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# **Particle Acceleration in Space and Laboratory Plasmas**

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# PARTICLE ACCELERATION IN SPACE AND LABORATORY PLASMAS

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## ABSTRACT

The general principle of charged particle acceleration in space and laboratory plasmas is illustrated by a discussion of particular types of acceleration mechanisms which can be classified as either deterministic processes or stochastic processes. Acceleration by parallel electric fields, produced in double layers is an example of a deterministic process. Fermi acceleration and acceleration by turbulent wave fields are examples of stochastic processes. The physical acceleration mechanism involved in each type of process is discussed and examples given for space and laboratory plasmas.

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## INTRODUCTION

The study of acceleration processes remains one of the most important areas of research in both laboratory and space plasmas. The subject is also vast, making it almost impossible to cover all aspects. We will concentrate here on a few processes that occur in both space and laboratory plasmas. The subject of acceleration mechanisms we will discuss can be categorised into either deterministic processes such as parallel electric fields, double layers and coherent single waves or stochastic processes such as wave turbulence and Fermi acceleration.

We begin by defining what we mean by acceleration in plasmas which can be summed up by the simple equation

$$\Delta T = q \int_0^{\Delta t} \underline{v}_0(t) \cdot \underline{E}(\underline{r}_0(t), t) dt$$

where  $\underline{E}$  is the electric field determined by currents and charges in the plasma,  $\Delta T$  is the change in the particle kinetic energy  $\underline{v}_0$  is the particle velocity and  $q$  is particle charge. A deterministic system is defined as one where  $\underline{E}(\underline{r}, t)$  does not change significantly during the acceleration phase, whereas for a stochastic system  $\underline{E}(\underline{r}, t)$  changes in a random fashion. We will only be concerned in this report with those acceleration processes which result in energetic particles which have been appreciably accelerated rather than processes that lead to bulk plasma heating. The processes considered are distinguished by the formation of energetic or runaway in the particle distribution function. Such energetic particle distributions are prevalent in all plasmas examples being:

Cosmic rays  $e^-$ ,  $p$ ,  $He$ , heavy ions,  $10^7$ - $10^{22}$ eV

Solar flares  $e^-$ ,  $10^5$ - $10^9$ eV

Aurorae in Planetary Magnetospheres  $e^-$ ,  $10^4$ - $10^6$ eV

Runaways in Tokamaks  $e^-$ ,  $10^5$ - $10^6$ eV

Laser Fusion  $e^-$ ,  $10^5$ - $10^6$ eV.



Although the above plasma environments and scales are vastly different (see table 1) a general trend appears, namely a sizeable fraction of the free energy goes into a small fraction of the available particles. The results is the formation of non-thermal tails in the distribution function. In all plasmas the fundamental question arises of how the particles are raised from the thermal level to higher energies. In space plasmas they are usually preceded by violent events with the energy coming from magnetic fields in, for example, magnetospheric substorms, solar flares, supernovae, etc. In laboratory plasmas the accelerated particles can be produced by injecting high power electro-magnetic radiation into the plasmas as, for example, in radio frequency current drive in tokamaks or laser fusion.

The report will also concentrate on acceleration processes that occur in magnetospheric plasmas. Although they are minute in energy content relative to astrophysical plasmas the magnetospheres of the planets are essential links in the chain of studies that connect plasmas in the laboratory and cosmic plasmas. They and the solar wind are the largest plasmas that can be studied in situ. Measurements made in the earth's magnetosphere are now so precise that in some cases they exceed laboratory measurements in resolution and detail.

## **DETERMINISTIC SYSTEMS**

### **1. DOUBLE LAYERS**

In discussing acceleration of charged particles one must start with the simplest process, i.e acceleration by parallel DC electric fields such that  $E$  in equation (1) is constant during the time it takes to accelerate the particle by substantial amount. Parallel electric fields can be created by the formation of electrostatic double layers<sup>[1]</sup> or by the effects of anomalous resistivity<sup>[2]</sup> in a plasma with field aligned currents. Such a system is deterministic since once the potential difference through which the particles fall is known then everything else is known. From a macroscopic point of view a double layer is a discontinuity surface where a potential difference,  $\phi$  exists across the surface. The magnitude of the voltage drop is at best the order of  $KTe/e$ , where  $K$  is Boltzmann's constant,  $T_e$  is the electron temperature and  $e$  is

the electron charge. In a strong double layer  $\phi \gg KTe/e$ . A double layer is collisionless with a transition region a few tens of Debye lengths,  $\lambda_{De}$ , thick in which there is violation of quasi-neutrality. The thickness of the layer is less than the mean free path. Double layers are usually associated with the Buneman two stream instability and are created when the electron drift velocity  $v_D$  is greater than the thermal velocity  $v_{Te}$  which is the condition required to satisfy the Buneman instability criteria. In other words an electron beam is usually necessary to generate double layers<sup>[3,4]</sup>. Figure 1 represents a typical set up to study laboratory double layers. One of the applications of double layers have been to planetary magnetospheres in particular as a possible mechanism for producing the high energy electron streams responsible for producing the aurora<sup>[5]</sup>. However the model has been seriously criticised by several authors<sup>[6,7,8,9]</sup> who have demonstrated that the observed electron and ion distributions with peaks in the range 1-20 keV are created by a stochastic process and not one based on a quasi-static potential difference, they have also pointed out that it is necessary to assume a source of particles which need constant replenishment while this is not a problem in the laboratory it is in the magnetosphere. It is also unlikely that such intense field aligned currents necessary to satisfy the Buneman instability can exist on auroral field lines. However, it is conceivable that the energetic electrons can themselves generate double layers.

## 2. COHERENT WAVE ACCELERATION

The acceleration of particles by coherent fields is another deterministic process and is important in laboratory plasmas to Physics where such fields are produced by high powered microwave or laser systems. One topic of intense study at present is the beat wave concept<sup>[10]</sup> where large amplitude coherent plasma waves are generated by the non-linear interaction of two high powered electromagnetic waves. The electric field of the plasma wave can be as large as  $10^9$  V/m which is  $10^3$  times greater than the electric fields produced in high energy accelerators which are limited by the breakdown voltage of solid material. The beat wave concept has many valuable applications in physics. Charged particle acceleration, current drive and plasma heating in tokamaks are examples. In the beat wave accelerator a large amplitude coherent plasma mode can be generated by beating together two laser beams of frequencies and wavenumbers ( $\omega_1$ ,



$\underline{k}_1$ ) and  $(\omega_2, \underline{k}_2)$  such that the beat wave  $(\omega_b, \underline{k}_b)$  where  $\omega_b = \omega_1 - \omega_2$ ,  $\underline{k}_b = \underline{k}_1 - \underline{k}_2$ , has a phase velocity  $\omega_b/\underline{k}_b$  equal to the group velocity of the original laser beams which in an underdense plasma ( $\omega_{1,2} \gg \omega_{pe}$ ) is very close to the velocity of light. The plasma mode is driven by the non-linear force or ponderomotive force of the laser beams. The amplitude of the plasma mode grows linearly in time with

$$\frac{\delta n(t)}{n_0} = \frac{\delta n(0)}{n_0} + \frac{e^2 E_1 E_2}{4 m_e^2 \omega_0 \omega_1 c^2} \omega_{pe} t \quad (1)$$

where  $\delta n$  is the density perturbation of the plasma mode,  $E_{1,2}$  are the electric field strengths of the laser,  $m_e$  is the electron rest mass and  $c$  is the velocity of light. In the above description the plasma wave amplitude will grow until the wave breaking limit is reached with  $\delta n/n = 1$ . However relativistic effects saturate the growth well before the wave breaking limit is reached. This technique will produce possibly the largest fields available in laboratory plasmas. Electrons which are injected into the longitudinal plasma wave frame will experience acceleration by an electric field  $E_p$  given by

$$E_p = \alpha m_0 c \omega_{pe} / e \quad (2)$$

where ( $\alpha \approx 0.1$ ) is determined by the saturation mechanism. For a typical design  $E_p = 10^9 \text{V/m}$  which is substantially larger than conventional accelerator fields.

In astrophysical plasmas very intense low frequency electromagnetic waves are produced by rotating pulsars which have dipole fields as strong as  $10^8$  Tesla. Gunn and Ostriker<sup>[11]</sup> have demonstrated that these fields can produce cosmic rays with energies in excess of  $10^{18} \text{eV}$  with a maximum at about  $10^{21} \text{eV}$ . A relativistic particle in a low frequency large amplitude electromagnetic field will ride the wave at essentially constant phase and see crossed fields producing a drift velocity  $v_D = E/B$  in a time of the

order of the cyclotron period  $m/eB$  where  $E$  and  $B$  are the electric and magnetic field components of the wave. The electron can reach energies equal to  $mc^2\omega_c^2/\omega_0^2$  [12], where  $\omega_c (= eB/m)$  is the cyclotron frequency,  $\omega_0$  is the wave frequency ( $\omega_0 \ll \omega_c$ ), which for low frequency waves can be extremely large. The characteristic time to accelerate the particle  $m/eB$  approaches infinity because  $m \rightarrow \infty$ , consequently the particle goes on accelerating indefinitely.

Although coherent wave acceleration processes are important in laboratory plasmas where large coherent fields such as lasers can easily be generated it is rather doubtful they play a major role in space plasmas where the majority of waves are observed to be random turbulent fluctuations. This however, may not be the case in pulsars where very large amplitude coherent fluctuations have been predicted [11]. There are exceptions also in space plasmas where coherent waves have been detected, an example being coherent electrostatic ion cyclotron waves on auroral field lines observed by the S3-3 polar orbiting satellite [13]. These waves may be responsible for perpendicularly heating the ions by wave trapping. Some of the perpendicular energy can then be converted into parallel energy by the dipole mirror forces of the earth.

## STOCHASTIC ACCELERATION

Stochastic acceleration by turbulent wave fields is an efficient mechanism for transferring plasma turbulence to fast particles, increasing their energy even further. Charged particles interacting with the turbulent fields gain a small amount of energy during each "collision". In astrophysics the best known stochastic acceleration mechanism is Fermi acceleration [14]. Fermi suggested that cosmic rays could "collide" with moving magnetised clouds. The collision is effective via the magnetic mirror effect and the particle suffers an energy gain or loss proportional to the velocity of the cloud and the energy of the particle illustrated in Fig. 2.

For relativistic particles the average gain is

$$\Delta T = \pm \frac{v}{c} T \quad (3)$$



where  $T$  is the kinetic energy of the particle the sign of the increment depends on the relative direction of motion between the cloud and particle +ve if the cloud and particle are moving towards each other (-ve if not). Statistically head on collisions predominate with particles gaining energy on average. In fact  $dT/dt = n(\frac{v}{c})^2 T$  where  $n$  is the number of collisions per unit time. If the particles are lost at a rate  $1/\tau$  the particles then take on a power law distribution. If  $f$  is the number of particles in range  $dT$  then

$$\frac{df}{dT} = - \frac{\partial}{\partial T} (\alpha T f) - \frac{f}{\tau}, \quad \alpha = \left(\frac{v}{c}\right)^2 n \quad (4)$$

and the steady state solution is

$$f = \text{const. } T^{-(1+1/\alpha\tau)} \quad (5)$$

This expression fits the data extremely well with a coefficient between 2 and 3. A number of things about Fermi acceleration are worth noting since they are common to other stochastic acceleration processes.

- a) The acceleration takes place in very small steps with many "collisions" off different clouds to reach the final energy.
- b) The particles are accelerated in a non-isotropic fashion - energy being put into the parallel direction.
- c) Energy distributed among a large number of particles is redistributed among a few of very high energy. The energy of the colliding particles always increases. Fermi acceleration although only relevant in astrophysical situations does illustrate the mechanism of stochastic acceleration in general.

In laboratory plasmas wave-particle interactions play a fundamental role in transferring energy and momentum accelerating particles through stochastic processes. In magnetic fusion devices radio frequency heating

is regarded as a major area of research. A number of schemes have been proposed such as electron (ion) cyclotron heating [E(I)CRF], Alfvén wave and lower-hybrid wave heating. In these schemes microwaves generated by powerful gyratons are mode converted in the plasma to a particular plasma wave which then heats the plasma. If the correct scheme is used it is not only heating that takes place but a tail develops which constitutes a beam. This beam can be represented by a current  $J_b = n_b e v_b$  which supplements the normal induced toroidal current  $J_T$ . It is hoped that the radio frequency current driven system will be able to sustain a steady state operation for tokamaks. In the sun, solar wind and planetary magnetospheres acceleration and heating by plasma turbulence is of fundamental importance. Phenomena such as the aurora, x-ray bursts in solar flares and radio wave generation, producing for example auroral kilometric radiation in the magnetosphere rely on wave-particle interaction processes where particles are stochastically accelerated by the broad spectrum of turbulent waves. These wave-particle interactions can take place through two main mechanisms Landau or gyroresonance. In Landau resonance, sometimes called a Cherenkov process, the longitudinal electric field component of the wave interacts with the particle and changes its parallel energy. The acceleration through absorption of longitudinal plasma waves is by no means the only mechanism by which particles can accumulate energy. Absorption through the cyclotron mechanism or gyroresonance increases the velocity component at right angles to the magnetic field. We will concentrate on the acceleration by longitudinal fields since we believe it is the one responsible for beam production on auroral field lines and certainly for beam production and current drive in tokamaks ([16]). In particular we consider the stochastic acceleration of electrons parallel to the magnetic field by lower hybrid turbulence. This is also an important wave for current drive in tokamaks where it has been shown<sup>[15]</sup> that beam currents of 1A per 1W of RF power are produced by lower-hybrid waves.

In the laboratory the waves are created by mode converting radio frequency power in the plasma. However, in natural plasmas the waves must be generated by an instability. The nature of the instability depends on the origin of the free energy (in a non-equilibrium plasma). In the magnetosphere such non-equilibrium plasma states are created during magnetospheric substorms resulting in bulk plasma motion, and the creation of



shocks and steep gradients. It has been shown<sup>[16]</sup> that such conditions exist on auroral field lines and can trigger either the modified two stream instability or the lower hybrid drift instability in either case the fluctuating fields are primarily electrostatic with  $k_{\perp} \gg k_{\parallel}$  where  $k_{\perp, \parallel}$  are the perpendicular, parallel wave number component these waves are lower

hybrid waves with frequencies  $\omega_{LH}^2 = \omega_{pi}^2 / (1 + \omega_{pe}^2 / \omega_{ce}^2)$ . Such waves are seen with large amplitudes in auroral regions of the magnetosphere<sup>[16]</sup>.

The nature of the waves are such that they have velocities along the field lines ranging from a cut-off above the electron thermal,  $v_{Te}$ , to greater than the speed of light. Thus lower-hybrid waves have the range of phase velocities necessary to accelerate electrons in the tail of the distribution function.

The stochastic acceleration of electrons in the tail of the distribution due to interaction with lower-hybrid turbulence is described by the quasi-linear Fokker-Planck or diffusion equation for the electrons

$$\frac{\partial f_e}{\partial t} = \frac{\partial}{\partial v_{\parallel}} D_{\parallel} \frac{\partial f_e}{\partial v_{\parallel}} \quad (6)$$

where  $f_e$  is the electron distribution function,  $|\Delta v|^2 (= D_{\parallel T})$ , is the change in particle velocity in time  $\tau$ ,  $D_{\parallel} = |\Delta v|^2 / \tau$  is the quasi-linear velocity diffusion coefficient given by

$$D_{\parallel} = 16\pi^2 \frac{e^2}{2} \frac{k_{\parallel}}{m_e \omega_{pe}} \epsilon_k \quad (7)$$

where  $\epsilon_k$  is the wave energy density per unit wave number. The diffusion equation describes the change in velocity of particles which resonate with waves with phase velocities close to the particle velocity. The particles

gain energy from those waves that slowly overtake them. On the bases of the Fokker-Planck model an almost flat plateau, over the region of velocities the turbulent waves exist, is produced as the end results of the acceleration. Figure 3 shows the change in the distribution function and wave spectrum as a result of wave-particle interactions. A number of interesting results such as the acceleration time or length, number of particles accelerated and approximate tail temperature can be derived. For example if the acceleration region is of length  $L$  then the effective range  $k_{\parallel}$  for the waves is  $k_m \leq k_{\parallel} \leq k_0$  where  $k_m \approx 2\pi/L$  and  $k_0$  is determined by the strong Landau damping that takes place for  $k_{\parallel} \approx \omega_{LH}/2v_{te}$ . The total energy density is thus

$$W_{Lh} = (k_0 - k_m) \epsilon_k \approx k_0 \epsilon_k$$

for  $L$  large. The time for electrons to be accelerated in the tail from  $v_1$  to  $v_2$  is  $\tau_{acc} = (v_2 - v_1)^2 / D_{\parallel}$  with the characteristic acceleration length  $L_{acc} = \langle v \rangle \tau_{acc}$ , where  $\langle v \rangle$  is the average electron velocity,  $L_{acc}$  can be written as [16]

$$L_{acc} = \frac{\omega_{Lh}^2}{4\pi\omega_{pe}^2} \frac{n_0}{W_{Lh}} \frac{KT_e}{m_e} \frac{m_i}{m_e} \frac{(v_2 - v_1)^2}{v_{Te}^2} \frac{k_0}{k_{\parallel}} \langle v \rangle \quad (3)$$

In the aurora this length,  $L_{acc}$ , is of the order of  $1-3 \times 10^6 m$  [16] which is a suitably short distance for auroral acceleration. Thus tail formation by lower-hybrid waves on auroral field lines is an efficient mechanism. Laboratory experiments [15] also demonstrate the efficiency of accelerating electrons to high energies with lower-hybrid waves.



## CONCLUSIONS

A number of different acceleration mechanisms have been given which are representative of the various processes that exist in both space and laboratory plasmas. The mechanisms have been shown to be of two types namely deterministic (coherent) or stochastic in nature. One important acceleration process, however, was not included this is acceleration by collisionless shocks. The bow shock would be such an example. Acceleration in a collisionless shock can be due to either a potential difference or turbulent wave fields so in this sense it was covered.

We have shown that apart from Fermi acceleration acceleration processes in laboratory and space plasmas are essentially very similar and a study of both will lead to a better understanding of these fundamental physical processes involved.

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T A B L E 1

## TYPICAL PLASMA PARAMETERS

|                                          | Gassecus Nebula           | Magnetosphere                | Tokamak                      | Laser                       |
|------------------------------------------|---------------------------|------------------------------|------------------------------|-----------------------------|
| Density<br>$n(\text{cm}^{-3})$           | $10^3$                    | 10                           | $10^{14}$                    | $10^{21}$                   |
| Temp<br>$T_e(\text{eV})$                 | 1                         | $10^3$                       | $10^3-10^4$                  | $10^3$                      |
| Plasma Frequ<br>$\omega_{pe}(5^{-1})$    | $2 \times 10^6 \text{Hz}$ | 30kHz                        | $2 \times 10^{12} \text{Hz}$ | $10^{15} \text{Hz}$         |
| Debye Length<br>$\lambda_{De}(\text{m})$ | .2m                       | 100m                         | $2 \times 10^{-5} \text{m}$  | $7 \times 10^{-9} \text{m}$ |
| $n\lambda_{De}^3$                        | $10^7$                    | $10^{13}$                    | $10^6$                       | $4 \times 10^2$             |
| $\beta (\frac{nokTe}{\beta^2})$          |                           | 5%                           | 1%                           | -                           |
| Relative Plasma<br>Dimensions            | $\gg \lambda_{De}$        | $3 \times 10^5 \lambda_{De}$ | $10^5 \lambda_{De}$          | $10^5 \lambda_{De}$         |

**Note**

- a) The magnetosphere has the largest number of particles in a Debye sphere.
- b) The relative dimensions of magnetosphere, tokamak and laser are all the same  $\sim 10^5 \lambda_{De}$ .

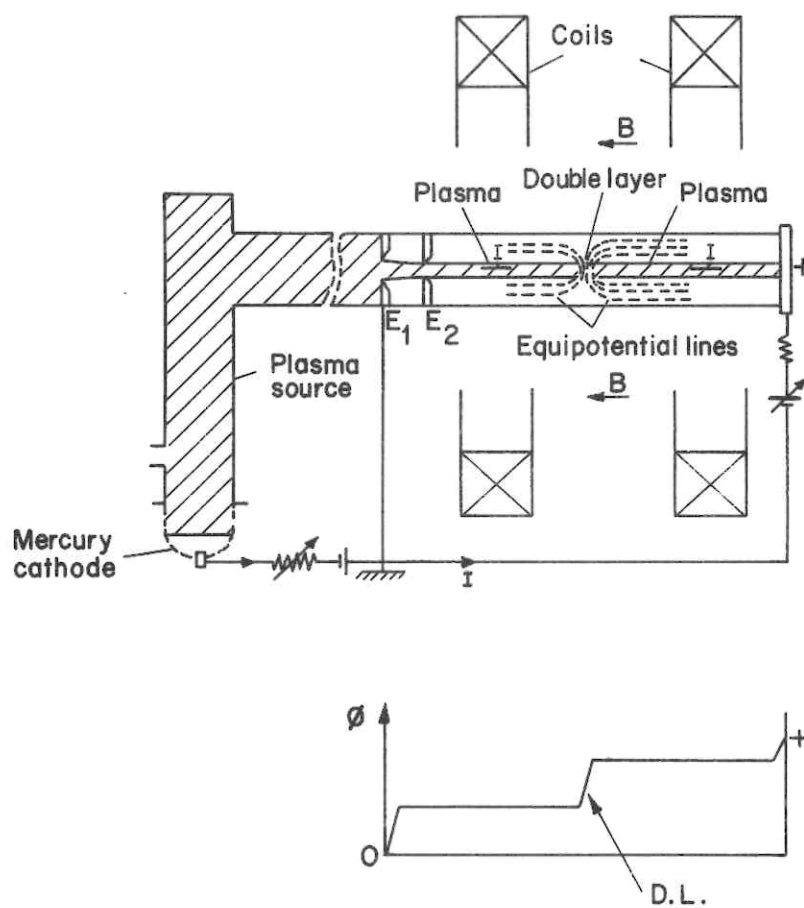


Fig.1 Typical Laboratory Set Up to Study Double Layers and Corresponding Potential Jump (Torvén 1979)



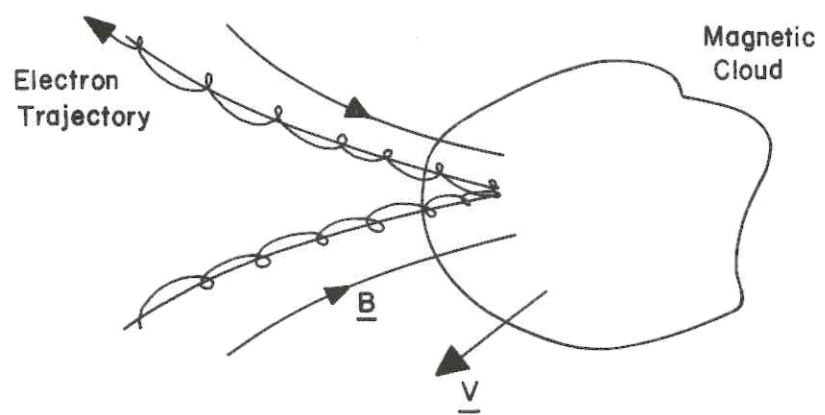


Fig.2 Fermi Acceleration of Charged Particle Colliding With a Moving Magnetic Cloud.

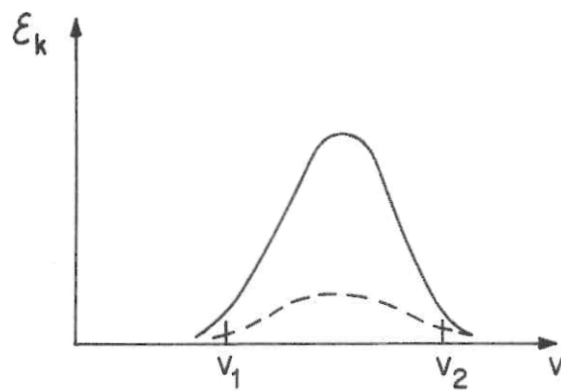
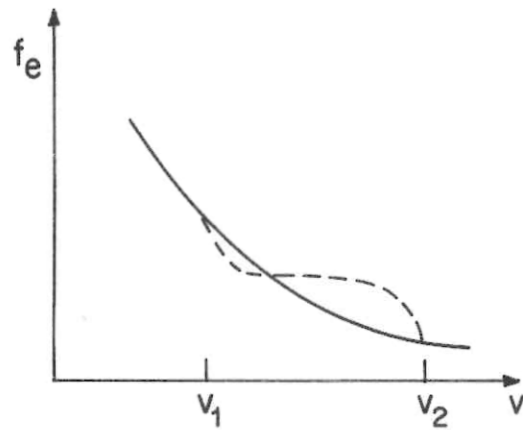


Fig.3 Electron Distribution Function  $f_e$  and Wave Energy Density Illustrating the Evolution of the Distribution Function as the Waves Damp. Solid Lines Indicate Initial State, Broken Lines Represent Final State