

Water: Two Liquids Divided by a Common Hydrogen Bond

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8 **Water: two liquids divided by a common hydrogen**
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Abstract

The structure of water is the subject of a long and ongoing controversy. Unlike simpler liquids, where atomic interactions are dominated by strong repulsive forces at short distances and weaker attractive (van der Waals) forces at longer distances, giving rise to local atomic coordination numbers of order 12, water has pronounced and directional hydrogen bonds which cause the dense liquid close-packed structure to open out into a disordered and dynamic network, with coordination number 4 - 5. Here I show that water structure can be accurately represented as a mixture of two identical, interpenetrating, molecular species separated by common hydrogen bonds. Molecules of one type can form hydrogen bonds with molecules of the other type, but cannot form hydrogen bonds with molecules of the same type. These hydrogen bonds are strong along the bond, but weak with respect to changes in the angle between neighbouring bonds. The observed pressure and temperature dependence of water structure and thermodynamic properties follow naturally from this choice of water model, and it also gives a simple explanation of the enduring claims based on spectroscopic evidence that water is a mixture of two components.

Introduction - mixture models of water and the three-body force

Mixture models have been a recurring theme throughout the history of water research, and have reappeared in different guises right up to the present time.¹ They have been inferred from both structural² and spectroscopic³⁻⁷ evidence. The justification for mixture models has sometimes been based on the observation of so-called “isosbestic” behaviour,^{2,3} in which a series of spectra as a function of some state parameter such as temperature, pressure or concentration are seen to vary about a common point, suggesting a transfer of population from one type of fluid to another. Other studies have argued that the claim for two-state behaviour in water is not justified on the basis of either spectroscopic^{8,9} or diffraction¹⁰ evidence: in both cases continuum computer simulation models of water could be shown to give the same isosbestic behaviour that was used to claim two-state behaviour. More recently x-ray emission spectroscopy on water⁷ has been used to claim water

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3 does indeed have two local structures, based on the observation of two peaks in these spectra, one
4 ice-like and therefore assumed to indicate tetrahedral order, and the other gas-like and therefore
5 assumed to indicate hydrogen-bond disordered structure. It has to be said however that a different
6 interpretation of similar data was given in an earlier paper.¹¹
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11 Computer simulations of water generally do not show two or more phases as distinct enti-
12 ties since the natural stochastic variations in density and structure, which occur in any disordered
13 system which is undergoing diffusion, are often as large as, over the relevant distance scales, or
14 larger than any supposed fluctuations in density arising from there being two or more components
15 present.^{12,13} It is a matter of some irony that at about the same time that Walrafen was stating
16 categorically that the observed temperature dependence of the Raman stretch spectrum of water
17 “provide strong support for the two-state model of water structure”,³ the first computer simulation
18 models of water appeared which naturally gave a continuum view of water interactions.^{14,15}
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28 A common feature of many models of the water interaction potential energy is that they do not
29 overtly specify a three-body interaction. Most of the simpler, point charge models of the water
30 interaction potential¹⁶⁻²⁰ rely on Coulomb repulsion to keep water molecules at their respective
31 positions. In these potentials a negative charge or charges are placed on or near the oxygen atom,
32 while positive charges are placed on or near the hydrogen atoms. Attractive Coulomb interactions
33 between the hydrogen and oxygen atoms on neighbouring water molecules give rise to the hy-
34 drogen bonds in these models, while repulsive forces between like-charged oxygen atoms prevent
35 them approaching one another closely unless they are joined by a hydrogen bond. A good example
36 of these forces at work is given in the results from Klein and colleagues.²¹ Innumerable refine-
37 ments and variations of these potentials have occurred, including the use of flexible or polarisable
38 molecules - see for example²² - but the basic format remains similar.
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50 One notable facet of two-body models of water however is that if the bond angle distribution
51 of the simulated liquid is calculated, that is the distribution of included angles of triplets of water
52 molecules two of which lie at the nearest neighbour hydrogen bond distance from the middle
53 molecule, there is a marked peak at angles $< 60^\circ$,^{21,23,24} angles which do not occur even in the
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dense form of ice, ice VII, which has a coordination number of 8, and nearly twice the density of
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hexagonal ice Ih.²⁵ This suggests there may be a three body term which is missing in these simple
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models of the water potential.

Recently Molinero and co-workers,¹ have shown that an alternate, effective potential which
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incorporates a variation of the Stillinger-Weber three-body potential for silicon²⁶ can be useful for
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studying many of the underlying features of water properties. In this potential hydrogen atoms
do not occur explicitly, but are replaced by a directional three-body potential which depends on
the relative separations of triplets of atoms and the included angle between them. This potential
appears to give a realistic view of water properties, such as a temperature of maximum density
and structure, even if an exact match is not achieved. Using this short ranged three-body potential
avoids the need to use long ranged Coulomb potentials to capture water properties.

In fact three-body forces can be incorporated within an effective two-body, pairwise additive
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framework.^{27,28} In that work the structures of elemental tetrahedral glasses such as silicon and
germanium are modelled as a mixture of two identical components. The atoms of one component
approach those of the other component at the known nearest neighbour distance, but atoms of one
component cannot approach other atoms of the same component at this distance. Instead atoms in
the same component are constrained to have a near-neighbour distance corresponding to the *second*
peak in the radial distribution function. In this way a model of the structure is built up which forces
the expected tetrahedrality in these materials. It must be emphasized that the use of two identical
but distinct components is a convenient device for generating a three-body body force within a
two-body framework but should not be used to claim the two components are distinct states: they
have identical structure and are fully interpenetrating.

Notwithstanding all the arguments and counter arguments given above about whether water is
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a mixture or not, I would like here to apply this method to the structure of water. Two types of
otherwise identical water molecules are created labelled 1 and 2. Water molecules of one type
can form hydrogen bonds with water molecules of the other type (unlike interactions), but not to
water molecules of the same type (like interactions). Instead water molecules of the same type are

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4 weakly held apart to a distance corresponding approximately to the second peak of the oxygen-
5 oxygen radial distribution function for water, namely $\sim 4.5\text{\AA}$, but there are no other interactions
6 specified at the outset between atoms on like molecules. It is the relative weakness of this like-like
7 interaction coupled with the much stronger hydrogen-bond interaction between unlike molecules
8 that gives this model its essential characteristic. The potentials governing these interactions are
9 put into a NVT Monte Carlo computer simulation of water at 295K and ambient pressure, and the
10 parameters needed to define the interaction potential energies are optimised by comparison of the
11 simulated x-ray and neutron differential scattering cross sections for water with new experimen-
12 tal measurements of the same quantities. This results in a mixture model of water where the two
13 components are identical and fully interpenetrating. The model demonstrates the feature that some
14 neighbours of any given water molecule will be hydrogen bonded to it, while others will not be
15 bonded at all, offering a natural explanation for the enduring claims, based spectroscopic data, that
16 water is a mixture of two components. Changes of temperature or pressure will cause the relative
17 populations of these bonded and non-bonded molecules to change. The simulations show remark-
18 ably good agreement with the structural data, and the same interaction potentials can be used to
19 predict the temperature and pressure dependence of both water structure and some thermodynamic
20 quantities, in particular the pressure and specific heat. Intriguingly the results demonstrate a hidden
21 periodicity in water which is closely comparable to that found in hexagonal ice.
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43 Monte Carlo simulation

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46 For the Monte Carlo computer simulation of water at 295K an NVT ensemble of water molecules
47 (500 of each type) is placed in a cubic simulation box of side 31.0516\AA , giving a number density
48 of $0.0334\text{ molecules/\AA}^3$. Site-site interaction potentials are defined, where specified in terms of a
49 modified Morse potential plus up to 3 additional Gaussian potentials:
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$$54 U(r) = E_0 \left[\exp\left(2\frac{r_0 - r}{w_0(r)}\right) - 2\exp\left(\frac{r_0 - r}{w_0(r)}\right) \right] + \sum_{k=1} E_k \exp\left[-\frac{1}{2}\left(\frac{r_k - r}{w_k}\right)^2\right] \quad (1)$$

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4 where

$$w_0(r) = \begin{cases} \Delta r_-, & r \leq r_0 \\ \Delta r_+, & r > r_0 \end{cases} \quad (2)$$

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10 and with r_k the position of the minimum (or maximum) of each potential term. The form (2) allows
11 the possibility that the Morse potential decays at a different rate with r beyond the position of the
12 minimum compared to before the minimum. For the present case four sets of such potentials are
13 defined, namely O-O (like), O-O (unlike), O-H (unlike) and H-H (unlike). The potentials for O-H
14 (like) and H-H (like) interactions are set identically to zero. The reason for this particular choice of
15 potential was simply for convenience, since different values of the Morse and Gaussian parameters
16 can give a very wide range of potentials, but this potential form is almost certainly not unique and
17 alternatives could have been considered.
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26 The parameters for these potentials (Table 1) are adjusted in an extended series of trial steps
27 so that the simulation of ambient water at 295K and 0.1MPa has a reasonable pressure ($\sim 0 \pm 100$
28 MPa) and energy ($\sim -45 \pm 5$ kJ/mole), as well as giving an accurate reproduction of new x-ray
29 and neutron diffraction data on heavy and light water and mixtures thereof (see Experimental).
30 The energy value applied is derived from the known heat of vaporisation of water, combined with
31 the likelihood that water molecules in the liquid are more strongly polarised than in the gaseous
32 state.¹⁹ An additional requirement is that when used in a simulation of water at 268K and num-
33 ber density of 0.0381 molecules/Å³, the simulated pressure is close to the experimental pressure
34 (400 MPa²⁹). The use of purely short range potentials, (1), precludes the need to perform long
35 range corrections for the energy and pressure, but otherwise the simulation follows standard pro-
36 cedures.³⁰ The resulting potential energy functions used in this work, Figure 1, show a strong
37 attraction (deep minimum in the potential energy) for O-H (unlike) interactions, but are primarily
38 repulsive for O-O (unlike) and H-H (unlike) interactions. The O-O (like) interaction has a weak
39 minimum near 4.5Å to encourage the formation of triplets of molecules at the tetrahedral angle, but
40 is otherwise repulsive. The simulations are performed with rigid but disordered molecules as with
41 previous simulations of this kind:³¹ these emulate the zero point disorder of the protons and are
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an essential prerequisite to obtaining a reasonable fit to the diffraction data, particularly at higher wave vector values.

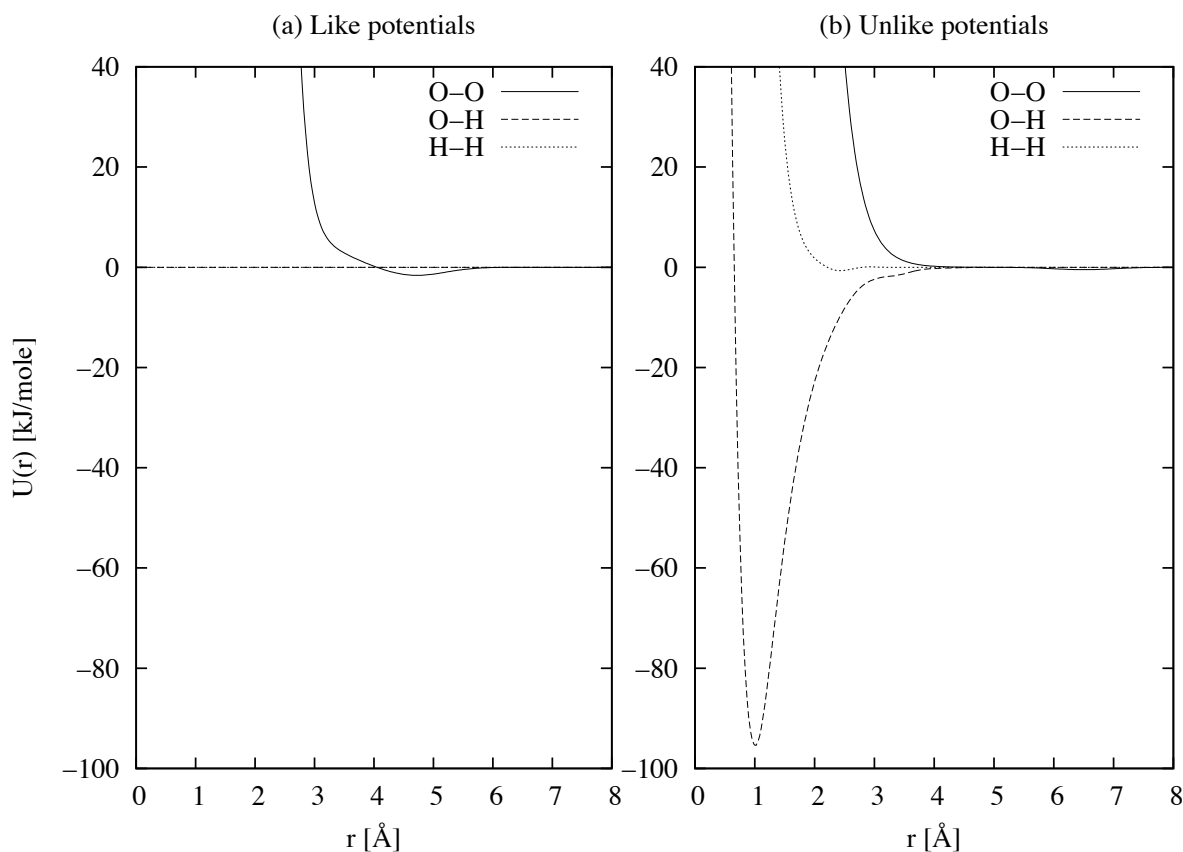


Figure 1: Inter-molecular potentials for like and unlike pairs of molecules. For interactions between molecules of the same type (a) the O-O potential energy has a weak minimum at the position of the *second* minimum of the corresponding radial distribution function. For interactions between molecules of unlike types, the potential energy is dominated by the hydrogen bond between oxygen and hydrogen atoms.

Table 1: List of parameters for the intermolecular potential, equation (1). Energies are given in units of kJ/mole, and distances in Å. It will be noted that there are no prescribed interactions between O-H and H-H on like molecules.

	Atom pair	r_0	Δr_-	Δr_+	E_0	r_1	w_1	E_1	r_2	w_2	E_2	r_3	w_3	E_3
Like	O-O	4.59033	0.34397	0.34397	0.001	3.20	0.50	2.9	4.70	0.5	-1.6	-	-	-
	O-H	-	-	-	-	-	-	-	-	-	-	-	-	-
	H-H	-	-	-	-	-	-	-	-	-	-	-	-	-
Unlike	O-O	5.58163	0.57874	0.57874	0.001	6.50	0.50	-0.5	-	-	-	-	-	-
	O-H	1.00883	0.50876	0.44974	95.15	2.00	0.46	-3.0	2.75	0.2	0.5	3.4	0.2	-0.6
	H-H	3.50063	0.39393	0.39393	0.001	2.38	0.20	-0.9	2.90	0.1	0.1	-	-	-

Experimental

X-ray scattering experiment

X-ray scattering data for water at 295K, ambient pressure, were recorded on a PANalytical x-ray diffractometer, using the white x-ray beam from a silver anode (K_{α} wavelength = 0.5609Å) using a 2.5mm silica glass thin walled capillary in transmission geometry. A Rh filter was used to eliminate K_{β} radiation. The scattering data were corrected for background, empty capillary scattering, attenuation, multiple scattering and Compton scattering, and put on an absolute scale of electron units using the Krogh-Moe method.³² In addition, using diffraction data from silicon crystalline powder as a calibrant, a correction was developed to remove the off-energy scattering that arose from the significant bremsstrahlung radiation in the incident x-ray spectrum. Finally the single atom scattering was subtracted from the diffraction data, which were then normalised to the same single atom scattering to give an x-ray interference differential scattering cross section (Fig. 2(a)), defined by:

$$F_x(Q) = \frac{\sum_{\alpha\beta\geq\alpha} (2 - \delta_{\alpha\beta}) c_{\alpha}c_{\beta}f_{\alpha}(Q)f_{\beta}(Q)H_{\alpha\beta}(Q)}{\sum_{\alpha} c_{\alpha}f_{\alpha}(Q)^2} \quad (3)$$

where c_{α} is the atomic fraction and $f_{\alpha}(Q)$ is the atomic form factor for component α , and the partial structure factor, $H_{\alpha\beta}(Q)$ between atom types α and β is defined as the Fourier transform of the corresponding site-site radial distribution function, $g_{\alpha\beta}(r)$:

$$H_{\alpha\beta}(Q) = 4\pi\rho \int r^2 (g_{\alpha\beta}(r) - 1) \frac{\sin Qr}{Qr} dr \quad (4)$$

with ρ the atomic number density and Q the wave vector change in the scattering experiment.

Neutron scattering experiment

Neutron scattering gives fundamentally the same information as the x-ray experiment, except that atomic form factors are replaced by numbers - neutron scattering lengths - one for each isotope.

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3 This means that heavy water has a completely different scattering profile compared to normal
4 light water. This can be exploited by measuring heavy and light water, and mixtures thereof, to
5 give direct information on the H-H and O-H correlations in the liquid. In the present instance,
6 as well as the pure liquids, measurements were made on mixtures of 50 mole% and 64 mole%
7 H₂O in D₂O, the latter sample being called “null” in the figures because the net coherent scattering
8 length of hydrogen in this sample is close to zero. Scattering data were corrected for background
9 scattering, container scattering, attenuation, multiple scattering, inelastic scattering, and put on an
10 absolute scale by comparison with the scattering from a known volume of vanadium, which has an
11 almost incoherent scattering cross section. The resulting interference differential scattering cross
12 section (Fig. 2(a)) for neutrons is:
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$$F_n(Q) = \sum_{\alpha\beta\geq\alpha} (2 - \delta_{\alpha\beta}) c_\alpha c_\beta \langle b_\alpha \rangle \langle b_\beta \rangle H_{\alpha\beta}(Q) \quad (5)$$

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29 where the angular brackets represent averages over the spin and isotope state of the respective
30 nuclei. The neutron scattering data in this work were recorded as part of the commissioning exper-
31 iments of the new NIMROD diffractometer at ISIS which is designed for looking at intermediate
32 range structure in liquids, complex fluids, and glasses.³³ Comparison of these datasets with those
33 measured previously on the SANDALS diffractometer at ISIS,^{31,34} shows excellent agreement in
34 general, though problems with inelastic scattering from light hydrogen appear to be more pro-
35 nounced on NIMROD due to the smaller scattered flight path to incident flight path ratio. Work on
36 trying to remove this inelastic scattering more reliably is ongoing.
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48 **Results and discussion**

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51 Figure 2 (a) and (b) shows the fits to the x-ray and neutron diffraction data at 295K that were
52 achieved in this work while Table 2 gives the densities and temperatures, and calculated mean
53 potential energies and pressures for the all the ambient and non-ambient simulations that were
54 performed in this work. It will be noted from Figure 2(b) that the present model is slightly over-
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estimating the height of the first peak in $g_{OO}(r)$: this is in line with other recent x-ray experiments on water^{31,35,36} that seem to indicate that the first peak in this function is lower than has traditionally been assumed.

Figure 2(c) shows the simulated O-O, O-H and H-H radial distribution functions separated into their like and unlike counterparts. It can be seen immediately, from the lack of a strong peak in the O-H distribution for like pairs, that there is no hydrogen bonding between like molecules, as expected, but strong hydrogen bonding between unlike pairs. Another feature to emerge in this representation is that the oscillations in $g(r)$, which almost disappear in the total functions, Figure 2(b) beyond $r \approx 8\text{\AA}$, actually proceed much further than this when separated into like and unlike functions. However they are almost exactly out of phase at longer distances and so cancel each other out in the total function.

Figure 3 shows the distribution of included angles of triplets of oxygen atoms for each of the cases of like-like pairs, unlike-unlike pairs and like-unlike pairs. In all cases two oxygens are considered bonded if their separation is 3.3\AA or less. Clearly the like-like and unlike-unlike triplets do not have any peaks below $\theta \approx 90^\circ$ corresponding to the strong repulsion between like atoms below 3\AA . However the like-unlike triplets do have quite a marked peak below 60° , so it seems that the presence of this peak in the triplet angle distributions as discussed in the introduction *must* arise from non-bonded molecules in the first coordination shell of water. This will happen even when a deliberate attempt is made, as is done here, to force non-bonded water molecules away from the central molecule. Note however that the coordination number of like oxygen atoms out to the distance used to define a bond, namely $r = 3.3\text{\AA}$, is only 0.3 atoms whereas that for unlike oxygen atoms at the same distance it is 4.1 atoms, so the occurrence of like-unlike triplets is more than 6 times less likely than for unlike-unlike triplets.

It should also be noted that the O-H coordination number for unlike pairs, integrated out to the first minimum in the unlike O-H $g(r)$, which occurs at $r \approx 2.45\text{\AA}$ is 1.90 hydrogen atoms around oxygen. Including the contributions from H-O as well as O-H interactions would imply the number of hydrogen bonds per water molecule is ≈ 3.8 which is smaller than the unlike O-O coordination

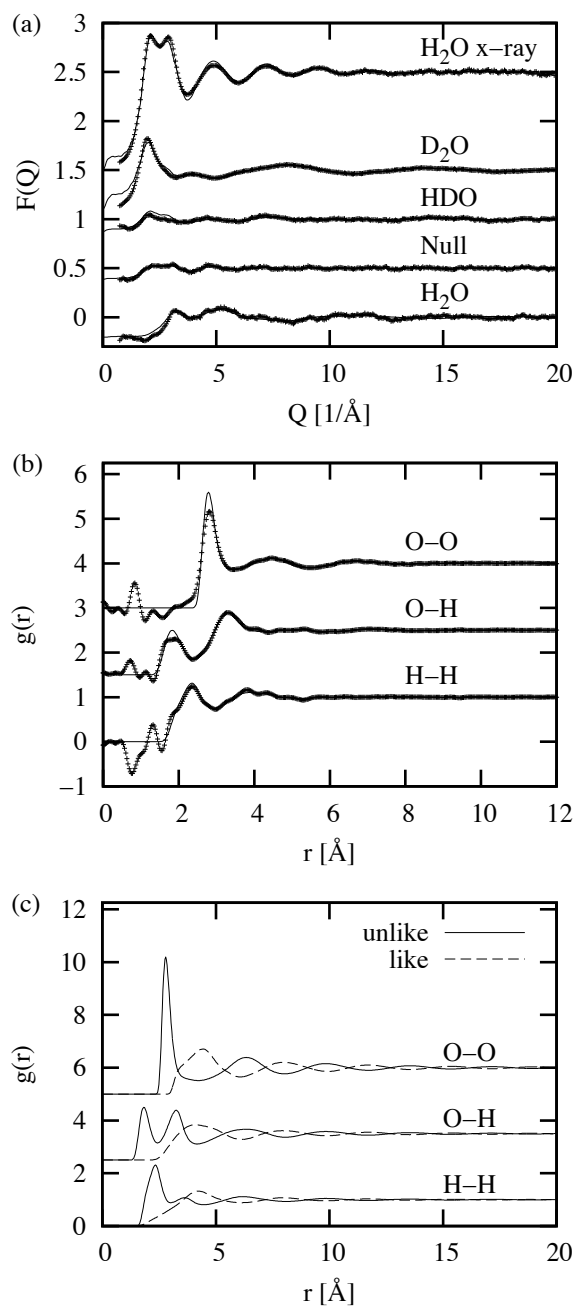


Figure 2: Structure of water using the two component model. (a) Fits (lines) to the x-ray (top) and neutron (lower 4 curves) scattering data (dots) from mixtures of heavy and light water as noted using the intermolecular potentials shown in Fig. 1. (b) Estimated total O-O, O-H and H-H radial distribution functions (rdf) as derived from the simulation (lines) and data (dots). (c) Breakdown of the O-O, O-H and H-H radial distribution functions into their like and unlike counterparts. Graphs (a) and (b) show that simultaneous fits to both x-ray and neutron scattering data can be achieved with the two component model. Graph (c) shows that hydrogen bonding (O-H rdf) is strong between unlike molecules, but non-existent between like molecules, as expected from the definition of the potential. All curves are shifted vertically for clarity.

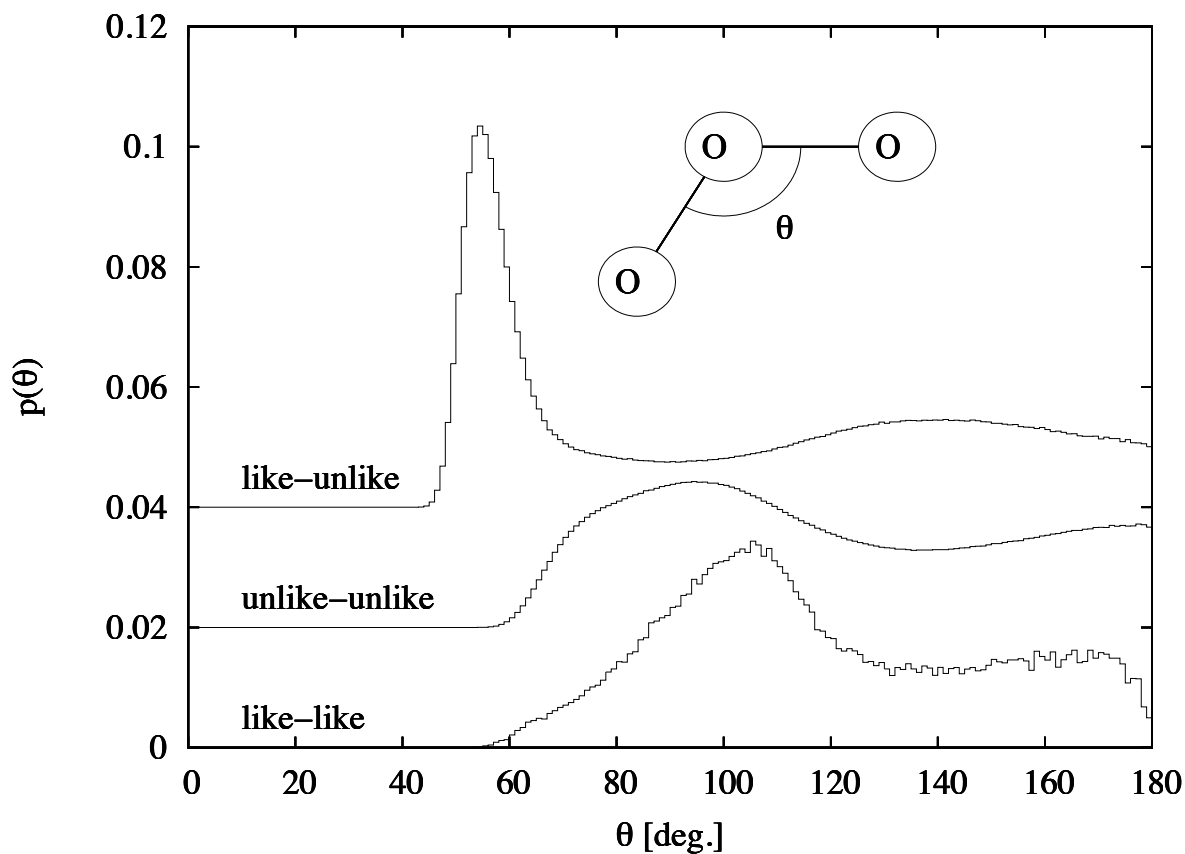


Figure 3: Distribution of included angles for triplets of oxygen atoms. Each triplet is divided into one of three kinds, where the three oxygens are of the same type (like-like), each pair is of the opposite type (unlike-unlike), and where one pair is of the same type and the other of opposite type (like-unlike). The three curves are shifted vertically for clarity.

number of 4.1 quoted above. Hence even in this model there will be some unlike water molecules in the first shell which are not hydrogen bonded to the central molecule.

Table 2: Temperatures, densities, experimental pressures, mean simulated pressures and mean simulated configurational potential energies for water using the two fluid model described in this paper. Pressures are rounded to the nearest 1 MPa (=10 atm.). All simulations are performed in the NVT ensemble, with the quoted energy and pressure values averaged over at least 5000 different simulation boxes. It is found the pressure of the simulations in the temperature range 280-365K increases slightly with increasing temperature, though the variation is not outside the uncertainty in its value.

T [K]	ρ [molecules/Å ³]	Pressure (expt.) [MPa]	Pressure (sim.) [MPa]	Energy (sim.) [kJ/mole]
268	0.03387	27	70	-45.4
268	0.03623	210	313	-46.0
268	0.03807	400	534	-46.8
280	0.03345	0	35	-45.0
288	0.03343	0	62	-44.6
295	0.03338	0	60	-44.1
313	0.03320	0	70	-43.4
343	0.03271	0	89	-41.6
365	0.03225	0	100	-40.4

Table 2 lists the pressures and potential energies that were obtained in the simulations of water using the same potential derived from the ambient data at non-ambient conditions. Moving along the coexistence curve at 0.1MPa there is a slight rise in pressure to near 100MPa at 365K. Note however even at 280K the pressure is 35MPa, so against that baseline the rise is not so significant, in particular when it is realised that the fluctuation of pressure from one simulation box to the next is of order 100MPa. Within this fluctuation the obtained pressures at each pressure and density are remarkably close to their experimental values.

Figure 4 shows the results from these simulations at different temperatures and pressures. Figure 4(a) shows the O-O structure factors as a function of temperature - these can be compared favourably with those shown by Narten *et al.* much earlier.³⁷ Figure 4(b) shows the corresponding

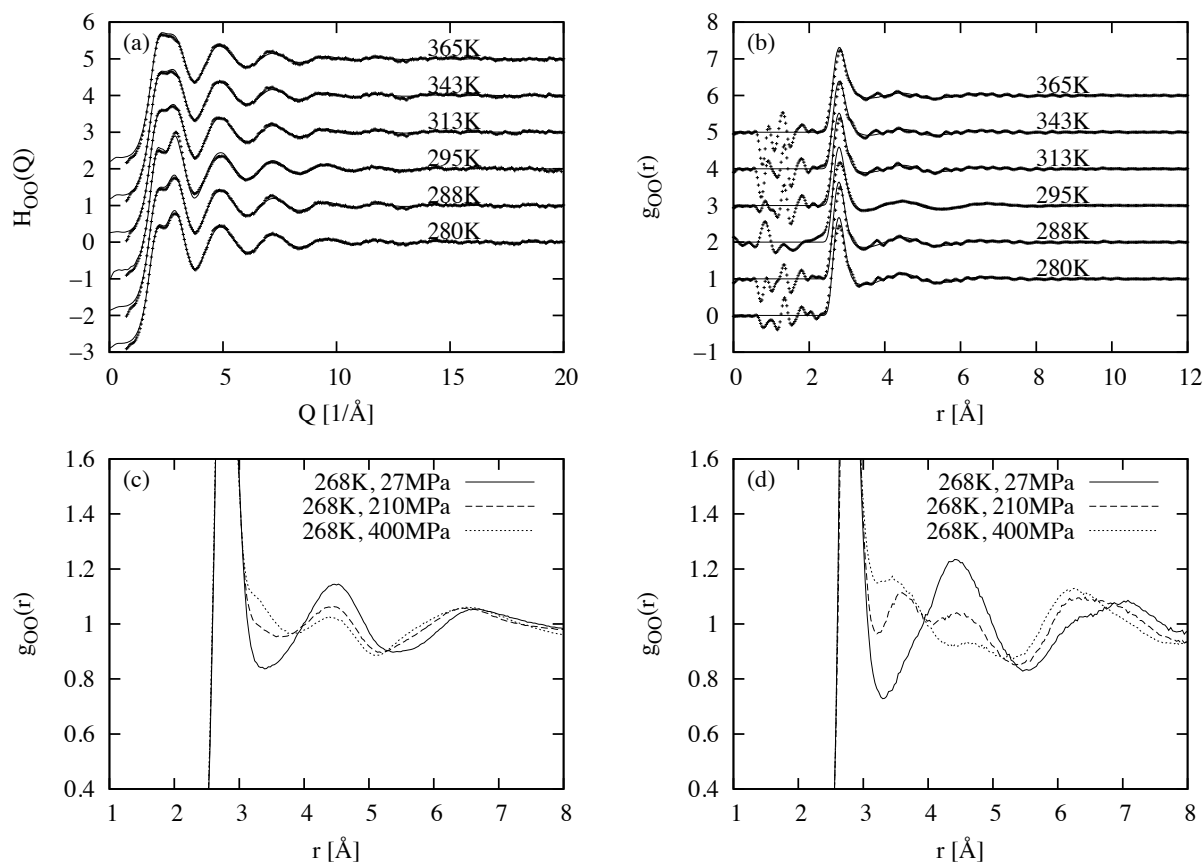


Figure 4: Temperature, (a) and (b), and pressure, (c) and (d), dependence of water structure with the two-fluid model. (a) O-O partial structure factors as a function of temperature for both simulation (lines) and data (dots), equation (4). (b) Corresponding O-O radial distribution functions. (c) Simulated O-O radial distribution function as a function of increased pressure at 268K within the two-fluid model compared to (d) the same functions obtained from the published data,²⁹ using a single molecular species. The dots are derived from neutron diffraction data measured at the listed temperatures.

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3 radial distribution functions with the gradual weakening with increased temperature of the main
4 peak and the second peak near 4.5Å. Note that these results were all obtained using the same
5 intermolecular potential energy function that was derived for ambient water.
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10 Figure 4(c) shows the simulated O-O functions with increasing density at 268K, and these
11 curves can be compared to what was estimated directly from the neutron scattering data using a
12 single molecular species (d).²⁹ Clearly the agreement is not perfect, but the qualitative behaviour
13 of these functions, in which the second peak weakens and extra intensity appears near 3.5Å is
14 closely similar to the experiment.^{29,38} In particular the isosbestic points - points where the curves
15 cross over each other - at ~3.9Å, ~5.2Å, and ~6.6Å - are reproduced quite accurately by the
16 two-component model. Bearing in mind that although this is a two component model there is
17 no segregation of the two components, which are identical and fully mixed, it will be seen that
18 isosbestic behaviour is not only a property of mixtures where distinct structural species occur.
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28 Further understanding of how water responds to changes in pressure and temperature is ob-
29 tained from Figure 5. Here, in (a), is shown the unlike-unlike bond angle distribution as a func-
30 tion of increasing pressure at 268K. The distribution shows a progressive increase in “interstitial”
31 molecules (bond angle ~ 70°) as the density increases. These would correspond to the typical
32 angles between unlike-unlike triplets found in higher density forms of ice. In (b) is shown the vari-
33 ation of angle distribution of the O-H vector about the O-O axis between unlike (hydrogen-bonded)
34 molecules. It is seen that the width of these distributions increases gradually with temperature, in
35 a manner that has been observed previously by NMR.³⁹ The actual standard deviations of these
36 widths are plotted in (c) as a function of temperature and pressure and are found to be somewhat
37 larger than those given by Halle *et al.*, but this might be due to the different methods used to extract
38 the bond angle distribution. In particular the NMR study apparently relies on a density functional
39 simulation of water for calibration, and it is well documented that such simulations can give too
40 strong an O-H correlation compared to what is measured experimentally.⁴⁰
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54 Finally Figure 5(d) shows the calculated specific heat for these models. Since the potential is
55 purely pairwise additive this can be calculated in the present case by noting that the total energy of
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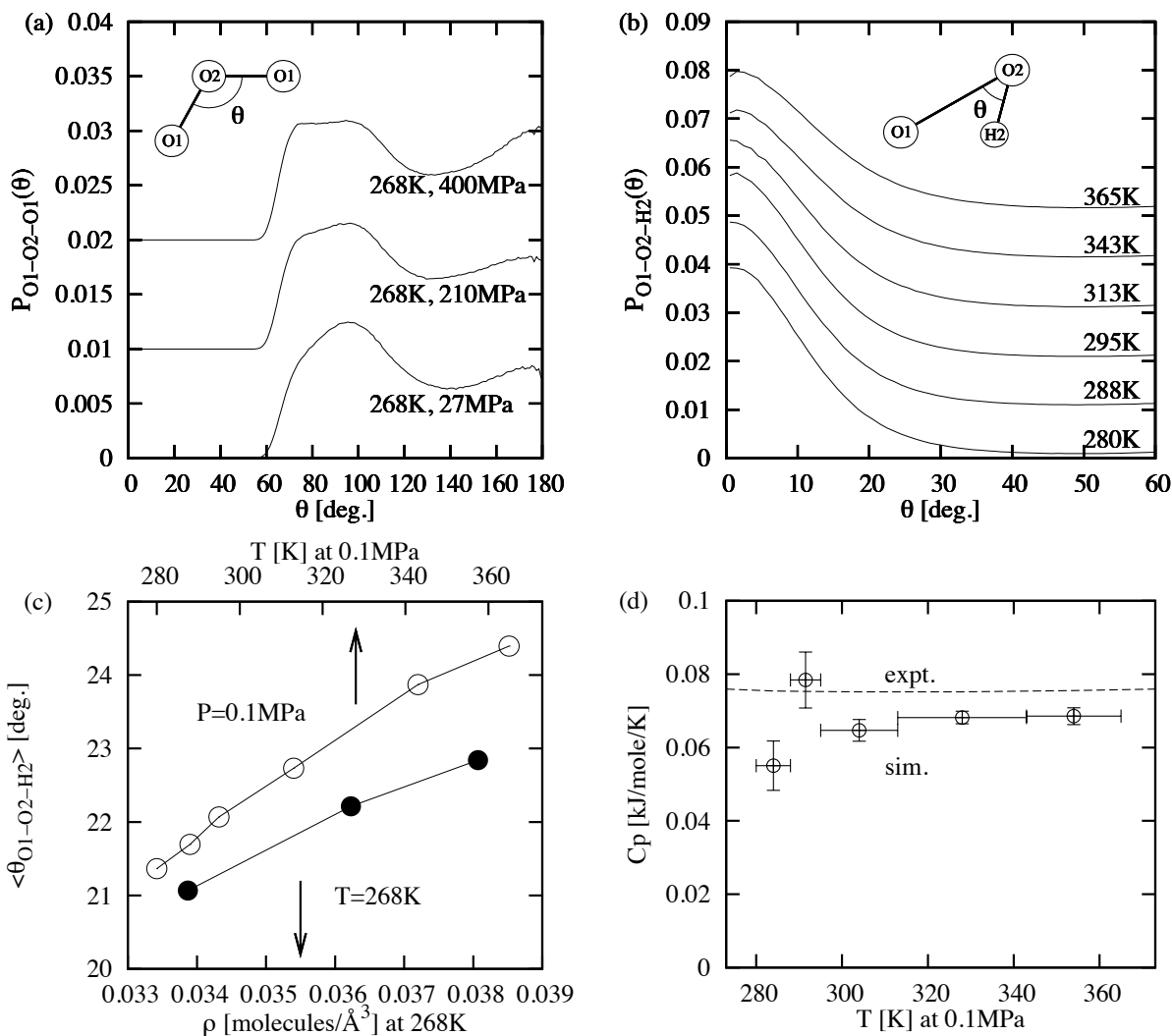


Figure 5: Bond angle distributions and estimated specific heats for the two-fluid model. (a) unlike-unlike bond angle distribution for the two fluid model as a function of increasing density - see Table 2 - at 268K. The distribution shows a progressive increase in “interstitial” molecules (bond angle $\sim 70^\circ$) as the density increases. (b) Bond angle distribution between the O-H vector on one molecule with the O-O vector between unlike pairs of molecules as a function of increasing temperature at a pressure of 0.1MPa. The distributions have been normalised to the $\sin \theta$ distribution that would occur for randomly oriented bonds. (c) Root mean square deviation of the O-O-H bond angle distribution for unlike pairs as a function of density at 268K (lower curve) and as a function of temperature at $P=0.1\text{MPa}$ (upper curve). Note how increased temperature and density have a similar effect on this deviation. (d) Simulated specific heat for the two fluid model, from equation 6. The vertical error bars show the fluctuations of this value, based on ~ 5000 molecular configurations, while the horizontal error bars show the temperature range over which each value is averaged. The dashed line shows the experimental values. The error bars become smaller at higher temperatures because the simulated difference is being averaged over a broader range of temperatures.

the system at any state point is the sum of the kinetic and potential energies, and since the pressure remains nearly constant for the simulations at ambient pressure, Table 2, $C_p \left(T \approx \frac{T_1+T_2}{2} \right)$ can be estimated approximately from the expression:³⁰

$$C_p(T) \approx \frac{3}{2}R + 2\pi N \sum_{\alpha\beta \geq \alpha} (2 - \delta_{\alpha\beta}) c_\alpha c_\beta \int r^2 U_{\alpha\beta}(r) \left[\frac{\left(\rho(T_2) g_{\alpha\beta}^{(T_2)}(r) - \rho(T_1) g_{\alpha\beta}^{(T_1)}(r) \right)}{(T_2 - T_1)} \right] dr \quad (6)$$

where R is the gas constant, $N (= 3)$ is the number of atoms per molecule, $\rho(T)$ is the number density and $g_{\alpha\beta}^{(T)}$ is the site-site radial distribution function at temperature T . Once again it is seen that the simulated specific heats are remarkably close to their experimental counterparts, noting that these values have not been fitted when determining the potential parameters. The experimental values have a slight dip near $\sim 310\text{K}$ but the simulated values have too large a variation to be able to see this dip.

It is interesting to discover where this specific heat comes from, so Figure 6 shows the kernel of the integral in (6), $\Delta E_{\alpha\beta}(r) = U_{\alpha\beta}(r) \left(\rho(T_2) g_{\alpha\beta}^{(T_2)}(r) - \rho(T_1) g_{\alpha\beta}^{(T_1)}(r) \right)$ for each of the 6 like and unlike distribution functions in this model. It can be seen that virtually all the contribution comes from a positive term from the OH unlike distribution combined with a negative term from the unlike O-O distribution.⁴¹ Since the O-H potential is strongly negative at the H-bond distance, a positive energy can only arise if bonds have been broken with increasing temperature. The negative contribution from the O-O unlike term arises because, as hydrogen bonds are broken, so the repulsive energy between oxygens which derives from the hydrogen bonding is relaxed. None of the other interactions make a significant contribution to the specific heat. Hence with this model it is seen that a large fraction of the specific heat in water arises directly from the hydrogen bonding, a result which may seem self-evident, but which is important nonetheless.

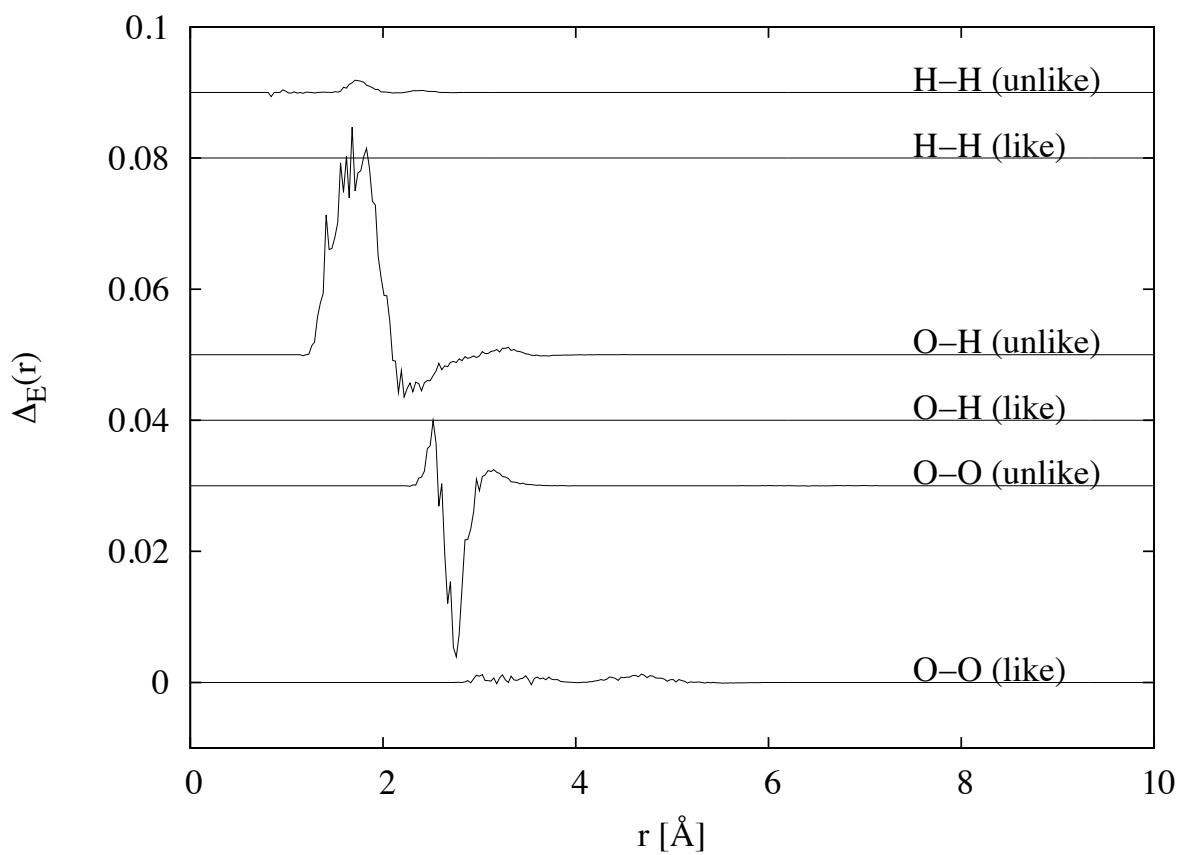


Figure 6: Kernel of the integral in (6) plotted as a function of radius. The difference data are derived from the simulations at 288 and 295K.

Conclusion

The underlying ansatz of the present work is that at any given time a water molecule in the liquid is surrounded by two types of water molecule, those to which it is hydrogen bonded, and those to which it is not bonded. I do not attempt to explain this feature of the model but treat it as fact, although it is worth remembering that standard models of the dense forms of ice include water molecules in the nearest neighbour shell which are not hydrogen-bonded to the central molecule. The two component model that emerges from this assumption is apparently able to capture the structure and thermodynamics of the liquid near ambient conditions surprisingly successfully without resorting to complicated polarisable molecular potentials. Various spectroscopies also strongly hint there are two types of environment around a water molecule in the liquid, bonded and non-bonded,^{3,4,7} and these results have been repeatedly used to claim that water is a mixture of two components, one more open and tetrahedral-like, the other more disordered and compressed. Yet serious counter arguments suggest that such spectroscopic results might also be consistent with the traditional continuum models of water that are usually produced by computer simulation methods.⁸ The present two component model of water is not a traditional mixture model since the two components have identical structures, are made up of identical molecules, and are fully interpenetrating. What distinguishes the two components is that hydrogen bonds are strong between unlike molecules, but non-existent between like molecules. As a result in this model, bonded and non-bonded water molecules occur quite naturally in the first coordination shell without the need to imagine there are two distinct structural components.

It is important to remember that the two-component model is introduced to generate a three-body force within a two-body framework. The result is that whereas the total density correlation in water damps out very quickly with distance, Figure 2(b), the density correlations associated with like and unlike pairs in the present model proceed to a much longer distance, Figure 2(c). These longer range correlations are hidden in the normal liquid, but one could imagine that if the like correlations are perturbed differently to the unlike correlations, as for example when under pressure or near a hydrophobic surface, a longer range total correlation might appear, such as a

freezing transition. This is of course highly speculative at this stage and has not been investigated in any detail. Strangely enough, for what it is worth, the partial structure factors associated with these like and unlike pairs have opposite sign but equal amplitude peaks in Q space at a Q value ($\approx 1.8\text{\AA}^{-1}$) very close to the first peaks in the O-O structure factor of ice Ih, Figure 7, yet there is no peak in the total O-O structure factor of the liquid at this Q value.

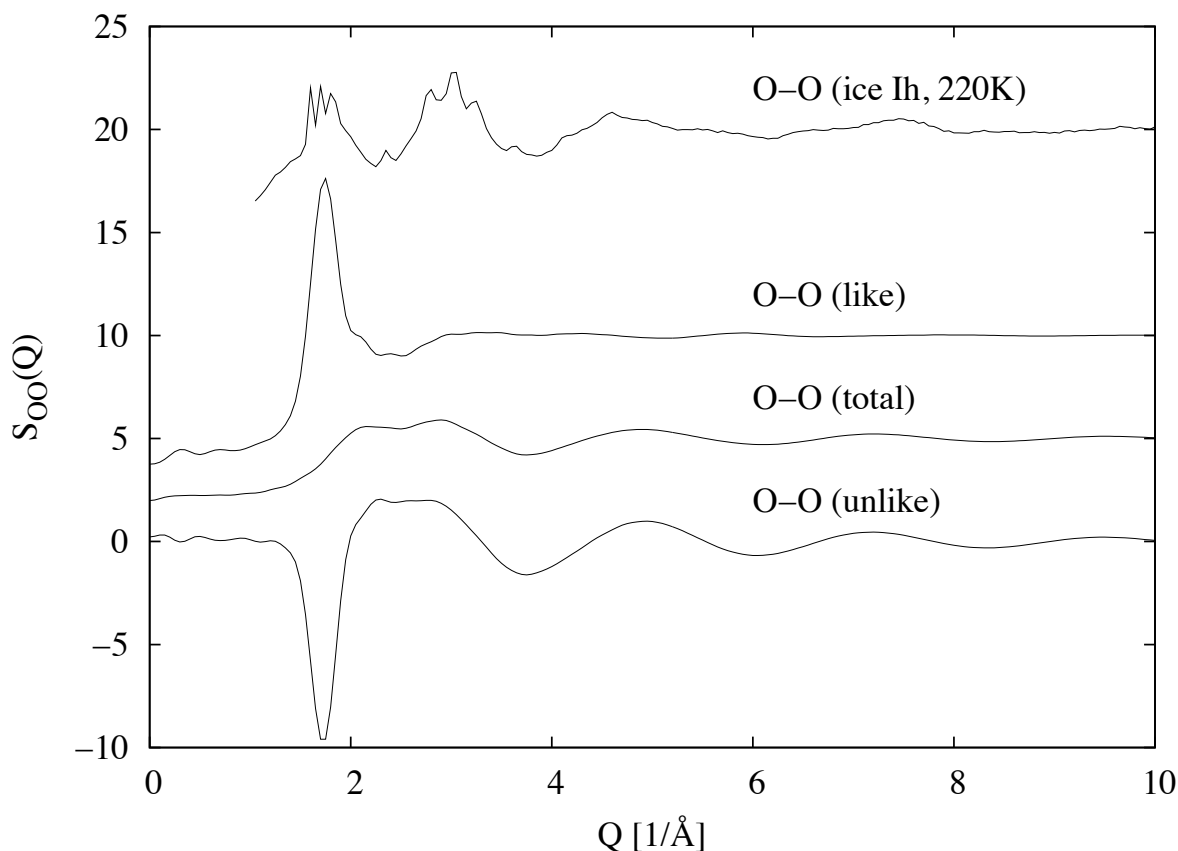


Figure 7: O-O partial structure factors for unlike, total and like pairs for the mixed component model of water. Also shown is the O-O partial structure factor for ice Ih as derived from the data in.³⁴

The presence of non-bonded water molecules in the first coordination shell, as exemplified by the presence of a peak below $\theta \approx 60^\circ$ in the like-unlike triplet angle distribution, Figure 3, would presumably be completely prohibited in an ideal ice Ih structure. Hence water structure has to be clearly distinguished from that of ice by the presence of these non-bonded molecules in the first shell.⁴² The ideas expressed by the present model therefore appear to fit well with those obtained

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3 in a recent *ab initio* calculation.⁴³
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6 Yet another intriguing feature of the model is that when compressed at constant temperature
7 the simulated water has *lower* potential energy than before compression, Table 2. The numbers
8 here are qualitatively consistent with the estimated enthalpy of water under pressure,⁴⁴ which if
9 anything would imply an even larger decrease in potential energy with pressure than simulated
10 here. To see how this is, the enthalpy is given in terms of the internal energy and pressure and
11 volume, $\Delta H = \Delta U + \Delta(PV)$. According to⁴⁴ the excess enthalpy of water at 273K and 400MPa
12 above ambient pressure is ≈ 2.4 kJ/mole. The pressure-volume contribution to the enthalpy change
13 is 6.3kJ/mole, so the net change in potential energy is -3.9kJ/mole. According to Table 2 the
14 change in potential energy in the simulation is -1.4kJ/mole, which is the correct sign even if the
15 wrong magnitude. This might also explain for example why many forms of high density crystalline
16 and amorphous ice can be recovered at ambient pressure: if the temperature is low enough there is
17 insufficient kinetic energy available to allow them to expand up to their (higher potential energy)
18 ambient pressure structures.
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32 Further refinements to this model could be envisaged to improve the fit to both the diffraction
33 and thermodynamic data over a wider range of temperatures and pressures, and it remains to be
34 seen whether this model will have a longer term impact on the overall understanding of water
35 properties.
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43
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47 ing the x-ray diffractometer.
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51 **Notes and References**

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24 (41) These terms are combined with their statistical weights to produce the final estimate so the
25 magnitudes shown here do not represent the actual contribution to the specific heat.
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28 (42) It should be clarified that the term “non-bonded” in this context does not imply the water
29 molecule is not hydrogen bonded to *any* other water molecules. It is simply intended to sig-
30 nify that the molecule is not bonded to the molecule at the centre of the coordinate system.
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