CLRC

Technical Report
RAL-TR-1999-066

Tri-Maximal vs. Bi-Maximal Neutrino Mixing

W G Scott

6th October 1999

COUNCIL FOR THE CENTRAL LABORATORY OF THE RESEARCH COUNCILS

The Central Laboratory of the Research Councils
Library and Information Services
Rutherford Appleton Laboratory
Chilton
Didcot
Oxfordshire
OX11 0QX
Tel: 01235 445384   Fax: 01235 446403
E-mail library@rl.ac.uk

ISSN 1358-6254

Topical Seminar on Neutrinos and Astroparticle Physics. San Miniato. May 1999. RAL-TR/1999 -066

Tri-Maximal vs. Bi-Maximal Neutrino Mixing

hep-ph/9909431

W. G. Scott
Rutherford Appleton Laboratory, Chilton, Didcot, UK.

It is argued that data from atmospheric and solar neutrino experiments point strongly to tri-maximal or bi-maximal lepton mixing. While ('optimised') bi-maximal mixing gives an excellent *a posteriori* fit to the data, tri-maximal mixing is an *a priori* hypothesis, which is not excluded, taking account of terrestrial matter effects.

1. TRI-MAXIMAL MIXING

Threefold maximal mixing, ie. tri-maximal mixing, undeniably occupies a special place in the space of all 3×3 mixings. In some weak basis the two non-commuting mass-matrices m_l^2, m_ν^2 ($m_i m_i^\dagger \equiv m_i^2$) may simultaneously be written [1] as 'circulative' matrices ('of degree zero') [2]:

$$\begin{pmatrix} a_l & b_l & \bar{b}_l\omega \\ \bar{b}_l & a_l & b_l\omega \\ b_l\bar{\omega} & \bar{b}_l\bar{\omega} & a_l \end{pmatrix} \qquad \begin{pmatrix} a_\nu & b_\nu & \bar{b}_\nu\omega \\ \bar{b}_\nu & a_\nu & b_\nu\omega \\ b_\nu\bar{\omega} & \bar{b}_\nu\bar{\omega} & a_\nu \end{pmatrix} \quad (1)$$

respectively invariant under monomial cyclic permutations ('circulation' matrices [2]) of the form:

$$C = \begin{pmatrix} . & 1 & . \\ . & . & \omega \\ \bar{\omega} & . & . \end{pmatrix} \quad \bar{C} = \begin{pmatrix} . & 1 & . \\ . & . & \bar{\omega} \\ \omega & . & . \end{pmatrix} \quad (2)$$

($C^\dagger m_l^2 C = m_l^2$, etc.) just as circulant matrices are invariant under simple cyclic permutations [3]. The mass matrices Eq. 1 are diagonalised by the (eg. circulant) unitary matrices V and $\bar{V}$:

$$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & \bar{\omega} & 1 \\ 1 & 1 & \bar{\omega} \\ \bar{\omega} & 1 & 1 \end{pmatrix} \qquad \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & \omega & 1 \\ 1 & 1 & \omega \\ \omega & 1 & 1 \end{pmatrix} \quad (3)$$

respectively, leading to threefold maximal mixing:

$$V\bar{V}^\dagger = \frac{1}{3}\begin{pmatrix} 2+\omega & 2\bar{\omega}+1 & 2\bar{\omega}+1 \\ 2\bar{\omega}+1 & 2+\omega & 2\bar{\omega}+1 \\ 2\bar{\omega}+1 & 2\bar{\omega}+1 & 2+\omega \end{pmatrix} \quad (4)$$

where in all the above and in what follows ω and $\bar{\omega}$ represent complex cube-roots of unity and the overhead 'bar' denotes complex conjugation.

After some rephasing of rows and columns the tri-maximal mixing matrix may be re-written:

$$U = \frac{1}{\sqrt{3}} \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \bar{\omega} \\ 1 & \bar{\omega} & \omega \end{pmatrix} \end{array} \quad (5)$$

where the matrix in the parenthesis is identically the character table for the cyclic group C_3 (group elements vs. irreducible representations). In threefold maximal mixing the CP violation parameter $|J_{CP}|$ is maximal ($|J_{CP}| = 1/6\sqrt{3}$) and if no two neutrinos are degenerate in mass, CP and T violating asymmetries can approach $\pm 100\%$.

Observables depend on the sqaures of the moduli of the mixing matrix elements [4]:

$$(|U_{l\nu}|^2) = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} \end{array} . \quad (6)$$

In tri-maximal mixing all survival and appearance probabilities are universal (ie. flavour independent) and in particular if two neutrinos are effectively degenerate tri-maximal mixing predicts for the locally averaged survival probability:

$$<P(l \to l)> = (1/3 + 1/3)^2 + (1/3)^2 = 5/9 \quad (7)$$

and appearnce probability:

$$<P(l \to l')> = 2 \times (1/3)(1/3) = 2/9. \quad (8)$$

If all three neutrino masses are effectively non-degenerate: $<P(l \to l)> = <P(l \to l')> = 1/3$.

2. ATMOSPHERIC NEUTRINOS

The SUPER-K [5] multi-GeV data, show a clear $\sim 50\%$ suppression of atmospheric ν_μ for zenith angles $\cos\theta \lesssim 0$, as shown in Fig. 1a. At the same time the corresponding distribution for ν_e seems to be very largely unaffected, as shown in Fig. 1b. The best fit is for (twofold) maximal $\nu_\mu - \nu_\tau$ mixing with $\Delta m^2 \simeq 3.0 \times 10^{-3}$ eV^2, as shown by the solid/dotted curves in Fig. 1.

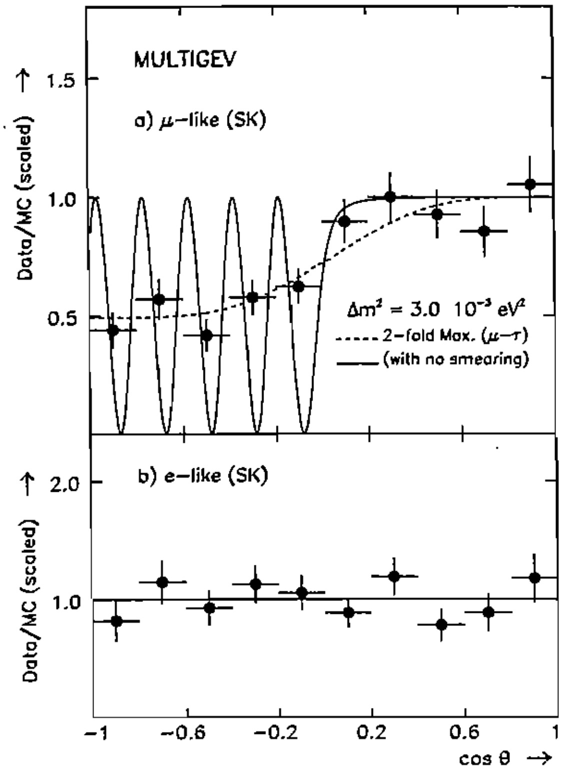

Figure 1. The multi-GeV zenith-angle distributions for a) μ-like and b) e-like events from the SUPER-K experiment. The solid curve is for maximal $\nu_\mu - \nu_\tau$ oscillations with $\Delta m^2 = 3.0 \times 10^{-3}$ eV^2 and the dotted curve shows the effect of energy averaging and angular smearing.

From the measured suppression we have:

$$(1 - |U_{\mu 3}|^2)^2 + (|U_{\mu 3}|^2)^2 \simeq 0.52 \pm 0.05 \qquad (9)$$

whereby the ν_μ must have a large ν_3 content, ie. $|U_{\mu 3}|^2 \simeq 1/2$, or more precisely:

$$1/3 \;\lesssim\; |U_{\mu 3}|^2 \;\lesssim\; 2/3 \qquad (10)$$

where the range quoted corresponds to the 1σ error above (which is largely statistical).

3. THE CHOOZ DATA

The apparent lack of ν_e mixing at the atmospheric scale, is supported by independent data from the CHOOZ reactor [6] (Fig. 2), which rules out large ν_e mixing over most of the Δm^2 range allowed in the atmospheric neutrino experiments.

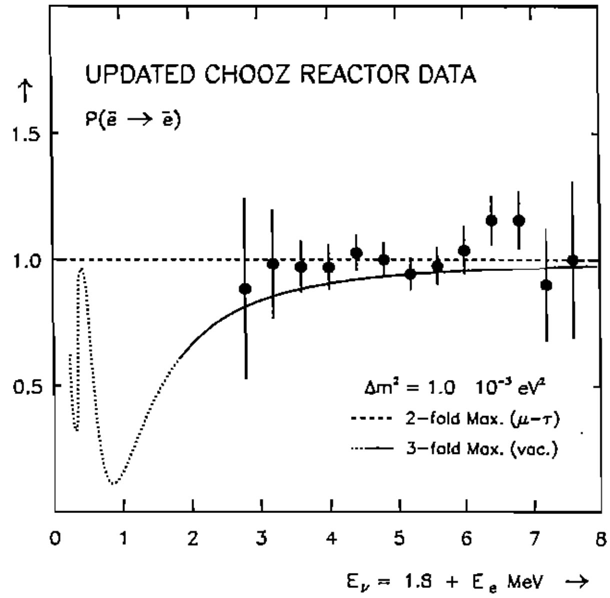

Figure 2. The latest data from the CHOOZ reactor, corresponding to the full data taking. (The solid curve is for tri-maximal mixing with $\Delta m^2 = 1.0 \times 10^{-3}$ eV^2.)

Taken together, the CHOOZ and SUPER-K data indicate that the ν_3 has no ν_e content, ie. there is a zero (or near-zero) in the top right-hand corner of the lepton mixing matrix, $|U_{e3}|^2 \lesssim 0.03$.

3.1. The Fritzsch-Xing Ansatz

Remarkably, the Fritzsch-Xing hypothesis [7] (published well before the initial CHOOZ data) predicted just such a zero:

$$(|U_{l\nu}|^2) \;=\; \begin{matrix} & \nu_1 & \nu_2 & \nu_3 \\ e \\ \mu \\ \tau \end{matrix}\!\!\begin{pmatrix} 1/2 & 1/2 & \cdot \\ 1/6 & 1/6 & 2/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} \qquad (11)$$

The Fritzsch-Xing ansatz (Eq. 11) is readily obtained from theeefold maximal mixing (Eq. 5) by the re-definitions: $\nu_e \rightarrow (\nu_e - \nu_\mu)/\sqrt{2}$ and

$\nu_\mu \to (\nu_e + \nu_\mu)/\sqrt{2}$ (up to phases), keeping the ν_τ tri-maximally mixed. While the *a priori* argument for these particular linear combinations [7] is far from convincing, it is clear that Eq. 11 is so far consistent with the atmospheric data:

$$<P(\mu \to \mu)> = (1/6 + 1/6)^2 + (2/3)^2 = 5/9 \quad (12)$$

(cf. Eq. 9), while beyond the second threshold:

$$<P(e \to e)> = (1/2)^2 + (1/2)^2 + (0)^2 = 1/2 \quad (13)$$

The famous 'bi-maximal' scheme [8] is very similar to the Fritzsch-Xing ansatz and likewise predicts a 50% suppression for the solar data.

4. THE SOLAR DATA

Taken at face value, the results from the various solar neutrino experiments imply an energy dependent suppression. In particular, taking BP98 fluxes (and correcting for the NC contribution in SUPER-K), the results from HOMESTAKE and SUPER-K: $P(e \to e) \simeq 0.3 - 0.4$, lie significantly below the results from the gallium experiments: $P(e \to e) \simeq 0.5 - 0.6$, as shown in Fig. 3.

4.1. 'Optimised' Bi-Maximal Mixing

Mindful of the popularity of the large-angle MSW solution and the undenied phenomenological promise of the 'original' bi-maximal scheme [8], we have ourselves proposed [4], a phenomenologically viable (and even phenomenologically favoured) *a posteriori* 'straw-man' alternative to tri-maximal mixing:

$$(|U_{l\nu}|^2) \quad = \quad \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \left(\begin{array}{ccc} 2/3 & 1/3 & . \\ 1/6 & 1/3 & 1/2 \\ 1/6 & 1/3 & 1/2 \end{array} \right) \end{array} \quad (14)$$

which we refer to here as 'optimised' bi-maximal mixing. This scheme is of course just one special case of the 'generalised' bi-naximal hypotheses of Altarelli and Feruglio [9] (and see also Ref. [10]). At the atmospheric scale Eq. 14 gives:

$$<P(\mu \to \mu)> = (1/6 + 1/3)^2 + (1/2)^2 = 1/2 \quad (15)$$

while beyond the second scale it gives:

$$<P(e \to e)> = (2/3)^2 + (1/3)^2 + (0)^2 = 5/9 \quad (16)$$

There is then the added possibility to exploit a large angle MSW solution with the base of the 'bathtub' (given by the ν_e content of the ν_2) given by $P(e \to e) = 1/3$, as shown in Fig. 3.

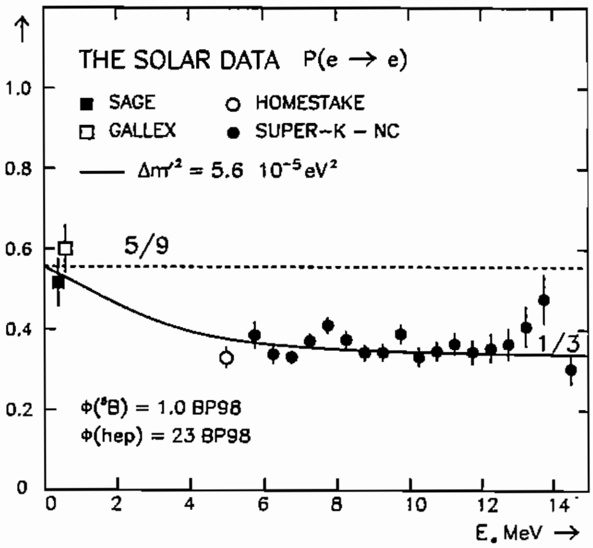

Figure 3. The SUPER-K solar data [11] plotted as a function of recoil electron energy E_e. The results from the two gallium experiments SAGE and GALLEX and the HOMESTAKE experiment are also plotted (at $<1/E>^{-1} \sim 0.5$ MeV and $<1/E>^{-1} \sim 5$ MeV respectively). Used BP98 [12] fluxes (with rescaled hep). The SUPER-K points are corrected for the NC contribution. The solid curve is Eq. 14 with $\Delta m'^2 = 5.6 \times 10^{-5}$ eV^2.

Although clearly the matrix Eq. 14 is readily obtained from the tri-maximal mixing matrix Eq. 5 by forming linear combinations of the heaviest and lightest mass eigenstates: $\nu_1 \to (\nu_1 + \nu_3)/\sqrt{2}$ and $\nu_3 \to (\nu_1 - \nu_3)/\sqrt{2}$ (up to phases), with the ν_2 remaining tri-maximally mixed, we emphasise that we claim no understanding of why these redefinitions should be necessary.

5. TERRESTRIAL MATTER EFFECTS IN TRI-MAXIMAL MIXING

In general, matter effects deform the mixing matrix and shift the neutrino masses, away from their vacuum values, depending on the local matter density. In the tri-maximal mixing scenario, the CHOOZ data require $\Delta m^2 \lesssim 10^{-3}$ eV^2, so that matter effects can be very important indeed. For 'intermeduate' densities [4], the matter

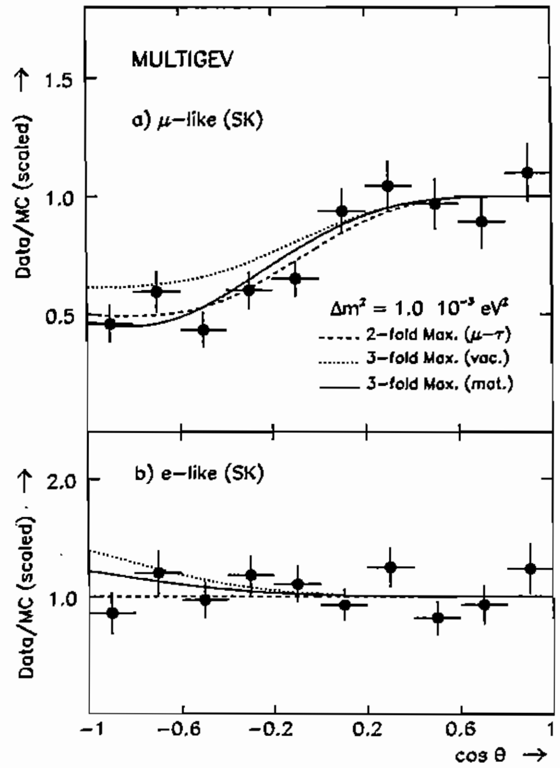

Figure 4. The multi-GeV zenith-angle distributions for a) μ-like and b) e-like events from the SUPER-K experiment. Tri-maximal mixing with matter effects (solid curve) is closer to twofold maximal $\nu_\mu - \nu_\tau$ mixing (dashed curve) than to vacuum tri-maximal mixing (dotted curve).

mass eigenstates become: $\nu_1 \to (\nu_1 - \nu_2)/\sqrt{2}$ and $\nu_2 \to (\nu_1 + \nu_2)/\sqrt{2}$ (up to phases) while the ν_3 remains tri-maximally mixed:

$$(|U_{l\nu}|^2) \;\; \to \;\; \begin{matrix} & \nu_1 & \nu_2 & \nu_3 \\ e & \cdot & 2/3 & 1/3 \\ \mu & 1/2 & 1/6 & 1/3 \\ \tau & 1/2 & 1/6 & 1/3 \end{matrix} \qquad (17)$$

As evidenced in Fig. 4, the phenomenology of Eq. 17 can be almost indistinguishable from that of 'optimised' bi-maximal mixing, Eq. 14. Beyond the 'matter threshold' we have for ν_μ:

$$<P(\mu \to \mu)> = (1/2)^2 + (1/6)^2 + (1/3)^2 = 7/18 \qquad (18)$$

while for ν_e:

$$<P(e \to e)> = (0)^2 + (2/3)^2 + (1/3)^2 = 5/9 \qquad (19)$$

For atmospheric neutrinos, account must be taken of the initial flux ratio. $\phi(\nu_\mu)/\phi(\nu_e)$. For $\phi(\nu_\mu)/\phi(\nu_e) \simeq 2/1$, the effective ν_μ suppression:

$$7/18 + 1/2 \times 2/9 = 1/2 \qquad (20)$$

(cf. Eq. 15) while the ν_e rate is fully compensated:

$$5/9 + 1/2 \times 2/9 = 1 \qquad (21)$$

so that ν_e appear not to participate in the mixing.

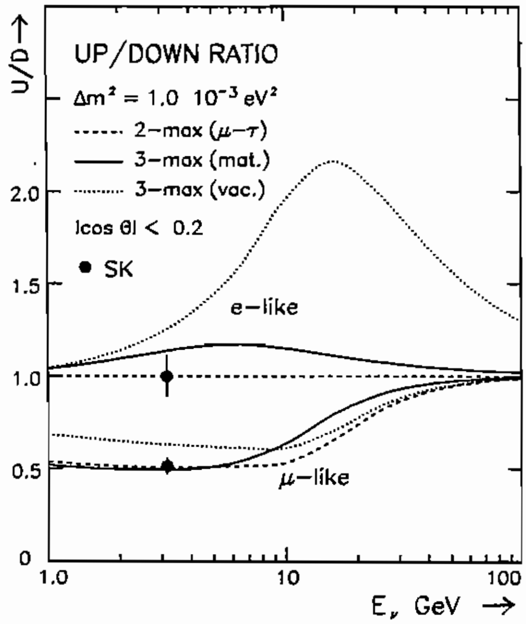

Figure 5. The up/down ratio for multi-GeV e-like and μ-like events as measured by SUPER-K. The curves are the various expectations plotted vs. neutrino energy, for $\Delta m^2 = 1.00 \times 10^{-3}$ eV^2.

The up/down ratio (Fig. 5) measures the effective supression. For energies $E \gtrsim 1$ GeV the initial flux ratio $\phi(\nu_\mu)/\phi(\nu_e) \gtrsim 2/1$ and the ν_e rate becomes 'over-compensated', while, for sufficiently high energies compensation effects vanish as the complete decoupling limit $\nu_e \to \nu_3$ is approached. The resulting maximum in the up/down ratio for ν_e (Fig. 5) is describerd as a 'resonance' by Pantaleone [13].

5.1. Matter Induced CP-violation

As regards the mixing matrix, CP and T violating effects are maximal in tri-maximal mixing, but in spite of this, due to the extreme hierarchy of Δm^2 values involved, for most accessible L/E values, observable asymmetries are expected to be unmeasurably small (in vacuum).

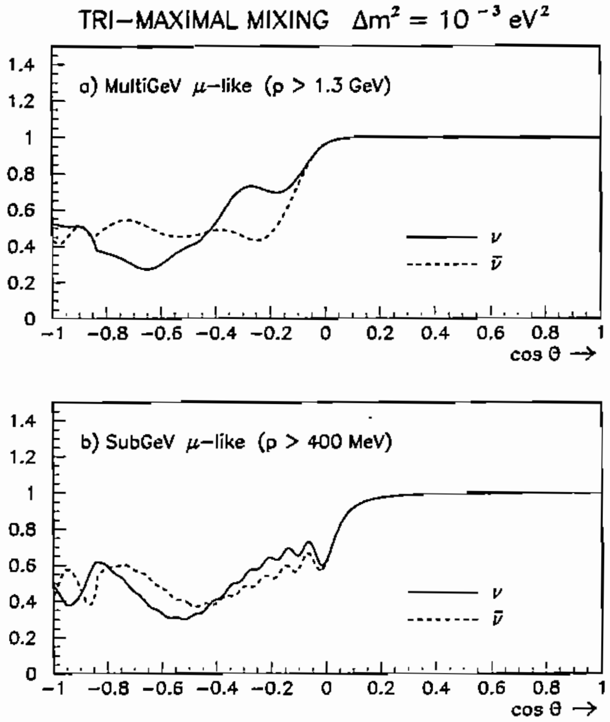

Figure 6. Predicted zenith-angle distributions in tri-maximal mixing, for a) multi-GeV and b) sub-GeV events in an atmopheric neutrinoexperiment, seperating ν (solid curve) and $\bar{\nu}$ (dashed curve) contributions. The curves plotted include energy averaging, but *not* angular smearing.

In the presence of matter (or indeed anti-matter) significant asymmetries can occur, however, 'enhanced' or 'induced' by matter effects. Thus for example if atmospheric neutrinos are separated $\nu/\bar{\nu}$, interesting matter induced asymmetries become observable (Fig. 6) in tri-maximal mixing. Such effects could be investigated using atmospheric neutrino detectors equipped with magnetic fields [14], and/or by using sign-selected beams in long-baseline experiments [15].

6. TRI-MAXIMAL MIXING AND THE SOLAR DATA

The vacuum predictions for the solar data in tri-maximal mixing are largely unmodified by matter effects in the Sun, as is well known, or, as we have seen, by matter effects in the Earth (Eq. 7 vs. Eq. 16). Thus we expect $P(e \to e) = 5/9$ in tri-maximal mixing with no energy dependence, as shown in Fig. 7. (Note that also the 'optimsed'

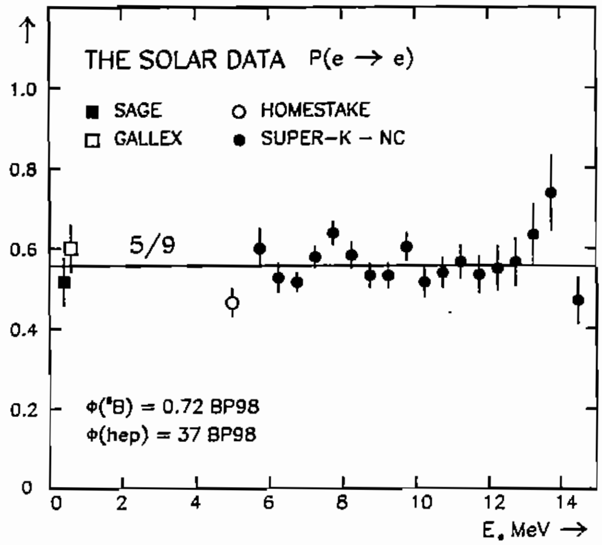

Figure 7. The SUPER-K solar data [11] plotted as a function of recoil electron energy E_e. The results from the two gallium experiments SAGE and GALLEX and the HOMESTAKE experiment are also plotted (at $<1/E>^{-1} \sim$ 0.5 MeV and $<1/E>^{-1} \sim$ 5 MeV respectively). Note that the SUPER-K points have been corrected for the NC contribution, and that the non-pp fluxes have been freely rescaled, with respect to BP98 [12], to test for an energy independent suppression.

bi-maximal scheme predicts $P(e \to e) = 5/9$ with no energy dependence for $\Delta m'^2$ outside the 'bath-tub'). In Fig. 7, the ^{8}B (and ^{7}Be) flux, affecting both the SUPER-K and HOMESTAKE data-points, has been rescaled by an arbitrary factor (0.72) for comparison to the energy-independent prediction. Given the flux errors ($\sim \pm 14\%$ for ^{8}B [12]), the fit (Fig. 7) seems not unreasonable.

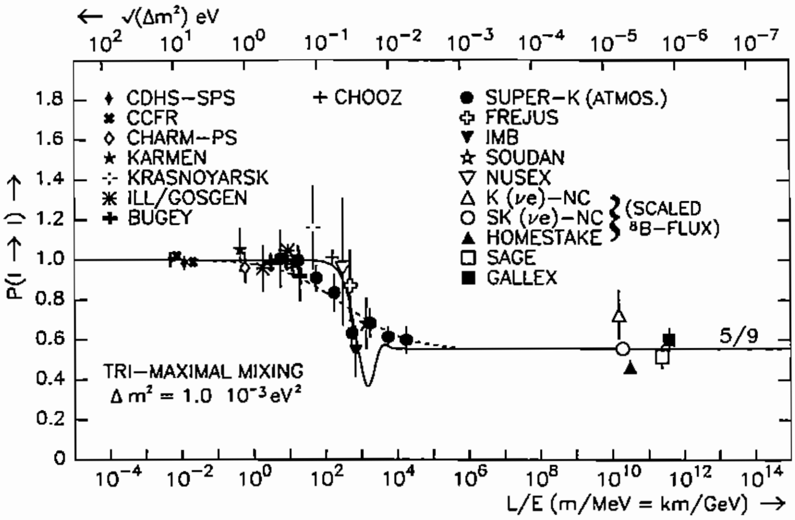

Figure 8. The updated L/E plot for disappearance experiments. The solid curve represents tri-maximal mixing with $\Delta m^2 = 1.0 \times 10^{-3}$ eV^2 and the dashed curve indicates the angular smearing in SUPER-K.

Fig. 8 shows the solar, atmospheric, accelerator and reactor data in overall perspective, within the tri-maximal context. Note that *dis*-appearance results *only* are represented: if the LSND appearance result [16] were ever to be confirmed, tri-maximal mixing would be instantly excluded.

7. CONCLUSION

The lepton mixing really does look to be either tri-maximal or bi-naximal at this point. Tri-maximal mixing is currently 'squeezed' in Δm^2 by CHOOZ vs. SUPER-K (Fig. 1-2). For some $\Delta m'^2$, bi-maximal mixing (in particular the 'optimised' version discussed here) clearly fits the data better. Tri-maximal mixing remains, however, the 'simplest' most 'symmetric' possibility.

Acknowledgement

I wish to thank Paul Harrison and Don Perkins for continued collaboration, and numerous helpful discussions relating to the above material.

REFERENCES

1 P. F. Harrison and W. G. Scott. Phys. lett. B 333 (1994) 471.
2 H-C. Chen. Siam J. Matrix Anal. Appl. 13 (1992) 1172.
3 S. L. Adler. Phys. Rev. D59 (1999) 015012. hep-ph/9806518.
4 P. F. Harrison, D. H. Perkins and W. G. Scott. Phys. lett. B 349 (1995) 137. B 374 (1996) 111. B 396 (1997) 186. B 458 (199) 79.
5 C. Walter. EPS HEP99 (1999) Tampere.
6 M. Appollonio et al. hep-ex/9907037.
7 H. Fritzsch and Z. Xing. Phys. Lett. B 372 (1996) 265; B 440 (1998) 313.
8 D. V. Ahluwalla. Mod. Phys. Lett. A 18 (1998) 2249. A. J. Bahz et al. Phys. Rev. Lett. 81 (1998) 5730. V. Barger et al. Phys. Lett. B 437 (1998) 107.
9 G Altarelli and F. Feruglio. Phys. Lett. B 439 (1998) 112. hep-ph/9807353.
10 C. Jarlskog et al. Phys. Lett. B 449 (1999) 240. hep-ph/9812282.
11 M. B. Smy. DPF99. hep-ex/9903034.
12 J. N. Bahcall, S. Basu and M. H. Pinsonneault. Phys. Lett. B 443 (1998) 1.
13 J. Pantaleone. Phys. Rev. Lett. 91 (1998) 5060. hep-ph/9810467.
14 M. Ambrosio et al. MONOLITH Letter of Intent. LNGS-LOI 20/99 (1999).
15 P. Adamson et al. MINOS Technical Design Report (1998).
16 C. Athanassopoulos et al. Phys. Rev. Lett. 75 (1995) 2650; 81 (1998) 1774.