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Barrier option pricing: modelling with neural nets

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Abstract

We report call option pricing for up-and-out style barrier options through the use of a neural net model. A synthetic data set was constructed from the real LIFFE standard option price data by use of the Rubenstein and Reiner analytic model (Risk September (1991) 28). Unbiased estimates at the 95% confidence level were achieved for realistic barriers (barrier 4% or more above $\max(S_0, X)$).

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Keywords: Up-and-out call option pricing; Neural net

1. Introduction

This work aims to find an effective way to assess the pricing of up-and-out barrier call options by models extracted from realistic data by using standard commercial neural net tools. Some success has already been reported for standard option pricing [1,2]. Barrier option data is not publicly available so we create a synthetic data set

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based upon LIFFE standard European call option data [1]. The synthetic barrier prices were derived using an analytic model [3]. We investigate a set of barriers applied to these data for up-and-out call barrier options. Previous investigators have identified the need to take special adaptive action near the barrier when deploying multinomial trees for barrier options [4,5]. We find a small fraction of predicted prices are disproportionately incorrect and we are investigating the source of this problem, e.g. many of the problematic prices are very close to maturity. Using two sample t-statistics at the 95% confidence level we find good agreement between target and predictions. The paired sample t-statistics accept most models of the test data or fail because of just a few data points; but the predictions for the 2% level barrier are emphatically rejected.

2. Barrier options

A standard call option gives the holder the right to buy an asset in the future at a previously agreed price X, known as the exercise price. The payoff of such a call is $\max(S_T - X, 0) - c_o$ where c_o is the price of the option and S_T is the value of the asset at expiry (see Fig. 1). An out-style barrier option limits the range S can take over the lifetime of the option. For $c_{up\text{-}and\text{-}out}$ the option ceases to have value if the barrier at H on S is crossed from below. The popularity of barrier options can be attributed to the following advantages: (1) lower cost compared with standard options; (2) flexibility in setting the barrier level; (3) ability to be linked to any underlying security; (4) precision to exposures (upside/downside potential).

3. The barrier option price modelling problem

Analytical models have limitations. Rubinstein and Reiner [3] defined an analytical model for the value of European-style barrier options on stock. The model is based on the Black–Scholes and Merton approach to pricing European-style ordinary options. Boundary conditions are added to obtain the path-dependent feature of barrier options. Appropriate formulae are given by Hull [6]. This analytical solution makes some assumptions such as the lognormal probability

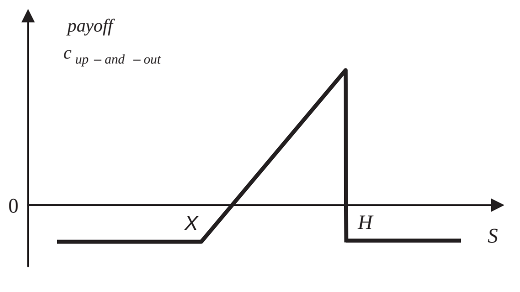

Fig. 1. Payoff diagram for an up-and-out call (with zero rebate and showing premium deducted).

distribution for the asset price, constant volatility and interest rate across the option's life span. These assumptions are rarely met in the real world. Moreover not all types of barrier options are amenable to this approach [6]. Asset prices are monitored at discrete and contractually specified times, t/m, for barrier crossing. With t as time to maturity, σ as the volatility, and m as number of timesteps. This means that actual crossings may not be detected in discrete samples. Broadie et al. [7] have proposed an effective barrier should be used for the case of discrete monitoring. For an up and out call the adjusted barrier value $H \to H\ \exp(-0.5826\sigma\sqrt{t/m})$. The binomial method for barrier options converges very slowly as the number of branches or lattice levels increases, often requiring unattainably high computing times for even a modest accuracy [4]. There is quantization error associated with fineness of the grid in time and specification error associated with the position of the barrier with respect to the asset price grid. Trinomial trees, implicit, explicit and other finite-difference methods suffer from similar problems. Furthermore, the numerical accuracy of these methods becomes an important issue. Special adaptive mesh methods and trees are deployed that require crafting to deal with the need for higher resolution near the barrier [5].

Barrier options are not exchange traded but depend on over the counter contracts so the prices are not publicly available. Hence we needed to generate a synthetic data set. We used a LIFFE standard index option daily prices data subset already described in a previous study to obtain S, X, t, and σ [1]. It covered the period March 1992 to April 1997. r was obtained from Datastream's LIBOR rate and synchronized with the LIFFE tabulation. Dividend, d, does not apply to LIFFE ESX European style FSTE100 index calls because it is an implied future. Our 6-input model includes t, r, σ, S, X, H. And our 7-input model has one more input, the trading date. Actual OTC (over-the-counter) prices may be affected by factors other than the standard option price and the level of the barrier. The LIFFE data were cleaned by removing records for which there were: (1) invalid or missing values; (2) omitted or mis-reported bid/offer prices; (3) zero or missing volume traded. For σ we used the implied volatility and omitted $\sigma <= 0.01$ or > 0.40. The data were randomly separated into two sets: 7083 option records in the training set and 7171 option records in the test set.

4. Findings

We show in Fig. 2 a scattergram and straight line fit between the actual synthetic prices and the predictions from the neural net for the $H_0 + 8\%$ for the 6-input model. Similar scattergrams for the other models were found. Over the whole price range the neural net gives reasonable predictions. For this case we found 84.8% of differences lie within the range of -10% to $+10\%$. Fig. 3 shows the overall %–age difference for $H_0 + 8\%$. We compared a range of barriers from $H_0 + 2\%$ to $H_0 + 10\%$. All models satisfy the 2-sample t-test for 95% confidence but the $H_0 + 2\%$ fails the paired t-test for 5% significance. A seven parameter model, with trading date was added to the input data, did not appear to give improved prediction. Table 1 reports

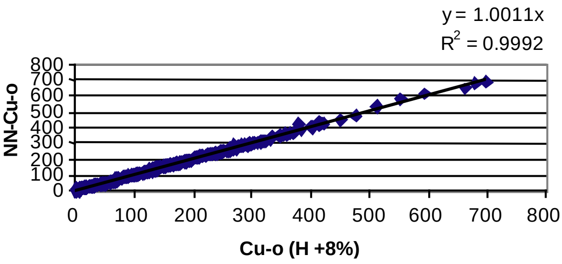

Fig. 2. Price plot of neural net prediction against synthetic actual level set at $H = \max(X, S_0) * (100\% + 8\%)$; $x = \text{Cu-o}(H + 8\%)$.

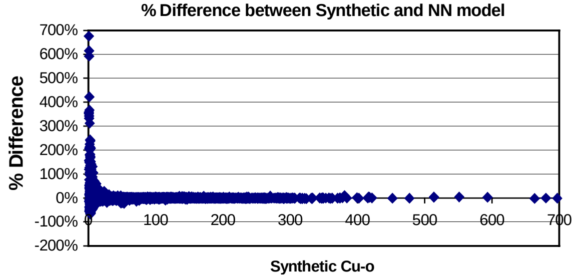

Fig. 3. Price plot % difference between neural net prediction and synthetic actual. % difference is percentage difference in option price calculated as 100%$*$ (Synthetic ($c_{up\text{-}and\text{-}out}$) $-$ NN($c_{up\text{-}and\text{-}out}$))/ Synthetic ($c_{up\text{-}and\text{-}out}$).

Table 1
Comparison of barrier option price fit $H = H_0 * (100\% + n\%)$ where $H_0 = \max(S_0, X)$

Barrier level H for $c_{up\text{-}and\text{-}out}$	6 Inputs (t, r, S, X, H, σ)	7 Inputs $(t, r, S, X, H, \sigma$ and trading date)
$(H_0 + 2\%)$	$y = 0.9925x$ $R^2 = 0.9940$ Probability under paired t-test: 0.0	$y = 0.9916x$ $R^2 = 0.9955$ Probability under paired t-test: 0.0
$(H_0 + 4\%)$	$y = 0.9865x$ $R^2 = 0.9902$ Probability under paired t-test: 0.47	$y = 0.9969x$ $R^2 = 0.9947$ Probability under paired t-test: 0.026
$(H_0 + 6\%)$	$y = 0.9994x$ $R^2 = 0.9981$ Probability under paired t-test: 0.34	$y = 0.9974x$ $R^2 = 0.9966$ Probability under paired t-test: 0.057
$(H_0 + 8\%)$	$y = 1.0011x$ $R^2 = 0.9992$ Probability under paired t-test: 0.05	$y = 0.9994x$ $R^2 = 0.9985$ Probability under paired t-test: 0.29
$(H_0 + 10\%)$	$y = 1.0016x$ $R^2 = 0.9982$ Probability under paired t-test: 0.44	$y = 0.9999x$ $R^2 = 0.9970$ Probability under paired t-test: 0.12

the line of best fit gradient for all the scattergrams, R^2, and the paired t-test probability with a significance target of 0.05.

These preliminary findings are encouraging. They show that standard commercial neural networks can be used to predict unbiased prices for up-and-out style barrier options defined on market traded standard options. Special adaptive mechanisms were not needed to deal with the barrier. Further investigations are needed of: (1) the derivation of prediction intervals for the predicted option prices; (2) the characteristics of prediction outliers; (3) The use of inputs consisting of combinations of variables e.g. S/X as moneyness to deal with ageing features found in predictions of standard models.

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