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Collective Plasma Processes and the Solar Neutrino Problem

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COLLECTIVE EFFECTS IN BREMSSTRAHLUNG IN DENSE PLASMAS

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ABSTRACT

The results of recent developments in the theory of fluctuations in a plasma shows that the previously used theory of bremsstrahlung is incomplete and the exact expressions for bremsstrahlung should include the transition bremsstrahlung. The collective effects in bremsstrahlung known previously as the Debye screening effect are changed to a qualitatively different structure, which removes the effect of the ion polarization in bremsstrahlung and introduces new effective polarization which depends on effective ion charge and electron velocity. The paper contains new explicit expressions for collective effects in bremsstrahlung replacing it by an effective polarization which depends both on average ion charge and electron velocity. The results may be relevant for application in dense plasma where the wavelength is larger than the Debye length. It is shown that for the problem of photon transport in the solar interior the correct collective corrections to the bremsstrahlung change the opacity only by $\sim -0.2\%$, less than was calculated previously when the collective effects in bremsstrahlung where estimated without taking the modern results of plasma fluctuation theory into account.

1. INTRODUCTION

The problem of collective effects in bremsstrahlung is rather old. The collective effects in bremsstrahlung were first considered in [1]. Independently a new effect called the transition bremsstrahlung was investigated for the first time in [2,3,4] it was not quite clear its relation with collective effects and the shielding effects considered in [1]. Later it was proved that to use the shielding effect in bremsstrahlung is wrong both from a physical point of view and from the point of view of a correct analytical description of the effect. These results are discussed in detail in the monograph [5]. Until the present time nevertheless in applications very often the old approach is used. For example in the recent paper [6] bremsstrahlung is treated with a so-called shielding factor due to electron-ion correlations. These results were applied to the problems of dense plasma in the solar interior [6].

We give a brief summary why the approach of the shielding factor is incorrect and what role is played by the new effect of transitional bremsstrahlung. The transitional bremsstrahlung from a physical point of view is a very simple effect described by coherent oscillation and wave emission of the ion screening cloud in the electron-ion collisions. This coherent oscillation almost cancels the effect of the screening of the ion field in the process of bremsstrahlung [5]. The last statement was illustrated in [5] by numerous examples and is true not only for bremsstrahlung in plasmas. The question then arises whether all collective effects in bremsstrahlung can be reduced to transition bremsstrahlung and what is the role of screening? The answer came from the recent development of the fluctuation theory of plasmas where it was realized that the proper description of bremsstrahlung can be obtained only with a nonlinear theory of fluctuations which was known (see [7]) but leads to a very cumbersome expressions. The general expressions are not so easy to use in practical applications and the physical meaning of the results obtained in fluctuation theory was not clear. Only recently a proof was obtained [8] for the general statement that the fluctuation theory gives exactly the sum of the matrix elements of the shielded usual bremsstrahlung and matrix element of the transition bremsstrahlung. Thus the fluctuation approach (which is only correct for such a system as a plasma) proves that there exists no other collective effects in bremsstrahlung but the transition bremsstrahlung if the screening effect is taken into account in the usual bremsstrahlung. The most important point is the possibility of cancelling parts of the matrix elements describing bremsstrahlung which is nothing but interference of two processes of emission and which leads to the cancelation of the effects of shielding by the transition bremsstrahlung. This means that all collective effects in bremsstrahlung should be reconsidered. The mathematical basis for this is given by the results of [4] in which explicit expressions for the matrix elements for both processes of bremsstrahlung were given and one can start to use them to derive explicit expressions which can be used in applications. This problem is the aim of the present paper. We will be able to give a simple and explicit expression for amplitudes of both processes and illustrate the effect of cancelation in amplitudes of bremsstrahlung. The results obtained are quite different from that used before for the bremsstrahlung in electron-electron and ion-ion collisions. In electron-ion collisions the collective effect appears to be determined by the dielectric function for electron velocity but not the ion velocity as it was previously. This leads to a stripping ie, not dressed, effect discussed in detail in [5]. The factor which determines the collective effects we obtain depends on the average ion charge and this reflects the new physical process of transition bremsstrahlung which also depends on the average ion charge. This paper contains for the first time a correct explicit expression for collective effects in bremsstrahlung which can be used in applications.

The collective effects are very important for transport of radiation in the solar interior where the maximum frequency of radiation (determined, for instance, by the Planck distribution in the solar interior) corresponds to a wavelength which is larger than the Debye length [12.13]. We have re-examine most of the possible collective plasma effects for different processes (alltogether about 10 processes) and find that previously existing calculations were not correct. The aim of this paper is not only to give a general expression for bremsstrahlung but also re-examine there role in the solar interior. Thus we re-examine here one of the collective effects among others which can be important in the solar interior. As a result we find that the collective effect in bremsstrahlung contributes less to the solar opacity as was previously thought, (this situation is not true for the other effects). Since from the point of view of plasma theory the previous investigation of collective effects in bremsstrahlung in the solar interior were not correct, the present paper presents the first proof that they are not large. On the other hand all the collective corrections to the opacity in the solar interior appears to be of the same sign, and in this case one needs to make a sum of all the corrections to have a realistic estimation of the total effect. In this case the calculated contribution of collective effects in bremsstrahlung to the value of the solar opacity should be counted among other contributions.

Obviously our general results will have other applications in plasmas, but we concentrate on the problem of photon transport in the solar interior where we find that the collective effects in bremsstrahlung are not as important as previously thought.

2. COLLECTIVE EFFECTS IN BREMSSTRAHLUNG

In a plasma each charge is screened by a cloud of equal and opposite charges distributed in a volume of linear dimension $\sim d$ (Debye length). The effect of the screening clouds (collective effects) change both the scattering cross-section and the cross-section of bremsstrahlung decreasing the scattering on electrons. The collective effect in scattering increases the scattering on ions (i.e. on the screening electron clouds of the ions) which becomes the dominating scattering process for $\lambda \gg d$ and is of order [9,10].

$$\sigma = \frac{< Z >}{1+ < Z >}\sigma_T \tag{1}$$

where

$$\sigma_T = \frac{8}{3}\pi\frac{e^4}{m_e^2 c^4}$$

is the Thomson cross-section and

$$< Z >= \frac{\sum_i n_i Z_i^2}{\sum_i n_i Z_i} \tag{2}$$

is the "mean charge" of the ions of charge Z_i and number density n_i.

We should expect similar qualitative changes also in bremsstrahlung since the physics is similar: not only is the central charge displaced in the field of another charge (the field of an incident wave in the case of scattering) but also its screening cloud [5]. If the field of a charge is decomposed into harmonics and considered in a sense as a superposition of (virtual) waves, the analogy of bremsstrahlung and scattering is obvious. In fact it is known that the probability of bremsstrahlung can be written as the product of the probability of particle collisions and the probability of scattering of the harmonics of the field (virtual waves) into electromagnetic radiation. It is clear therefore that we should expect the same qualitative changes appearing in bremsstrahlung.

It is known that identical particle collisions in vacuum give a negligible contribution since (in the dipole approximation) the rate of bremsstrahlung is proportional to $\left(\frac{e_\alpha}{m_\alpha} - \frac{e_\beta}{m_\beta}\right)$ and therefore vanishes for like particles.

But in a plasma for the ion-ion or electron-electron collisions the collective effects (the displacements of their screening clouds in the collision) can give an appreciable contribution, expecially for $\lambda \sim d$ when the dipole approximation is not valid (notice that the case $\lambda \sim d$ is of particular relevance in the sun's interior where the condition $\lambda \gg d$ is valid only for the lowest frequencies of the radiation spectrum, close to the plasma frequency).

In the following we shall therefore consider not only the emission due to electron-ion collisions but also due to ion-ion and electron-electron collisions.

In collisions all types of waves can be emitted, including longitudinal plasma waves [1].

In the present paper we start with a general approach using matrix elements of bremsstrahlung given in [4]. A big advantage in using the results of [4] is that they can lead to explicit expressions for collective effects in bremsstrahlung of electromagnetic waves which was previously not given in the literature in the form useful for applications.

Here we will consider the case of electromagnetic waves using an approach valid for any polarization of the emitted wave.

We shall make the following assumptions to simplify the general expression of ref.[4];

1) emission is treated as a classical process for the small impact parameters when the collective effects are important.

2) the velocities v_α, v_β of the colliding particles are non-relativistic;

3) the phase velocities of the waves are much larger than the particle velocity

$$\frac{\omega_k}{k} \gg v_\alpha, v_\beta \tag{3}$$

The first assumption is valid for

$$\hbar k \ll mv \tag{4a}$$

$$\hbar q \ll mv \tag{4b}$$

where q is the momentum transferred from one colliding particle to the other colliding particle during the process of bremsstrahlung (q is the momentum of a virtual wave).

From (4a) we have

$$\hbar\omega_k \ll mv^2 \left(\frac{\omega_k}{kv}\right) \tag{5}$$

For electromagnetic waves $\omega_k/k \sim c$ and for thermal particles equation (5) becomes

$$\hbar\omega_k \ll T\left(\frac{c}{v_T}\right)^{\frac{1}{2}} \tag{6}$$

where T is the thermal energy (temperature in energy units) and $v_T = (T/m)^{\frac{1}{2}}$ the thermal velocity.

Eq. (5), combined with eq. (3), means that the energy of an emitted photon can be of the order of the particle energy and condition (3) is still valid. For instance in the sun's interior where $\hbar\omega \sim T$ at the maximum of the Planck distribution,eq. (6) shows that condition (3) is still valid. Our assumption (1) is therefore really a consequence of the assumptions (2) and (3), but only for the emitted waves (i.e. (4a)). Condition (4b) is an important assumption. The results obtained here are expressed as an integral over all virtual momentum q and lead for electron-ion collisions to a logarithmic divergence which we will treat by introducing q_{max}. The lowest possible q is determined by the conservation law in the process of bremsstrahlung and for electron-ion collision is of the order of $\frac{\omega_k}{v}$, where v is the relative electron-ion velocity (practically the electron velocity since the ion velocity is usually much slower than the electron velocity). Thus our results for bremsstrahlung in electron-ion collisions will contain the logarithm

$$\ln\frac{q_{max}v}{\omega}$$

An important point for applications is that in the case $q_{max} \gg \frac{1}{d}$ (as it is for the solar interior) and for q of the order of q_{max} the collective effects in bremsstrahlung are important. Thus we can join our results with the known results for bremsstrahlung when the collective effects are neglected. This gives q_{max} of the order of $\frac{mv}{\hbar}$ (the quantum uncertainty principle gives an estimate even without any deep considerations). But in principle there is no problem to join the two results to find an exact value of the numerical coefficient under the logarithm i.e. the logarithm becomes

$$\ln\frac{mv^2}{\hbar\omega}$$

This problem can thus be solved in a simple way for electron-ion collisions.

For bremsstrahlung electron-electron or ion-ion collisions the problem is even simpler since the final integral with collective effects included converges in q and thus q_{max} does not play a significant role.

The second assumption ($v/c \ll 1$) makes all calculations much easier since in this case only the electrostatic virtual fields are important. The corresponding total probability of bremsstrahlung is given in reference [4] (p. 260), for an emitted wave of wave-number $\underline{k}$, frequency ω_k and unit polarization vector $\hat{e}^\sigma_k$ (σ stands for the type of polarization) it is given by

$$W_{\alpha\beta}(\underline{k},\underline{q}) = 2(2\pi)^2 \frac{\left|\hat{e}^\sigma_{\underline{k}} \cdot (\underline{M}^\alpha + \underline{M}^\beta + \underline{M}^{\alpha\beta})\right|^2}{\hbar \left[\frac{\partial\omega^2\epsilon^\sigma_{\omega,k}}{\partial\omega}\right]_{\omega=\omega^\sigma_k}} \delta(\omega^\sigma_k - \underline{k}\cdot\underline{v}_\alpha + \underline{q}\cdot(\underline{v}_\alpha - \underline{v}_\beta)) \quad (7)$$

where $\underline{q}$ is the wave-vector of the momentum transferred in the collision and

$$\epsilon^\sigma_{\omega,k} = \epsilon_{ij}(\omega,k) e^{\sigma *}_{\underline{k}i} e^\sigma_{\underline{k}j} \quad (8)$$

where $\epsilon_{ij}(\omega,k)$ is the plasma dielectric tensor. For electromagnetic waves

$$\omega^t_k = (c^2k^2 + \omega^2_{pe})^{\frac{1}{2}} \ ; \ \omega^2_{pe} = \frac{4\pi e^2 n_e}{m_e} \quad (9)$$

and, neglecting the thermal effects in the dielectric tensor we simply take

$$\epsilon^\sigma_{\omega,k} \equiv \epsilon(\omega) = 1 - \frac{\omega^2_{pe}}{\omega^2} \quad (10)$$

The vectors $\underline{M}^\alpha$ and $\underline{M}^\beta$ in eq. (3), given in reference [4] (see eq.s (5.60) and (5.61)), account for the oscillation (displacement of trajectory) of one charge (α or β) in the screened field of the other (β or α). The oscillations and emission of the screening clouds of particles α and β are taken into account in $\underline{M}^{\alpha\beta}$ (eq. (5.66) of reference [4]).

Here we shall derive expressions for these vectors using our assumptions (1-3). For $\underline{M}^\alpha$ from ref. [4] we write

$$M^\alpha_i = (2\pi)^3 e_\beta \wedge^\alpha_{ij} (\underline{q}) G_{jl}(q) v^\beta_l \quad (11)$$

where e_β is the charge of particle β with velocity $\underline{v}^\beta$, G_{jl} is the Green function of the field and we use here only its longitudinal part given by

$$G_{jl}(\underline{q}) = -i \frac{4\pi q_j q_l}{(\underline{q}\cdot\underline{v}_\beta) q^2 \epsilon_{\underline{q}\cdot\underline{v}_\beta, q}} \quad (12)$$

and then

$$G_{jl} v^l_\beta = -i \frac{4\pi q_j}{q^2 \epsilon_{\underline{q}\cdot\underline{v}_\beta, q}} \quad (13)$$

where $\epsilon_{\omega,k}$ is the longitudinal part of the dielectric permittivity. Using eq.(13) eq.(11) becomes

$$M_i^\alpha = -i\frac{(4\pi)(2\pi)^3 e_\beta}{q^2 \epsilon_{\underline{q}\cdot\underline{v}_\beta,q}} \wedge_{ij}^\alpha (q_\alpha) q_j \tag{14}$$

The tensor $\wedge_{ij}^\alpha(\underline{q})$ is given in reference [4] (eq. 4.143) and using the conservation law

$$\underline{q}\cdot\underline{v}_\alpha = \underline{q}\cdot\underline{v}_\beta - (\omega_k^\sigma - \underline{k}\cdot\underline{v}_\alpha) \tag{15}$$

from the δ-function in eq. (7), we find

$$\wedge_{ij}^\alpha(\underline{q})q_j = \tag{16}$$

$$i\frac{e_\alpha^2\left(1-\frac{v_\alpha^2}{c^2}\right)^{1/2}}{(2\pi)^3 m_\alpha(\omega_k^\sigma - \underline{k}\cdot\underline{v}_\alpha)^2}\left[v_\alpha^i\left(\underline{k}\cdot\underline{q} - \frac{\omega_k^\sigma}{c^2}\underline{q}\cdot\underline{v}_\beta\right) + (\omega_k^\sigma - \underline{k}\cdot\underline{v}_\alpha)\left(q_i + \frac{v_\alpha^i}{c^2}\omega_k^\sigma\right)\right]$$

where e_α, v_α are the charge and velocity of particle α.

A further simplification comes from using our assumptions (2) and (3); from eq. (15) for $\omega_k^\sigma \gg \underline{k}\cdot\underline{v}_\alpha$ it follows that

$$q > \frac{\omega_k^\sigma}{|\underline{v}_\alpha - \underline{v}_\beta|} \geq \frac{\omega_k^\sigma}{v_\alpha}$$

i.e.

$$q \gg \frac{v_\alpha \omega_k^\sigma}{c^2} \simeq \frac{v_\alpha^2}{c^2}\frac{\omega_k^\sigma}{v_\alpha}$$

Using these simplifications in eq.(16) the second term in square brackets is $\omega_k^\sigma q_i$, the first term can be neglected since it is of order $kv_\alpha \ll \omega_k^\sigma$ and $v_\alpha v_\beta/c^2 \ll 1$: then we find from eq.(14)

$$\underline{M}^\alpha = \frac{4\pi e_\alpha^2 e_\beta}{\omega_k^\sigma m_\alpha q^2 \epsilon_{\underline{q}\cdot\underline{v}_\beta,q}}\underline{q} \tag{17}$$

This matrix element contains the screening in $\epsilon_{\underline{q}\cdot\underline{v}_\alpha,q}$ and is often used in calculations of collective effects in bremsstrahlung. We will see that due to interference effects the correct factor entering in the collective effect contains the electron, but not the ion velocity in the dielectric permittivity ϵ and thus describes the stripping of electron screening for high electron velocities.

With the same simplifications we also find for the other colliding particle

$$\underline{M}^{\beta} = \frac{4\pi e_\alpha e_\beta^2}{\omega_k^q m_\beta |\underline{k}-\underline{q}|^2 \epsilon_{(\underline{k}-\underline{q})\cdot\underline{v}_\alpha, |\underline{k}-\underline{q}|}} (\underline{k}-\underline{q}) \tag{18}$$

In this case the α particle is light (electron) and the β particle is heavy (ion) and therefore this matrix element can be neglected. But this is not the case of ion-ion and electron-electron bremsstrahlung where the collective effects change drastically the emission power.

In the absence of the screening effects $\epsilon_{\omega k} \simeq 1$, $q \gg k$ we have

$$\underline{M}^{\alpha} + \underline{M}^{\beta} \sim e_\alpha e_\beta \left(\frac{e_\alpha}{m_\alpha} - \frac{e_\beta}{m_\beta} \right) \underline{q} \tag{19}$$

which reproduces the known result of negligible bremsstrahlung for $e_\alpha/m_\alpha \simeq e_\beta/m_\beta$.

For $\underline{M}^{\alpha\beta}$ (see eq. 5.66 and 6.43 in reference [4]) and using our assumptions we can substitute δ_{ij} for $\delta_{ij}(1 - \underline{k} \cdot \underline{v}/\omega) + k_i v_j/\omega$ in all factors in eq. (6.43) of ref [4] and ω for $\omega - \underline{k} \cdot \underline{v}$ in the denominator of the first factor, if we also assume that only the electrons contribute to the nonlinear current (i.e. ignore the ion contribution which is of the order of m_i^{-1}) we find after a lengthy calculation

$$\begin{aligned} \underline{M}^{\alpha\beta} = & \frac{4\pi e e_\alpha e_\beta \underline{q}}{\omega_k^q m_e q^2 \epsilon_{\underline{q}\cdot v_\beta, q}} \left(\frac{\epsilon^{(e)}_{(\underline{k}-\underline{q})\cdot\underline{v}_\alpha, |\underline{k}-\underline{q}|} - 1}{\epsilon_{(\underline{k}-\underline{q})\cdot\underline{v}_\alpha, |\underline{k}-\underline{q}|}} \right) + \\ & + \frac{4\pi e e_\alpha e_\beta (\underline{k}-\underline{q})}{\omega_k^q m_e |\underline{k}-\underline{q}|^2 \epsilon_{(\underline{k}-\underline{q})\cdot\underline{v}_\alpha, |\underline{k}-\underline{q}|}} \left(\frac{\epsilon^{(e)}_{\underline{q}\cdot\underline{v}_\beta, q} - 1}{\epsilon_{\underline{q}\cdot\underline{v}_\beta, q}} \right) \end{aligned} \tag{20}$$

where e is the electron charge, $\epsilon^{(e)}_{\omega,k} = 1 + \chi^{(e)}_{\omega,k}$ is the electron susceptibility and

$$\epsilon_{\omega,k} = 1 + \chi^{(e)}_{\omega,k} + \sum_i \chi^{(i)}_{\omega,k} \tag{21}$$

is the plasma susceptibility with $\chi^{(i)}_{\omega k}$ giving the contribution of type-i ion. The expression (20) is completely new and was not given previously in the literature. From (20) it can be seen that it gives the results very similar to (17) and (18) and can never be neglected.

A comparison of eq. (17) and (18) with eq. (20) shows that in general the contribution of the screening clouds $\underline{M}^{\alpha\beta}$ to bremsstrahlung is important since the expressions in brackets can be of the order of one.

Next we shall find expressions for the total $\underline{M}^{\alpha} + \underline{M}^{\beta} + \underline{M}^{\alpha\beta}$ entering eq. (7) for the probability, for the three possible collisions.

a) electron-electron collisions: ($\underline{v}$ and $\underline{v}'$ are the velocities of two colliding electrons).

$$\underline{M}^{(ee')} = \underline{M}^{e} + \underline{M}^{e'} + \underline{M}^{ee'} = -\frac{4\pi e^3 \underline{q}}{m_e \omega_k^\sigma q^2 \epsilon_{\underline{q}\cdot\underline{v}',q}} \frac{1 + \sum_i \chi^{(i)}_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}{\epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}} + \\ -\frac{4\pi e^3 (\underline{k}-\underline{q})}{m_e \omega_k^\sigma |\underline{k}-\underline{q}|^2 \epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}} \frac{1 + \sum_i \chi^{(i)}_{\underline{q}\cdot\underline{v}',q}}{\epsilon_{\underline{q}\cdot\underline{v}',q}} \tag{22}$$

It should be noted that this expression is quite different from that used in the present literature. It can also be directly found from the fluctuation theory of plasmas (see [8,11]).

b) ion-ion collisions.

Due to the ion mass in the denominator we can neglect eq. (17) and (18) and only the contribution of the screening clouds eq. (20) is important ($\underline{v}$ and $\underline{v}'$ are the velocities of two colliding ions)

$$\underline{M}^{(ii')} = \underline{M}^{ii'} = \frac{4\pi e^3 Z_i Z_i' \underline{q}}{m_e \omega_k^\sigma q^2 \epsilon_{\underline{q}\cdot\underline{v}',q}} \frac{\chi^{(e)}_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}{\epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}} + \\ + \frac{4\pi e^3 Z_i Z_i' (\underline{k}-\underline{q})}{m_e \omega_k^\sigma |\underline{k}-\underline{q}|^2 \epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}} \frac{\chi^{(e)}_{\underline{q}\cdot\underline{v}',q}}{\epsilon_{\underline{q}\cdot\underline{v}',q}} \tag{23}$$

c)electron-ion collisions

Let α be the electron and β the ion, then we can neglect eq. (18) (due to the ion mass in the denominator) and, denoting the electron velocity $\underline{v}$ and the ion velocity $\underline{v}'$, we find

$$\underline{M}^{(ei)} = \underline{M}^{e} + \underline{M}^{ei} = \frac{4\pi e^3 Z_i \underline{q}}{m_e \omega_k^\sigma q^2 \epsilon_{\underline{q}\cdot\underline{v}',q}} \frac{1 + \sum_i \chi^{(i)}_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}{\epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}} + \\ -\frac{4\pi e^3 Z_i (\underline{k}-\underline{q})}{m_e \omega_k^\sigma |\underline{k}-\underline{q}|^2 \epsilon_{\underline{q}\cdot\underline{v}',\underline{q}}} \frac{\chi^{(e)}_{\underline{q}\cdot\underline{v}',q}}{\epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}} \tag{24}$$

For longitudinal waves ($\hat{e}^{\sigma}_{\underline{k}} = \underline{k}/k, \omega^{\sigma}_{k} = \omega^{e}_{k} \simeq \omega_{pe}$) these expressions lead to the well known results of [8]. Here we shall consider the case of transverse waves ($\omega^{\sigma}_{k} = \omega^{t}_{k}, \hat{e}^{\sigma}_{\underline{k}} \cdot \underline{k} = 0$) and use them in eq. (7) to find the emission in the following section.

We conclude this section with some comments on equations (22-24).

First notice that these expressions satisfy the known relation between scattering and bremsstrahlung discussed in the introduction. In fact the scattering cross-section (taking into account the collective effects) is proportional to $|M_{scat}|^2$ where [9,10]

$$M^{(e)}_{scat} \simeq \frac{e^2}{m_e}(\hat{e}^{\sigma *}_{\underline{k}} \cdot e^{\sigma'}_{\underline{k'}})\frac{1 + \sum_i \chi^{(i)}_{\omega-\omega',(\underline{k}-\underline{k}')\cdot\underline{v}}}{\epsilon_{\omega-\omega',(\underline{k}-\underline{k}')\cdot\underline{v}}}$$

for scattering on electrons, and

$$M^{(i)}_{scat} \simeq \frac{e^2}{m_e}(\hat{e}^{\sigma *}_{\underline{k}} \cdot e^{\sigma'}_{\underline{k'}})\frac{\chi^{(e)}_{\omega-\omega',(\underline{k}-\underline{k}')\cdot\underline{v}}}{\epsilon_{\omega-\omega',(\underline{k}-\underline{k}')\cdot\underline{v}}}$$

for scattering on ions.

Thus the bremsstrahlung matrix elements can be expressed through the scattering matrix elements. From $M^{(ee')}$ in eq. (22) we can see that the two terms describe two effects. The first term gives the appearance of the harmonic $\underline{q}$ from the field of the electron with velocity $\underline{v}'$ and scattering on the other electron (with velocity $\underline{v}$). The second term gives the appearance of the harmonic $(\underline{k} - \underline{q})$ from the field of the electron with velocity $\underline{v}$ and scattering on the other electron (with velocity $\underline{v}'$).

The same can be seen in $M^{(ii')}$ (eq. 23) where either one of the ions scatters the harmonic appearing from the field of the other ion.

Notice that both $\underline{M}^{(ee')}$ and $\underline{M}^{(ii')}$ are invariant for $\underline{q} \leftrightarrow \underline{k} - \underline{q}$. The term $\underline{M}^{(ei)}$ (eq.24) describes both the scattering on an electron (first term) and the scattering on an ion (second term) of the harmonics in the field of an ion or an electron respectively.

Finally without collective effects only the electron-ion collision term survives (see eq. 19) with the first term in which the collective effects in scattering are also neglected ($\underline{q} \simeq \underline{k}$) and $v' \simeq 0$, i.e.

$$\underline{M}^{(ei)}_{non-collective} \sim \frac{4\pi e^3 Z_i \underline{q}}{m_e \omega_k q^2 \epsilon_{0,q}} \tag{25}$$

The comparison of this expression with eq's (22-24) shows how important the changes can be in bremsstrahlung due to properly taking into account collective effects.

3.SPONTANEOUS AND STIMULATED EMISSION

We are interested in the total emission rate of electromagnetic radiation from a unit plasma volume particularly in the frequency domain

$$\omega_{pe} < \omega \leq \frac{T}{\hbar} \ll \frac{T}{\hbar}\left(\frac{c}{v_{Te}}\right)^{\frac{1}{2}} \tag{26}$$

which is the frequency range relevant to the solar interior.
The total intensity of emission is given by [3]

$$Q = Q_{ee} + \sum_i Q_{ei} + \sum_{i,i'} Q_{ii'} \tag{27}$$

where

$$Q_{\alpha\beta} = \int \hbar\omega_k W_{\alpha\beta}(\underline{k},\underline{q}) \frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\, f_\alpha(\underline{v}_\alpha) f_\beta(\underline{v}_\beta) d^3v_\alpha d^3v_\beta \tag{28}$$

and where $f_\alpha(\underline{v})$ is the distribution function of particles of type-α in the plasma, normalized to the number density n_α

$$n_\alpha = \int f_\alpha(\underline{v}_\alpha) d^3v_\alpha \tag{29}$$

For transverse waves we use the identity (in the following we use the notation $\omega_k = \omega$)

$$k^2 dk = \frac{\omega^2}{c^3}\sqrt{\epsilon(\omega)}d\omega \tag{30}$$

and defining the emission $Q^{(\omega)}_{\alpha\beta}$ in the frequency range $d\omega$ as

$$Q_{\alpha\beta} = \int Q_{\alpha\beta}(\omega)d\omega \tag{31}$$

we find from eq's (28) and (30) that

$$Q_{\alpha\beta}(\omega) = \frac{\hbar\omega^3}{c^3}\sqrt{\epsilon(\omega)} \int W_{\alpha\beta}(\underline{k},\underline{q}) \frac{d\Omega_{\underline{k}}}{(2\pi)^3} f_\alpha(\underline{v}_\alpha) f_\beta(\underline{v}_\beta) d^3v_\alpha d^3v_\beta \frac{d^3q}{(2\pi)^3} \tag{32}$$

where $d\Omega_k$ is the solid angle associated with $\underline{k}$.

Then from eq. (27) we get

$$Q(\omega) = Q_{ee}(\omega) + \sum_i Q_{ei}(\omega) + \sum_{i,i'} Q_{ii'}(\omega) \tag{33}$$

for the total intensity in the frequency interval $d\omega$.

The stimulated emission (or inverse bremsstrahlung) produces an absorption of waves with a damping rate (see ref. [3])

$$\gamma_{\underline{k}} = \gamma_{\underline{k}}^{ee} + \sum_{ei} \gamma_{\underline{k}}^{ei} + \sum_{ii'} \gamma_{\underline{k}}^{ii'} \tag{34}$$

where

$$\gamma_{\underline{k}}^{\alpha\beta} = \frac{1}{2} \int W_{\alpha\beta}(\underline{k}, \underline{q}) \left[\left(f(\underline{v}_\beta) - f(\underline{v}_\beta - \frac{\hbar \underline{q}}{m_\beta}) \right) f(\underline{v}_\alpha) + \right.$$

$$\left. \left(f(\underline{v}_\alpha) - f(\underline{v}_\alpha - \frac{\hbar(\underline{k} - \underline{q})}{m_\alpha}) \right) f(\underline{v}_\beta) \right] d^3 v_\alpha d^3 v_\beta \frac{d^3 q}{(2\pi)^3} \tag{35}$$

where $f(\underline{v}_\alpha)$ is the distribution function and the probability $W_{\alpha\beta}$ is given by eq.(7).

Energy conservation requires

$$\frac{p_\alpha^2}{2m_\alpha} + \frac{p_\beta^2}{2m_\beta} = \frac{\left[\underline{p}_\alpha - \hbar(\underline{k} - \underline{q})\right]^2}{2m_\alpha} + \frac{(\underline{p}_\beta - \hbar \underline{q})^2}{2m_\beta} + \hbar\omega_k$$

where $\underline{p}_\alpha = m_\alpha \underline{v}_\alpha$ is the particle momentum. For the electron-ion case, neglecting the small transfer of momentum to the ion, and for Maxwellian distributions the term in square brackets in the integral eq. (35) becomes

$$f(\underline{v}_e) f(\underline{v}_i) \left[1 - e^{\frac{\hbar\omega_k}{T}}\right]$$

and $\gamma_{\underline{k}}^{ei}$ can be written in the form

$$\gamma_{\underline{k}}^{ei} = \frac{1}{2}(1 - e^{\frac{\hbar\omega_k}{T}}) \int W_{ei}(\underline{k}, \underline{q}) f_e(\underline{v}_e) f_i(\underline{v}_i) d^3 v_e d^3 v_i \frac{d^3 q}{(2\pi)^3} \tag{35a}$$

or, comparing with (32) integrated over the solid angle, it is given by

$$\gamma_k^{ei} = \frac{1}{2} \frac{(1 - e^{\frac{\hbar\omega_k}{T}})}{(\hbar\omega_k / T)} \left[\frac{2\pi^2 c^3}{T \omega_k^2 \sqrt{\epsilon(\omega_k)}} Q_{ei}(\omega_k) \right] \tag{36}$$

where

$$\gamma_k^{ei} = \frac{1}{4\pi} \int \gamma_{\underline{k}}^{ei} d\Omega_{\underline{k}}$$

For the e-e and i-i cases the relation between $\gamma^{\alpha\beta}$ and $Q^{\alpha\beta}$ can only be found for $\hbar k \ll p$ and $\hbar q \ll p$ (which means $\hbar\omega_k \ll T$), in this limit we can expand

$$f(\underline{v}) - f(\underline{v} - \frac{\hbar k}{m}) \simeq \frac{\hbar \underline{k}}{m} \cdot \frac{\partial f(\underline{v})}{\partial \underline{v}} \simeq -\frac{\hbar}{T}(\underline{k} \cdot \underline{v}) f(\underline{v})$$

such that in the integrand of eq.(35) we find

$$\gamma_k^{\alpha\beta} = -\frac{2\pi^2 c^3}{2T\omega_k^2 \sqrt{\epsilon(\omega_k)}} Q_{\alpha\beta}(\omega_k)$$

which only coincides with eq.(36) in the limit $\hbar\omega_k/T \ll 1$.

In the following we shall find that only the e-i bremsstrahlung is important and then use the more general relation given by eq.(36) to relate emission to absorption.

4. COLLECTIVE EFFECTS IN SPONTANEOUS AND STIMULATED BREMSSTRAHLUNG

We consider separately the three cases.

1) electron-ion bremsstrahlung.

Using eq.(24) in eq.(7) and then eq.(32) we get

$$Q_{ei}(\omega) =$$

$$= \frac{\sqrt{\epsilon(\omega)} e^6 Z_i^2}{m_e^2 c^3 \pi^2} \int f^{(e)}(\underline{v}) f^{(i)}(\underline{v}') d^3q d\Omega_k d^3v d^3v' \delta[\omega - \underline{q} \cdot \underline{v}' - (\underline{k} - \underline{q}) \cdot \underline{v}] F_{k,q}(\underline{v}, \underline{v}') \tag{37}$$

where

$$F_{k,q}(\underline{v}, \underline{v}') = \frac{\frac{1}{2}|\underline{k} \times \underline{q}|^2}{k^2 |\epsilon_{\underline{q}\cdot\underline{v}',q} \epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}|^2} \left| \frac{1 + \sum_i \chi^{(i)}_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}{q^2} + \frac{\chi^{(e)}_{\underline{q}\cdot\underline{v}',q}}{|\underline{k} - \underline{q}|^2} \right|^2 \tag{38}$$

We now make the following simplifications; the ion susceptibility corresponding to the electron velocity ($\underline{v}$) is small and can be ignored with respect to 1; the electron susceptibility corresponding to the ion velocity ($\underline{v}'$) is practically the Debye screening, ie.,

$$\chi^{(e)}_{\underline{q}\cdot\underline{v}',q} \simeq \frac{\omega_{pe}^2}{q^2 v_{Te}^2}$$

We can also neglect the ion velocity v' in the δ-function.

Then we have, using Maxwellian distributions for $f^{(e)}(v), f^i(v')$ and integrating over the component of velocity perpendicular to $\underline{q}$ and to $(\underline{k}-\underline{q})$ respectively

$$Q_{ei}(\omega) =$$

$$= \frac{\sqrt{\epsilon(\omega)}e^6 Z_i^2 n_i n_e}{m_e^2 c^3 \pi^2} \int \frac{e^{-y^2}}{\sqrt{\pi}} \frac{e^{-y'^2}}{\sqrt{\pi}} d^3q d\Omega_{\underline{k}} F_{\underline{k}\underline{q}}(y,y') \delta(\omega - |\underline{k}-\underline{q}| v_{Te}\sqrt{2}y) dy dy' \tag{39}$$

where

$$F_{\underline{k}\underline{q}}(y,y') = \frac{\frac{1}{2}|\underline{k}\times\underline{q}|^2 \left|1 + \frac{\omega_{pe}^2}{|\underline{k}-\underline{q}|^2 v_{Te}^2}\right|^2}{q^4 k^2 \left|1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2} + \sum_i \frac{\omega_{pe}^2 W_i^2 n_i}{q^2 v_{Te}^2 n_e} W(y')\right|^2 \left|1 + \frac{\omega_{pe}^2}{|\underline{k}-\underline{q}|^2 v_{Te}^2} W(y)\right|^2}$$

where y and y' are the normalized velocity components parallel to $\underline{k}-\underline{q}$ and $\underline{q}$ respectively, i.e.

$$y = \frac{v_\parallel}{\sqrt{2}v_{Te}}, v_\parallel = \frac{(\underline{k}-\underline{q})\cdot\underline{v}}{|\underline{k}-\underline{q}|};\ y' = \frac{v'_\parallel}{\sqrt{2}v_{Ti}}, v'_\parallel = \frac{\underline{q}\cdot\underline{v}'}{q} \tag{40}$$

and $W(y)$ is the plasma function, from the susceptibilities can be written as

$$W(y) = 1 - 2ye^{-y^2}\int_0^y e^{t^2}dt + i\sqrt{\pi}ye^{-y^2} \tag{41}$$

From the δ-function in eq.(39) we see that for $k \sim q$

$$\omega = ck \sim v_{Te} q y$$

i.e. $y \sim c/v_{Te}$ and e^{-y^2} becomes negligible, we can then assume $k \ll q$ in the integral and simplify it to find

$Q_{ei}(\omega) =$

$$= \frac{16}{3}\frac{\sqrt{\epsilon(\omega)}e^6 Z_i^2 n_i n_e}{m_e^2 c^3}\int dq \int \frac{e^{-y^2}}{\sqrt{\pi}}\delta(\omega - qv_{Te}\sqrt{2}y)\frac{\left(1+\frac{\omega_{pe}^2}{q^2 v_{Te}^2}\right)^2}{\left|1+\frac{\omega_{pe}^2}{q^2 v_{Te}^2}W(y)\right|^2}dy I_q \tag{42}$$

where

$$I_q = \int \frac{e^{-y'^2}}{\sqrt{\pi}}\frac{dy'}{\left|1+\frac{\omega_{pe}^2}{q^2 v_{Te}^2}+\sum_i \frac{\omega_{pe}^2 Z_i^2 n_i}{q^2 v_{Te}^2 n_e}W(y')\right|^2} \tag{43}$$

The y'-integral (the ion contribution) is well known from the theory of scattering and is given by

$$I_q = \frac{1}{\left(1+\frac{\omega_{pe}^2}{q^2 v_{Te}^2}\right)\left[1+\frac{\omega_{pe}^2}{q^2 v_{Te}^2}(1+<Z>)\right]} \tag{44}$$

where $<Z>$ is given by eq.(2).Then from eq.(42) we have

$\sum_i Q_{ei}(\omega) =$

$$\frac{\sqrt{\epsilon(\omega)}e^6 n_e^2 <Z>}{m_e^2 c^3}\int \left(1+\frac{\omega_{pe}^2}{q^2 v_{Te}^2}\right) dq \int \frac{e^{-y^2}}{\sqrt{\pi}}\frac{\delta(\omega - qv_{Te}\sqrt{2}y)}{\left[1+\frac{\omega_{pe}^2}{q^2 v_{Te}^2}(1+<Z>)\right]\left|1+\frac{\omega_{pe}^2}{q^2 v_{Te}^2}W(y)\right|^2}dy$$

Performing the y-integral using the δ-function and then introducing the new variable to transform the q-integral

$$x = \frac{\omega}{\sqrt{2}v_{Te}q} \tag{45}$$

we finally find

$\sum_i Q_{ei}(\omega) =$

$$= \sigma_T \frac{2}{\pi\sqrt{2\pi}}\left(\frac{c}{v_{Te}}\right) n_e^2 e^2 \sqrt{\epsilon(\omega)} <Z> \int_{x_{min}}^{\infty}\frac{dx}{x}\frac{e^{-x^2}\left(1+2\frac{\omega_{pe}^2}{\omega^2}x^2\right)}{\left[1+2\frac{\omega_{pe}^2}{\omega^2}(1+<Z>)x^2\right]\left|1+2\frac{\omega_{pe}^2}{\omega^2}x^2 W(x)\right|^2} \tag{46}$$

This result differs from previous ones on collective electron-ion bremsstrahlung (see e.g. Bekefi [1], Ichimaru [14]) for the contribution of the screening clouds of the colliding particles (transition bremsstrahlung) which is included here for the first time.

In the classical description for the probability the q-integral is taken between q_{min} and q_{max} (the Bethe-Heitler limit). In our case we point out that since for $q \sim q_{max}$ it is $\epsilon \sim 1$ (negligible collective effects) then we can take the known results from single particle emission to define q_{max} the same Bethe-Heitler value of the non-collective treatment and join the collective and single-particle results for some $q^* < q_{max}$.

Starting from the expression for the emitted power (eq.(37) of ref. [8]) but not performing the integration over the perpendicular component of the electron velocity (as done in ref.[8]) but only over the ion distribution and the angle between $\underline{v}$ and $\underline{q}$ using the δ-function, the result is

$$Q_{ei}(\omega) = \frac{16}{3}\frac{e^6 n_e^2 \langle Z\rangle}{m_e^2 c^3} \int d^3v \frac{e^{-\frac{v^2}{2v_{Te}^2}}}{(2\pi)^{3/2} v_{Te}^3} \frac{1}{v} \int_{q_{min}}^{q_{max}} \frac{dq}{q} \mathcal{H}(\omega, q) \tag{47}$$

where

$$\mathcal{H}(\omega, q) = \frac{1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2}}{\left[1 + (1 + \langle Z\rangle)\frac{\omega_{pe}^2}{q^2 v_{Te}^2}\right] |1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2} W(\frac{\omega}{\sqrt{2} q v_{Te}})|^2}, \tag{48}$$

$< Z >$ in the effective ion charge, and the limits of integration for the transferred momentum q can be found from the quantum conservation laws (v' is the electron velocity after the emission)

$$\underline{q} = \frac{m_e}{\hbar}(\underline{v} - \underline{v}') \; ; \; q = \frac{m_e}{\hbar}\sqrt{v^2 + v'^2 - 2vv' \cos\theta} \tag{49}$$

and

$$\frac{m_e}{2} v^2 = \frac{m_e}{2} v'^2 + \hbar\omega \; ; \; v' = \sqrt{v^2 - \frac{2\hbar\omega}{m_e}} \tag{50}$$

Then

$$q_{min} = \frac{m_e}{\hbar}\left(v - \sqrt{v^2 - \frac{2\hbar\omega}{m_e}}\right) \; ; \; q_{max} = \frac{m_e}{\hbar}\left(v + \sqrt{v^2 - \frac{2\hbar\omega}{m_e}}\right) \tag{51}$$

Notice that in most applications, including the solar interior we have the result

$$\frac{\omega_{pe}^2}{q_{max}^2 v_{Te}^2} \simeq \frac{\omega_{pe}^2 \hbar^2}{m_e^2 v_{Te}^2 v^2} \simeq \left(\frac{\hbar\omega_{pe}}{T}\right)^2 << 1 \tag{52}$$

and therefore near the upper limit ($q \simeq q_{max}$) $\mathcal{H}(q,\omega) \simeq 1$ (no collective effects).The corrections are therefore essential only for $\omega << m_e \frac{v^2}{\hbar}$ and this is the most important domain for the contribution to the opacity. Integrating by parts eq.(47), using $\mathcal{H}(\omega, q_{max}) = 1$ (see 52), introducing $y = \frac{v}{\sqrt{2}v_{Te}}$ and changing variable to

$$x = \frac{1}{2}[y + \sqrt{y^2 - z}] \tag{53}$$

the result is

$$Q_{ei} = \frac{16}{3}\sqrt{\frac{2}{\pi}}\frac{e^6 n_e^2 \langle Z\rangle \sqrt{\varepsilon(\omega)}}{m_e^2 c^3 v_{Te}}\mathcal{F}_{col}(\omega) \tag{54}$$

where $\mathcal{F}_{col}(\omega)$ is given by

$$\mathcal{F}_{col}(\omega) = \int_{\frac{\sqrt{z}}{2}}^{\infty} \frac{dx}{x} e^{-(x+\frac{z}{4x})^2} \left(1 + \frac{1 + 2\frac{\omega_{pe}^2}{\omega^2}x^2}{\left[1 + 2\frac{\omega_{pe}^2}{\omega^2}(1+ < Z >)x^2\right] |1 + 2\frac{\omega_{pe}^2}{\omega^2}x^2 W(x)|^2}\right) \tag{55}$$

where $z = \hbar\omega/T$

The absorption coefficient is given in terms of the emitted power (eq.36) and the result is

$$2\gamma_k = \frac{8\sqrt{\pi}}{\sqrt{2}}\sigma_T \frac{e^2 < Z > n_e^2 c^4}{v_{Te}\hbar\omega^3}(1 - e^z)\mathcal{F}_{col}(\omega) \tag{56}$$

When collective effects are neglected ($\mathcal{H} = 1$) the q-integral in (56) can be performed and the function $\mathcal{F}_{col}(\omega)$ of (54) becomes

$$\mathcal{F}_o(\omega) = 2\int_{\frac{\sqrt{z}}{2}}^{\infty} \frac{dx}{x} e^{-(x+\frac{z}{4x})^2} = \int_{\sqrt{\frac{2\hbar\omega}{m_e}}}^{\infty} \frac{e^{-\frac{v^2}{2v_{Te}^2}}}{v_{Te}^2} v \ln\frac{v + v'}{v - v'} dv \tag{57}$$

where v' is given by eq.(49) and the second equality can easily be proved by integrating by parts and using the substitution (52), eq.(54) with $\mathcal{F}_0(\omega)$ is the known non-collective result for electron-ion bremsstrahlung and Maxwellian electrons.

Equation (54) represents one of the main results of this paper and differs significantly from previous attempts such as the treatment of Ichimaru's [14] to include plasma collective effects in bremsstrahlung. In particular equation 9.132 of Ichimaru [14] does not contain $< Z >$, the effective Z value for the plasma, in the denominator. He also has $W(x) = 1$ for all values of x. The result obtained in [14] only includes the screening of the ion field for all electron velocities. This is the effect of permanent shielding irrespective of the electron velocity which is not correct for high electron velocity. The result obtained in the present paper shows that the screening effect is completelly cancelled by the effect of emission from the oscillating cloud and what is left is a new effect of redressing the screening cloud as soon as the electron velocities become larger than the electron mean thermal velocity. The oscillation of the electron cloud is also included in $\left[1+2\frac{\omega_{pe}^2}{\omega^2}(1+<Z>)x^2\right]$ in the denominator. The result of [14] can be written by changeing $\mathcal{F}_{col}$ to $\mathcal{F}_{scr}$ where

$$\mathcal{F}_{scr}=\int_{\sqrt{z}/2}^{\infty}\frac{dx}{x}e^{-(x+\frac{z}{4x})^2}\left(1+\frac{1}{1+2\frac{\omega_{pe}^2}{\omega^2}x^2}\right)$$

Fig.1 shows the difference between the result obtained from Ichimaru [14] and our result for $<Z>=1,4,7$ and 10 (Figures 1a, 1b, 1c, 1d respectively) by plotting

$$f_{col}=\frac{\mathcal{F}_{col}(\omega)}{\mathcal{F}_0\omega)}\qquad f_{scr}=\frac{\mathcal{F}_{scr}(\omega)}{\mathcal{F}_0(\omega)}$$

as function of ω/ω_{pe}.

The solid curve is our case and the dotted line is obtained from equation 9.132 of Ichimaru. In all cases there is a significant difference for small values of $\frac{\omega}{\omega_{pe}}$, as ω/ω_{pe} increases both curves asymptotically approach 1. The main difference is that Ichimaru's result overestimates the value of radiation emitted and thus reduces the opacity more than our result.

2) electron-electron bremsstrahlung.

We now use eq.(22) in eq.(7) and then from eq.(32) we have

$$Q_{ee}(\omega)= \tag{58}$$
$$=\frac{\sqrt{\epsilon(\omega)}e^6}{\pi^2m_e^2c^3}\int f_e(\underline{v})f_e(\underline{v}')d^3vd^3v'd^3qd\Omega_k\delta[\omega-\underline{q}\cdot\underline{v}'-(\underline{k}-\underline{q})\cdot\underline{v}]F_{\underline{k},\underline{q}}(\underline{v},\underline{v}')$$

where

$$F_{\underline{k},\underline{q}}(\underline{v},\underline{v}')=$$

$$=\frac{\frac{1}{2}|\underline{k}\times\underline{q}|^2}{k^2|\epsilon_{\underline{q}\cdot\underline{v}',q}\epsilon_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}|^2}\left|\frac{[1+\sum_i\chi^{(i)}_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}]}{q^2}-\frac{[1+\sum_i\chi^{(i)}_{\underline{q}\cdot\underline{v}',q}]}{|\underline{k}-\underline{q}|^2}\right|^2 \tag{59}$$

Again neglecting the ion susceptibilities (calculated for electron velocities) and taking

$$\epsilon_{\underline{x}\cdot\underline{v},x} \sim \epsilon^{(e)}_{\underline{x}\cdot\underline{v},x} = 1 + \frac{\omega_{pe}^2}{x^2 v_{Te}^2} W(y) \tag{60}$$

for $\underline{x} = \underline{q}$ or $\underline{x} = \underline{k} - \underline{q}$, the normalized parallel components of electron velocities being given again by eq.(40), we find with similar calculations

$$Q_{ee}(\omega) =$$

$$= \frac{\sqrt{\epsilon(\omega)} e^6 n_e^2}{\pi^2 m_e^2 c^3} \int \frac{e^{-y^2}}{\sqrt{\pi}} \int \frac{e^{-y'^2}}{\sqrt{\pi}} dy' \delta(\omega - q\sqrt{2} v_{Te} y' - |\underline{k} - \underline{q}|\sqrt{2} v_{Te} y) F_{\underline{k},\underline{q}}(y, y') d^3 q \tag{61}$$

with

$$F_{\underline{k},\underline{q}}(y, y') = \frac{\frac{1}{2}|\underline{k} \times \underline{q}|^2 |k^2 - 2(\underline{k}\cdot\underline{q})|^2}{k^2 q^4 |\underline{k} - \underline{q}|^4 \left| \left(1 + \frac{\omega_{pe}^2}{q^2 v_{Te}^2} W(y')\right) \left(1 + \frac{\omega_{pe}^2}{|\underline{k}-\underline{q}|^2 v_{Te}^2} W(y)\right) \right|^2} \tag{62}$$

Changing $\underline{q} \to \underline{q} + \frac{1}{2}\underline{k}$ in the q-integral it is then easy to see that (for $\omega \sim ck$) from the conservation law (δ-function) it is

$$k \sim \frac{v_{Te}}{c} q \ll q$$

and we can then neglect k with respect to q; the integral over angles can be done such that

$$\int \frac{(\underline{k}\cdot\underline{q})^2}{k^2} (\underline{k} \times \underline{q})^2 d\Omega_k = \frac{8}{15} \pi k^2 q^4$$

and we finally have

$$Q_{ee}(\omega) =$$

$$= \frac{64}{15} \frac{\sqrt{\epsilon(\omega)} e^6 n_e^2 k^2 v_{Te}}{m_e^2 c^3 \omega_{pe}} \int_0^\infty q^6 dq \int \frac{e^{-y^2}}{\sqrt{\pi}} dy \int \frac{e^{-y'^2}}{\sqrt{\pi}} dy' \frac{\delta[\omega - q\sqrt{2}\omega_{pe}(y + y')]}{|q^2 + W(y)|^2 |q^2 + W(y')|^2} \tag{63}$$

where we have introduced the new variable

$$q = \left(\frac{v_{Te}}{\omega_{pe}}\right) q$$

It can be seen from eq.(53) that compared to $Q_{ei}(\omega)$, $Q_{ee}(\omega)$ is of order $(v_{Te}/c)^2\omega_{pe}/\omega$ and can therefore be neglected.

3) ion-ion bremsstrahlung.

Using eq.(23) we find that the integrand of $Q_{ii}(\omega)$ contains the factor

$$\left|\frac{\chi^{(e)}_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|}}{q^2} - \frac{\chi^{(e)}_{\underline{q}\cdot\underline{v}',q}}{|\underline{k}-\underline{q}|^2}\right|^2$$

Using the same approximations as in previous calculations

$$\chi^{(e)}_{(\underline{k}-\underline{q})\cdot\underline{v},|\underline{k}-\underline{q}|} \simeq \frac{\omega_{pe}^2}{|\underline{k}-\underline{q}|^2 v_{Te}^2}; \qquad \chi^{(e)}_{\underline{q}\cdot\underline{v}',q} \simeq \frac{\omega_{pe}^2}{q^2 v_{Te}^2}$$

the result is zero for the ion-ion bremsstrahlung.

To next order in the susceptibilities, i.e.

$$\chi^{(e)}_{\omega,k} \simeq \frac{\omega_{pe}^2}{k^2 v_{Te}^2}\left(1 + i\sqrt{\pi}\frac{\omega}{\sqrt{2}kv_{Te}}\right)$$

we find

$$\left|\underline{M}^{(ii')}\right|^2 \propto \frac{|y-y'|^2}{v_{Te}^2}$$

where y, y' are defined in eq.(40). When integrated with the distribution functions of the ions to find Q_{ii} the result is $v_{Ti}^2/v_{Te}^2 = m_i/m_e$ smaller then Q_{ei}. The ion-ion bremsstrahlung can therefore be neglected.

5. Discussion of the results

We have succeeded for the first time to give a complete analytical description of collective effects in bremsstrahlung. It is shown that the collective effects in electron-ion collisions can not be described by some factor due to the screening effects as used previously. An important difference between ours and previous results is that we take into account the oscillation of the ion screening cloud. The results then become dependent on the effective ion charge $< Z >$. This dependence is absent in previous work. The partial cancelation in the matrix elements due to the oscillation of the ion cloud diminishes the actual contribution of collective effects in the total bremsstrahlung. The difference with previous results is also that the screening factor (see (48)) contains the electron thermal velocity in W. For all electrons with velocities larger than the average thermal velocity the screening becomes negligible (the stripping effect of [5]).

In the case of a Maxwellian distribution the contribution of the tail thermal electron appears without screening. But even for electrons with velocities much less than the mean thermal velocity the other factor $\left(1+ < Z > \frac{\omega_{pe}^2}{q^2 v_{Te}^2}\right)$ survives and describes the changes due to transition bremsstrahlung. The contribution of collective effects in total bremsstrahlung absorption becomes very important for the frequency close to the electron plasma frequencies. For example for parameters in the solar interior the reduction of the total bremsstrahlung coefficient is about 30%. But the effective frequencies responsible for energy transfer in the solar interior are $\frac{\hbar\omega}{T} \simeq 3.8$ and $\frac{\hbar\omega_{pe}}{T} \simeq 0.2$. The collective effects as can be seen from eq.(55) decrease with ω. We make an accurate numerical investigation of the possible change to the solar opacity due to collective bremsstrahlung corrections found in this paper. The numerics in general need precise calculations of complex integrals with high accuracy. Our results show that the change in the solar opacity due to collective bremsstrahlung is only -0.2% which is less than in previous calculations. The decrease of the total change in opacity is due to the correct treatment of collective effects including the effect of particle redressing (stripping of the shielding cloud). For a complete description of all collective effects in the solar interior that we are studying the result obtained is important as a first proof that the collective effects in bremsstrahlung in the solar interior are small. Other results concerning a substantial increase of ion-ion and electron-electron bremsstrahlung should also be included in further applications of the collective effects in bremsstrahlung. An important point is that the ion-ion bremsstrahlung is proportional to Z^4. For high Z plasmas this bremsstrahlung can exceed the usual one.

The main difference between our results and those of previous calculations which only includes screening of the ion field for all electron velocities that do not take into account the fact that screening decreases for energetic particles. In fact for electron velocities larger than the thermal velocity these electrons are not screened. Previous results calculated using static screening overestimate the effect of bremsstrahlung resulting in a smaller opacity. Our result which takes into account the velocity dependence on screening. There is a significant difference for small values of $\frac{\omega}{\omega_{pe}}$ as one would expect since collective effects are more important.

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Figure Captions

Figure 1. (a,b,c,d) represent the scattering including the full collective effects f_{col} solid line and including only the screening of the ion field, f_{scr} dashed line, for different effective values of $< Z >$ a)$< Z >= 1$, b) $< Z >= 4$, c) $< Z >= 7$, and d) $< Z >= 10$. The scattering is normalized to $\mathcal{F}_0(\omega)$ when collective effects are neglected. the result of Ichimaru's corresponds to the dashed curve ie for screening.

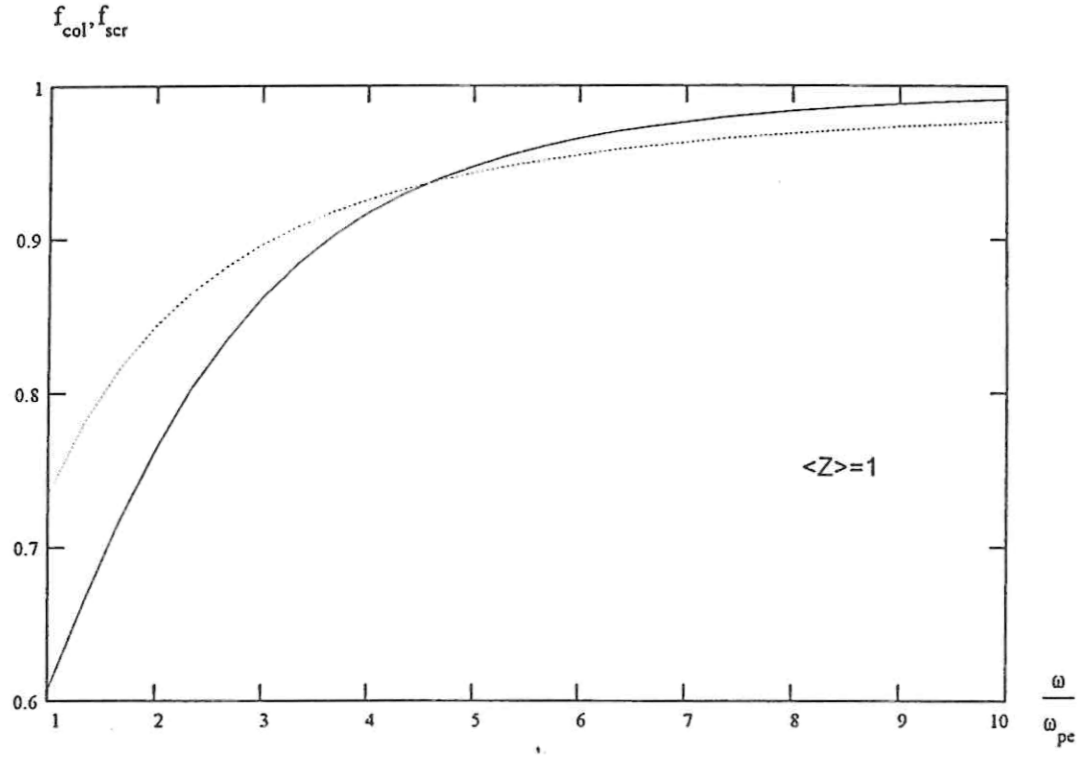

Fig 1a

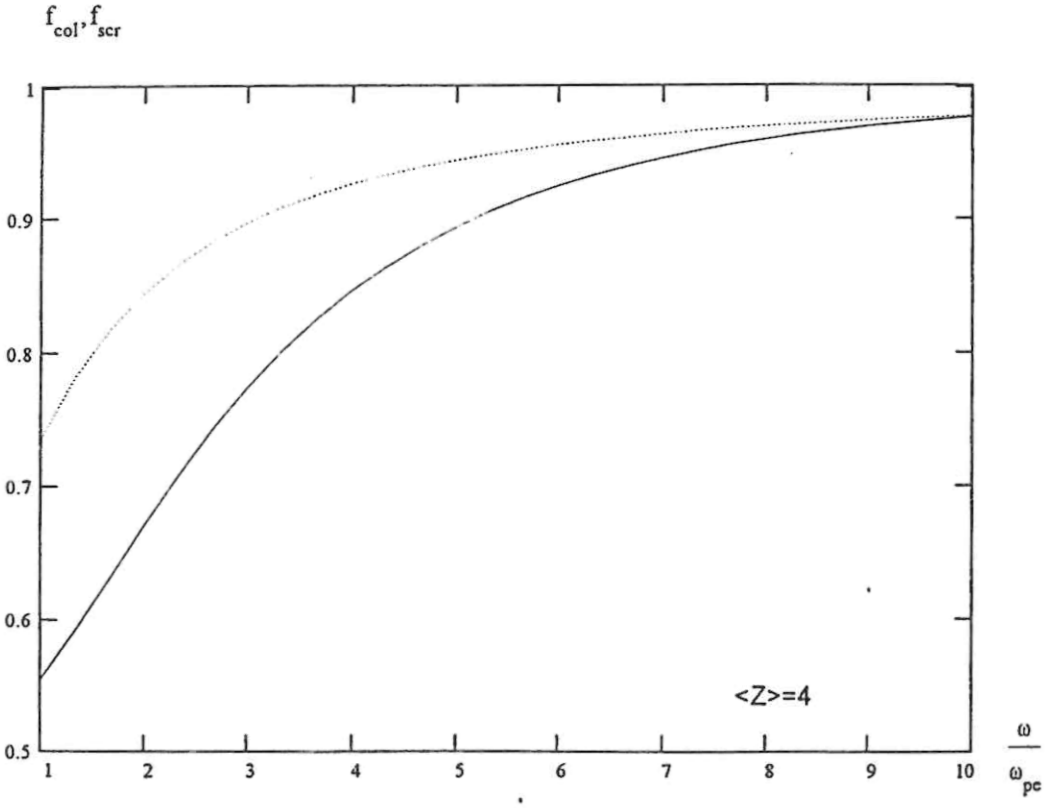

Fig 1b

$1+\frac{\wedge}{3}$

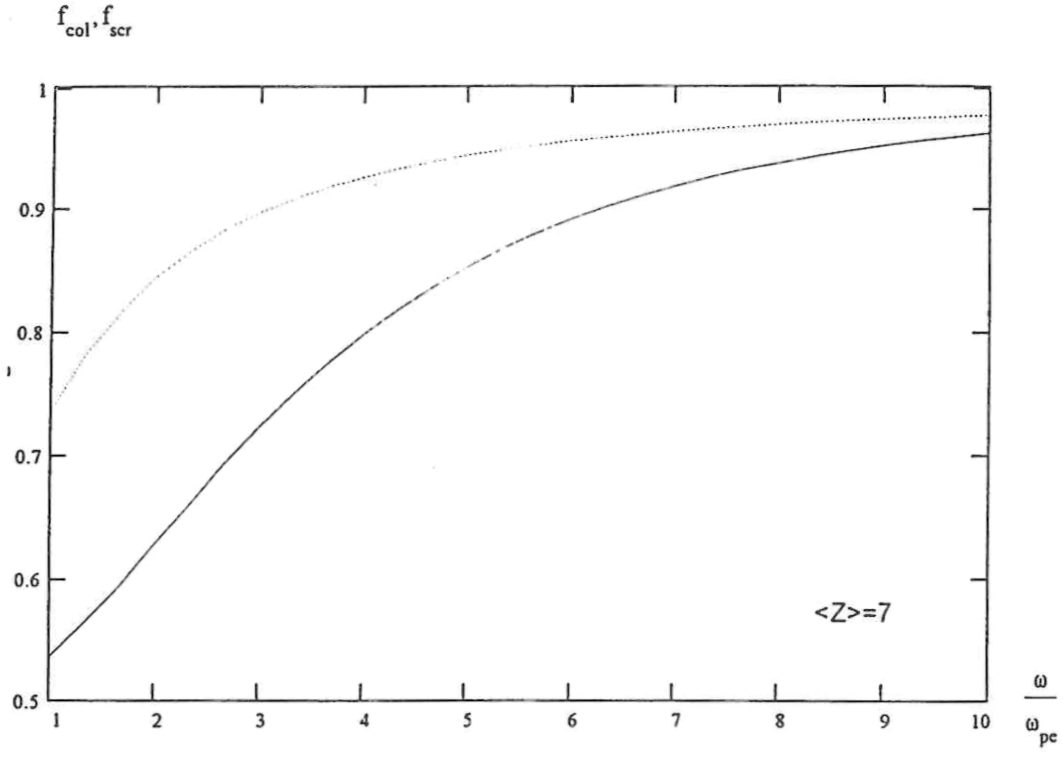

Fig 1c

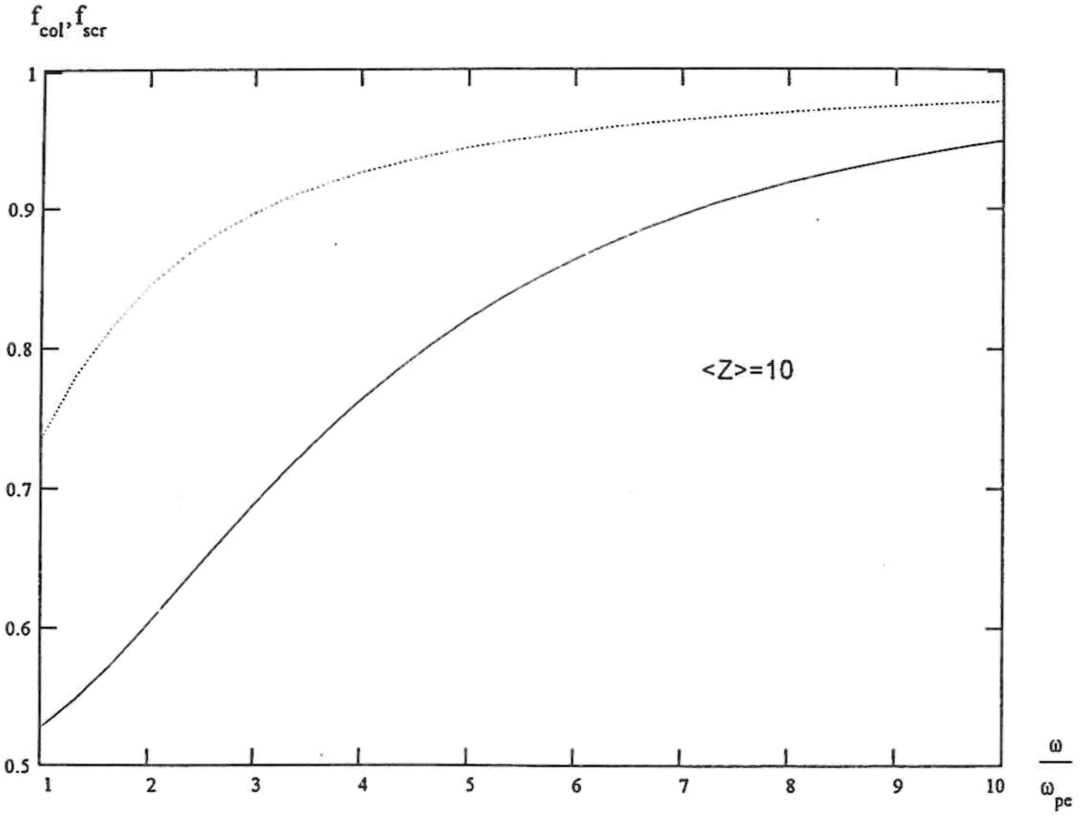

Fig 1d