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Abstract

Silicon Photomultipliers (SiPM) are interesting detectors for beam diagnostics applications due to their reduced dimensions and costs, and higher photon detection efficiency. Possible applications include longitudinal beam profile measurements by synchrotron light imaging, detection of optical transition radiation for energy spectrum measurements and medical imaging. However, quantitative measurement with SiPMs are jeopardized by the systematic reading error due to Optical Cross-talk (OC), i.e. optical coupling between neighbouring diodes in the array. OC results in overestimation of the impinging light level, and reflects the probability of a triggered avalanche creating a photon of suitable energy and direction to fire a second avalanche in another diode. In this paper, we derive a generalized response distribution for SiPM in presence of OC noise, which overcomes the limitations of assumptions currently made in literature and provides a correction of the SiPM response distribution valid for arbitrary large levels of OC.

INTRODUCTION

Sensors capable of detecting single photons have found different applications in diverse fields, including beam loss detection in particle accelerators [1]. The need to reduce the detector dimensions requires the use of small area, highly sensitive detectors that combine integrated readout circuitry functionality in a cheap fabrication process. In addition, small area detectors can be easily integrated in a dense arrays and be easily coupled with optical fibers.

In the last decade, bi-dimensional, closely packed arrays of up to 500 independent Single Photon Avalanche Diodes (SPAD) per square millimetre have been reported and are now widely available commercially. The SiPM retains the photon counting ability of the SPAD while outperforming it dramatically for dynamic range and recovery time due to the large number of independent cells. Nevertheless, SiPM do suffer from erroneous counting due to dark noise effects, mainly caused by three phenomena:

1. electron-hole pairs created in the depletion layer by random thermal ionization (dark count).
2. parasitic avalanche triggering by photons created during a primary avalanche and migrated to a neighbouring cell (OC). OC has been reported to be sensibly reduced for SiPM featuring optical trenches: strips of material with different refraction index placed between neighboring cells, which deflect photons away from the active area [2] .
3. time delayed release of a hot carrier by a trap level due to imperfections in the lattice, leading to a time delayed second avalanche phenomenon (after-pulsing).

Of these noise sources OC in particular is neither negligible for relatively large signals which cause 10 or more cells to fire, as for dark noise, nor linked to the timing of the acquisition system used, as for afterpulsing. Furthermore, it modifies the response of the SiPM leading to an overestimation of the impinging light level. Only one main study has been published to date, by Vinogradov and co-workers [3], which tackles the issue of creating a reliable theoretical cross talk model to correct for the light level overestimation.

In that same work, Vinogradov and co-workers show that the event spectrum of a SiPM is not distributed in a Poissonian fashion, but undergoes a shift towards higher order events (where by *order* of an event is intended the number of avalanches which form it), leading to an overestimation of the light signal, and a set of equations is provided which describes the modified probability distribution which originates the spectrum.

However the model relies on the assumption that each primary avalanche can only create one single secondary avalanche with a probability p. This secondary can in turn create a tertiary avalanche with the same probability p and so on until, with probability 1-p no additional avalanches are created: this path of reasoning leads to the establishment of a coupled Poisson-binomial distribution for the expected spectrum of the device. The physics of the OC phenomenon, however, lies in the creation of secondary avalanches due to optical photons created in the depletion layer of a single SPAD cell by a recombining charge pair while an avalanche event is unfolding. There is no physical reason why each avalanche should create only one OC photon; instead, being a discrete random event depending on a very high number of trials (the number of charged pairs which can possibly recombine) each with a low probability of success, its description fits perfectly the Poisson statistics definition, and a full theory of OC should allow for each SPAD cell to create n secondary avalanches via optical photons with n being Poisson distributed. This model would then reduce to the binomial model for low OC rates, as the Poisson distribution approximates the binomial distributions for λ approaching zero.

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THEORETICAL MODEL

In what follows, we present a more realistic theoretical model for OC which improves the precision on light level measurement. The model makes use of the following assumptions:

1. Each primary avalanche can create any number of secondary avalanches. The distribution of the random variable indicating the number of secondary avalanches created will be Poissonian, with average value γ. γ is considered to be characteristic of the particular device and have no further dependencies. Each secondary avalanche will also have the same Poissonian probability of triggering more avalanches, which will be inturn able to produce more avalanches with a Poissonian probability distribution, until all avalanches spontaneously create 0 further avalanches.
2. The light level is low enough that saturation effects (i.e. lowering of the probability of cross talk p due to the fact that neighbouring cells have already fired and are therefore insensitive) can be neglected.

In the following, D_i indicates the probability of an avalanche to create i secondary avalanches. Since this is Poissonian distributed we will have for D_i:

$$D_i = \frac{\gamma^i e^{-\gamma}}{i!} \tag{1}$$

We can then calculate the probability P_{m-n} that m initial avalanches create n more avalanches. P_{m-n} will be composed of several addends, each given by the product of the probabilities of each of the m initial avalanches to create a number of additional avalanches so that the total number of secondary avalanches produced amounts to n. This is expressed by a product of m terms (one for each initial avalanche), each of which being given by $D_{a(i)}$, $1 \le i \le m$; with the indexes a(i) representing the number of secondary avalanches created by the i^{th} primary avalanche. Since P_{m-n} expresses the probability of n secondary avalanches being created, the indexes a(i) need to add up to n. Furthermore, one has to take into account in a summation one of these products for each of the possible sets of indexes a(i) giving n as total. These conditions can be written as:

$$\begin{aligned} P_{m-n} &= \sum_{\sum_{j=1}^{m} a(j)=n} \prod_{i=1}^{m} D_{a(i)} = \\ &\gamma^n e^{-m\gamma} \sum_{\sum_{j=1}^{m} a(j)=n} \left(\prod_{i=1}^{m} a_i! \right)^{-1} = \\ &\gamma^n e^{-m\gamma} S_{m-n} \end{aligned} \tag{2}$$

This expression is valid for every m>0 and every n≥0. We note that:

$$P_{0-k} = \begin{cases} 1 \rightarrow k = 0 \\ 0 \rightarrow k > 0 \end{cases} \tag{3}$$

P_{m-n} expresses the probability that m initial avalanches create n more, but says nothing on what these n more will do. We therefore introduce the probability C_{m-n} that m initial avalanches create *exactly* n more avalanches, meaning that all the secondary avalanches created are then extinguished by not creating any more secondaries in turn. This can be expressed in terms of the probabilities P_{m-n} as the product of the probability of m events creating a_1 more, times the probability of these a_1 more events to create a_2 more and so on, until n more avalanches have been created, say at the k^{th} step; and finally times the probability of a_k events to create 0 more, hence ending the avalanche process. We note that should, at any time in this cascade, 0 further events be created, the process would stop. Thus the k steps can never be more than n, in which case all indexes a_i, $1 \le i \le k$ would be 1. We can moreover assume these steps to be always exactly n, provided that if one of the indexes is 0, all the following will be 0 as well: this way, all the probabilities following the first null index will be expressed as $P_{0-0} = 1$, and hence do not contribute to the product. Finally, the probabilities relative to each possible combination of indexes a_i have to be added together. Thus, the probability C_{m-n} can be expressed as:

$$\begin{aligned} C_{m-n} &= \sum_{\sum_{i=1}^{n} a(i)=n \;\; s.t. \;\; a(k)=0 \Rightarrow a(h)=0 \; \forall h>k} P_{m-a_1} \left(\prod_{i=1}^{n-1} P_{a_i - a_{i+1}} \right) P_{a_n - 0} = \\ &= \gamma^n e^{-(m+n)\gamma} \sum_{\sum_{i=1}^{n} a(i)=n \;\; s.t. \;\; a(k)=0 \Rightarrow a(h)=0 \; \forall h>k} S_{m-a_1} \left(\prod_{i=1}^{n-1} S_{a_i - a_{i+1}} \right) S_{a_n - 0} \\ &= \gamma^n e^{-(m+n)\gamma} G_{m-n} \end{aligned}$$

Where the coefficients G_{m-n} can be calculated with a computer program making use of partition mathematics. We can now express the probability distribution expected in presence of strong OC noise, in terms of the parameters λ and γ of the two Poisson distributions for light input and cross talk noise respectively.

The probability of observing an n^{th} order event will be given by the probability of n events arising from primary avalanches, times the probability of these events not to create any OC secondary, plus the probability of m primary events arising times the probability of these m primary events to generate exactly n-m secondary avalanches, summed over all the values of m s.t. $1 \le m < n$.

$$\begin{aligned} P_n &= \frac{\lambda^n e^{-\lambda}}{n!} \cdot P_{n-0} + \sum_{k=1}^{n-1} \frac{\lambda^k e^{-\lambda}}{k!} \cdot C_{k-(n-k)} = \\ &\frac{\lambda^n e^{-\lambda}}{n!} e^{-n\gamma} + \gamma^n e^{-\lambda} e^{-n\gamma} \sum_{k=1}^{n-1} \frac{\lambda^k}{k!} \cdot G_{k-(n-k)} \end{aligned} \tag{4}$$

Note that this distribution correctly reduces to the Poisson distribution in λ if the probability of creating secondary avalanches is null (i.e. $\gamma = 0$).

In order to compare this description of the cross talk effect, which we will refer to as *double-Poissonian* with the one presented by Vinogradov and co-workers, which we will refer to as Poisson-Binomial, it is necessary to choose the values of γ and p so that the expected values of the two distributions are equal, calculating the expected value of the distribution in eqn. 4 directly from the values of the computed distribution.

Fig. 1 shows examples of this corrected distribution for different values of γ, whilst Fig. 2 shows a comparison with the corresponding uncorrected Poissonian distribution and with the Poisson-Binomial distribution derived in the first part of the paper, with p chosen as described above.

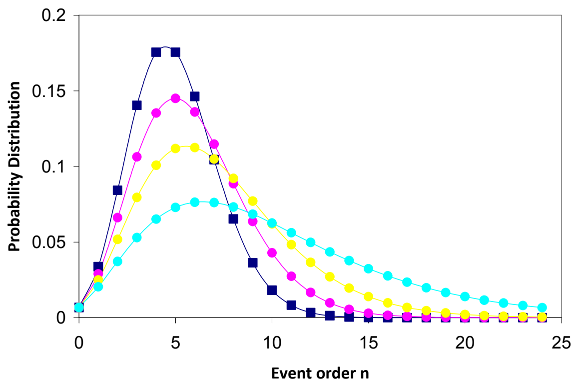

Figure 1: Uncorrected Poisson distribution (square markers) and three double-Poissonian distributions (round markers) for increasing value of γ: 0.15 (pink), 0.3 (yellow), 0.5 (light blue). For all cases $\lambda = 5$.

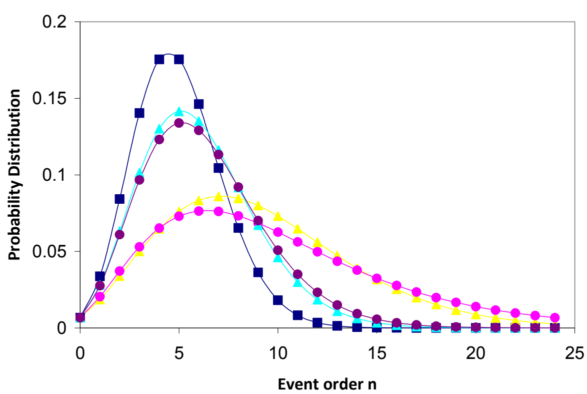

Figure 2: Uncorrected Poisson distribution (square markers) with $\lambda = 5$ and both Poisson-binomial (triangular markers) and double-Poissonian (round markers) distributions, with $\gamma = 0.2$ (violet) and 0.5 (pink) parameters p and γ chosen so that the expected value of each distribution is equivalent.

It can be seen from the plots how the double Poissonian distribution features a behaviour qualitatively similar to the binomial-Poisson distribution, in that it has a longer higher order events tail as compared to the uncorrected distribution, with this difference being even more pronounced for the double Poissonian than for the binomial-Poissonian.

In order to evaluate which difference to the measurement is made by the use of the more complex formulae for the double Poissonian, we fitted a double-Poissonian distribution of known parameters λ and γ with a binomial-Poisson distribution with parameters λ_b and p_b. The percentage difference $|\lambda-\lambda_b|/\lambda$ gives then the error on λ, and hence on the measurement of the impinging light intensity, due to use of the simplified cross talk effects theory based on the binomial Poisson distribution. It is observed that the error increases both with increasing lambda and with increasing OC probability γ, and it can be concluded that the binomial-Poisson distribution is a good enough approximation of the actual SiPM spectral distribution at medium light levels operation (about 50 cells firing on average) only for values of γ (expected value of the number of secondary avalanches created by a single primary) lower than 0.3. Beyond these values, the error in estimation of light level due to use of the binomial-Poisson distribution is larger than 10%.

Therefore, it can be concluded that the simplified model suggested by Vinogradov and co-workers constitutes a good enough approximation for small area SiPM, whose OC has been reported to be as low 1% ($\gamma \approx 0.01$) thanks to improved manufacturing technologies; whilst the same model presents significant errors for the analysis of large area SiPM, which are attracting much interest for applications in Positron Emission Tomography, where the OC can be as high as 30-50% (γ up to 0.5) [4].

OUTLOOK AND CONCLUSION

In this contribution we have presented an original theory accounting for the effects of OC on the spectral distribution of SiPM signal. We have shown this theory to rely on less stringent assumptions than the previously available theory by Vinogradov and co-worker. The suggested theory leads to shallower probability distributions, whose deviation from the mathematically simpler theory of Vinogradov becomes relevant for medium levels of impinging radiation and devices subject to higher OC effects. Examples of distributions derived from the proposed theory have been shown and compared with the alternative theory.

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