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HIGH FREQUENCY EFFECTS IN CAPACITORS

by

M. M. PRZYBYLSKI, Daresbury Laboratory

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Science Research Council

Daresbury Laboratory

Daresbury, Warrington WA4 4AD

1. INTRODUCTION

Although the frequency dependance of capacitor impedance is a well known phenomenon, its implications are becoming more serious with the universal application of high speed switching circuits and wideband amplifiers. A theoretical and empirical examination has been made of typical capacitors used in high speed switching and analogue circuits.

A capacitor as a circuit element should ideally present a purely reactive impedance

$$Z = \frac{1}{jwC} = \frac{1}{2\pi fC} e^{-j\pi/2} \qquad (1)$$

In applications such as a.c. coupling of electronic circuits, and high frequency bypass or decoupling circuits, the designer is not usually interested in the phase, but only in the magnitude of a capacitor's impedance, and requires that it be lower than a specified value $|Z|$ for all frequencies $f > \frac{1}{2\pi|Z|C}$. On the other hand, in filter circuits and feedback amplifiers where capacitors are used to provide phase compensation to ensure stability, the designer may require the phase of the impedance to remain a constant $-\pi/2$ over a wide range of frequencies.

However at high frequencies, typically greater than 1 MHz, both the magnitude and phase of the impedance of a capacitor can change, so that it presents an effectively inductive impedance. This is caused by the inductance of the interconnecting leads.

As a result decoupling capacitors lose their effectiveness at high frequencies and compensating or feedback capacitors in wideband amplifiers may cause instability.

2. LEAD INDUCTANCE

A wire lead in proximity to a conducting structure, which in a practical circuit may be the instrument case or the circuit ground plane, forms a transmission line. Figure 1 shows a cross section of a conductor over a ground plane.

For non ferromagnetic conductors in air the characteristic impedance of such a line is given by[1]

$$Z_0 = 60 \ln \frac{4H}{D} \qquad (2)$$

where H is the distance between the conductor and the ground plane and D is the diameter of the conductor. Figure 2 shows the characteristic impedance as a function of the transmission lines geometry.

In an electronic circuit H and D would be typically 5 mm and 0.5 mm respectively, giving $Z_0 \approx 220\Omega$.

A capacitor with one of its leads connected to ground is shown in fig. 3(a).

For frequencies $f \gg f_0 = \frac{1}{2\pi Z_0 C}$ the impedance of the capacitor is negligible compared with Z , the characteristic impedance of its leads, thus an equivalent circuit used for calculating the input impedance is shown in fig. 3(b).

The input impedance of the resulting length l of transmission line is given by:

$$Z_{in} = j Z_0 \tan \frac{2\pi l}{\lambda} \qquad (3)$$

where l is the total length of capacitor leads, and λ is the wavelength at a given frequency.

For short leads $l << \lambda$, the above can be written as:

$$Z_{in} \approx j Z_0 \frac{2\pi l}{\lambda} = jw L_0 l \qquad (4)$$

where w is the angular frequency (= $2\pi f$) and L_0 is the inductance per unit length of line.

Thus the effective input impedance of a capacitor connected to ground is inductive for frequencies

$$f_0 << f << \frac{\lambda}{l}$$

The inductance for a line structure shown in fig. 1 is given by:

$$L_0 = \frac{1}{2\pi} \ln \frac{4H}{D} = 2 \ln \frac{4H}{D} \text{ nH/cm} \qquad (5)$$

Figure 4 shows the above graphically.

3. MEASUREMENT OF FREQUENCY CHARACTERISTICS OF CAPACITORS

A simple test network was connected between the output of a tracking generator and the input of a spectrum analyser as shown in fig. 5. The capacitor was connected so that its leads had a total length of 2 cm and were 5 mm above a copper ground plane. A detailed analysis of the circuit performance is given in the Appendix and fig. 6 shows the calculated and measured values for a range of capacitors. Also shown is the transfer characteristic of a short length of copper conductor.

The measured and calculated results are in good agreement.

To demonstrate the inductive impedance of capacitors used for decoupling short lived transients a test circuit shown in fig. 7 was used. The input was a 5V step with a 1 ns rise time. Figure 8 shows the voltage waveform at the output for a 0.1 μF metalised polyester test capacitor (the type popularly used for decoupling power supply lines in TTL logic circuits). The effect of varying the capacitor lead length can be seen and in particular for virtually zero lead lengths the inductive effect of the capacitor's internal inductance can be seen.

4. CONCLUSIONS

For applications where capacitors are required to present a low impedance for high frequency signals as for example in decoupling circuits it is important to minimize their lead inductance. This should be done by keeping the interconnecting leads as short as possible and arranging the layout to minimize the H/D ratio, by providing a good ground plane.

In wideband feedback amplifiers it may be necessary to use low inductance chip capacitors.

ACKNOWLEDGEMENT

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REFERENCES

1. Reference data for radio engineers, Fourth edition, Standard Telephones and Cables Ltd.

APPENDIX

CALCULATION OF TRANSFER FUNCTION OF THE TEST NETWORK

Figure 9 shows the equivalent circuit of the test system where r is the resistance of the capacitor leads and connections, L is the series inductance of the leads, V_0 is the output voltage of the tracking generator and v_i is the input voltage of the spectrum analysis.

With the capacitor disconnected the transfer function is:

$$G_0 = \frac{v_i}{v_0} = \frac{50}{200} = \frac{1}{4} \qquad (6)$$

With the test capacitor the transfer function is:

$$G = \frac{1}{4} \; \frac{r + jwL + 1/jwC}{r + 50 + jwL + 1/jwC} \qquad (7)$$

In a practical circuit $r << 50\Omega$ and the above simplifies to:

$$G \doteqdot \frac{1}{4} \; \frac{r + jwL + 1/jwC}{50 + jwL + 1/jwC} = G_0 \; \frac{r + jwL + 1/jwC}{50 + jwL + 1/jwC} \qquad (8)$$

For practical circuits where L < 100 nH and C > 100 pF the transfer function will have four frequency regions in which the following approximations will hold:

A. $$f < \frac{1}{100\pi C}, \quad G \doteqdot G_0 \qquad (9)$$

B. $$\frac{1}{100\pi C} < f < \frac{1}{2\pi\sqrt{LC}}, \quad G \doteqdot G_0 \; \frac{1}{jw50C} \qquad (10)$$

C. $$\frac{1}{2\pi\sqrt{LC}} < f < \frac{2r}{\pi L}, \quad G \doteqdot G_0 \; \frac{jwL}{50} \qquad (11)$$

D. $$f > \frac{2r}{\pi L}, \quad G \doteqdot G_0 \qquad (12)$$

At a frequency $f = \frac{1}{2\pi\sqrt{LC}}$ series resonance occurs and the transfer function has a minimum

$$G_{min} = G_0 \; \frac{r}{50} \qquad (13)$$

Figure 10 shows the magnitude of the transfer function as a function of frequency. It shows the capacitor impednace to be inductive for frequencies

$$f > \frac{1}{2\pi\sqrt{LC}} \qquad (14)$$

Equations (9 - 12) were used to calculate the transfer function for capacitors of 0.001, 0.01, 0.1 and 1 μF with total lead lengths of 2 cm with D = 0.5 m and H = 5 m.

If instead of the test capacitor a 2 cm length of 0.5 m diameter copper wire 5 mm above the ground plane is connected, equation 8 simplifies to:

$$G \doteqdot G_0 \; \frac{r + jwL}{50 + jwL} \qquad (15)$$

This result as well as the measured transfer function is shown in fig. 6. The results are in good agreement.

FIGURE CAPTIONS

Fig. 1 Cross section of a conductor over a ground plane.

Fig. 2 Characteristic impedance of a conductor over a ground plane as a function of H/D.

Fig. 3(a) A capacitor with one of its leads connected to ground.

(b) The equivalent circuit when $f \gg f_0$.

Fig. 4 The inductance of a conductor over a ground plane as a function of H/D.

Fig. 5 Circuit of a capacitance test rig.

Fig. 6 Calculated and measured values as a function of frequency $l = 2$ cm, D = 0.5 mm and H = 5 mm.

A - 1 μF metalised polyester

B - 0.1 μF metalised polyester

C - 0.01 μF ceramic

D - 1000 pF polystyrene

E - 2 cm long conductor, D = 0.5 mm, H = 5 mm.

Fig. 7 Circuit of imepdance test network

Fig. 8(a) Output voltage for capacitor with a total of 2 cm lead length

(b) as (a) but with 1 cm lead length

(c) as (a) but with minimum lead length

(Note, the output voltage is 0.1 of the voltage across the capacitor.)

Fig. 9 Equivalent circuit of capacitance test rig shown in fig. 5

Fig. 10 Transfer function as a function of frequency.

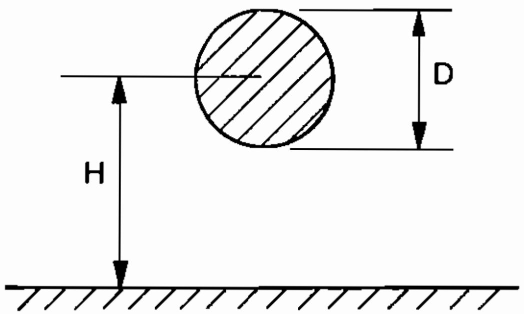

Fig. 1

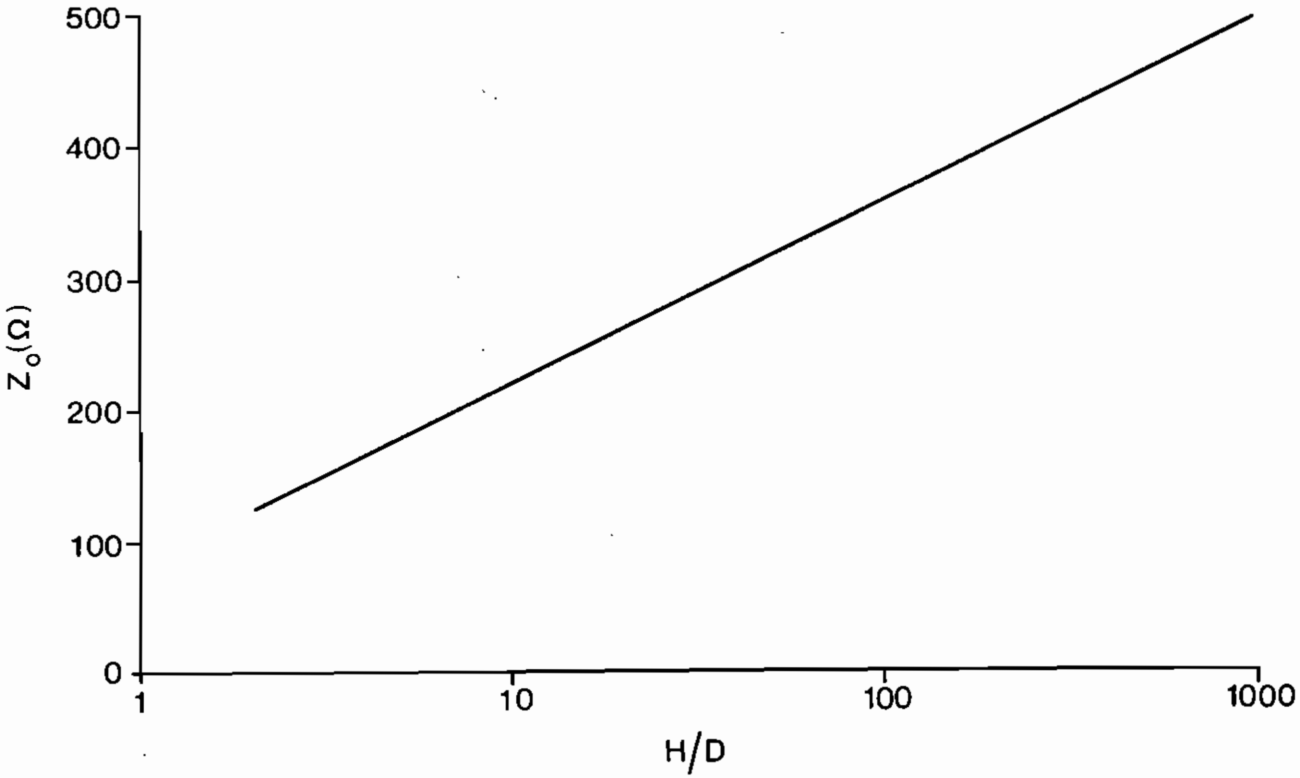

Fig.2

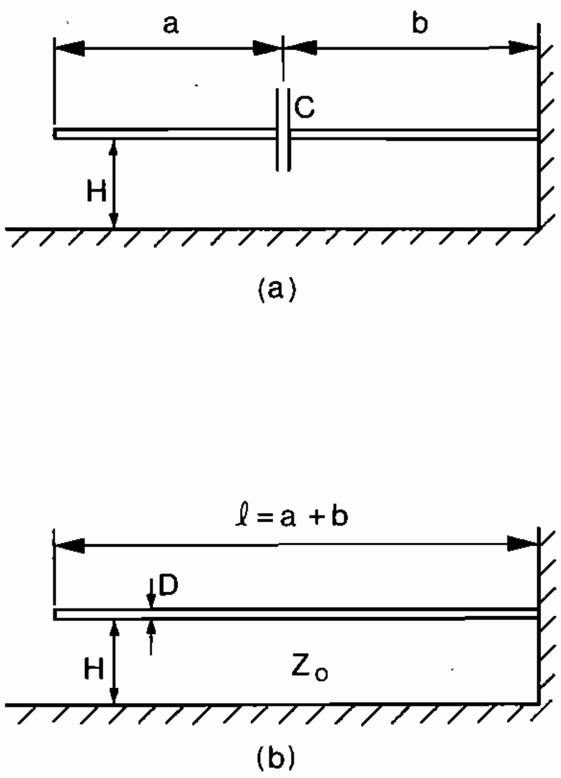

Fig. 3

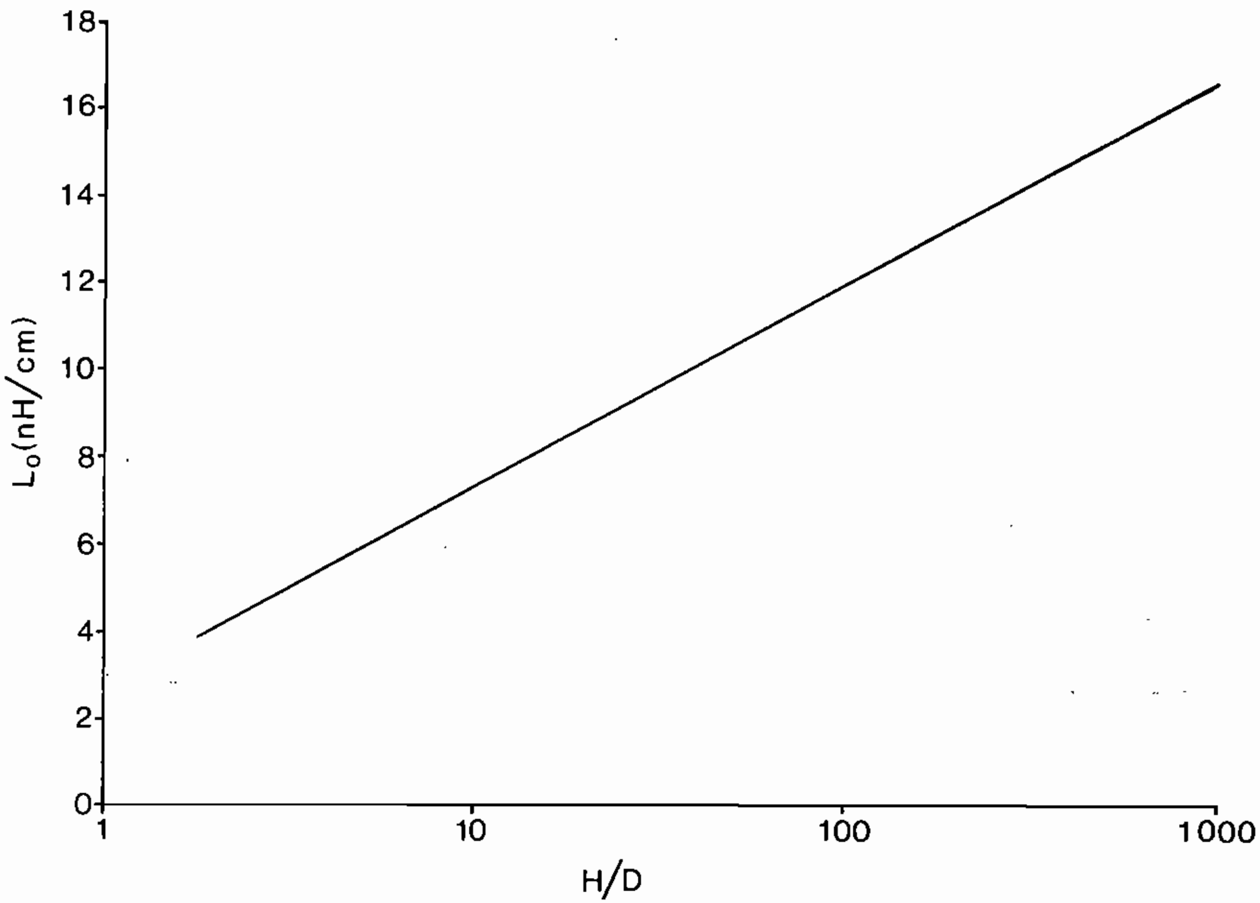

Fig.4

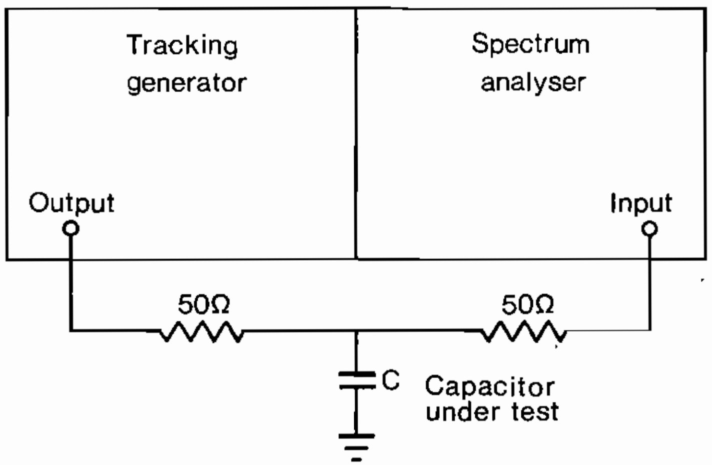
Tracking generator
Spectrum analyser
Output
Input
50Ω
50Ω
C
Capacitor under test

Fig. 5

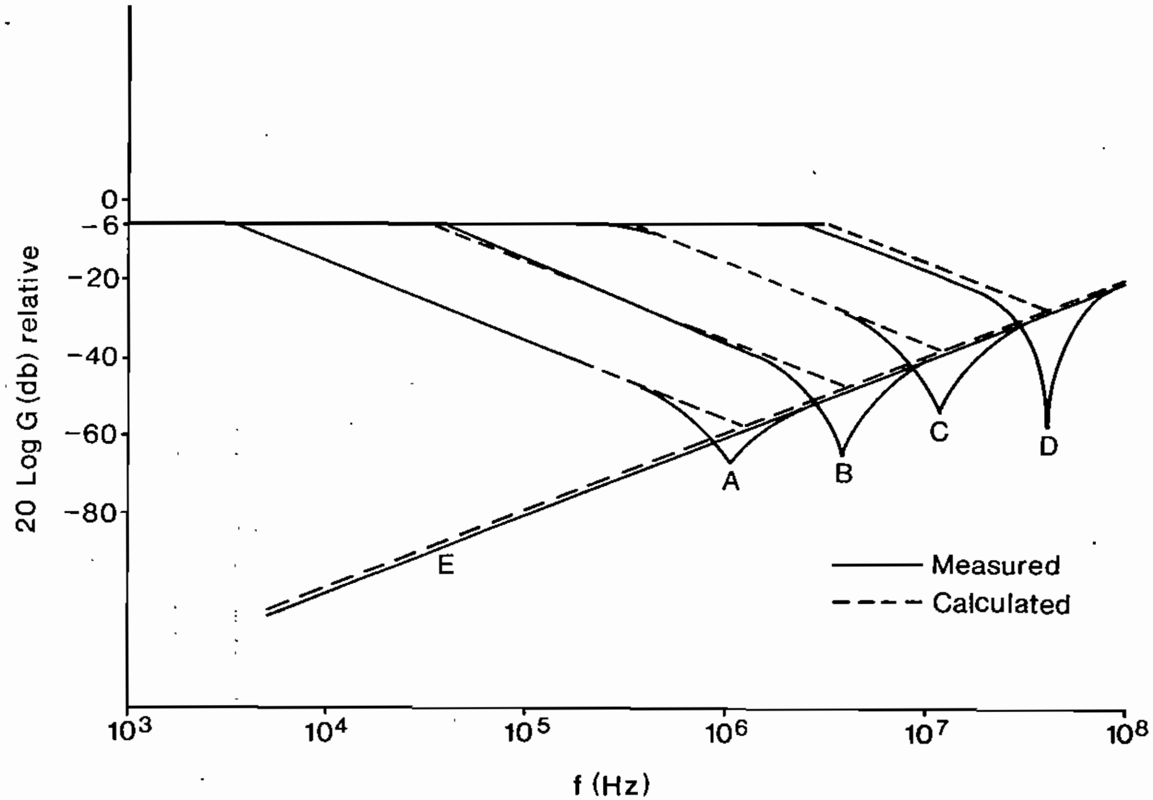
0
-6
-20
-40
-60
-80
20 Log G (db) relative
A
B
C
D
E
Measured
Calculated
10^3
10^4
10^5
10^6
10^7
10^8
f (Hz)

Fig. 6

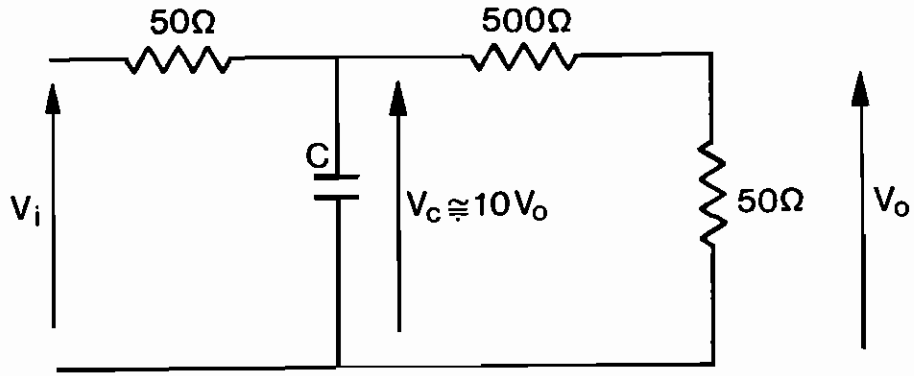

Fig.7

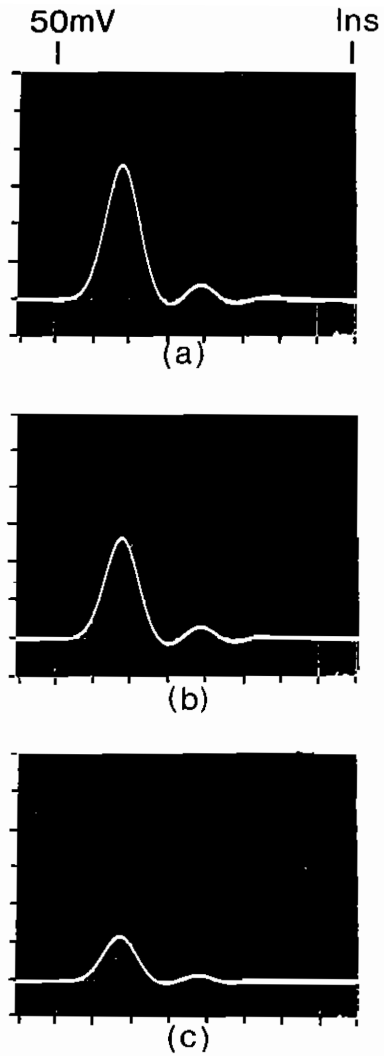

Fig. 8

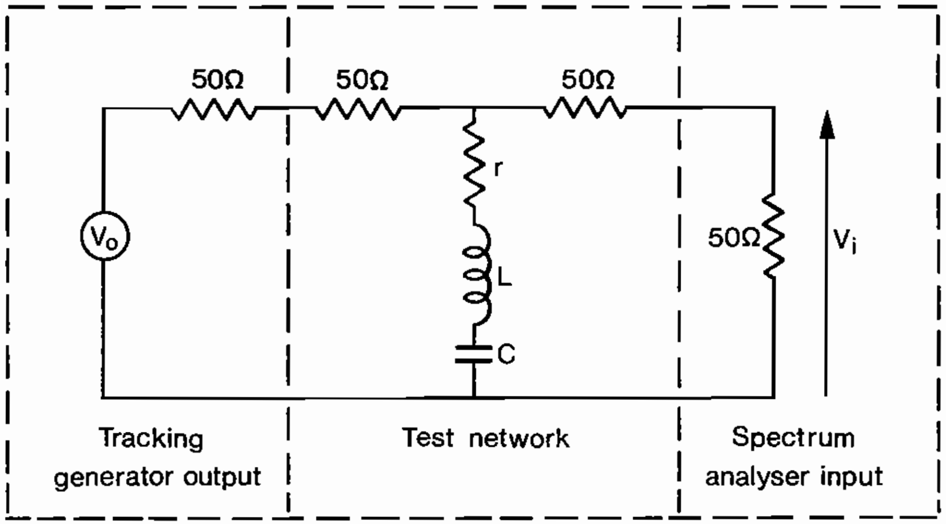
50Ω
50Ω
50Ω
r
L
C
50Ω
Vo
Vi
Tracking
generator output
Test network
Spectrum
analyser input

Fig.9

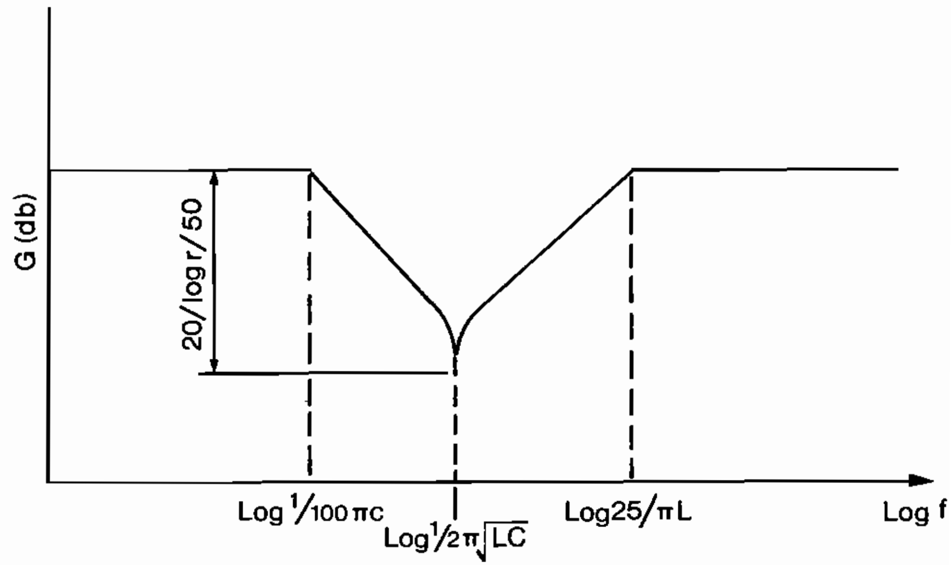
G (db)
20/log r/50
Log 1/100 πC
Log 1/2π√LC
Log25/πL
Log f

Fig.10