Science and Engineering Research Council
**Rutherford Appleton Laboratory**
CHILTON, DIDCOT, OXON, OX11 0QX



RL-83-079

# Interaction between the Precipitating Electrons and the Auroral Plasma



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August 1983

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## ABSTRACT

The excitation of Langmuir turbulence due to the presence of a directed high energy electron stream in the aurora plasma can modify the behaviour of ion acoustic waves and lead to a modulational type instability. The effect of the turbulence on the plasma is to provide a low frequency force, the ponderomotive force, this force interacts with the low frequency modes changing their dispersion relations. The high frequency turbulence undergoes a modulational type instability leading to a bunching of the Langmuir turbulence in space. An effective temperature $T_{eff}$ due to Langmuir turbulence can be derived.

## INTRODUCTION

One of the important features of the aurora plasma is the existence of a high energy electron beam with a velocity directed along the ambient magnetic field $B_o$. Such a situation of streaming electrons, with an average directed velocity $v_o > v_{Te}$, where $v_{Te}$ is the thermal velocity of the plasma with a velocity spread $\Delta v < v_o$, leads to a beam - plasma instability (Krall and Trivelpiece, 1973) leading to the excitation of Langmuir waves with wave numbers around $k_o \approx \omega_{pe}/v_o$ in an interval $\Delta k/k_o \approx \Delta v/v_o$ and with a growth rate :

$$\gamma_g \approx \frac{\pi}{2} \frac{n_B}{n} \left( \frac{v_o}{\Delta v} \right) \omega_{pe}$$

where $n_B$ and $n$ are the densities of the beam and the aurora plasma and $\omega_{pe}$ is the plasma frequency. The excited waves are directed along the direction of the streaming electrons. According to quasi-linear theory (Krall and Trivelpiece 1973) the instability is driven by the portion of the beam with a positive slope in the velocity distribution function ($\partial f/\partial v > 0$). The particles in this region lose energy to the waves, resulting in a flattening of the distribution function to form a plateau with $\partial f/dv = 0$. The energy of the plasma waves at this point is given by

$$W_\ell \simeq n_B m_e v_o \Delta v \qquad (1)$$

In the aurora which has definite boundaries the maximum value of the wave energy density is greater than that given by the usual quasi-linear theory equation (1). The reason for the increase is caused by the slow group velocity $v_{gr}$ ($= 3\, v_{Te}/v_o \ll v_o$) of the waves generated by the beam. Fast electrons then interact not only with the waves they themselves generate but also with waves generated earlier by other particles, this creates an oscillation pile-up resulting in a substantial increase in the level of the maximum wave energy density over that expected from simple quasi-linear theory. Tsytovich (1970) has shown that the maximum wave energy density in a plasma with finite boundaries is

$$W_{\ell MAX} = W_\ell \, v_o/v_{gr} \qquad (2)$$

The diffusion of particles in velocity space is described by the quasi-liner equations

$$\begin{aligned} \frac{\partial \mathcal{E}_k}{\partial t} &= \gamma \mathcal{E}_k \\ \frac{\partial f}{\partial t} &= \frac{\partial}{\partial v} D \frac{\partial f}{\partial v} \end{aligned} \qquad (3)$$

where $\gamma = \pi \dfrac{\omega_{pe}^2 \omega_R}{k^2} \left.\dfrac{\partial f}{\partial v}\right|_{v=\omega_r/k}$

$\omega_R$ is the resonant frequency ($= kv_{PARTICLE}$) and D is the diffusion coefficient along the magnetic field given by

$$D(v) = \frac{4\pi\omega_{pe}^2}{mnv}\,\mathcal{E}_k(k)\Big|_{k=\omega_r/v}$$

with $W_l \approx \int\varepsilon_k dk$, and $\varepsilon_k$ is the energy density per unit wavenumber.

The turbulent wave energy density in a typical aurora plasma can be very strong. From the data obtained by Bryant et al (1983) of a rocket flight through the aurora with $n_B \simeq 50\ \mathrm{cm}^{-3}$, $v = 6 \times 10^7$ m/sec, $\Delta v_o = 1.5 \times 10^7$ m/sec, the normalized wave energy density $W_l/nKTe \simeq O(1)$, which is an extremely strong wave capable of producing a number of nonlinear effects.

Coupling of the Langmuir turbulence to plasma modes.

Vedenov and Rudakov (1965) have demonstrated the possibility of Langmuir turbulence coupling to low frequency plasma modes the so-called modulational instability. Most of the previous papers on this subject considered to development of the Langmuir turbulence in configuration space in particular the break-up of the turbulence into discrete wave-packets called solitons and the creation of plasma density cavities or cavitors (Zhakharov 1972; Rudakov 1973; Nicholson and Goldman 1976, Bingham and Lashmore-Davies 1979). In this paper we shall study the effect of broad band Langmuir turbulence on low frequency plasma waves such as ion-acoustic waves using an approach similar to the one developed by Rudakov (1967) that is describing the high-frequency Langmuir turbulence by the "kinetic equation" for the action density occupation numbers $N_k$

$$N_k = \frac{\mathcal{E}_k}{\omega_k} \quad ; \quad \mathcal{E}_k = \frac{1}{8\pi}|E_k|^2$$

where $E_k$ is the electric field strength.

The interaction between Langmuir waves and ion-acoustic waves is due to the ponderomotive force. The expression for this force is obtained from the fluid equations and is given by

$$f = -\frac{\nabla e^2|E_k|^2}{2m_e\omega_{pe}^2} \qquad (5)$$

this is the mean force acting on an electron in the field of a Langmuir wave representing a gradient of the radiation pressure.

The equation describing the coupling between the high frequency Langmuir turbulence and the low frequency ion-acoustic modes is the linearized Liouville equation, describing the number of plasmons (Vedenov (1967))

$$\frac{\partial N_k}{\partial t} + \frac{\partial \omega_k}{\partial k} \cdot \nabla N_k - \frac{\partial \omega_k}{\partial \underline{x}} \cdot \nabla_k N_k \qquad (6)$$

where $N_K$ (= $\frac{|E_k|^2}{4\pi\omega_k}$ ) is the wave action or plasmon number density.

In a homogeneous plasma changes in the frequency $\omega_k$ arise only from the ion-acoustic wave since

$$\frac{\partial \omega_k}{\partial x} = \omega_k \frac{1}{n} \frac{\partial \delta n}{\partial x} = -\omega_k \underline{\nabla}(\underline{\nabla} \cdot \underline{\xi}) \qquad (7)$$

where $\delta n$ represents the ion-acoustic density perturbation and $\xi$ is the fluid displacement. The equation describing the ion-acoustic mode in the presence of Langmuir turbulence is obtained from the fluid equations and is given by

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2 \nabla^2\right) \delta n = -\frac{\omega_{pe}}{2m_i} \nabla^2 \sum_k \frac{|E_k|^2}{4\pi\omega_k} \qquad (8)$$

If we include ion wave Landau damping , $\gamma_s$, this equation becomes

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2 \nabla^2 + \gamma_s \frac{\partial}{\partial t}\right) \delta n = -\frac{\omega_{pe}}{2m_i} \nabla^2 \sum_k \frac{|E_k|^2}{4\pi\omega_k} \qquad (9)$$

This density perturbation causes a change in the action density $N_k$ which can now be written as $N_{k_o} + \delta N_k$ where $N_{k_o}$ is the equilibrium value and $\delta N_k$ is the change due to the presence of the ion wave. Using this expression for $N_k$ the kinetic equation for $\delta N_k$ is given by

$$\frac{\partial \delta N_k}{\partial t} + \frac{\partial \omega_k}{\partial \underline{k}} \cdot \frac{\partial \delta N_k}{\partial \underline{x}} - \frac{\partial \omega_k}{\partial \underline{x}} \cdot \frac{\partial N_{k_o}}{\partial \underline{k}} = 0 \qquad (10)$$

Using equation (7) we can write equation (9) as

$$\frac{\partial^2 \xi}{\partial t^2} - c_s^2 \underline{\nabla}(\underline{\nabla} \cdot \xi) + \gamma_s \frac{\partial \xi}{\partial t} = -\frac{\omega_{pe}}{\rho} \underline{\nabla} \sum_k \delta N_k \qquad (11)$$

where $\rho = m_i n$ is the mass density of the fluid Equations (10) and (11) describe self consistently the interaction of high frequency Langmuir plasmons and low frequency ion-acoustic waves. To determine the effect of Langmuir plasmons on the ion acoustic mode we linearize equation (10) by assuming $\delta N_k$ varies as $e^{i(qx - \omega t)}$ yielding

$$\delta N_k = \frac{i\,\omega_{pe}}{\omega - \underline{q} \cdot \frac{\partial \omega}{\partial \underline{k}}} (\underline{q} \cdot \underline{\xi}) \left( \underline{q} \cdot \frac{\partial N_{k_o}}{\partial \underline{k}_o} \right) \qquad (12)$$

Substituting this expression for $\delta N_k$ in equation (11) and changing the summation to an integral over k we obtain the following equation describing the ion acoustic mode

$$\left( \frac{\partial^2}{\partial t^2} - c_s^2 \nabla\nabla + \gamma_s \frac{\partial}{\partial t} \right) \xi = \frac{\omega_{pe}^2}{\rho} q^2 \xi \int \frac{\underline{q} \cdot \frac{\partial N_{k_o}}{\partial \underline{k}_o}}{\omega - \underline{q} \cdot \frac{\partial \omega}{\partial \underline{k}}} \, d\underline{k} \qquad (13)$$

For the case where $\zeta$ varies as $e^{i(q \cdot x - \omega t)}$ the dispersion relation for the ion-acoustic wave is given by :

$$\omega^2 - c_s^2 q^2 - i \gamma_s \omega - \frac{\omega_{pe}^2}{\rho} q^2 \int \frac{\underline{q} \cdot \frac{\partial N_{k_o}}{\partial \underline{k}}}{\omega - \underline{q} \cdot \frac{\partial \omega}{\partial \underline{k}}} \, d\underline{k} = 0 \qquad (14)$$

The effect of the Langmuir turbulence on the propagation of the ion acoustic wave is to produce an extra pressure (radiation pressure of the plasmons) term in the plasma fluid. This extra pressure term can be described in terms of an effective temperature $T_{eff}$ given by

$$K T_{eff} = \frac{\omega_{pe}^2}{n} \int \frac{\underline{q} \cdot \frac{\partial N_{k_0}}{\partial \underline{k}}}{\omega - \underline{q} \cdot \frac{\partial \omega}{\partial \underline{k}}} d\underline{k} \qquad (15)$$

and an effective pressure = n $KT_{eff}$, K is Boltzann the constant. This term is in general complex with the singularity being treated according to the Landau rule with

$$\frac{1}{\omega - \underline{q} \cdot \frac{\partial \omega}{\partial \underline{k}}} = P \frac{1}{\omega - \underline{q} \cdot \frac{\partial \omega}{\partial \underline{k}}} - i\pi \delta\left(\omega - \underline{q} \cdot \frac{\partial \omega}{\partial \underline{k}}\right) \qquad (16)$$

Using this expression the dispersion relation (14) describing the propagation of ion acoustic waves in the presence of Langmuir turbulence becomes

$$\omega^2 - q^2 \left( KT_e - KT_{eff} \right) / M_i - i \left( \gamma_s - \gamma_L^{\pm} \right) \omega = 0 \qquad (17)$$

$\gamma^{\pm}$ is given by the imaginery part of equation (16) becomes

$$\gamma_L^{\pm} = \pi \frac{\omega_{pe}^2}{\rho c_s} q \int \left( \underline{q} \cdot \frac{\partial N_{k_0}}{\partial \underline{k}} \right) \delta\left( \pm q c_s - \underline{q} \cdot \frac{\partial \omega_k}{\partial \underline{k}} \right) . d\underline{k} \qquad (18)$$

and represents growth (+) or damping (-) of the ion acoustic mode in the presence of Langmuir turbulence. The analogy with the usual Landau damping (growth) of waves on particles is obvious. The similarity is even more evident if we write the equation for the low frequency density perturbation in terms of the electron and ion susceptibility where, normally it is (Fried and Conte 1961)

$$\left( 1 + \chi_e + \chi_i \right) \zeta = 0 \qquad (19)$$

where $\chi_e$ and $\chi_i$ are the electron and ion susceptibilities given by

$$\chi_e = \frac{\omega_{pe}^2}{k^2}\int_c \frac{\underline{k}\cdot\frac{\partial f_e}{\partial \underline{v}}}{\omega - \underline{k}.\underline{v}}\, d^3\underline{v} \quad ; \quad \chi_i = \frac{\omega_{pi}^2}{k^2}\int_c \frac{\underline{k}\cdot\frac{\partial f_i}{\partial \underline{v}}}{\omega - \underline{k}.\underline{v}}\, d^3 v \qquad (20)$$

In the presence of Langmuir turbulence we can introduce a new susceptibility $\chi_W$ which is due to Langmuir turbulence (- or any other type of wave turbulence) equation (19) becomes

$$\left(1 + \chi_e + \chi_i + \chi_w\right)\zeta = 0 \qquad (21)$$

where

$$\chi_w = \frac{\omega_{pe}^2}{\omega^2}\frac{q^2}{\rho}\int \frac{\underline{q}\cdot\frac{\partial N_{k_o}}{\partial \underline{k}}}{\omega - \underline{q}\cdot\frac{\partial\omega}{\partial \underline{k}}}\, d\underline{k} \qquad (22)$$

is the plasmon susceptibility due to the prescence of plasmon turbulence with a distribution $N_{k_o}$.

As an example assume $N_{k_o} = k^3 N_o \delta\,(k-k_o)$, substituting this into (15) and integrating we obtain the dispersion relation :

$$\omega^2 - q^2 c_s^2\left(1 + \frac{3q^2\omega_{pe} N_o k_o^3}{\rho\left(\omega - \underline{q}\cdot\frac{\partial\omega}{\partial \underline{k}}\right)^2}\right) = 0$$

**DISCUSSION**

We have demonstrated the effects of Langmuir turbulence on the propagation of ion-acoustic waves. In particular we found that the effect of turbulence could be described by a wave susceptibility $\chi_W$ and its effect on ion-waves could be described in terms of an effective temperature $T_{eff}$. These results will have some bearing on laser scattering in laboratory plasmas where turbulence is induced and also in incoherent scattering experiments in the ionosphere where turbulence is produced by the precipitating aurora electrons. In particular incoherent scatter radars such as Eiscat should be able to monitor the effects of turbulence on ion acoustic modes by observing changes in the form factor.

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