

## NEW, HIGH POWER, SCALING, FFAG DRIVER RING DESIGNS

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### Abstract

High power driver rings are examined, using new FFAG designs, based on cells of five, symmetrical, scaling pumplet magnets. Apertures are minimized by using large, betatron phase shifts per cell, typically  $\mu_h \sim 280^\circ$  and  $\mu_v \sim 130^\circ$ . Key aspects are the lengths of the long straight sections, particularly if  $H^-$  charge exchange injection is required. Rings are considered for ISIS upgrades and Neutrino Factory proton and muon drivers, both with and without insertions.

### INTRODUCTION

FFAG rings of pumplet cells have previously been thought better in a non-scaling than a scaling form [1]. However, scaling pumplets may be simpler and have smaller apertures if operation is in a higher stability region of Hill's equation [2], at betatron phase shifts per cell of  $\mu_h \sim 280^\circ$  and  $\mu_v \sim 130^\circ$ . Scaling triplet cells, in such a mode, using similar non-linear magnet field profiles, are found to have *beta*-functions which are significantly larger than those of the pumplet cells.

The cell forms are: O f(+) o D(-) o F(+) o D(-) o f(+) O (when scaling) and : O d(-) o F( $\pm$ ) o D(+ ) o F( $\pm$ ) o d(-) O (when non-scaling), where the  $\pm$  refer to bend directions. Long (OO) and short (o) straight sections interleave with vertical focusing (D) and defocusing (f, F) units. Scaling cells have the same normalized field gradients and bend radius in all magnets, whereas isochronous and non-isochronous, non-scaling cells do not, having more complex, non-linear magnet field profiles.

Field gradients and unit spacings may be varied to adjust the cell tunes. To minimize misalignment and field error effects, the scaling cells are set with the tunes above the fourth-order betatron resonances,  $4q_v = 1$  and  $4q_h = 3$  ( $\mu_h = 270^\circ$ ) but below the fifth-order,  $5q_h = 4$ . The stable area is wide, but the vertical *beta*-values,  $\beta_v$ , need optimization, as the clearances required for extraction set the acceptances for injection above the typical values used in synchrotrons.

Fields are modified from traditional scaling forms, as discussed later in the report. Also described are the ways in which various sequences of long and short straight sections may be realized in rings of pumplet cells. More non-linear cell resonances may be excited than in a typical high current linac focussing structure.

### SCALING PUMPLET CELLS

A scaling pumplet cell has the sequence of combined-function magnets: O f(+) o D(-) o F(+) o D(-) o f(+) O, where (+) and (-) represent normal and reverse bending and (OO) and (o) are the long and short straight sections. The cell has mirror symmetry about the centre of the F(+).

The traditional magnet field profile for a scaling cell and its local, normalized field gradient are described by:  $B_{y=0} = B = B_o (1 + x/r_o)^K$ , and  $B'/B\rho_o \approx K/\rho_o(r_o + x)$ , where  $x$  defines a radial offset from a  $B_o$  reference orbit, at distance  $r_o$  from the centre of the FFAG ring, and the common values of the parameter  $K$  define the magnetic field gradients.

Parallel edged magnets are used, offset relative to one another. Vertical guide fields are then modified, from the traditional scaling form given above, to an exponential form, to produce a smaller variation of  $B'/B\rho_o$  values over the poles of the high  $k/\rho_o$  ( $=K/r_o$ ), parallel edged magnets:  $B_{y=0} = B = B_o \exp(kx/\rho_o)$  and  $B'/B\rho_o \approx k/\rho_o^2$ .

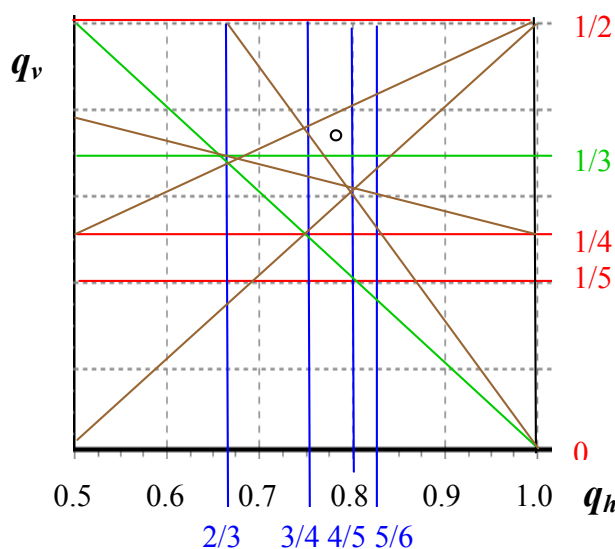


Figure 1: Pumplet cell,  $q_h, q_v$  resonance diagram.

Cell tunes for Figure 1 are at  $q_h = 11/14$  and  $q_v = 5/14$ . The adjacent resonances are shown in blue (horizontal), red (vertical), brown (for coupled, sextupole ( $q_h - 2q_v = 0$ ) or octupole ( $2q_h - 2q_v = 1$ ) or for decapole ( $q_h + 4q_v = 2$ ,  $3q_h + 2q_v = 3$ )) and in green ( $q_h + q_v = 1$ ,  $3q_v = 1$ ). The two cell resonances in green are due to the combined effect of vertical orbit errors and the sextupole or octupole field components in the pumplet magnets.

Tunes are set to optimize lattice parameters, avoid as many of the cell and ring resonances as possible, and also provide straights, approximately two cell lengths apart, for the  $3\pi$ -horizontal and  $\pi$ -vertical, orbit bump units. Typical choices that are provided for the cell tunes are: ( $q_h = 11/14$ ,  $q_v = 5/14$ ) or ( $q_h = 10/13$ ,  $q_v = 5/13$ ). These have the form of a ratio of two integers, as this is often found advantageous in the lowering of cell resonance excitations.

Due to the strong focusing, lattice parameters for small beam amplitudes exhibit low  $\beta$ - $h$  and  $\beta$ - $v$  values near the cell centre and large, maximum to minimum,  $\beta$ -ratios. On lowering the tunes, there is an increase of  $\beta$ -values and ratios, but there is little initial change in the values of momentum dispersion.

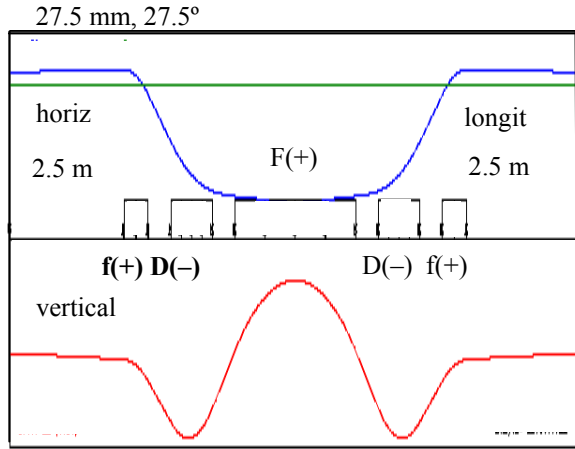


Figure 2: Typical beam amplitudes in a pumplet cell.

Approximate beam amplitudes may be found with a linear beam envelope code, by including quadrupole and dipole magnet fields but omitting all the higher multipole field components. In Figure 2 are given approximate, rms amplitudes for a 800 MeV beam with rms emittances of  $27(\pi)$  mm mr rms, in a pumplet cell of length 12.5664 m. Linear space charge tune depressions of  $\sim 0.25$  do not significantly affect the beam envelopes.

The ratio,  $B_y/B_o = k/\rho_o = K/r_o$ , is a better indicator of the strength of focusing of the pumplet cell than is either of the field indices,  $k$  or  $K$ . Beam dynamics for the exponential pumplet field is similar around each closed orbit, as all normalized dipole, quadrupole and off-axis multipole fields are given by:

$$B_y/B_o = 1 + (k/\rho_o) x + (k/\rho_o)^2 (x^2 - y^2)/2 + (k/\rho_o)^3 (x^3 - 3xy^2)/6 + (k/\rho_o)^4 (x^4 - 6x^2y^2 + y^4)/24 + \dots$$

$$B_x/B_o = 0 + (k/\rho_o) y + (k/\rho_o)^2 (x y) + (k/\rho_o)^3 (3x^2y - y^3)/6 + (k/\rho_o)^4 (x^3y - xy^3)/6 + \dots$$

Beam dynamic apertures are set by the excitations of resonances, caused by alignment and field errors and the amplitude-dependent tune-shifts. Major areas requiring study are sensitivity to errors, beam losses for multi-turn  $H^-$  injection, beam loss protection, effects of space charge fields and the component cost and reliability.

The Zgoubi code [3] is proposed for the beam tracking needed to study effects of the large non-linear fields. Of interest is the  $h$ - $v$  coupling due to the fixed and adjustable orbit bumps used for  $H^-$  injection and beam collimation. Coupling while bump magnet currents are collapsed during a machine cycle may affect foil and collimator interceptions and painting of injected beam distributions.

## LONG STRAIGHT SECTIONS

A ring's circumference may become too large if each cell has a long straight. It is thus of interest to see if cells may be designed with identical pumplet magnets but with different straight section lengths. This possibility has been examined in three ways, and examples are given for the driver designs outlined later.

Firstly, a ring of regular pumplet cells is modified by extending the straight section in a few of the cells (two, for example). Lattice parameters are enhanced but the increases may be reduced by minor adjustments of the short inter-magnet spaces.

Next, the magnet sets from two adjacent cells are moved towards each other to form two different length straights, and this is repeated around the ring. The dual pumplet cells may also have the ripple of the  $\beta$ -values reduced by inter-magnet space adjustments.

Finally, bending pumplet cells of low dispersion are arranged to form insertions with integer tunes, for automatic arc cell matching. The cell tunes are chosen as outlined previously. Arc and insertion cells have different length straights, but they may be arranged to have the same  $q_h$  and  $q_v$  values, by adjusting the tune-sensitive, inter-magnet spaces. A small ripple for the  $\beta$ -values results over the insertion.

## ISIS UPGRADE, 0.8-3.2 GEV $H^+$ DRIVER

The main parameters for a potential, ISIS upgrade FFAG proton driver are a 50 Hz pulse repetition rate, a 0.8 to 3.2 GeV energy range, a 2 to 5 MW average beam power, a 52 m mean ring radius (twice that of ISIS ring), a  $h=4$ , harmonic number rf system, and  $2 \cdot 10^{14}$  accelerated protons per pulse. Table 1 gives possible parameters.

Table 1: Data for 26 Pumplet Cells with 5 m Straights

Mean and bend radius (m)	52.000,	7.92846
Short and long straights (m)	0.500,	5.00236
Length, bend angle for f (m, r)	0.540,	0.06810
Length, bend angle of D (m, r)	0.912,	-0.11502
Length, bd. angle of F/2 (m, r)	1.330,	0.16775
$B'/B = k/\rho_o$ ( $m^{-1}$ ), $B'/B\rho_o$ ( $m^{-2}$ )	$\pm 8.827$ ,	$\pm 1.11335$
Cell betatron tunes ( $q_h, q_v$ )	0.778,	0.36120
Ring betatron tunes ( $Q_h, Q_v$ )	20.215,	9.39000
Gamma-t, dispersion $D_h$ (m)	21.425,	0.11300
Maximum $\beta$ - $h, \beta$ - $v$ in f (m)	18.261,	12.91800
Maximum $\beta$ - $h, \beta$ - $v$ in D (m)	5.979,	25.07500
Maximum $\beta$ - $h, \beta$ - $v$ in F (m)	1.125,	7.96800
Minimum $\beta$ - $h, \beta$ - $v$ in F (m)	0.781,	1.02200
Orbit sep'n. (m), max.B (T)	0.113,	1.69570
V inj, ext accept ( $\pi$ mm mr)	760.000,	100.00000

The Table 1 data is for a suitable, scaling FFAG ring of 26 identical pumplet cells, each with a 5 m long straight section. These allow direct fast injection of the bunches from the ISIS ring, at the required rate of 50 Hz.

The maximum  $\beta$ - $\nu$  in the D magnet is a key item for comparing cell designs, as beam extraction, and hence injection, requires a large vertical acceptance. Though it is large, the maximum  $\beta$ - $\nu$  of 25.08 m is less than half that obtained for similar rings designed with high- $k$ , scaling cells of combined-function, triplet magnets.

The scaling ring has to be modified, however, to allow for multi-turn, charge exchange injection from a 0.8 GeV,  $H^-$  upgrade linac. An 8 m straight is required and, for this, the second and the first schemes that have been outlined earlier are considered.

A ring of 26 cells may have 13 dual pumplet cells and 26 long straights, which alternate in length around a set circumference. For 3 and 7 m straights,  $\beta$ - $\nu$  rises from 25.1 to 30.0 m and  $\beta$ - $h$  from 18.3 to 25.5 m, but small adjustments of the inter-unit spaces and  $k$ -value reduce the maximum  $\beta$ - $\nu$  values to 26.8 m and the  $\beta$ - $h$  to 24.8 m. For alternate, 2 and 8 m straights, however, the resulting  $\beta$ -values are not acceptable.

In the other scheme, the straights are shortened in 24 of the cells, increased to 8 m in two, diametrically opposite cells, and the tunes are re-adjusted. Also required are increases from 0.5 to 0.503 and 0.547 m, respectively, in spacing between the two nearest and two, next-nearest magnets in the cells next to the 8 m straights. For the mean radius of 52 m needed, the final ring requires 24, 4.7424 m straights and two 8.022 m straights. Peak  $\beta$ - $\nu$  is 26.0 m, peak  $\beta$ - $h$  is 19.7 m, and modified ring tunes are  $Q_v = 9.765$  and  $Q_h = 20.386$ . A scaled-down model for this preferred solution is outlined next, and is followed by outlines for Neutrino Factory proton and muon drivers.

### MODEL FOR A $H^-$ INJECTION STUDY

A 40 MeV, ten cell ring of scaling pumplet cells, with two straights of 5.0 m and eight of 2.6 m, is proposed for error sensitivity and  $H^-$  charge exchange injection studies. Cell tunes are retained at the previous values, but the bend angles have to increase due to the fewer number of cells. The model has a circumference of 60.78 m; the orbit fields are 0.6995 T, and the ring tunes, for the ten cell design, are  $Q_h = 7.774$  and  $Q_v = 3.700$ .

Orbit lengths in the f, D and F units are, respectively, 0.220, 0.3396 and 1.0688 m, and  $B'/B\rho_o$  is 6.6724  $m^{-2}$  for the design orbit. The gamma-transition value is 9.233, and the  $D_h$  functions vary from 0.106 to 0.117 m. The maxima of the  $\beta$ - $\nu$  vary between 7.98 and 13.8 m and those of the  $\beta$ - $h$  between 6.00 and 7.47 m.

### NEUTRINO FACTORY, $H^+$ DRIVER

Required for the protons are a 3-10 GeV energy range, a 50 Hz repetition rate and a 4 MW of beam power. There are 16 dual pumplet cells with superconducting magnets in a 400.792 m, circumference ring (one half that of an earlier design). The long straight sections alternate in length from 4 to 6 m; the orbit fields are 1.295 to 3.687 T; the cell tunes are  $q_h = 0.65$ ,  $q_v = 0.32$ , and the ring tunes are  $Q_h = 20.80$  and  $Q_v = 10.24$ .

Orbit lengths in the f, D and F units are, respectively, 0.500, 0.8924 and 2.720 m, for bend radii of 9.85589 m. The common  $B'/B\rho_o$  value is  $\pm 1.114525 m^{-2}$ , for a  $k$  value of 108.3. The maximum of  $\beta$ - $\nu$  is 27.5 m and of  $\beta$ - $h$  is 26.0 m. Gamma-transition has a value of 26.47, and orbit separations are 0.0953 m

### NEUTRINO FACTORY MUON DRIVER

A 6.2-16 GeV, 684.71 m circumference,  $\mu^\pm$  ring has two arcs of 20 cells with 2.5 m long straights and two, 14 cell insertions with 5.5 m long straights. Superconducting magnet orbit fields are 1.28 to 3.26 T. All cell tunes are set at  $q_h = 11/14$  and  $q_v = 5/14$ , so the arcs are matched and the ring tunes are  $Q_h = 53.429$  and  $Q_v = 24.286$ .

Orbit lengths of the f, D and F units are, respectively, 0.41, 0.69544 and 2.0928 m. Bending radii are 16.471 m and  $B'/B\rho_o$  are  $\pm 1.16943 m^{-2}$ . The maxima of  $\beta$ - $\nu$  and  $\beta$ - $h$  are, respectively, 22.3 and 16.6 m in the arcs, and 26.2 and 19.4 m in the insertions. Acceleration for half a phase oscillation is above the transition-gamma of 55.43.

### REFERENCES

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