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THE GEOMETRY OF CONSTANT PERIMETER DIPOLE WINDINGS

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ABSTRACT: The geometry of a simple constant perimeter end shape is described. A method of assessing its suitability (by means of a computer program) is mentioned.

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1. INTRODUCTION

A recent publication[1] from this Laboratory referred to a type of dipole winding known as "constant perimeter". The requirement is that it should be possible to wind each layer with no tendency for turns to slip sideways when being laid in place.

Probably there is an infinite number of end shapes which would satisfy this requirement. This report describes the geometry of one shape only.

The shape can be made cheaply since it is generated by a cylindrical milling cutter whose axis is maintained at a constant angle to, and intersects, the axis of the magnet former, while the former is simultaneously rotated and traversed beneath it. In this way the cutter axis is made to intersect an imaginary semi-circle (referred to as the generating circle) wrapped around an imaginary cylinder (referred to as the "generating cylinder") whose axis is coincident with the former axis.

The nomenclature used is given in Fig. 1.

The surface is a ruled surface, and may be developed onto a flat plane (hereafter called the "development"). We take as axiomatic that a wire stretched tightly between two points on such a surface will be represented by a straight line on the development. Since we want the first turn of each layer to lie snugly against the base cylinder (or outer cylinder), all we need do is to make the intersection of each layer with the base cylinder (or outer cylinder) such that the line of intersection is straight on the development. This line should be a continuation of the line of intersection of the straight sides of the layer on the same development (see Fig. 2).

2. THEORY

Axiom: The tangent to the direction of motion (relative to the former axes) of a point on the cutter axis is parallel to the tangent plane at the line of contact between cutter and the shape being generated.

The co-ordinates of a point on the generating circle are given by

$$x_1 = r_1 \sin\theta \quad (1)$$

$$y_1 = r_1 \cos\theta \quad (2)$$

$$z_1 = r_2 \cos\phi \quad (3)$$

where $\theta = \frac{r_2}{r_1} \sin\phi$

The direction cosines of the cutter axis are

$$l_c = \sin\theta\cos\beta \quad (4)$$

$$m_c = \cos\theta\cos\beta \quad (5)$$

$$n_c = -\sin\beta \quad (6)$$

The line of contact has the same direction cosines.

Direction cosines for the tangent to the generating circle may be obtained: Consider a point on the generating circle moving a small distance ds, then

$$\frac{d\phi}{ds} = \frac{1}{r_2} \; ; \quad \frac{d\theta}{d\phi} = \frac{r_2}{r_1} \cos\phi$$

$$\frac{dx_1}{d\theta} = r_1\cos\theta; \quad \frac{dy_1}{d\theta} = -r_1\sin\theta; \quad \frac{dz_1}{d\phi} = -r_2\sin\phi$$

$$\therefore \quad l_t = \frac{dx_1}{ds} = \cos\theta\cos\phi \quad (7)$$

$$m_t = \frac{dy_1}{ds} = -\sin\theta\cos\phi \quad (8)$$

$$n_t = \frac{dz_1}{ds} = -\sin\phi \quad (9)$$

A point in the (x', y', z') system is transformed to the (x, y, z) system by

$$x = x_1 + l_{x'}x' + l_{y'}y' + l_{z'}z' \quad (10)$$

$$y = y_1 + m_{x'}x' + m_{y'}y' + m_{z'}z' \quad (11)$$

$$z = z_1 + n_{x'}x' + n_{y'}y' + n_{z'}z' \quad (12)$$

where $l_{x'}$ is the direction cosine between x' axis and x axis, $m_{x'}$ is that between

x' axis and y axis and so on.

We put $z' = 0$, and insert the appropriate values of direction cosines in (10), (11) and (12), and obtain

$$x = x_1 - x'\sin\theta\sin\beta - y'\cos\theta$$

$$y = y_1 - x'\cos\theta\sin\beta + y'\sin\theta$$

$$z = z_1 - x'\cos\beta$$

Since $x' = r\cos\alpha$ and $y' = r\sin\alpha$, then

$$x = x_1 - r\cos\alpha\sin\theta\sin\beta - r\sin\alpha\cos\theta \qquad (13)$$

$$y = y_1 - r\cos\alpha\cos\theta\sin\beta + r\sin\alpha\sin\theta \qquad (14)$$

$$z = z_1 - r\cos\alpha\cos\beta \qquad (15)$$

which relate a point on the cutter cross-section perimeter to the (x, y, z) system.

Now consider a small element ds of this perimeter.

$$\frac{d\alpha}{ds} = \frac{1}{r} \qquad (16)$$

Differentiating (13), (14) and (15) with respect to α, and using (16) gives

$$\frac{dx}{ds} = \sin\alpha\sin\theta\sin\beta - \cos\alpha\cos\theta \qquad (17)$$

$$\frac{dy}{ds} = \sin\alpha\cos\theta\sin\beta + \cos\alpha\sin\theta \qquad (18)$$

$$\frac{dz}{ds} = \sin\alpha\cos\beta \qquad (19)$$

which are direction cosines of a line in the tangent plane.

This line is perpendicular to the line of contact, of which the direction cosines are given by (4), (5) and (6). Equations (4), (5), (6), (17), (18) and (19) therefore define the plane. The angle made by the line defined by (7), (8) and (9) with the line (4), (5), (6) is equal to the compliment of the angle between (7), (8), (9) and (17), (18), (19). Applying the usual rule for finding the angle between two lines with known direction cosines gives

$$\cos\psi = \sin\phi\sin\beta \tag{20}$$

$$\sin\psi = -\cos\phi\cos\alpha - \sin\phi\cos\beta\sin\alpha \tag{21}$$

(21) may be written

$$f(\alpha) = \cos\phi\cos\alpha + \sin\phi\cos\beta\sin\alpha + \sin\psi = 0$$

Fortunately the root of this equation occurs at its maximum, hence

$$f'(\alpha) = -\cos\phi\sin\alpha + \sin\phi\cos\beta\cos\alpha = 0$$

(ϕ and β are fixed, hence ψ is fixed by equation (20))

or $$\tan\alpha = \tan\phi\cos\beta \tag{22}$$

or $$\sin\alpha = \tan\phi\cos\beta\cos\alpha \tag{23}$$

(13), (14) and (15) are used to determine the point of contact of the cutter cross-section with the generated shape, and with the aid of (23), (1), (2) and (3):

$$x_2 = r_1\sin\theta - r\cos\alpha\sin\theta\sin\beta - r\cos\alpha\cos\theta\cos\beta\tan\phi \tag{24}$$

$$y_2 = r_1\cos\theta - r\cos\alpha\cos\theta\sin\beta + r\cos\alpha\sin\theta\cos\beta\tan\phi \tag{25}$$

$$z_2 = r_2\cos\phi - r\cos\alpha\cos\beta \tag{26}$$

A line drawn through x_2, y_2, z_2 with direction cosines given by (4), (5), (6) intersects the base cylinder at x_3, y_3, z_3 such that

$$\frac{x_3 - x_2}{l_c} = \frac{y_3 - y_2}{m_c} = \frac{z_3 - z_2}{n_c} \tag{27}$$

Substituting

$$x_3 = r_3 \sin\theta_3 \tag{28}$$

$$y_3 = r_3 \cos\theta_3 \tag{29}$$

and the appropriate expressions for l_c and m_c into (27) gives

$$\sin(\theta_3 - \theta) = \frac{x_2\cos\theta - y_2\sin\theta}{r_3} \tag{30}$$

By using (24), (25) and (23), this reduces to

$$\sin(\theta_3 - \theta) = \frac{-r\sin\alpha}{r_3} \tag{31}$$

Having determined α from (22)*, θ_3 from (31), x_3 and y_3 are got from (28) and (29). z_3 is obtained from the last two expressions of (27), by substituting (25) and (26) for y_2 and z_2. The result is

$$z_3 = r_2\cos\phi + \tan\beta\left(r_1 - \frac{y_3}{\cos\theta}\right) - \frac{r\cos\alpha}{\cos\beta}\ (1 - \sin\beta\cos\beta\tan\theta\tan\phi) \tag{32}$$

In which we see that the first term represents the intersection of the cutter axis with the generating circle, the second term represents the "offset" due to the inclination of the cutter axis, and the third term allows for the displacement of the contact line due to the radius of the cutter.

3. DEVELOPMENT

A simple Fortran program, TRUE (Trigonometrical Routine for Unwinding Ends), has been written to develop the shape. It divides the surface into elements bounded by the intersection with the base cylinder and lines of contact with the cutter:

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* Equation (22) gives the value of α for the point of contact with the inner surface. Contact with the outer surface is diametrically opposite, i.e. add 180° to the value from (22)

these elements are "placed side by side" on a flat plane. Results are presented graphically on a visual display unit (see Fig. 3), satisfactory lines of intersection are straight and parallel to the horizontal axis. Parameters (e.g. cutter angle β) are varied until satisfactory intersections have been achieved.

4. COMMENTS

The shape cannot be perfect for every layer of a winding. This is evident from the results of the computer program (Fig. 3). Probably the best approach is to achieve a perfect shape for the middle layer: discrepancies in other layers can be tolerated because of the friction which exists between wires wound under tension.

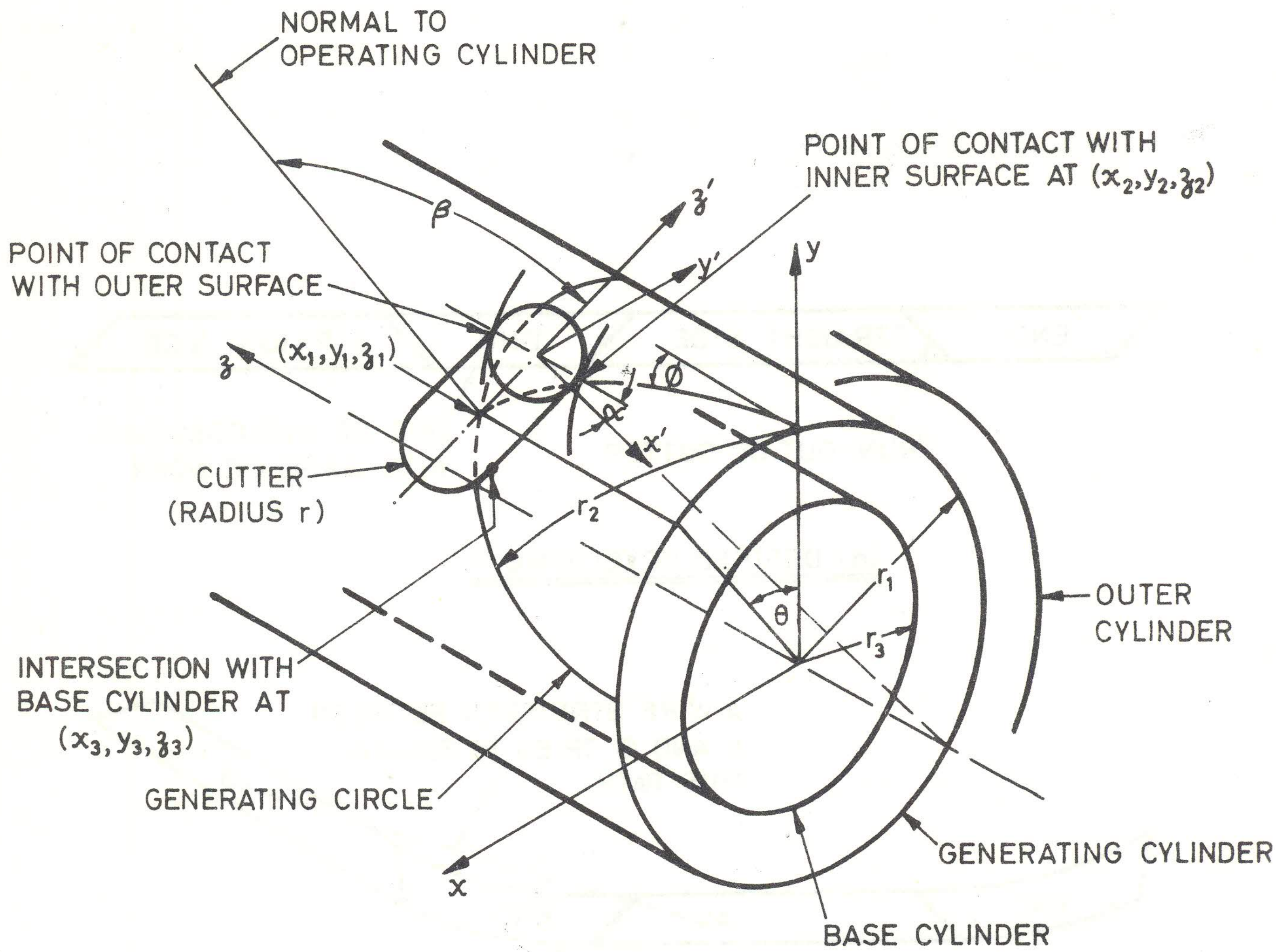
Experience has shown that it is best to make the generating cylinder coincide with the mid-height of the winding along the straight sides. Increasing the base cylinder radius beyond this merely lengthens the ends: decreasing the radius towards that of the base cylinder gives a sharp radius at the transition from end to straight side, which increases the difficulty of winding.

5. ACKNOWLEDGEMENTS

R.B. Hopes and R. Eagle , whose practical work on shapes and winding stimulated this paper, are thanked gratefully.

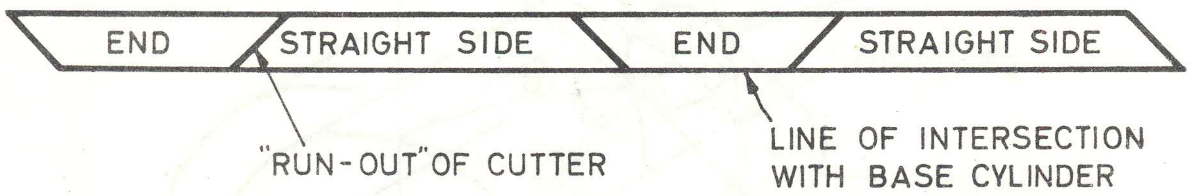
6. REFERENCE

1. M.N. Wilson. "Superconducting Dipoles for Beam Transport" RL-73-108.

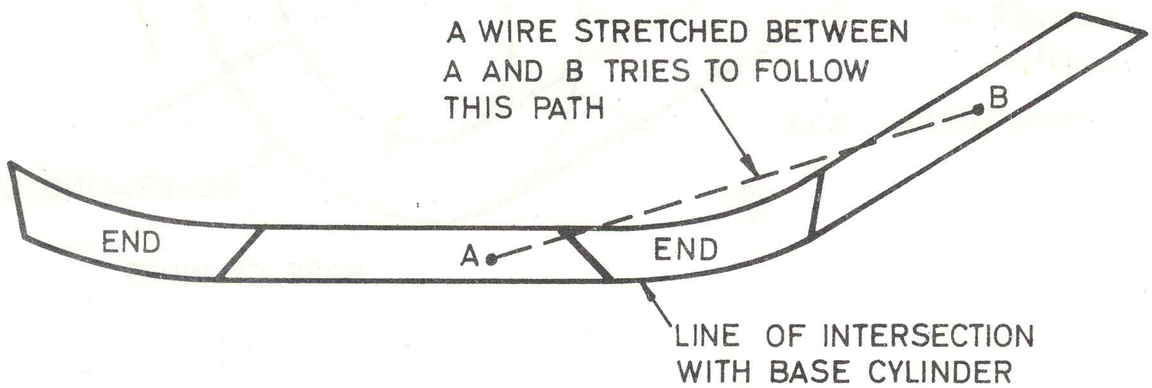


NOTE:- THE THEORY IS DERIVED BY MAKING THE ORIGIN OF THE SYSTEM OF AXES (x', y', z') COINCIDENT WITH THE GENERATING CIRCLE, NOT AS HAS BEEN DRAWN ABOVE (FOR THE SAKE OF CLARITY). z' IS THE AXIS OF ROTATION OF THE CUTTER. x AND z AXES INTERSECT.

Fig. 1. NOMENCLATURE



(a) DESIRED DEVELOPMENT



(b) UNSATISFACTORY DEVELOPMENT CAUSED BY MAKING β TOO LARGE

Fig. 2. DEVELOPMENTS

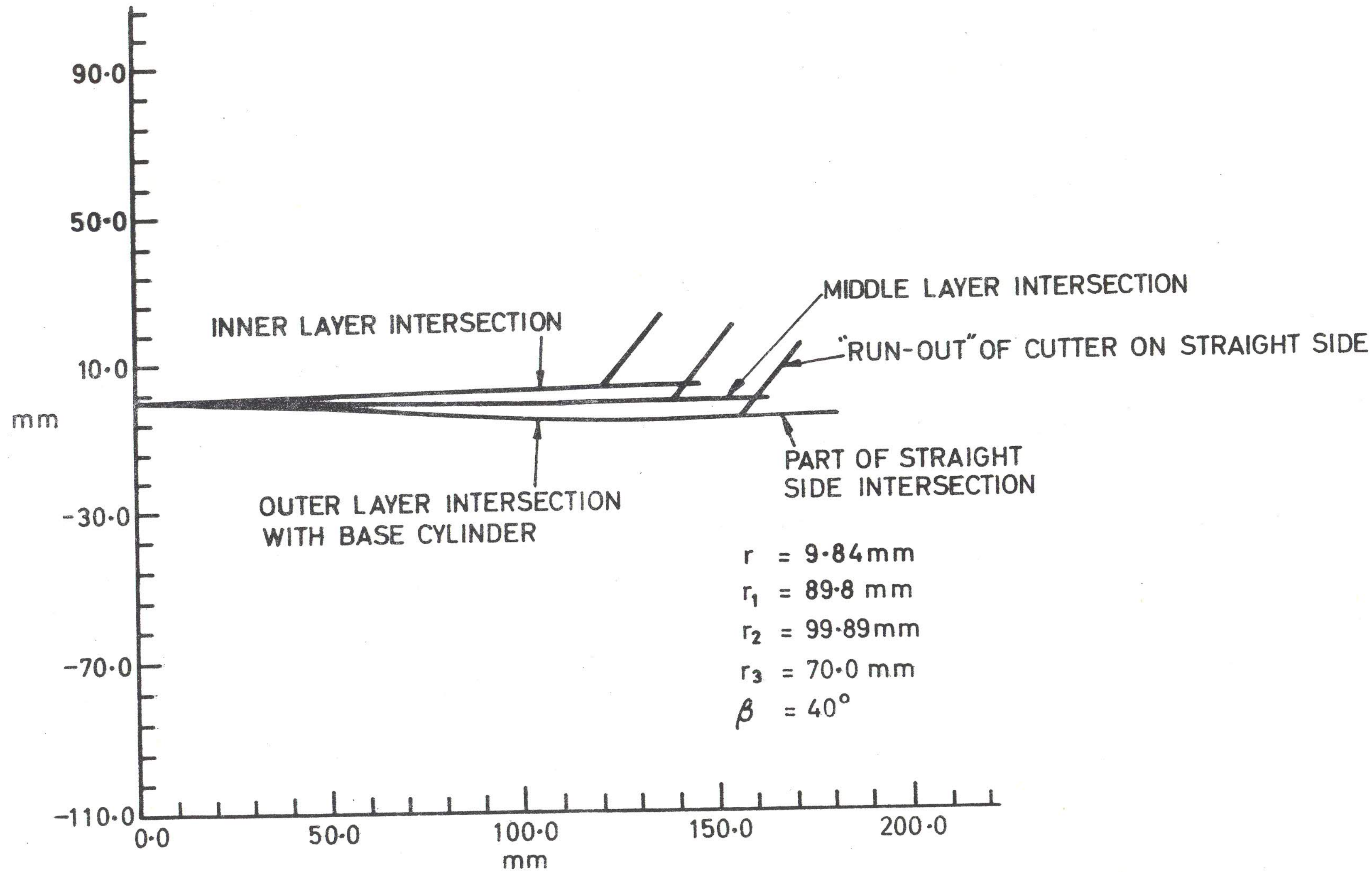


Fig. 3. GRAPHICAL DISPLAY FROM PROGRAM