a result, a minimum mass flow rate is reached.

Based on the above analysis, it appears that the Knudsen minimum of micro-scale gas flow plays a crucial role in intensifying the wedge gap effect of lubrication gas flow at the Kn value of about unity. This, in turn, can lead to further congestion of lubrication gas flow. In hydrodynamic lubrication, the lifting force generated from the lubrication flow is primarily due to the wedge gap effect.

Understanding the Knudsen maximum effect in micro-scale gas lubrication would make more optimal design of gas bearings, seals and MEMS gas devices. For micro-scale gas bearings and seals, the dimension of the gas film can be designed appropriately to achieve the maximum load capacity or pressure value. For MEMS resonators[28], the distance between the electrode plates can be determined properly to obtain the optimal squeeze film damping performance.

5 Conclusions

The Knudsen maximum effect in micro-scale gas lubrication has been investigated. When the reference Knudsen number Kn_o is about unity, there exists a maximum value of load capacity for micro-scale gas lubrication. The exact value of Kn_o where the maximum value of load capacity occurs would change if the pitch angle of the slider bearings changes. The larger the pitch angle of the slider bearings is, the closer to the unity of the exact value of Kn_o where the maximum value of load capacity is. It implies that for each pitch angle, there exists an optimum value of h_o for maximum loading capacity.

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