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Description of the non-linear behaviour of Fourier filtering wavefront sensor using matrix formalism: the specific matrix

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ABSTRACT

Fourier Filtering WaveFront Sensors (FFWFS) are extremely sensitive wavefront sensors that will improve the wavefront estimation of large ground-based facilities such as European Extremely Large Telescope (ELT) or Giant Magellan Telescope (GMT). Yet they suffer from inherent non-linear behaviour that prevents their optimised use.

We propose a modification of the calibration process of these sensors to implement their non linearities in the matrix formalism (linear formalism). The new approach called the specific matrix tackles the loss of sensitivity and the modal confusion of the FFWFS. It allows the measurement of the absolute phase and to close the loop using a non-modulated pyramid. We demonstrate this method by use of numerical simulations.

Keywords: Adaptive Optics, wavefront sensor, high performances

1. INTRODUCTION

Fourier Filtering WaveFront Sensors (FFWFS) are at the heart of the next generation of adaptive optics (AO) systems. Among the FFWFS class we count the Pyramid[1] and the Zernike[2] wavefront sensors respectively PWFS and ZWFS. FFWFS are attractive for Adaptive Optics (AO) applications, as they allow higher sensitivity compared to the historical Shack-Hartmann WFS[3].[4] This sensitivity means that the noise propagation is better in FFWFS. For a given guide star flux, the estimation is therefore better. Contrarily, a given estimation precision can be reached with a fainter guide star flux.

The high sensitivity of these class of WFS that allow to work in a very low flux regime comes at the price of a very limited dynamic range in term of wavefront amplitude, making difficult the use of these sensors in strong turbulence regime. The practical solution when using a PWFS is to add a tip/tilt modulation in the pupil plane to increase the dynamic range. However, this solution is not suitable for all types of FFWFS as for the ZWFS. Furthermore, when adding the modulation to linearize the PWFS, it also reduces its sensitivity to low order modes and specific modes such as low wind effect or differential piston. These two modes are induced by the telescopes structure and will limit their resolution if not corrected and controlled[5 6].[7]

This paper aims at introducing a mathematical formalism that includes the non-linearities of FFWFS, while still using the Matrix-Vector-Multiplication (MVM) for the inversion procedure. The advantage of this formalism is that no hypothesis is made about the nature of the FFWFS, therefore, it is valid for the whole FFWFS

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class. This article is organised as follows, the first section (section 2) introduces the mathematical formalism that led to the construction of a new reconstructor. Then we demonstrated its ability to provide information about the non-linear terms such as the optical gain and the coupling terms in section 3. Finally, we focus on a practical implementation of this new tool to improve the frame-by-frame estimation of phase residual with a non-modulated PWFS.

2. THE SPECIFIC MATRIX

Considering a given wavelength for WFS, several strategies can be used to mitigate the non-linear behaviour of FFWFS. On one side one can modify the phase mask in the focal plane. On the other side one can modify the reconstructor. The first strategy led to the modulated PWFS that allows to increase in the dynamic range of the PWFS and use it on sky in[8] and.[9] When considering the ZWFS, the use of two ZWFS in the focal plane allows an estimate of both phase and amplitude aberration[10] which is a substantial advantage when at a picometer level of aberration.[11] It also enables to increase the dynamic range of the classical ZWFS using a non-linear phase reconstructor[12].[13]
The second strategy that consists in modifying the reconstructor is the heart of this paper. The classical approach consists in calibrating the AO system to build an interaction matrix. It is used to calibrate the response of the WFS to a modification of the Deformable Mirror (DM) surface. In this manner, the light propagates through the entire AO system. In this manner all the additional optical aberration induced by the optics are taken into account. Although this approach has the advantage of describing accurately the optical path and the output/input relation, it is based on a linear assumption, both spatial and in amplitude. Yet this linear relation is not always true. It is valid for SH-type WFS and depends on the phase regime for Fourier-type WFS. Indeed FFWFS are inherently non-linear with the phase. For the linear assumption to be valid, the calibration uses small phase amplitude $\phi \ll 1$. This calibration in a small phase regime will induce reconstruction error because the on-sky regime is too different from the calibration regime. In that context, we aim at building a reconstructor in a matrix formalism that will cancel out the modelling and reconstruction error for one given phase mask independently of the phase regime. A Linear Parameter Varying approach (LPV) is used to take into account the non-linear behaviour of FFWFS. The approach developed and presented here is valid for all FFWFS.

2.1 Mathematical definition of the specific matrix

We use the mathematical formalism developed by[14] to unify the expression of the intensity signals for all the different FFWFS. For simplicity and uniformity, we will use the reduced intensity formalism as in equation (1):

$$\Delta I(\delta\phi;\phi_r) = I(\phi) - I(\phi_r), \tag{1}$$

where $\delta\phi$ is the phase-to-be-measured, i.e $\delta\phi = \phi - \phi_r$ where ϕ is the incoming phase and ϕ_r the reference phase. $I(\phi_r)$ is thus the reference intensity. $\Delta I(\delta\phi;\phi_r)$ is the output signal from the wavefront sensor after subtracting the reference signal.

In the linear regime, the signal can be directly linked to the wavefront decomposition using the equation (2):

$$\widehat{\mathbf{a}}(\delta\phi) = \mathbf{M}^{\dagger}\Delta\mathbf{I}(\delta\phi;\phi_r). \tag{2}$$

where $\mathbf{M}^{\dagger}$ is the pseudo-inverse of the interaction matrix $\mathbf{M}$, i.e. interaction matrix computed around the reference wavefront and $\widehat{\mathbf{a}}(\delta\phi)$ is the decomposition of the estimated phase $\widehat{\delta\phi} = \sum_{i=1}^{n} a_i\Phi_i$ on the control basis (Φ_i). When $\phi_r = 0$, $\mathbf{M}$ is called the interaction matrix working in diffraction-limited conditions.

The matrix formalism describes the linear evolution of the signal from the reference to the turbulence regime. It captures the linear link between the wavefront and the signal. However, this linear relation between the signal and the phase is only true in the small phase regime. When the amplitude of the phase to be measured is too large, the interaction matrix $\mathbf{M}$ will provide an underestimation of the wavefront (cf blue curve in Fig.1). It is a direct consequence of the loss of sensitivity of FFWFS in a strong turbulence regime. It is commonly accepted and research has been done in the case of the modulated PWFS that one solution is to build the interaction matrix around the operational point $\mathbf{M}(0;\phi_r+\delta\phi)$, represented in red in Fig.1. However, this matrix does not establish the connection between the reference wavefront and the phase to be measured. To effectively use this matrix, one information is absent: the reference intensity. This information depends on the operational point. Consequently, when using the signal of the WFS given by equation (1), the reference intensity is systematically leading to a wrong estimation of the phase.

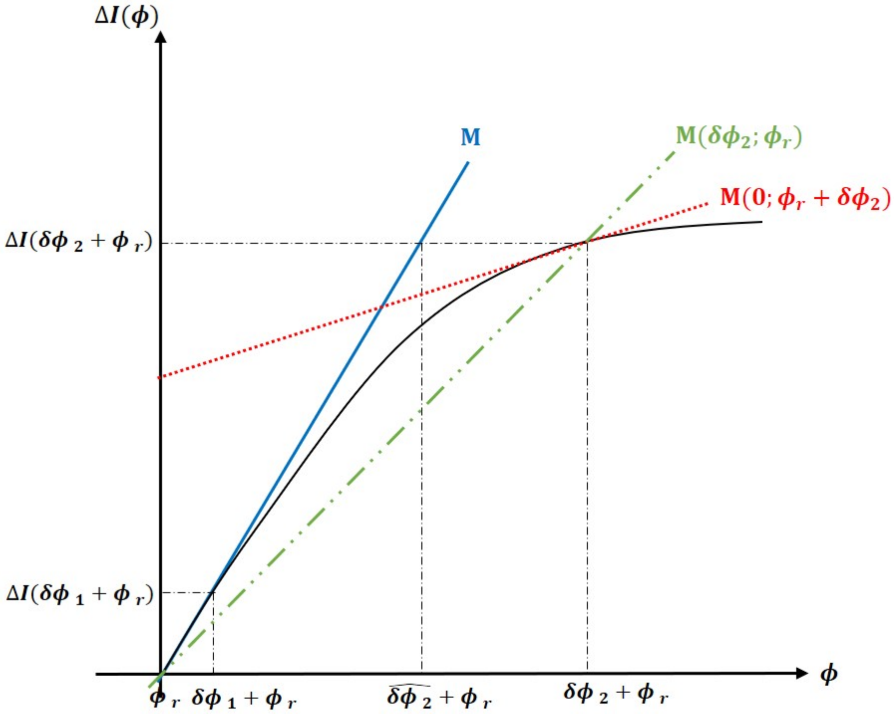


Figure 1: One-Dimension analogy

The only linear relation that link the two points $(\phi_r;\Delta I(\phi_r))$ and $(\delta\phi+\phi_r;\Delta I(\delta\phi+\phi_r))$ is described by the green dotted-dashed line. Following the 1-D analogy, this matrix represents the exact slope:

$$\frac{f(x_r+\delta x)-f(x_r)}{\delta x} \tag{3}$$

where the function f represents the intensity and the variable x, the phase. The function f exhibits a linear behaviour around the reference value x_r and non-linear behaviour for larger value of x.

$$\begin{aligned} \phi &\iff x \\ I(\phi) &\iff f(x) \\ \Delta I(\phi) &\iff f(x_r+\delta x)-f(x_r) \end{aligned}$$

When $\delta x \to 0$, $\lim\limits_{\delta x\to 0} \frac{f(x_r+\delta x)-f(x_r)}{\delta x} = f'(x_r)$, so the slope is equal to the derivative of the function around x_r which is the equivalent of the pseudo-inverse of the interaction matrix $\mathbf{M}^{\dagger}$. In this context, considering the connection between the 1-D analogy and the matrix formalism:

$$\begin{aligned} \mathbf{M} &\iff f'(x) \\ \mathbf{M}^{\dagger} &\iff \frac{1}{f'(x)} \end{aligned}$$

Taking a close look at the expression (3), one realises it can also be written as the integral of the derivative of the function f between x_r and $x_r + \delta x$:

$$\frac{f(x_r + \delta x) - f(x_r)}{\delta x} = \int_0^1 f'(x_r + t\delta x)\mathrm{d}t \tag{4}$$

Outside the linear domain of the function f, the perfect slope was expressed by the rate of change in equation (4). Following exactly the same logic, the matrix that will provide the perfect phase estimation could be written:

$$\mathbf{M}(\delta\phi; \phi_r) = \int_0^1 \mathbf{M}(0; \phi_r + t\delta\phi)\mathrm{d}t \tag{5}$$

where $\mathbf{M}(\delta\phi; \phi_r)$ is the only matrix that links these two points: $(\phi_r; \Delta I(\phi_r))$ and $(\delta\phi; \Delta I(\delta\phi))$. It creates a linear relation between the reference point and the one of interest. The matrix $\mathbf{M}(0; \phi_r + t\delta\phi)$ is a push-pull matrix computed around the phase $\phi_r + t\delta\phi$. The matrix $\mathbf{M}(\delta\phi; \phi_r)$ will be called the *specific matrix*. Its name come from its dependency with the incoming wavefront.

In practice, the integral of equation (5) is approximated by a discretised sum such as:

$$\mathbf{M}_m(\delta\phi; \phi_r) = \frac{1}{m}\sum_{i=1}^{m} \mathbf{M}\left(0; \phi_r + \frac{i}{m+1}\delta\phi\right) \tag{6}$$

We define m as the degree of the discrete specific matrix $\mathbf{M}_m$. As m increases, $\mathbf{M}_m$ is expected to gradually tend towards the real specific matrix $\mathbf{M}(\delta\phi; \phi_r)$.

The physical interpretation of the specific matrix can be linked to the modulated PWFS. Since, the signal from the modulated PWFS is the sum of incoherent propagation of the wavefront and the tip/tilt phases, the resulting modulated interaction matrix is the average of push-pull matrices around variable operational points. In the same way, the specific matrix is the average of interaction matrices built around moving operational points from ϕ_r to the incoming wavefront $\phi = \phi_r + \delta\phi$. These moving operating points can be interpreted as modulation. Therefore, the specific matrix can be interpreted as the reconstructor resulting from the self-modulation of the wavefront.

2.2 Characterization of the specific matrix

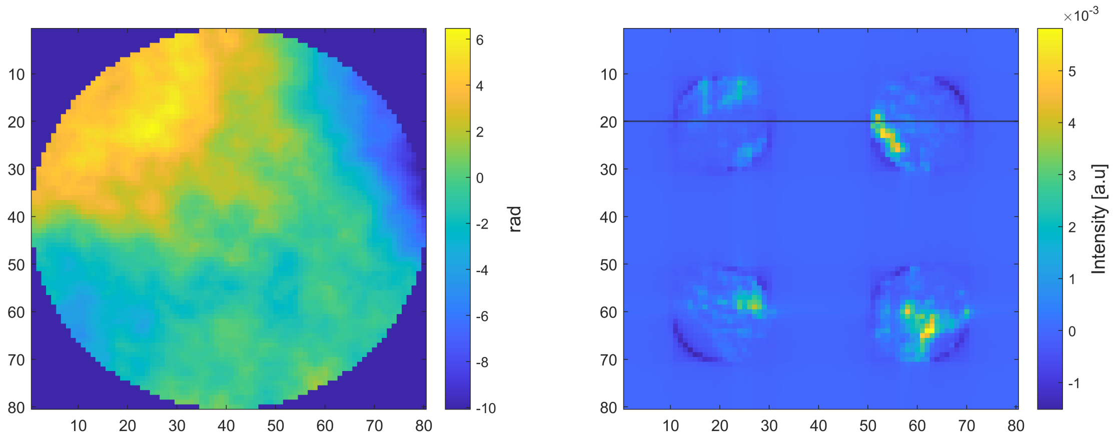
To assess the performances of our approximation, we consider a turbulent uncorrected wavefront (therefore of high amplitude) $\delta\phi$. The phase screen follows the Kolmogorov power-law. The WFS considered is a non-modulated PWFS. In the following figures, we compare the E2E intensity and the ones obtained with the different linear models :

1. Interaction matrix: $\Delta I = \mathbf{M}\mathbf{a}$
2. Push-pull around the operational point: $\Delta I = \mathbf{M}(0; \phi_r + \delta\phi)\mathbf{a}$
3. Specific matrix of degree $m = 1$ and $m = 10$: $\Delta I = \mathbf{M}_i\mathbf{a}$ with $i = 1$ or 10

Telescope	1.52m (80 pixels resolution)
DM	17x17 correcting 200 Zernike modes
PWFS	sub-aperture 20x20 pixels, $r_{mod} = 0\lambda/D$
λ	640nm

Table 1: Simulation parameters

As demonstrated in Fig.3, the classical interaction matrix does not provide an accurate description of the actual behaviour of the WFS. The same conclusion can be made regarding the matrix computed around the operational point. The specific matrix of degree $m = 1$ provides a good estimation of the intensity pattern. The



(a) Input phase $\delta\phi$: amplitude 2.9 rad RMS

(b) E2E intensity

Figure 2: Phase and E2E intensity of the turbulent phase screen of 2.9rad RMS.

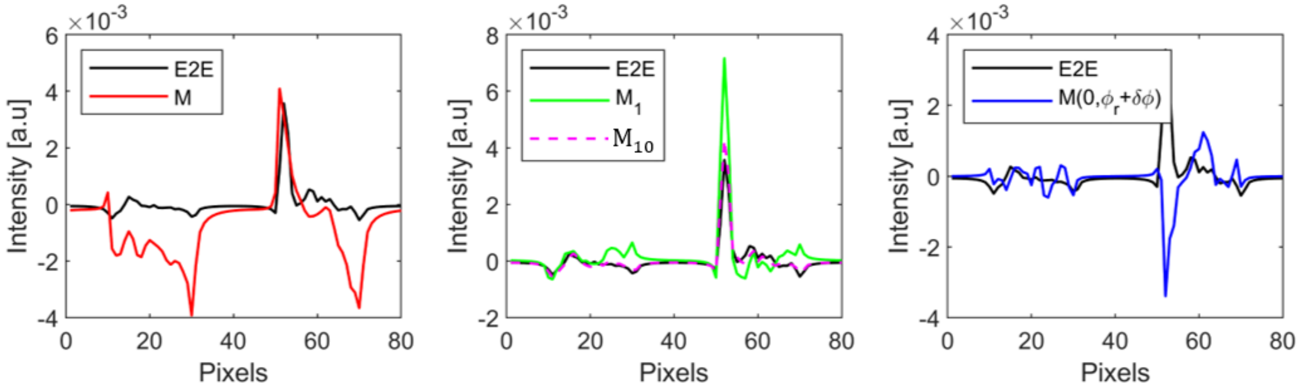


Figure 3: Linear model obtained for the phase screen plotted in Fig.2a. Left: E2E intensity. Centre: linear model using $\mathbf{M}_1$ and $\mathbf{M}_{10}$. Right: linear model using the push-pull matrix around the operational point

specific matrix of degree $m = 1$ corresponds to the push-pull matrix computed around the wavefront divided by two, $\delta\phi/2$. The best result is obtained using the matrix of degree $m = 10$ we can consider this matrix as a good approximation of the specific matrix. Nonetheless, a good alternative will be the median matrix $\mathbf{M}_1$.

Another important aspect of this reconstructor is its specificity, indeed, this matrix is built to provide a proper estimation of one given phase $\delta\phi$ of a given amplitude. When estimating another phase screen, the reconstructor has to be changed. One way to understand this is to consider a situation where the phase is wrapped. The wrapping being unique to each phase screen, a given specific matrix built for this phase will no longer provide a proper estimation of another phase screen because the specific matrix unwraps the phase. This situation is illustrated in Fig.4 where one can see the intensity cut from the E2E simulation and the one provided by our linear model $\mathbf{M}(\delta\phi\text{''}; \phi_r)$ is far from reproducing the E2E simulation intensity.

It is important to assess the performances of these matrices in the estimation of phase residuals as it is the regime in which the AO system is working. For this demonstration we will use aberrations constructed from a power spectral density (PSD) with a power-law exponent of -2.5. By comparing the performances of the full matrices (specific matrix and the one around the phase residual) we highlight the accuracy in the measurement provided by the specific matrix. Indeed, Fig.5a shows that the push-pull matrix around the residual struggles, when the amplitude of the residual increases, to provide an accurate measurement of the residual itself because it can not measure the point for which this matrix was calibrated for. To find a trade-off between the specific

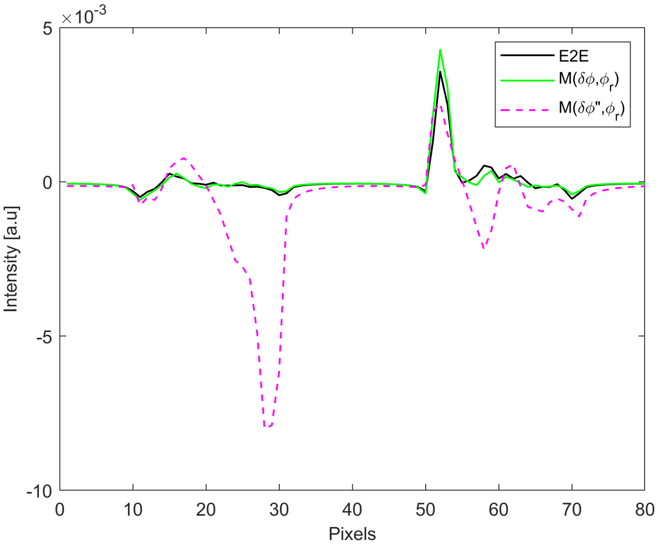
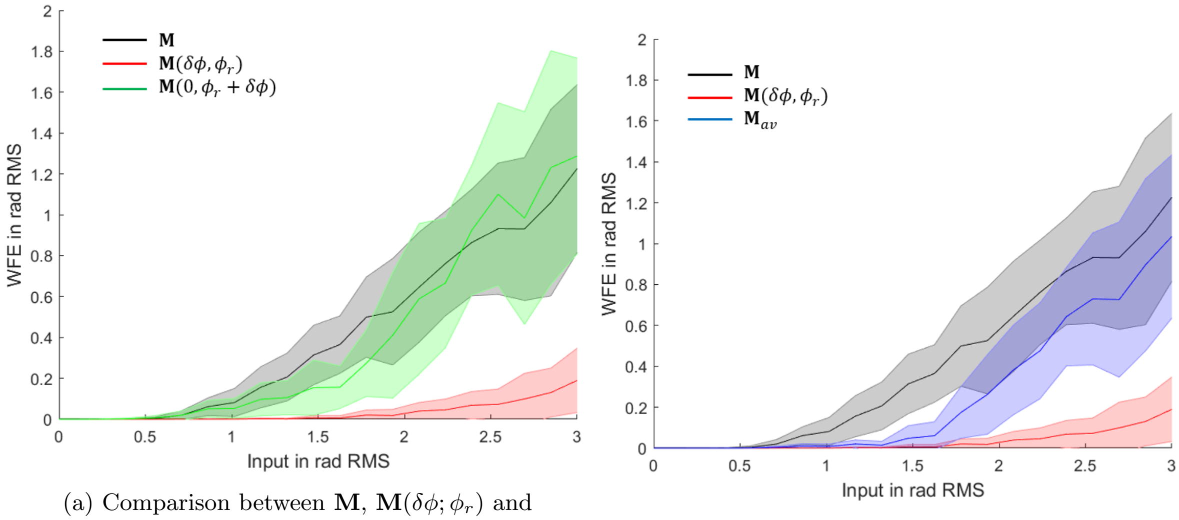


Figure 4: Specificity of the reconstructor

matrix and the push-pull around the phase and still improve the phase estimation one should use the average matrix ($\mathbf{M}_{av} = (\mathbf{M} + \mathbf{M}(0; \phi_r + \delta\phi))/2$ which is the average between the interaction matrix and the push-pull around the phase). It improve the measurement of the phase in any phase residual regime in comparison with the classical approach (see Fig.5b).



(a) Comparison between $\mathbf{M}$, $\mathbf{M}(\delta\phi; \phi_r)$ and $\mathbf{M}(0; \phi_r + \delta\phi)$

(b) Comparison between $\mathbf{M}$, $\mathbf{M}(\delta\phi; \phi_r)$ and $\mathbf{M}_{av}$

Figure 5: Frame-by-frame estimation of phase residual. The residual screens are equivalent to a Strehl ratio of 71% in H band

3. IN SEARCH OF THE TRUE OPTICAL GAINS

The specific matrix is a great tool that demonstrates the possibility of using the matrix formalism in any phase regime to have an accurate estimation of the wavefront. However, its dependency on the phase-to-be-measured makes it impossible to use in practice. Therefore, it is necessary to explore ways to approximate this perfect reconstructor through the gain formalism.

When it comes to optical gain formalism, one thinks about the modulated PWFS. Indeed, the gain formalism considers the non-linear behaviour of the PWFS to be a first-order non-linear problem. It consideres that the linearity of the FFWFS varies from one mode to the other, independently one from another, hence the modal gain compensation. Yet, this way of explaining the non-linearities of the sensor is incomplete. In this section we look for the true optical gains (OG). Our concern is to find the real modal gain **and** the coupling terms that translate optimally the non-linear response of the WFS. To do so we will use the following strategy for our demonstration. We will consider the estimation of one Zernike mode Z_i in two different phase regimes: linear phase regime and the strong phase regime. In the strong phase regime, we will consider first the modal gain compensation and then the full gain compensation (OG + coupling). To do so we will consider a modulated PWFS. The simulation parameters can be found in table 1 and the modulation radius is $r_{mod} = 3\lambda/D$. The different reconstructors we are considering are the interaction matrix around the flat wavefront $\mathbf{M}$, the specific matrix $\mathbf{M}(\delta\phi;\, \phi_r)$ and the push-pull matrix around the considered wavefront$\mathbf{M}(0;\, \phi_r + \delta\phi)$.

In the linear regime, as expected, the three matrices give similar results, they all provide an accurate estimation of the amplitude of Z_i.

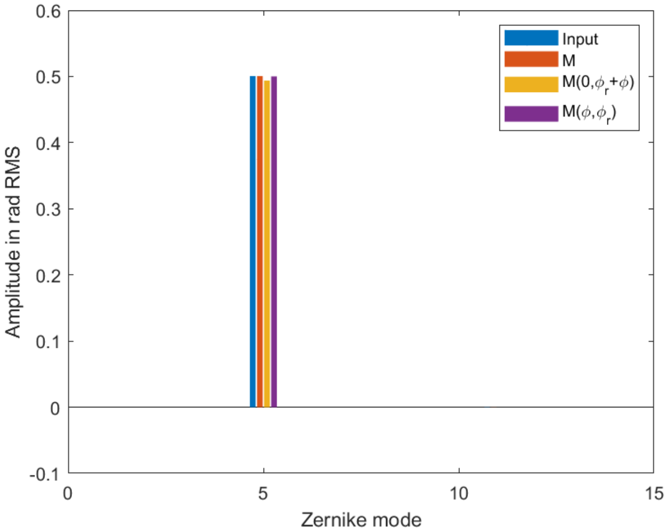


Figure 6: Phase coefficients using different reconstructors in the linear regime

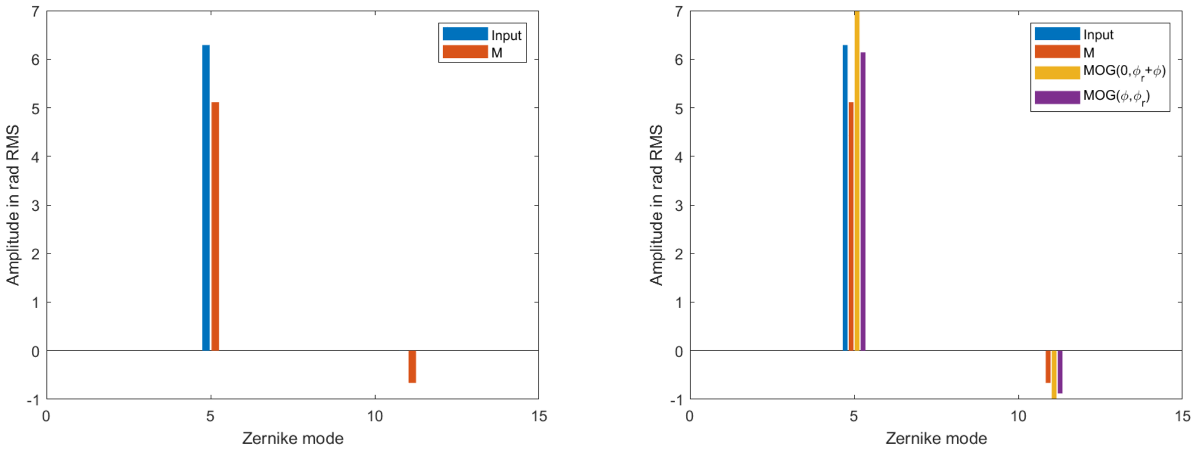
In the strong regime, the input amplitude of the Zernike mode Z_i is 2π rad RMS. We can directly see in the estimation provided by the interaction matrix $\mathbf{M}$ an underestimation of the input mode (see Fig.7a). When considering only a modal gain compensation, the ideal gain would be the ratio of the amplitude of the input wavefront and the one of the estimation provided by the interaction matrix $\mathbf{M}$. In our case this gain is equal to 0.81. We use the approach described in[15] and[16] to compute the OG and summarise in equation (7). With this approach, the OG comes from the diagonal of the matrix that results from the projection of the push-pull matrix around the wavefront and the interaction matrix. The resulting matrix should provide information about how far from the linear regime the system is.

$$\mathbf{T}(0;\phi_r + \delta\phi) = \mathbf{M}^{\dagger}\mathbf{M}(0;\phi_r + \delta\phi) \tag{7}$$

$$\mathbf{OG}(0;\phi_r + \delta\phi) = \mathbf{diag}(\mathbf{T}(0;\phi_r + \delta\phi)) \tag{8}$$

where $\mathbf{T}(0;\, \phi_r + \delta\phi)$ is the gain matrix obtained from the interaction matrix computed around the wavefront. The diagonal terms of $\mathbf{T}(0;\, \phi_r + \delta\phi)$ are the classical optical gains, $\mathbf{OG}(0;\, \phi_r + \delta\phi)$, and the non-diagonal terms are the coupling terms.

We use the same formalism for the specific matrix and we build another gain matrix $\mathbf{T}(\delta\phi;\phi_r)$. By com-paring the diagonal of the two matrices, we realise that the OG from the specific matrix are the true one, the gain for the considered mode is 0.83 when the one from the push-pull around the phase is 0.23. Therefore, if the non-linearities of the PWFS were only an underestimation of the wavefront or a gain issue, the OG should be the one from the specific matrix. Another alternative consists in using the gain from the average between the push-pull around the wavefront and the interaction matrix $\mathbf{OG}_{av}$ (see Fig.8a).
If considering the coupling between the modes induced by the PWFS response, we need to compare the performances of the matrix $\mathbf{T}(0;\phi_r+\delta\phi)$ and $\mathbf{T}(\delta\phi;\phi_r)$. In this scenario, the new phase reconstructor is $\mathbf{R}_\phi = \Big(\mathbf{MT}(0;\phi_r+\delta\phi)\Big)^\dagger$ or $\mathbf{R}_m = \Big(\mathbf{MT}(\delta\phi;\phi_r)\Big)^\dagger$. One can see that using the matrix $\mathbf{T}(\delta\phi;\phi_r)$ helps to both compensate for the OG by increasing the amplitude of the mode of interest Z_i and cancel out the coupling induced by the interaction matrix around the flat wavefront (see Fig.8b).

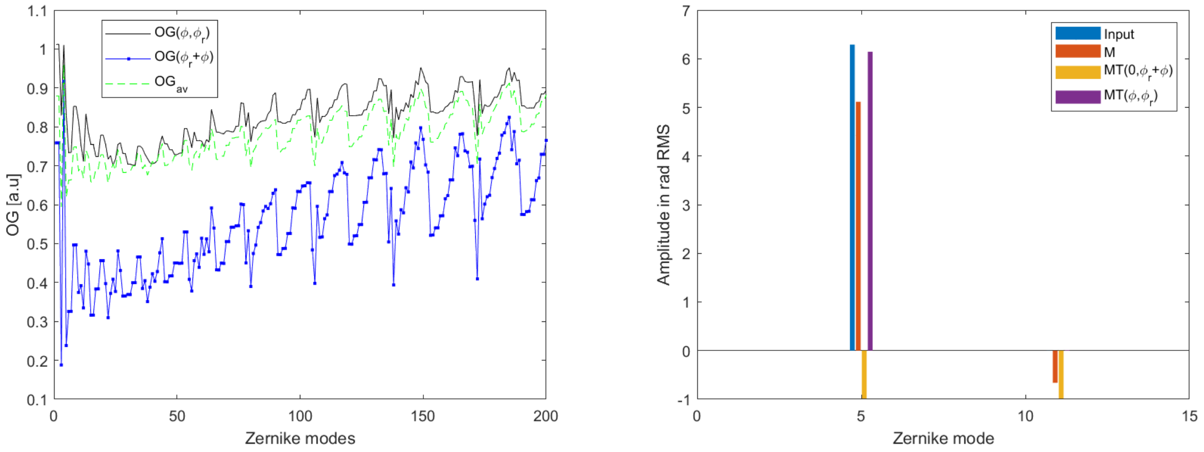


(a) Estimation with **M** (b) After OG compensation

Figure 7: Projected coefficients. Left: without gain compensation. Right: After OG compensation.

With the example of the reconstruction of one phase mode Z_i using a PWFS with a modulation radius of $3\lambda/D$, we prove the usefulness of the specific matrix and its approximation to measure the absolute phase, in particular in strong phase regime. This example tackles the loss of sensitivity of the sensor due to the amplitude of the phase itself. The specific matrix should be seen as the target reconstructor and all the attempts to build the best phase estimator in the matrix formalism should go toward that matrix. However it is not an easy task, as shown in the previous section, one matrix $\mathbf{M}(\delta\phi;\phi_r)$ is specific to the phase screen $\delta\phi$ and cannot be used for another one $\delta\phi'$. Moreover, if we consider a regime where the modal gain is the main issue how can we approximate those gains?

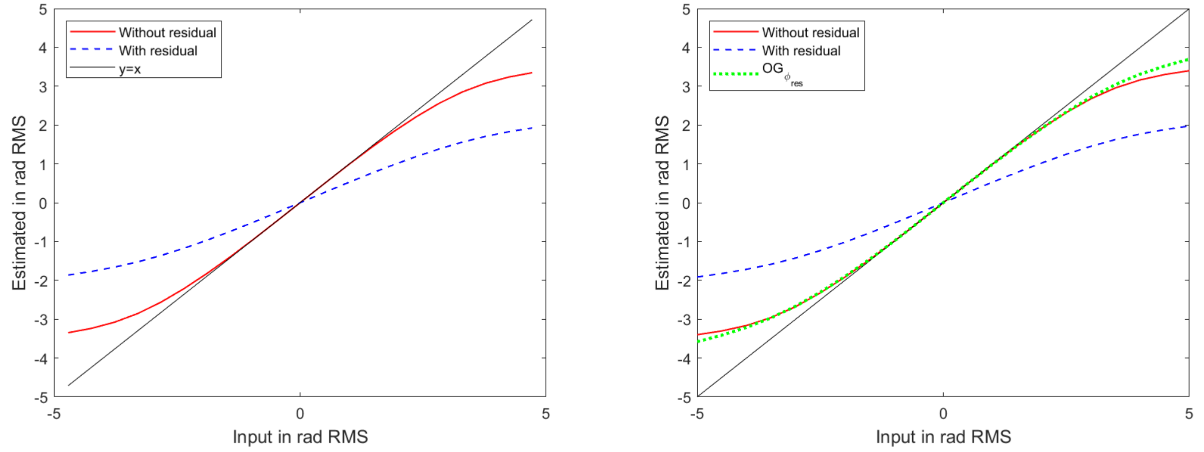
Nonetheless, it is important to highlight the use of the interaction matrix computed around the operating point. Indeed, this matrix $\mathbf{M}(0;\phi_r+\delta\phi)$ was thought to mitigate the loss of sensitivity that is observed when using FFWFS around a non-zeros operational point. Since their response depends on the phase regime, when using an interaction matrix calibrated around the flat wavefront, one systematically observes an underestimation of the wavefront when this reconstructor is used on-sky. This is illustrated in Fig.9a with the example of the reconstruction of one Zernike mode. One can see that the underestimation of the mode Z_5 when trying to estimate it in the presence of phase residual. The slope difference between the red curve (estimation of Z_5 without residual) and the dashed-blue curve (in the presence of residual) represents the loss of sensitivity due to the difference between the calibration regime and the on-sky regime. The information regarding this loss



(a) Optical gain (b) After full gain compensation

Figure 8: Left: Optical gains (OG). Right: projected coefficients after full gain compensation.

of sensitivity can be found in the projection of the interaction matrix calibrated on-sky $\mathbf{M}(0; \phi_r + \delta\phi)$ on the one calibrated around the flat wavefront $\mathbf{M}$. After compensation with the diagonal of the resulting matrix, we manage to retrieve the curve in the absence of phase residual.



(a) Reconstruction curves of Z_5 (b) After OG compensation

Figure 9: Left: Reconstruction curves of Z_5 with and without phase residual respectively the dashed-blue curve and the red curve. Right: After optical gain compensation green dotted curve

Therefore, the fundamental difference between the interaction matrix around a given operating point and the specific matrix is that the latter provides an exact measurement of a given phase independently from the phase regime by construction. It will systematically provide an estimation that will enable to go back around the reference operating point. The link created by this matrix between the reference point and the one to be measured ensures an accurate compensation of the wavefront. Hence the specific matrix should be seen as the ideal reconstructor one can get and all the attempts to find a proper reconstructor in the matrix formalism should tend toward that matrix.

4. DISCUSSION AND CONCLUSION

This article explores the measurement of the true phase in any phase regime using FFWFS. As a demonstration tool, we use the PWFS to understand which non-linear behaviour we are trying to mitigate. We used the matrix formalism, a linear formalism to prove the feasibility of measuring wavefront outside the non-linear regime of the PWFS. The specific matrix that enables this measurement is the new targeted reconstructor as it measures the phase in any regime.

By comparing the performances of the specific matrix and the classical approach which consists in calibrating the system around the new operating regime, we demonstrated the limitation of this approach to estimate the operational point and the optical gain necessary to improve the wavefront estimation. We also pointed out the necessity of knowing the coupling terms to improve the wavefront measurement because the non-linear behaviour of a FFWFS is more complex than a simple gain issue. This lead us to a good approximation of the specific matrix by using the average between the push-pull matrix around the phase and the interaction matrix. The gain provided by this matrix are closer to the one of the specific matrix which get us closer to our target.

The research on the frame-by-frame compensation of the OG leads to the use of a focal plane camera.[16] When considering a modulated PWFS this approach lead to better performances of the AO loop. Following this technique, we suggest the use of a hybrid method to improve the phase estimation and get closer to the measurement of the true phase at each iteration of the loop. This approach could consist in a hybrid method using the focal plane camera for a frame-by-frame estimation of the modal gains and a statistical approach on the phase regime to build a gain matrix containing the coupling terms using the specific matrix formalism. In this manner at each iteration of the closed-loop we update the diagonal of the gain matrix which will automatically scale the coupling terms, hence improving the wavefront estimation. This approach complete the diagonal OG approach to adapt the actual response of the FFWFS.

Finally we emphasise the use of the specific matrix formalism for the whole class of FFWFS. No hypothesis on the nature of the phase mask was made when developing this formalism which signify that it can be use for any FFWFS. For instance, this can be adapted to the ZWFS although the chromaticity of this mask will lead to few adjustments. In important effort will have to be made on the phase mask substrate to limit the chromatic effect. Technologies such as liquid crystal to manufacture the ZWFS[10] would be preferred.

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