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COMPARATIVE SYNCHROTRON RADIATION SPECTRA

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1. INTRODUCTION

In designing a purpose built accelerator for synchrotron radiation use it is often difficult to determine which accelerator parameter can be most suitably adjusted to produce a given effect on the spectrum of the synchrotron radiation. In this note it is shown that when the spectrum is expressed in terms of the number of photons emitted into a bandwidth of a fixed fractional size and also in terms of the instantaneous circulating beam current then a universal spectral curve can be generated which is only dependent on the accelerator energy E and a wavelength ratio λ/λ_c (which will be defined later). This spectrum can be applied to an accelerator with any combination of overall radius and magnetic bending radius and also to special insertions ('wigglers') of any field strength.

This universal spectrum is then used to produce a comparison between the synchrotron radiation spectra of several contemporary accelerators.

2. THEORETICAL FORMULATION OF SPECTRA

The spectrum of a source of synchrotron radiation may be calculated from a formula given by Schwinger. (See ref. 1). In terms of the wavelength of the emitted radiation this may be written as:

$$P(\lambda).d\lambda = \frac{3^{5/2}}{16\pi^2} \cdot \frac{e^2 c}{R^3} \cdot \left(\frac{E}{m_0 c^2}\right)^7 \left(\frac{\lambda_c}{\lambda}\right)^3 \cdot \int_{\lambda_c/\lambda}^{\infty} K_{5/3}\,(u)\,du.\,d\lambda \qquad (1)$$

cms^{-1}; in cm; e is in esu units

where $P(\lambda)$. $d\lambda$ is the instantaneous power flow per orbiting electron at

$e_{(esu)} = 10\,e_{(C)}\,c_{(ms^{-1})}$

= energy radiated/sec by an e^- over 4π steradians in a wavelength range $d\lambda$ centred at λ. (Lowe et al) a wavelength λ into a wavelength band $d\lambda$. The electron energy is E and the bending radius of the orbit, which is assumed to be completely circular, is R.

The spectrum is characterised by a wavelength λ_c given by:

$m_0c^2 = 0.511\,\text{MeV}$

SRS, $\lambda_c = 3.88\,\text{Å}$

R = bending radius.

$$\lambda_c = \frac{4\pi R}{3} \cdot \cdot \left(\frac{m_0 c^2}{E}\right)^3 \qquad (2)$$

It is convenient to substitute the universal function $G(^{\lambda}/_{\lambda_c})$ in eqn. 1, where:

$$G(^{\lambda}/_{\lambda_c}) = \left(\frac{\lambda_c}{\lambda}\right)^3 \int_{\lambda_c/\lambda}^{\infty} K_{5/3} \; (u) \; du \qquad (3)$$

and $K_{5/3}$ is a modified Bessel function.

The universal function $G(^{\lambda}/_{\lambda_c})$ depends only upon the wavelength ratio $^{\lambda}/_{\lambda_c}$ and in the limiting cases of long and short wavelengths reduces to:

$\Gamma\left(\frac{2}{3}\right) = 1{\cdot}35412$

$$^{\lambda}/_{\lambda_c} >> 1 \quad G(^{\lambda}/_{\lambda_c}) = 2^{2/3}\,\Gamma\left(\tfrac{2}{3}\right)\left(^{\lambda_c}/_{\lambda}\right)^{7/3} \qquad (4)$$

$$^{\lambda}/_{\lambda_c} << 1 \quad G(^{\lambda}/_{\lambda_c}) = \sqrt{\tfrac{\pi}{2}}\; e^{-\lambda_c/\lambda} \left(^{\lambda_c}/_{\lambda}\right)^{3/2} \qquad (5)$$

In the region where $^{\lambda}/_{\lambda_c}$ is near unity the function has been evaluated numerically. See refs. 1 and 2 for example. A representative set of values for the function is given in Table 1.

3. GENERAL FEATURES OF THE SPECTRUM

The overall shape of the spectrum is determined by $G(^{\lambda}/_{\lambda_c})$. For short wavelengths this function decreases exponentially and produces essentially a sharp cut off for wavelengths shorter than about $0.1\ \lambda_c$.

At long wavelengths the behaviour is obtained by substituting eqns. (4), (3) and (2) in (1) to give:

$$P(\lambda).d\lambda = \left(\frac{4\pi}{3}\right)^{\frac{1}{3}} \sqrt{3}\ .2^{2/3}\ \Gamma\left(\tfrac{2}{3}\right) \frac{e^2 c}{R^{2/3}}\ \lambda^{-7/3}\ d\lambda \qquad (6)$$

The important thing to understand in this expression is that in this region of the spectrum the intensity is independent of the energy of the electrons and that the intensity decreases with increasing radius.

4. APPLICATION TO PRACTICAL ACCELERATORS

Equation (1) as it stands is not directly applicable to a real accelerator because it has been assumed in its derivation that the electron orbit is completely circular. The expression must be diluted by a factor which is the ratio of the electron orbit length in the bending magnets to the total circumference of the accelerator, in order to allow for the straight sections which exist in any practical accelerator. Thus the overall radius of the accelerator is introduced into calculations of the spectrum intensity.

This introduction of the overall radius is awkward in that it makes more complex the comparison of spectra from accelerators of differing dimensions. It may be removed, however, by converting the expression to being given in terms of the instantaneous current circulating in the accelerator instead of the total number of electrons. This has a further advantage in that the circulating current is remarkably similar between accelerators of greatly differing dimensions. Most electron synchrotrons, for example, operate with circulating currents in the range 10 - 50 mA and electron storage rings in the range 50 - 250 mA.

In this way eqn. (1) can be reduced to:

$$N(\lambda)\, d\lambda = \frac{3^{5/2}\, 10^{-33}}{16\pi^2} \cdot \frac{ec}{hR^2} \cdot \left(\frac{E}{m_o c^2}\right)^7 .\lambda . G(\lambda/\lambda_c).d\lambda \qquad (7)$$

where $N(\lambda)\, d\lambda$ is the number of photons per second at a wavelength λ in a bandwidth $d\lambda$ emitted into a horizontal acceptance angle of one milliradian for a circulating electron beam of one milliamp.

E is the energy of the electrons in GeV

R is the bending radius in metres

e is the electronic charge in Coulombs

h is Planck's constant in Joule seconds

$m_o c^2$ is the electron rest mass in GeV

λ and $d\lambda$ are in Angstrom units (10^{-8} cm)

c is the velocity of light in metres/s

A realistic bandwidth for experimental work in many regions of the spectrum is one in which

$$\frac{d\lambda}{\lambda} = 10^{-3}$$

and for this equation (7) becomes

$$N(\lambda) \text{ (photons/s/mR/mA in 0.1\% bandwidth)} = 7.15\ 10^{-15}\ \frac{\lambda^2}{R^2}\left(\frac{E}{m_o c^2}\right)^7\ G(^{\lambda/}\lambda_c) \tag{8}$$

Converting eqn. (2) to the same practical units results in

$$\lambda_c = 5.59\ \frac{R}{E^3} \tag{9}$$

and when this is substituted in eqn. (8) a further simplification is achieved.

$$N(\lambda) \text{ (photons/s/mR/mA in 0.1\% bandwidth)} = 2.46\ 10^{10}\left(\frac{\lambda}{\lambda_c}\right)^2\ E.G\ (^{\lambda/}\lambda_c) \tag{10}$$

It should be noted that this final expression is a universal spectrum in $^{\lambda/}\lambda_c$ and is directly proportional only to the electron energy. Figure 1 is a presentation of this spectrum expressed per GeV of electron energy. The spectrum for any accelerator is simply produced from eqn. (9) and this figure.

In the vertical plane the synchrotron radiation is emitted with a divergence which is dependent on the particular wavelength. This angle decreases with wavelength but is of the order

$$\frac{m_o c^2}{E} \text{ radians}$$

Because this is a very small angle for electron energies greater than 100 MeV it has been assumed in the above formulae that at each wavelength all the radiation is collected in the vertical plane.

5. SPECTRA OF CONTEMPORARY ACCELERATORS

In fig. 2 the synchrotron radiation spectra of several contemporary electron accelerators are shown. For each accelerator the characteristic wavelength λ_c was calculated from the accelerator parameters given in Table 2. This was then used with the universal spectrum of fig. 1 together with the accelerator energy and beam current to produce the spectra shown in fig. 2. For the NINA synchrotron it was also necessary to perform a graphical integration to allow for the sinusoidal variation of electron energy during the acceleration process.

REFERENCES

1. J. Schwinger. Phys. Rev. 75, (1949) 1912

2. R.A. Mack. CEA report CEAL - 1027 (1966)

3. D.H. Tomboulian and P.L. Hartman. Phys. Rev. 102, (1950) 1423

FIGURE CAPTIONS

Fig. 1 Universal synchrotron radiation spectrum

Fig. 2 Spectra of contemporary accelerators

TABLE 1

λ/λ_c	$G(\lambda/\lambda_c)$
0.100	1.80.10 [illegible]
0.143	$1.58.10^{-1}$
0.200	$5.32.10^{-1}$
0.333	1.16
0.400	1.24
0.500	1.21
1.00	$6.52.10^{-1}$
5.00	$3.61.10^{-2}$
10.00	$8.18.10^{-3}$
50.0	$2.33.10^{-4}$
100	$4.63.10^{-5}$
500	$1.08.10^{-6}$
1000	$2.15.10^{-7}$
5000	$5.03.10^{-9}$
10000	$9.98.10^{-10}$

TABLE 2

	Energy (GeV)	Bending Radius (metres)	Beam Current (mA)	λ_c (Å)
Daresbury Storage Ring	2.0	5.55	1000	3.88
DORIS	3.0	12.19	1000	2.52
SPEAR	2.5	12.7	250	4.54
EPIC	14.0	168.1	22	0.34
NINA Synchrotron	5.0	20.77	20	0.93
NINA Storage Ring	3.4	20.77	50	2.95

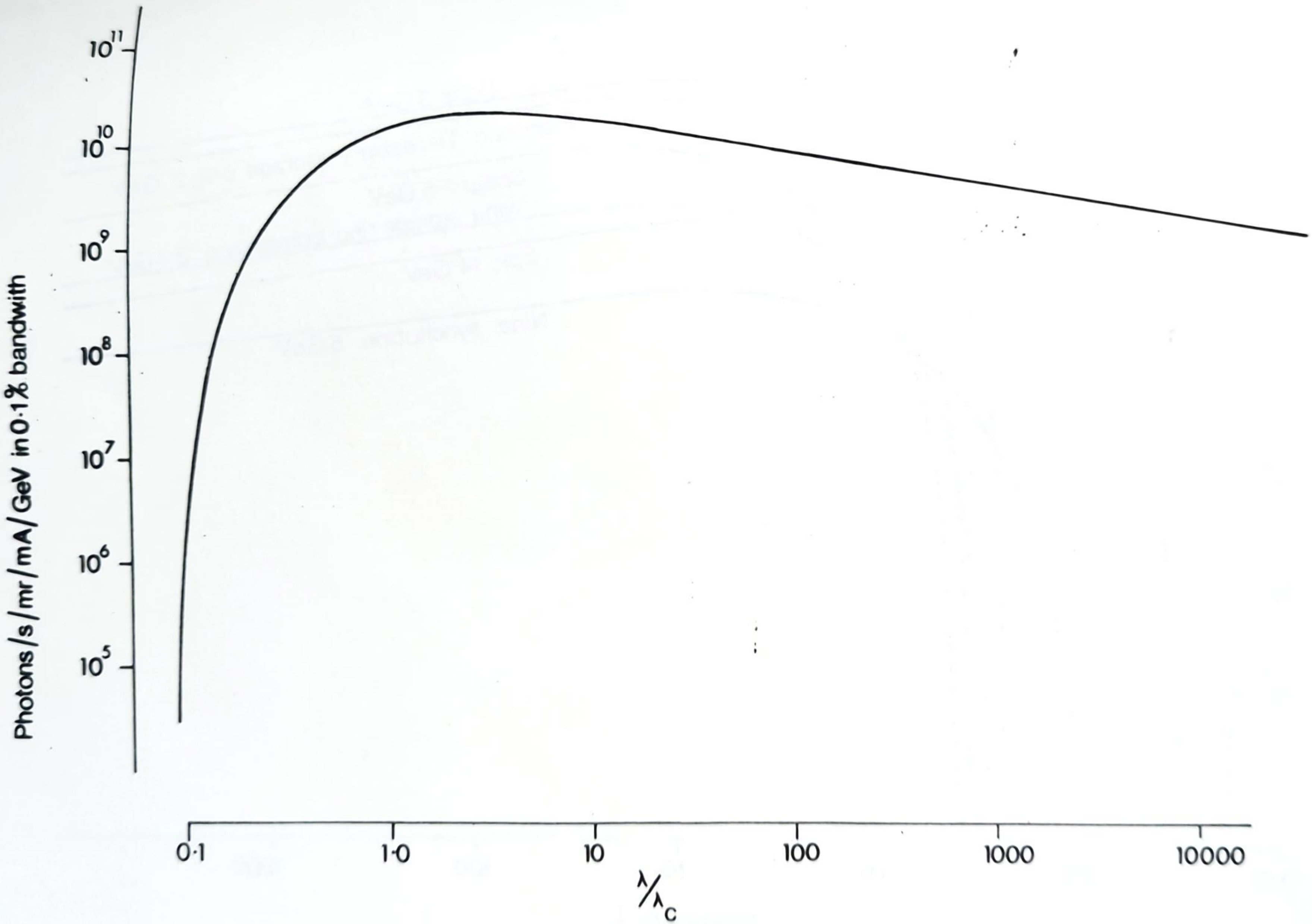
Photons/s/mr/mA/GeV in 0·1% bandwith
10^11
10^10
10^9
10^8
10^7
10^6
10^5
0·1
1·0
10
100
1000
10000
λ/λc

Fig 1

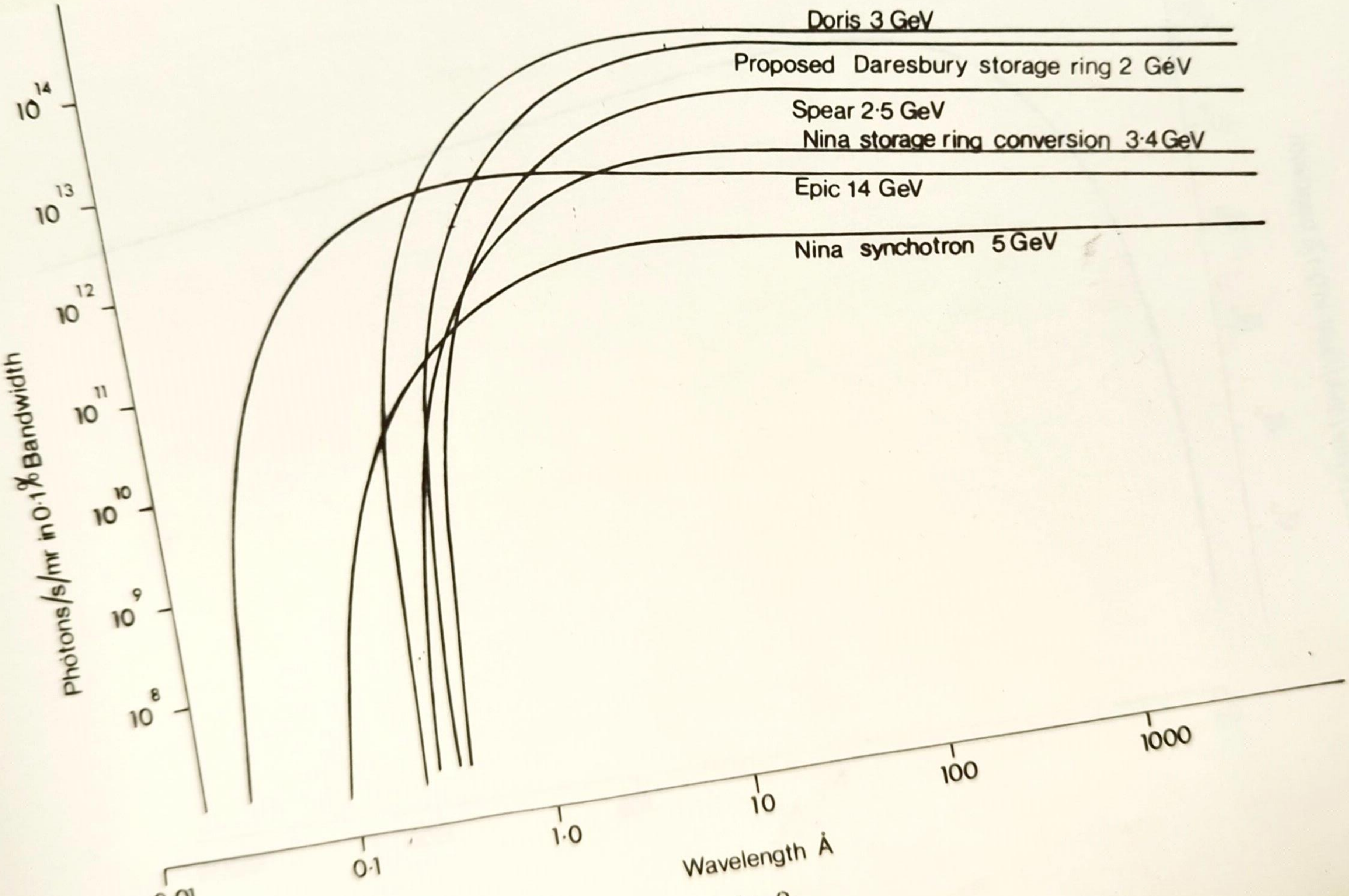
Doris 3 GeV
Proposed Daresbury storage ring 2 GeV
Spear 2·5 GeV
Nina storage ring conversion 3·4 GeV
Epic 14 GeV
Nina synchotron 5 GeV
Photons/s/mr in 0·1% Bandwidth
10^14
10^13
10^12
10^11
10^10
10^9
10^8
0·01
0·1
1·0
10
100
1000
Wavelength Å

Fig 2