A FEA report for applying autofrettage technology on plain cylinders

Compiled by Yanling Ma on 270709

1. Introduction

In order to provide supportive information for the development of high pressure cells for the JRA project, further FEA simulations were conducted with a view to validating a hand calculation results (carried out by Rob Done) on plain cylinders. Geometries/diameters of the plain cylinders for hand calculation are shown in Table 1. Formulas used for the hand calculation are shown in Equations (1) to (4). Some of the hand calculation results are listed in Table 2 of the following section 2. Different diameters and lengths of the plain cylinders are used for FEA simulation, respectively. The results are presented and discussed in sections 3. Additional information is given in Section 4 and conclusions are drawn in section 5.

2. Hand calculation and FEA requirement on plain cylinders

2.1 Hand calculation (conducted by Rob Done starting and ended with *)

<u>*JRA – cylindrical pressure vessel calculations – 18th May 2009</u>

<u>Background</u>

Analysis of thick-walled cylindrical pressure vessels can be predicted by use of the theories of elasticity and plasticity. The pressure limitations of such vessels can be determined to a well known level of accuracy based on the assumption that it is sealed at both ends and therefore has a uniform longitudinal stress distribution. In addition, during the process known as autofrettaging, non-uniform plastic flow will progress through the wall of the vessel, allowing subsequent use to be undertaken safely in the resultant elastic region. A maximum autofrettaging pressure exists however, above which reverse yielding of the vessel bore will occur. The applicable equations for this manufacturing technique are shown below.

Specification

There are principally only two upper pressure limit requirements within the JRA project:

- 10 Kbar seal test assembly requiring a test pressure of 14 Kbar
- 8 Kbar inert/hydrogen gas pressure vessel requiring a test pressure of 11 Kbar

For the purposes of this report, analysis has only been undertaken using a beryllium-copper alloy grade 25HT. The critical objective of the analysis is to ensure that in both cases the calculated maximum autofrettaging pressure is above the required test pressure.
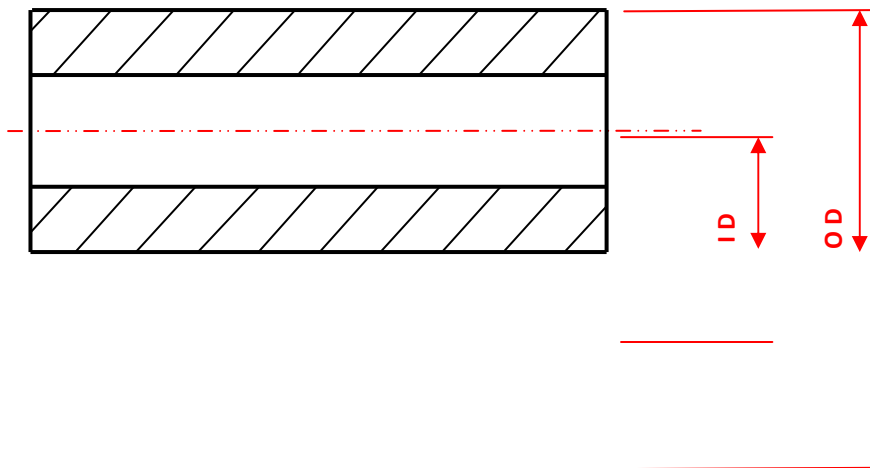
Nominal material properties

Beryllium copper – grade 25HT

UTS $\quad$ (σ_{uts}) $\qquad$ = $\qquad$ **1400 x 10^6 N/m^2**

0.2% Proof stress (σ_p) $\qquad$ = $\qquad$ **1200 x 10^6 N/m^2**

Cylindrical pressure vessel geometry.

(Figure 1 The plain cylinder added by Yanling)

$$Ratio\ of\ diameters = K = \frac{OD}{ID}$$

Three values of K have been considered with the bore of each pressure vessel remaining constant at 7.0 mm diameter throughout.

(Table 1 Diameters of the plain cylinders added by Yanling)

ID	OD	K
7.0 mm	21.0 mm	3
7.0 mm	28.0 mm	4
7.0 mm	35.0 mm	5

Reference source of equations: High pressure technology – volume 1 – chapter 7 – high pressure containment in cylindrical monobloc vessels.

Burst pressure:

$$Pb = \frac{2 \times \sigma_p}{\sqrt{3}} \ln K$$ (1) [1] (added by Yanling)

Maximum autofrettage pressure:

$$P_A = \frac{2 \times \sigma_p}{\sqrt{3}} \left[\frac{K^2 - 1}{K^2} \right]$$ (2) [1] (added by Yanling)

Yield pressure:

$$P_y = \frac{\sigma_p}{\sqrt{3}} \left[\frac{K^2 - 1}{K^2} \right]$$ (3) [1] (added by Yanling)

For K=3.0

$$Pb = \frac{2 \times 1200 \times 10^6}{\sqrt{3}} \ln 3.0 \quad = \quad 15200 \, bar$$

$$P_A = \frac{2 \times 1200 \times 10^6}{\sqrt{3}} \left[\frac{3.0^2 - 1}{3.0^2} \right] \quad = \quad 12300 \, bar$$

$$P_y = \frac{1200 \times 10^6}{\sqrt{3}} \left[\frac{3.0^2 - 1}{3.0^2} \right] \quad = \quad 6150 \, bar$$

For K=4.0

$$Pb = \frac{2 \times 1200 \times 10^6}{\sqrt{3}} \ln 4.0 \quad = \quad 19300\, bar$$

$$P_A = \frac{2 \times 1200 \times 10^6}{\sqrt{3}} \left[\frac{4.0^2 - 1}{4.0^2} \right] \quad = \quad 13000\, bar$$

$$P_y = \frac{1200 \times 10^6}{\sqrt{3}} \left[\frac{4.0^2 - 1}{4.0^2} \right] \quad = \quad 6500\, bar$$

For K=5.0

$$Pb = \frac{2 \times 1200 \times 10^6}{\sqrt{3}} \ln 5.0 \quad = \quad 22300\, bar$$

$$P_A = \frac{2 \times 1200 \times 10^6}{\sqrt{3}} \left[\frac{5.0^2 - 1}{5.0^2} \right] \quad = \quad 13300\, bar$$

$$P_y = \frac{1200 \times 10^6}{\sqrt{3}} \left[\frac{5.0^2 - 1}{5.0^2} \right] \quad = \quad 6650\, bar$$

Calculating the diameter where maximum autofrettage would occur:

$$\phi_A = \left[\left(\frac{P_A - P_y}{P_b - P_y} \right) \times (OD - ID) \right] + ID \qquad \textbf{(4 Rob's formula) (added by Yanling)}$$

For K=3.0

$$\phi_A = \left[\left(\frac{12300 - 6150}{15200 - 6150} \right) \times (21.0 - 7.0) \right] + 7.0 \quad = \quad 16.5\, mm$$

For K=4.0

$$\phi_A = \left[\left(\frac{13000 - 6500}{19300 - 6500} \right) \times (28.0 - 7.0) \right] + 7.0 \ = \ 17.66 \ mm$$

For K=5.0

$$\phi_A = \left[\left(\frac{13300 - 6650}{22300 - 6650} \right) \times (35.0 - 7.0) \right] + 7.0 \ = \ 18.9 \ mm$$

Summary of results for pressure vessels with 7.0 mm ID (Table 2 added by Yanling)

K	P_Y	P_A	P_B	ϕ_A	OD
3	6150 bar	12300 bar	15200 bar	16.5 mm	21.0 mm
4	6500 bar	13000 bar	19300 bar	17.66 mm	28.0 mm
5	6650 bar	13300 bar	22300 bar	18.9 mm	35.0 mm

Conclusions

10 Kbar seal test assembly requiring a test pressure of 14 Kbar

The theoretical analysis shown in these results indicates that a pressure vessel with a K value of 4.0 or even 5.0 would be safe to operate at 10 Kbar. The test pressure of 15 Kbar would be above the maximum autofrettaging pressure for this ratio. Consideration would have to be made to whether this test pressure can be lowered since this is a test vessel and would never be used outside of the controlled test area.

8 Kbar inert/hydrogen gas pressure vessel requiring a test pressure of 11 Kbar:

The theoretical analysis shown in these results indicates that a pressure vessel with a K value of only 3.0 would be safe to operate at 8 Kbar. The test pressure of 11 Kbar would be below the maximum autofrettaging pressure for this ratio.

Compiled by Rob Done – 18th May 2009 *

2.2 FEA requirement (proposed by Rob Done starting and ended with *)

* FEA requirements – 15/5/09

Be/Cu plain cylinder:
55 mm long
7 mm internal diameter
21 mm outside diameter

- 6150 bar – yield should be at 7.0 mm diameter
- 12300 bar – yield should be at 16.5 mm diameter
- 15200 bar – yield should be at 21.0 mm diameter

Repeat the above three for 100 mm long and 150 mm long

Be/Cu plain cylinder:
55 mm long
7 mm internal diameter
28 mm outside diameter

- 6500 bar – yield should be at 7.0 mm diameter
- 13000 bar – yield should be at 17.66 mm diameter
- 19300 bar – yield should be at 28.0 mm diameter

Repeat the above three for 100 mm long and 150 mm long

Be/Cu plain cylinder:
55 mm long
7 mm internal diameter
35 mm outside diameter

- 6650 bar – yield should be at 7.0 mm diameter
- 13300 bar – yield should be at 18.9 mm diameter
- 22300 bar – yield should be at 35.0 mm diameter

Repeat the above three for 100 mm long and 150 mm long*

3. FEA Simulations

3.1 Material properties

Material used in the following FEA simulation is virtually same as what was used in Section 2, i.e. Beryllium Copper 25HT. For FEA simulation of autofrettage process/technology, it was assumed that, the material possesses a perfect elastic plastic property with a young's modulus of 134GPa, a Poisson ratio of 0.285 and yield strength of 1200/1206MPa. It should be noted that the yield strength value of 1206MPa (rather than 1200MPa which was used in hand calculation) was used here in the FEA simulation with a view to comparing the results obtained previously (before end of April 2009). Also the difference between the two values is believed to be negligible.

3.2 FEA results

Based on the above material properties and the FEA requirement described in Section 2.2, FEA simulations were conducted on three different K values (of 3, 4 and 5), three different plain cylinder lengths (of 55mm, 100mm and 150mm) and three different pressure levels for each of the three K values, respectively. In addition, FEA simulation was also conducted on the plain cylinder of with a K=4 and a length of 100mm, with the autofrettage pressure of 1288.18MPa (which was used in the previous FEA simulation). All the 28 (in total) FEA simulation results are grouped and presented in the following Sections 3.2.1 to 3.2.3.

3.2.1 FEA results for K=3

As listed in Table 2 and section 2.2, three pressure levels of 615MPa/6150bar, 1230MPa/12300bar and 1520MPa/15200bar, were considered for three different plain cylinder lengths of 55mm, 100mm and 150mm, respectively, for the K value 3. The expected results are: (1) when the plain cylinder internally pressurised to the pressure level of 615MPa/6150bar, the inner surface of the cylinder should just start to yield with a zero plastic deformation depth; (2) when the plain cylinder internally pressurised to the pressure level of 1230MPa/12300bar (i.e. the maximum autofrettage pressure), the inner surface of the cylinder should be yield to a depth 4.75mm (from the inner surface, i.e. to a $\emptyset_A$=16.5mm); (3) when the plain cylinder further internally pressurised to the pressure level of 1520MPa/15200bar, the entire wall thickness of the cylinder should be yield.

To validate the above hand calculation results with FEA results, two points, one is on the inner surface and another is on the outer surface of the plain cylinder, as shown in Figure 2 (points A and B) , were created within the FEA model. So that the variation of the equivalent stress along the entire wall thickness (7mm in this case) can be quantified.

It however should be noted that although numerous (more than 10) two points models (similar to Figure 2) were created and used in the real FEA simulations to cope with different K values of 3, 4, 5 and different plain cylinder lengths of 55mm, 100mm, and 150mm, respectively, only Figure 2 will be used throughout this report to present the above different variables due to their similarity and also with a view to avoiding the repetitive figures within this report.
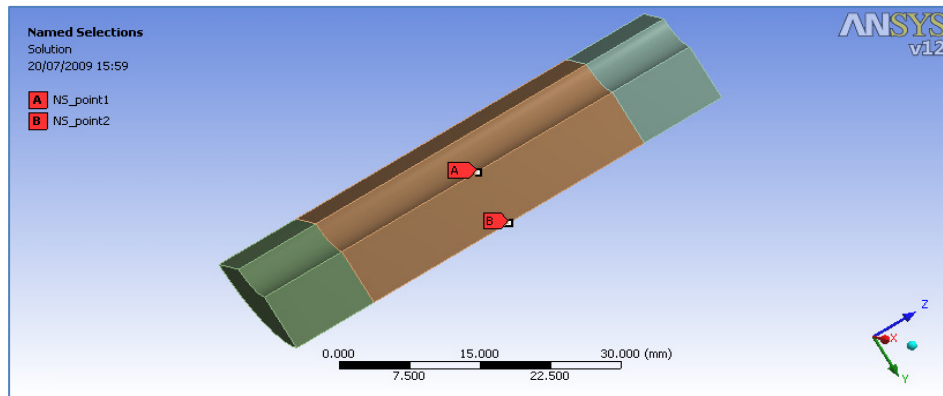

Figure 2 The two points model for post processing

Following these exercises, FEA results for K value 3 are presented in Figures 3 to 5, 6 to 8 and 9 to 11, for three different pressure levels of 615MPa, 1230MPa and 1520MPa, respectively. Under each of the three pressure levels, results for three different plain cylinder lengths of 55mm, 100mm and 150mm, are also depicted respectively.

To quantify, compare and facilitate the FEA results discussions to be carried out in the next Section 3.3, the results for all the variables are also sub-summarised at the end of the following sections 3.2.1 to 3.2.3 for the three K values of 3, 4, and 5, respectively, i.e. they are tabulated in Tables 3 to 5 respectively.
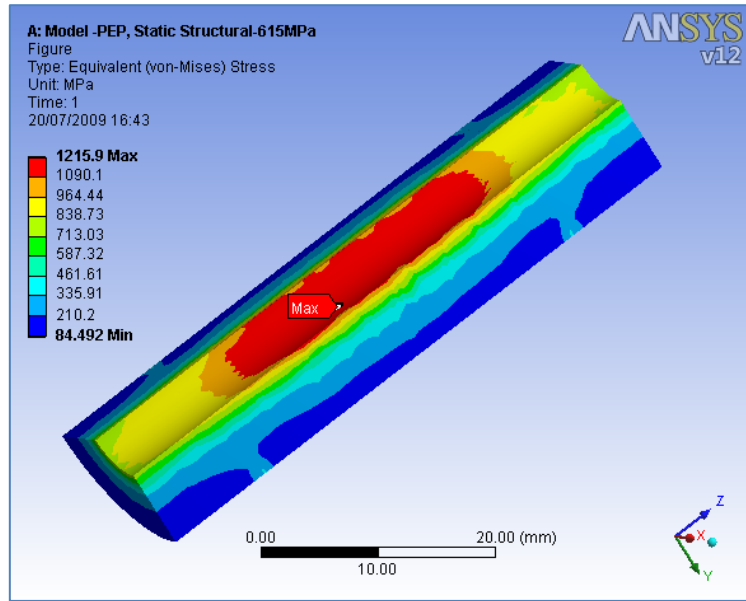
(a) The contour plot of equivalent stress
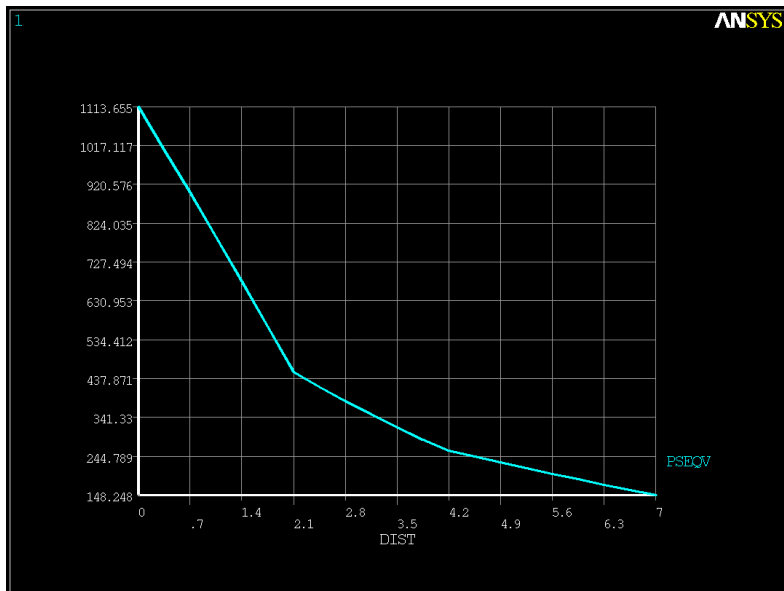
(b) The stress variation along wall thickness

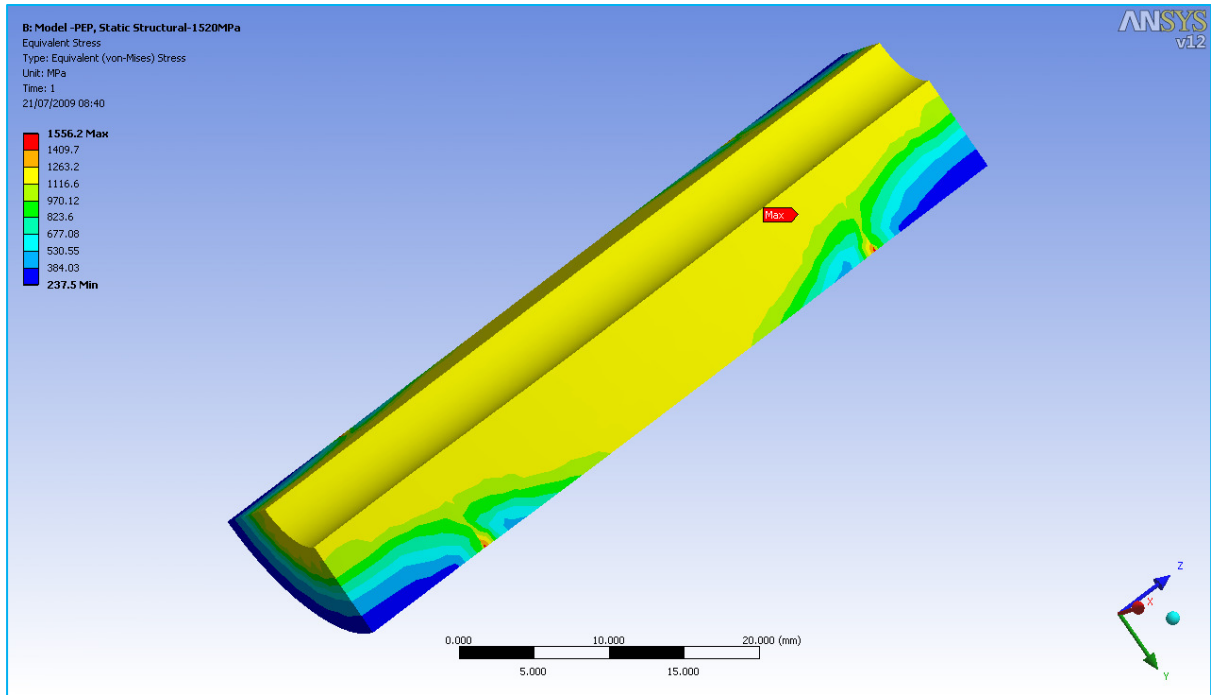**Figure 3 Equivalent stress of plain cylinder (L=55mm) under pressure level of 615MPa (6150bar)**
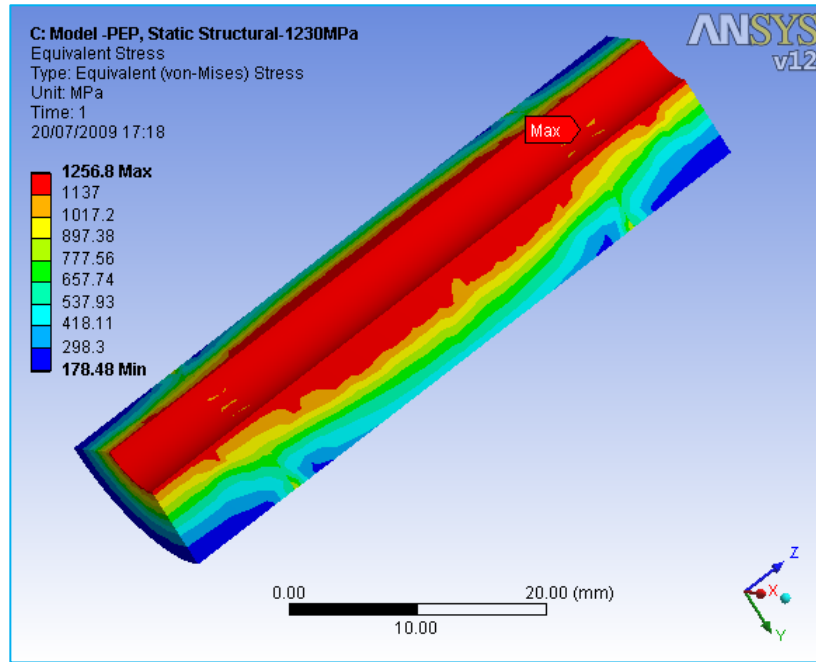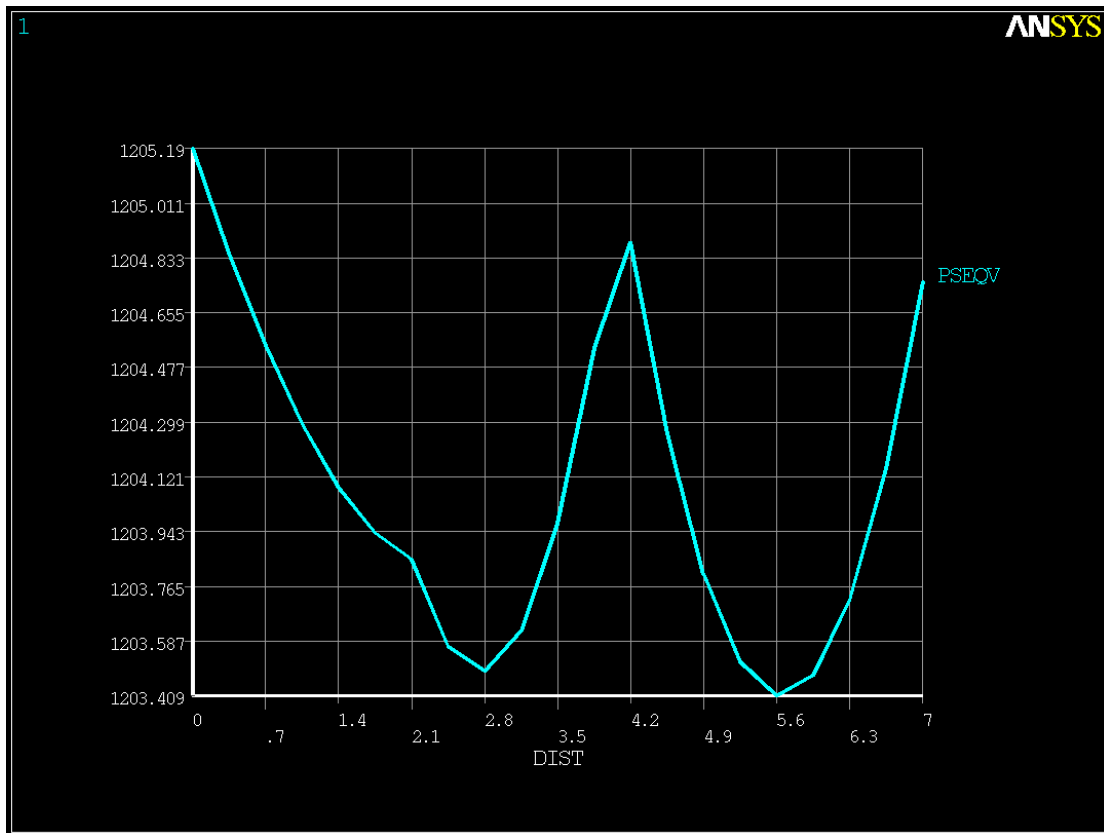
(a) The contour plot of equivalent stress
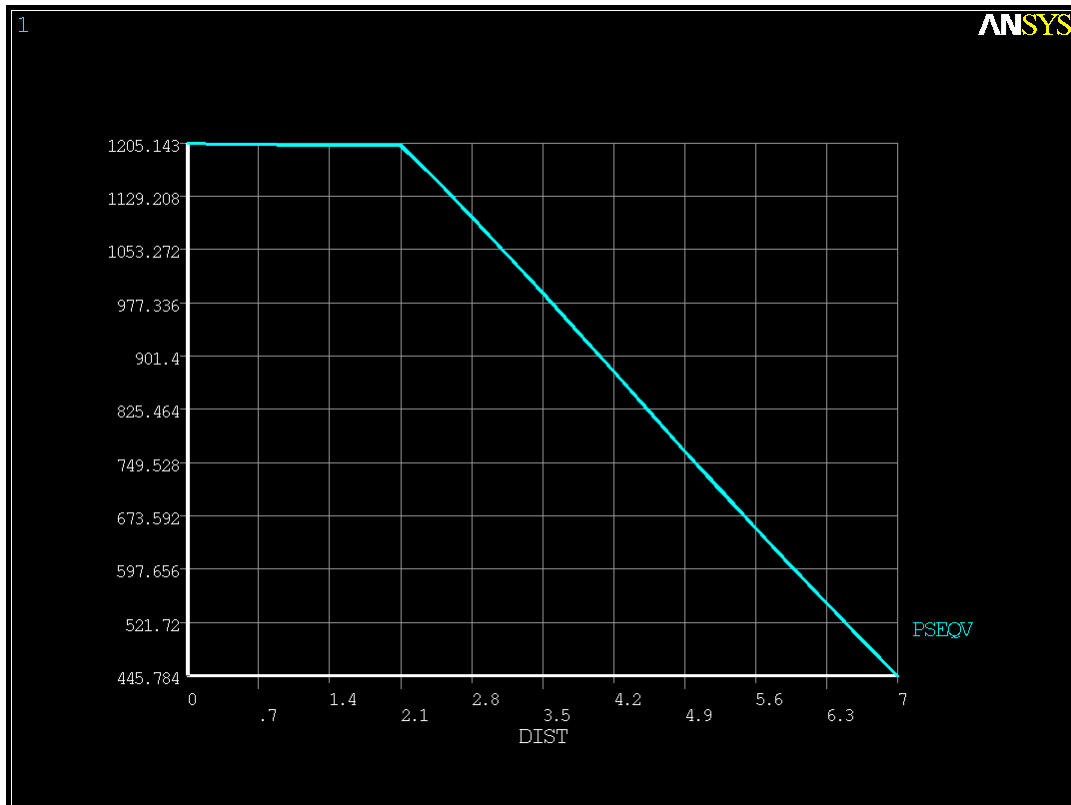
(b) The stress variation along wall thickness

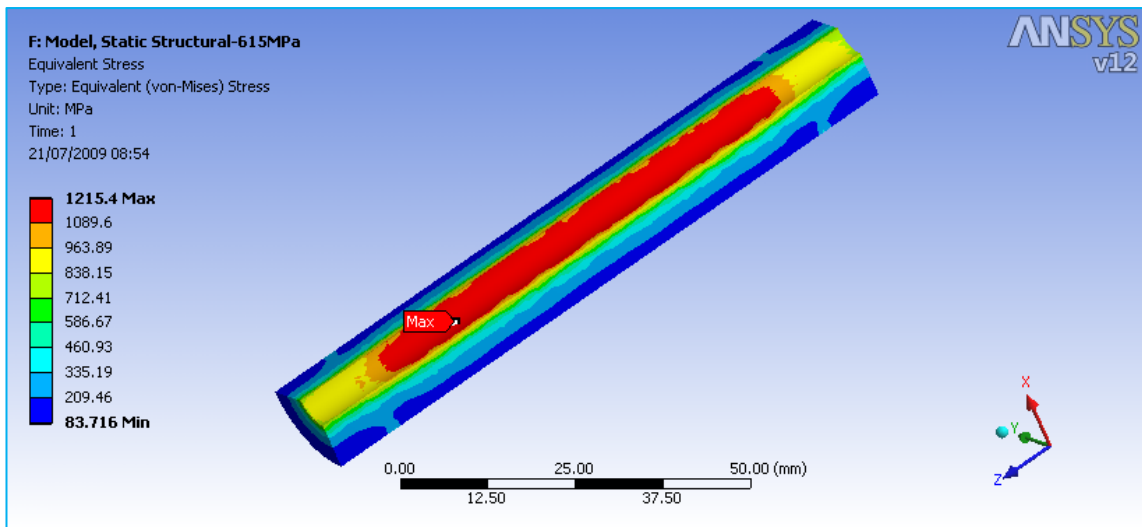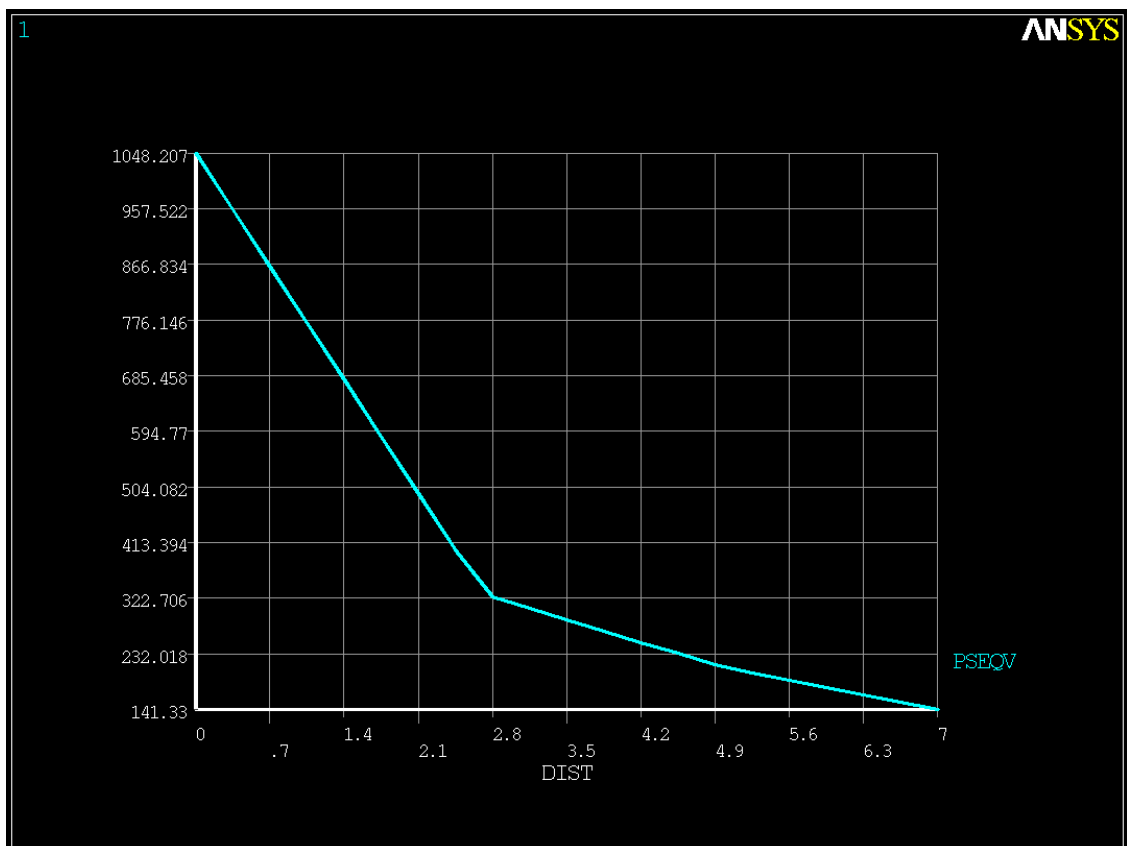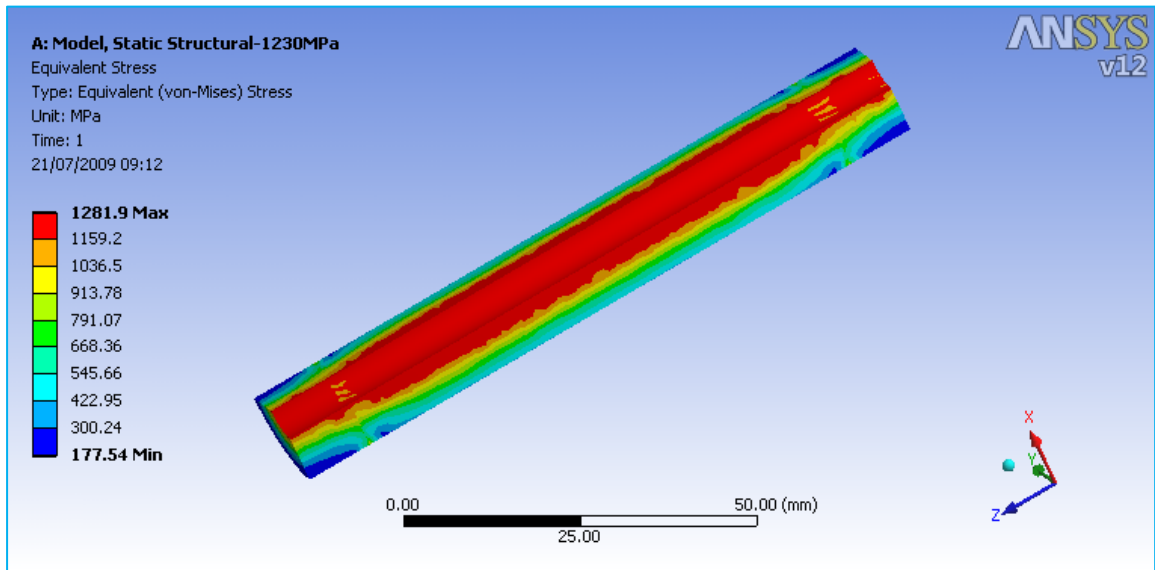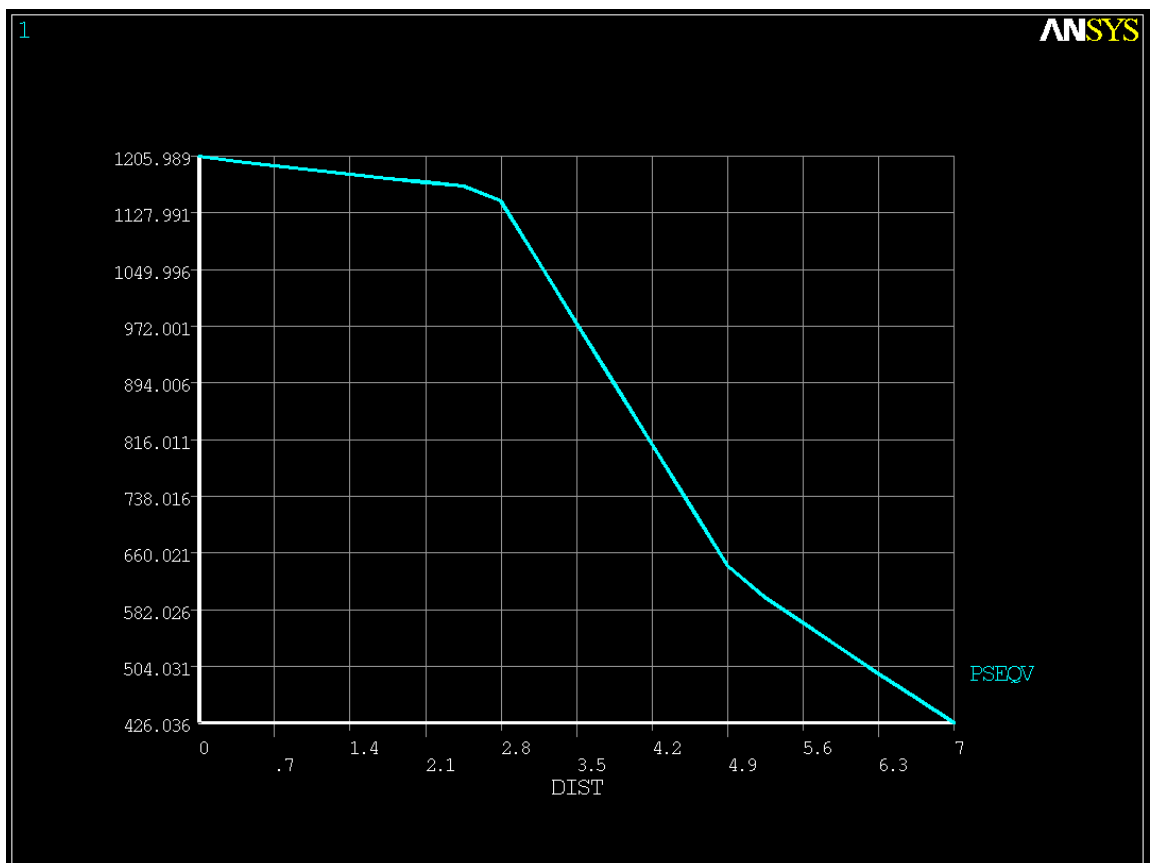**Figure 4 Equivalent stress of plain cylinder (L=55mm) under pressure level of 1230MPa (12300bar)**

(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

Figure 5 Equivalent stress of plain cylinder (L=55mm) under pressure level of 1520MPa (15200bar)

(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

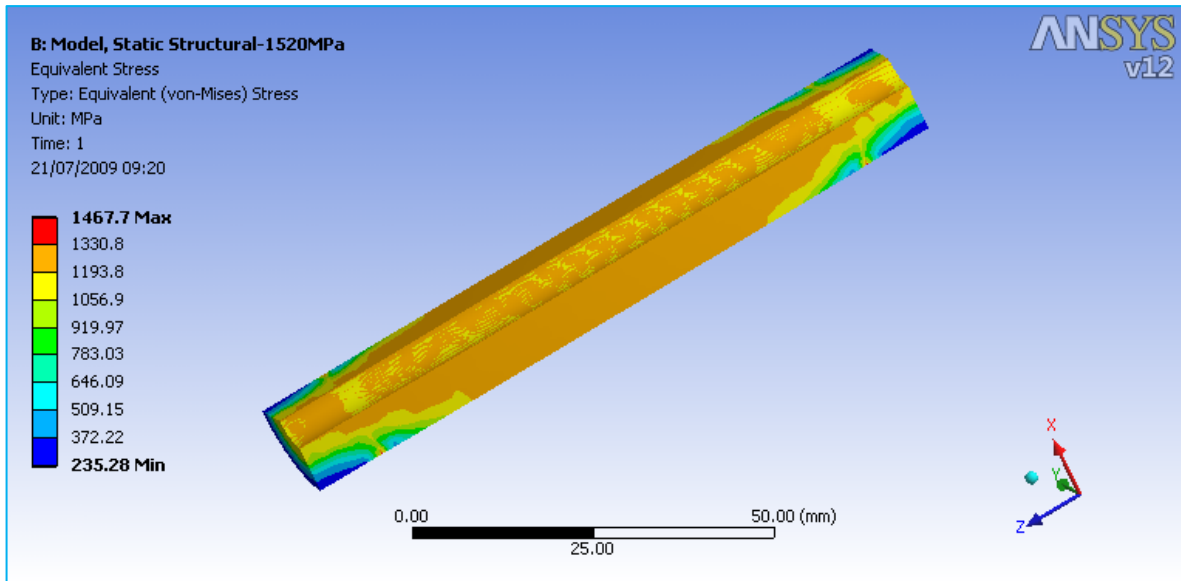**Figure 6 Equivalent stress of plain cylinder (L=100mm) under pressure level of 615MPa (6150bar)**
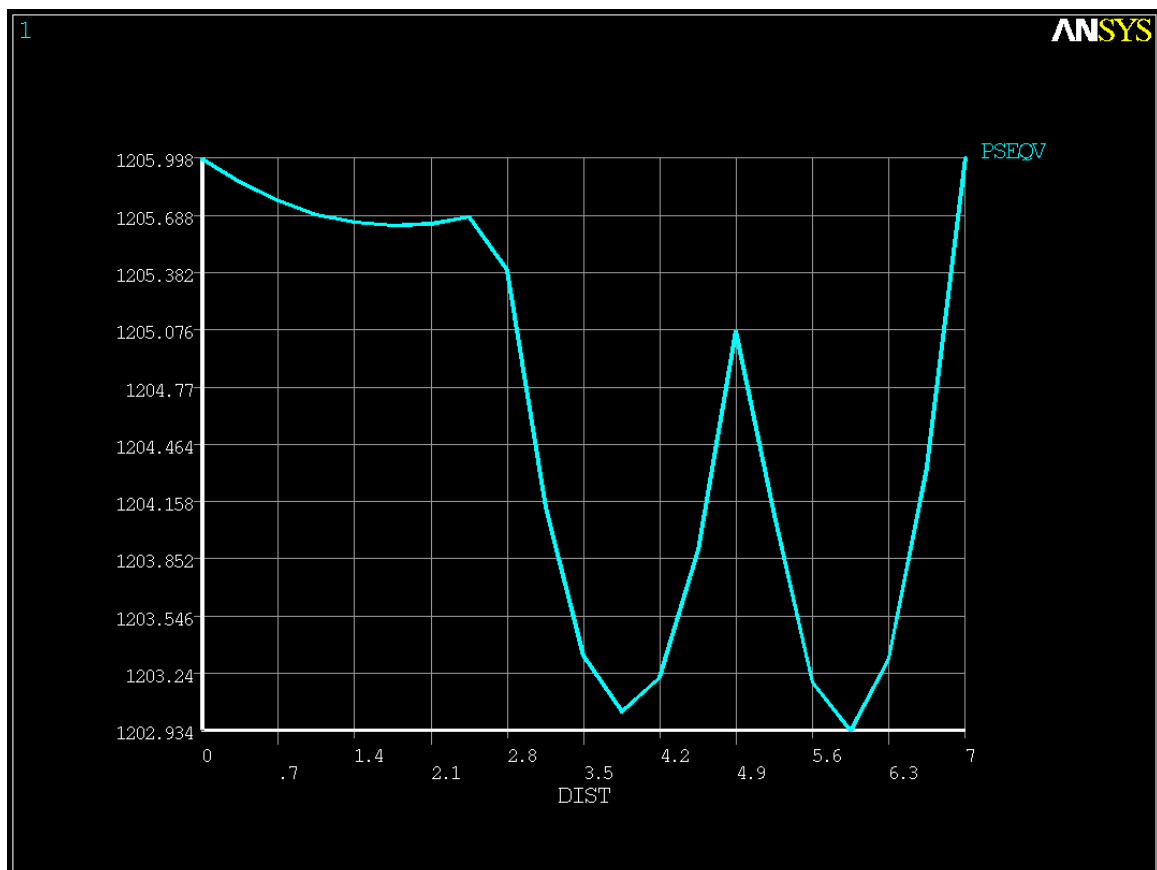
(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

Figure 7 Equivalent stress of plain cylinder (L=100mm) under pressure level of 1230MPa (12300bar)

(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

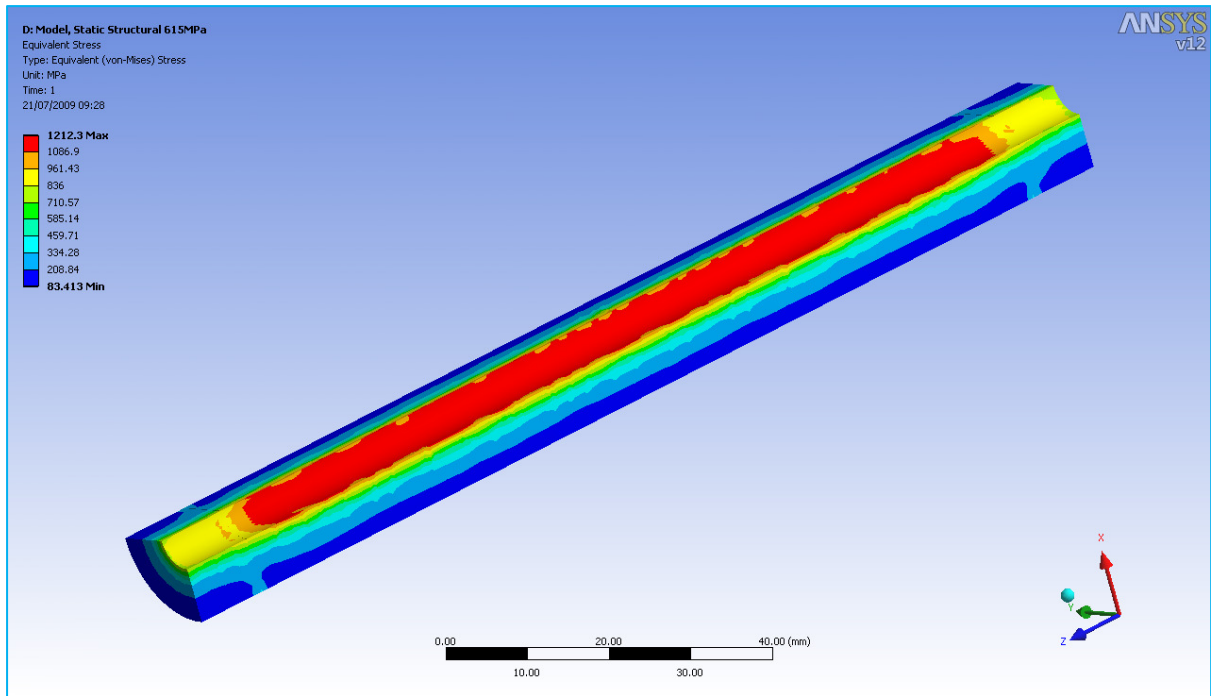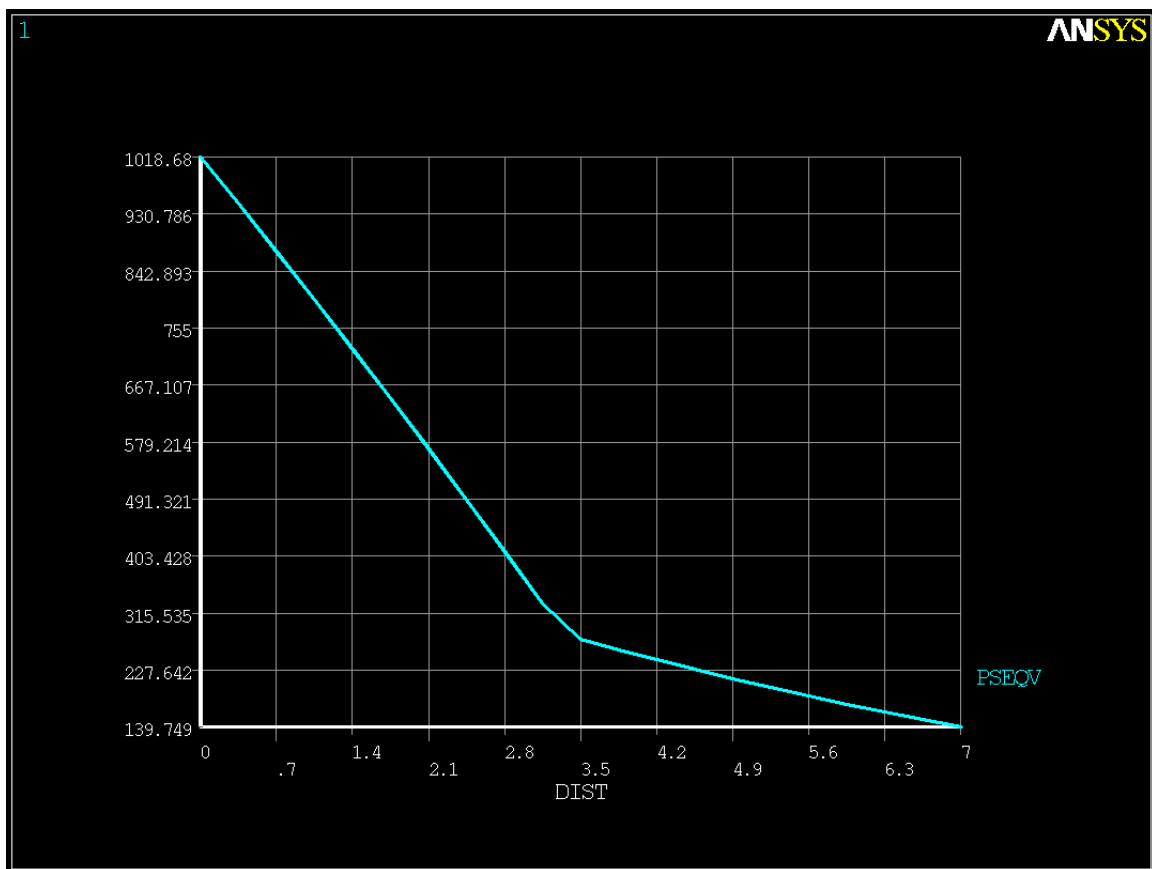**Figure 8 Equivalent stress of plain cylinder (L=100) under pressure level of 1520MPa (15200bar)**
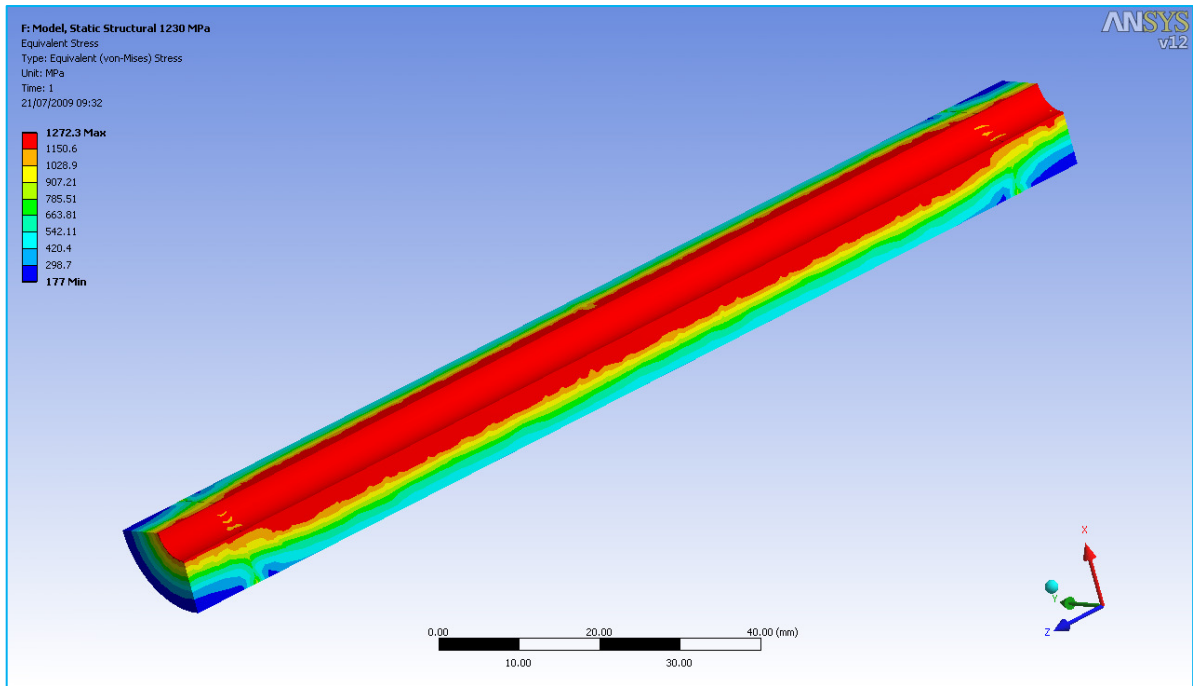
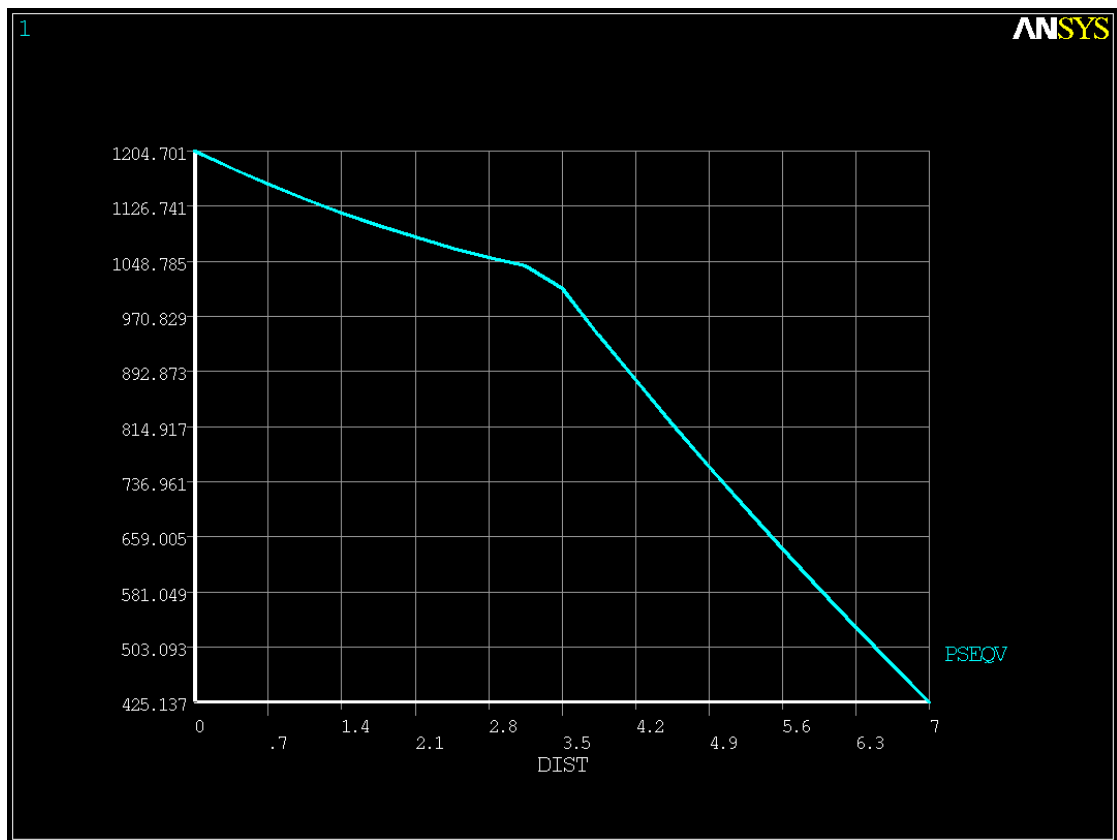(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

Figure 9 Equivalent stress of plain cylinder (L=150mm) under pressure level of 615MPa (6150bar)
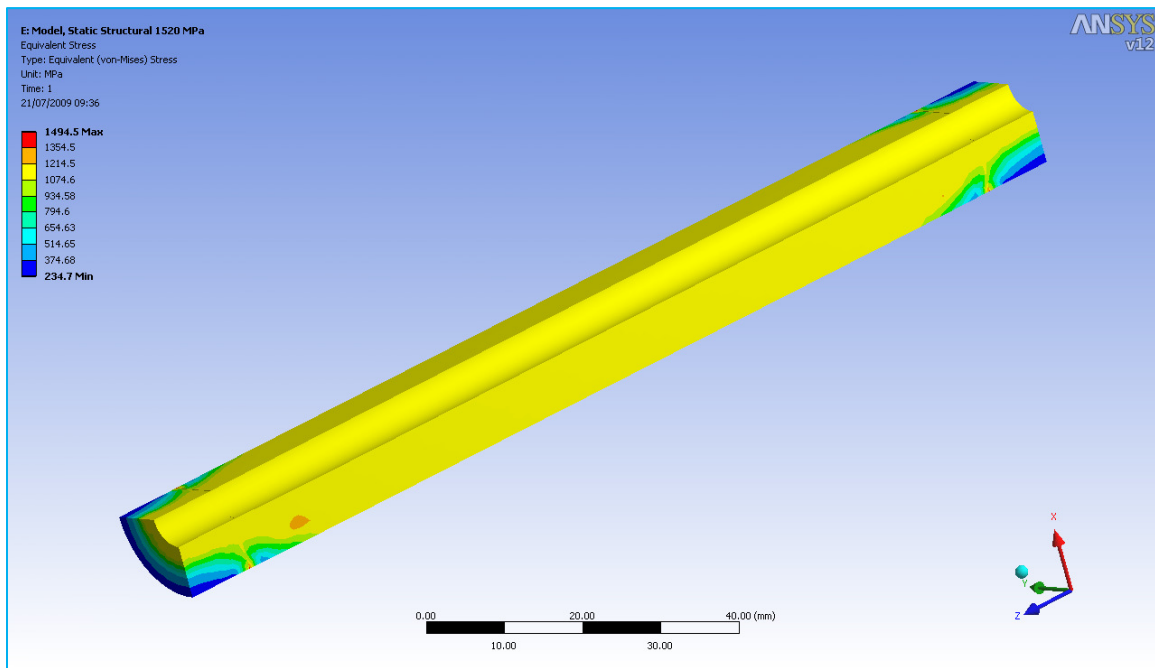
(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

Figure 10 Equivalent stress of plain cylinder (L=150mm) under pressure level of 1230MPa (12300bar)
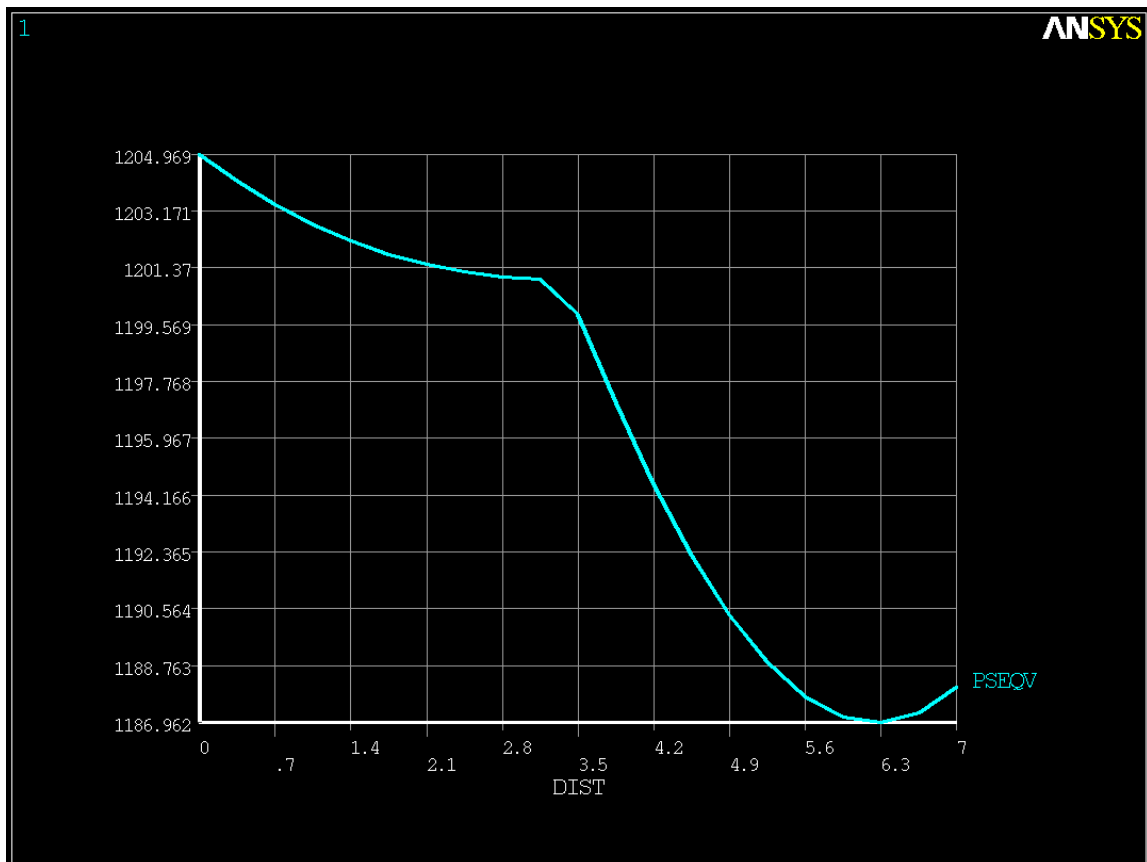
(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

Figure 11 Equivalent stress of plain cylinder (L=150mm) under pressure level of 1520MPa (15200bar)

Table 3 Sub-summary of FEA results for K value 3

Loading level and Comparison items	615 MPa/6150Bar		1230MPa/12300bar		1520 MPa/15200Bar	
	SEQV range (MPa)	PDDmax (mm)	SEQV range (MPa)	PDDmax (mm)	SEQV range (MPa)	PDDmax (mm)
ID7OD21L55mm	1113.655-148.248	0	1205.143-445.784	2.1	1205.19-1203.409	7
ID7OD21L100mm	1048.207-141.33	0	1205.989-426.036	1.8	1205.998-1202.934	7
ID7OD21L150mm	1018.68-139.749	0	1204.701-425.137	0.9	1204.969 -1186.962	7

Note: PDDmax stands for maximum plastic deformation depth from the inner surface of the plain cylinder

3.2.2 FEA results for K=4

For the plain cylinder with K value 4, three different pressure levels of 650MPa/6500bar, 1300MPa/13000bar and 1930MPa/19300bar, were considered for three different plain cylinder lengths of 55mm, 100mm and 150mm, respectively, as listed in Table 2 and section 2.2. Similarly, the expected results are: (1) when the plain cylinder internally pressurised to the pressure level of 650MPa/6500bar, the inner surface of the cylinder should just start yielding; (2) when the plain cylinder internally pressurised to the pressure level of 1300MPa/13000bar, (i.e. the autofrettage pressure) the inner surface of the cylinder should be yield to a depth 5.33mm (from the inner surface, i.e. to a $\emptyset_A$=17.66mm); (3) when the plain cylinder further internally pressurised to the pressure level of 1930MPa/19300bar, the entire wall thickness of the cylinder should be yield.

To validate these hand calculation results with FEA results, two points, one is on the inner surface and another is on the outer surface of the plain cylinder, as shown in Figure 2 (points A and B), were created within the FEA model for the same purpose of plotting the variation of the equivalent stress along the entire wall thickness of 10.5mm in this case.

Following these exercises, FEA results for K value 4 are presented in Figures 12 to 14, 15 to 17 and 18 to 20, for three different pressure levels of 650MPa, 1300MPa and 1930MPa, respectively. Similarly, under each of the three pressure levels, results for three plain cylinder lengths of 55mm, 100mm and 150mm, are also depicted respectively.

In addition and as mentioned previously, one more FEA simulation results for K value 4, cylinder length of 100mm and under the pressure level of 1288.18MPa, which was used in the previous FEA simulation, are also shown in Figure 21 for comparison purpose.
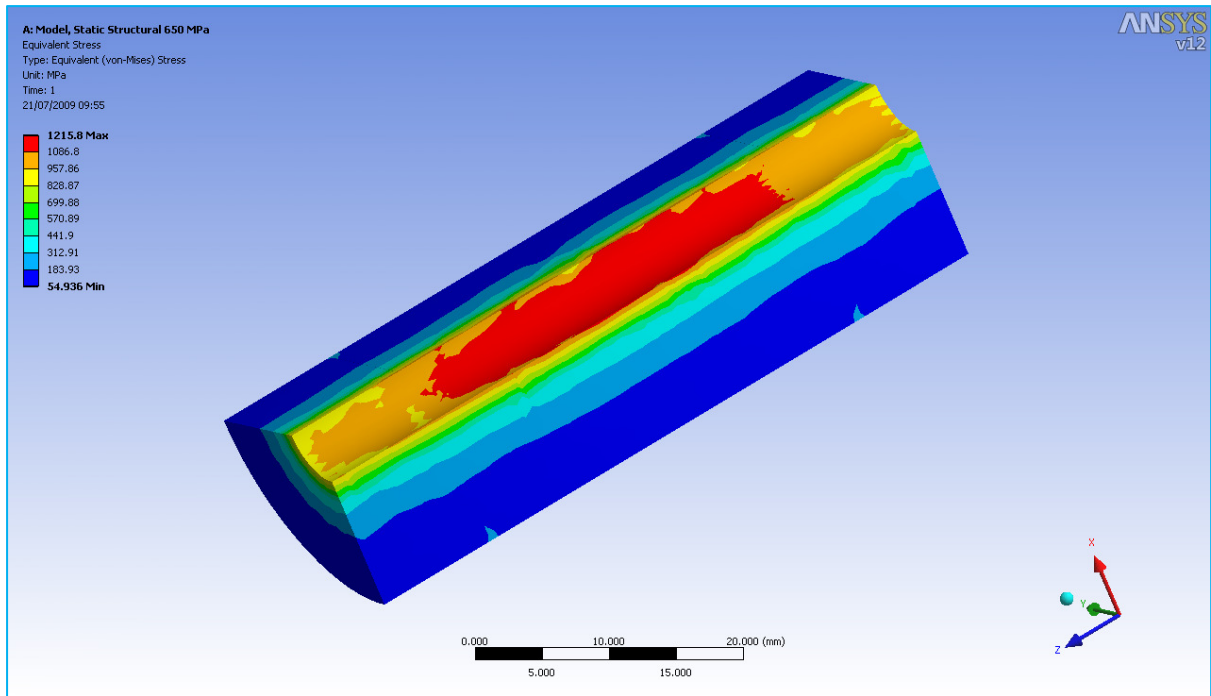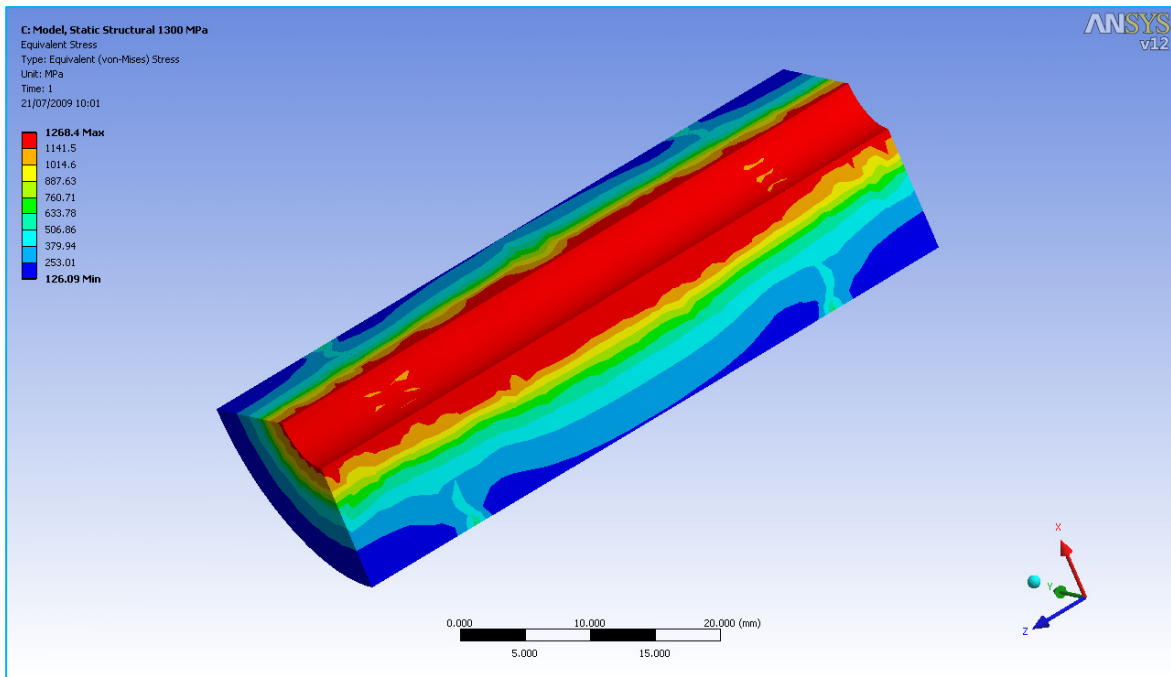
(a) The contour plot of equivalent stress
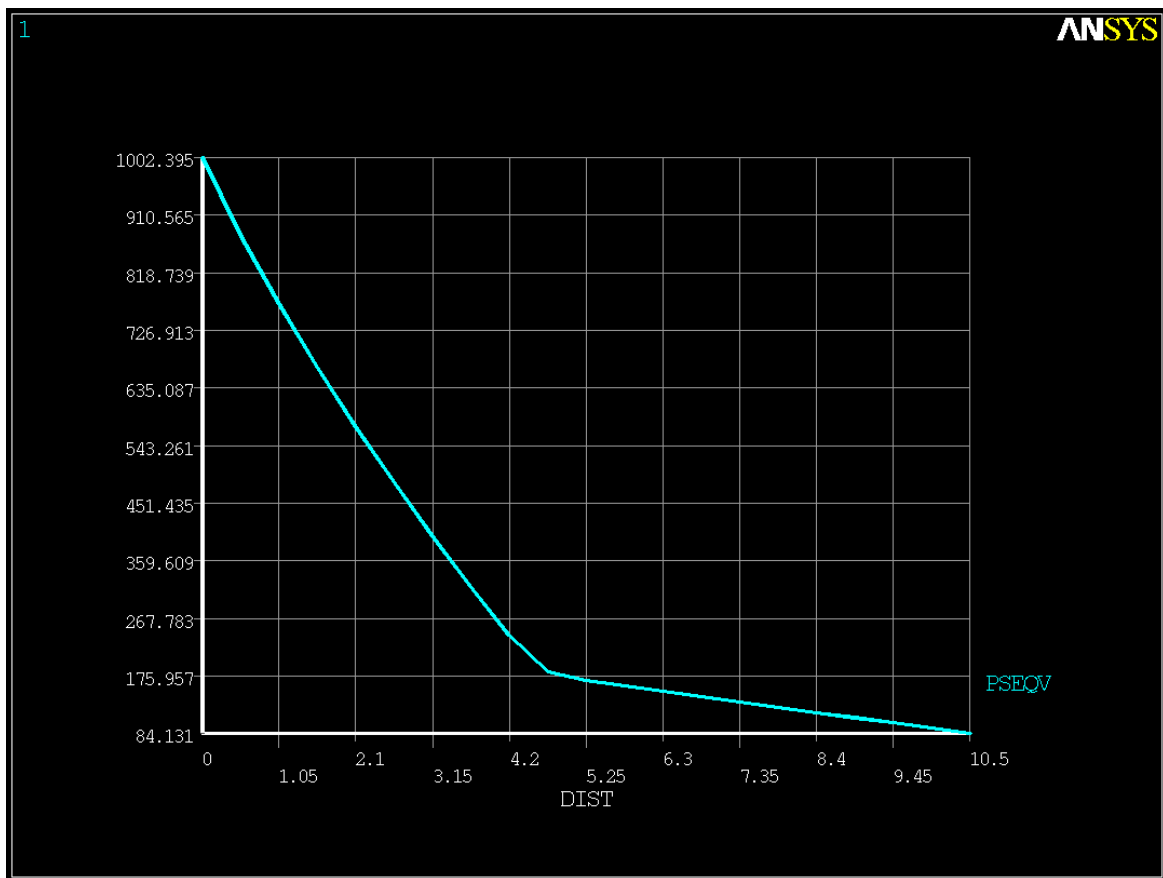
(b) The stress variation along wall thickness

Figure 12 Equivalent stress of plain cylinder (L=55mm) under pressure level of 650MPa (6500bar)

(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

Figure 13 Equivalent stress of plain cylinder (L=55mm) under pressure level of 1300MPa (13000bar)

(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

Figure 14 Equivalent stress of plain cylinder (L=55mm) under pressure level of 1930MPa (19300bar)
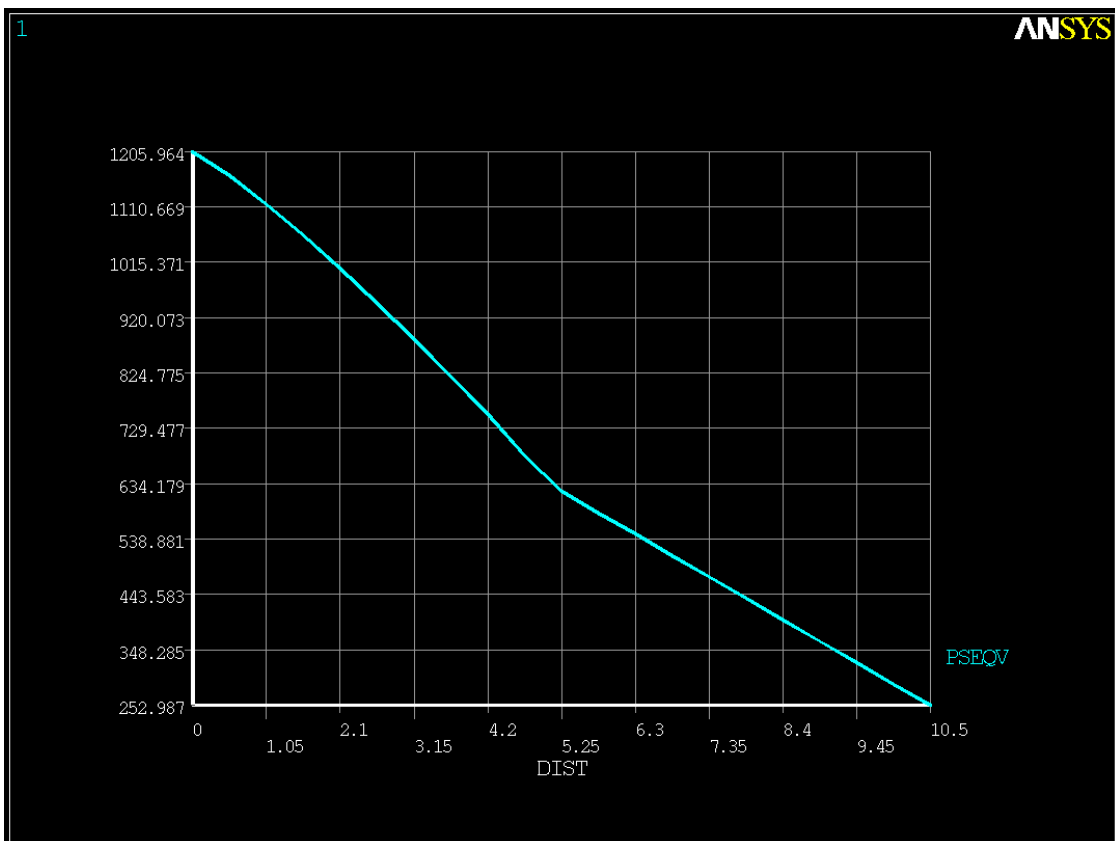
(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

Figure 15 Equivalent stress of plain cylinder (L=100mm) under pressure level of 650MPa (6500bar)

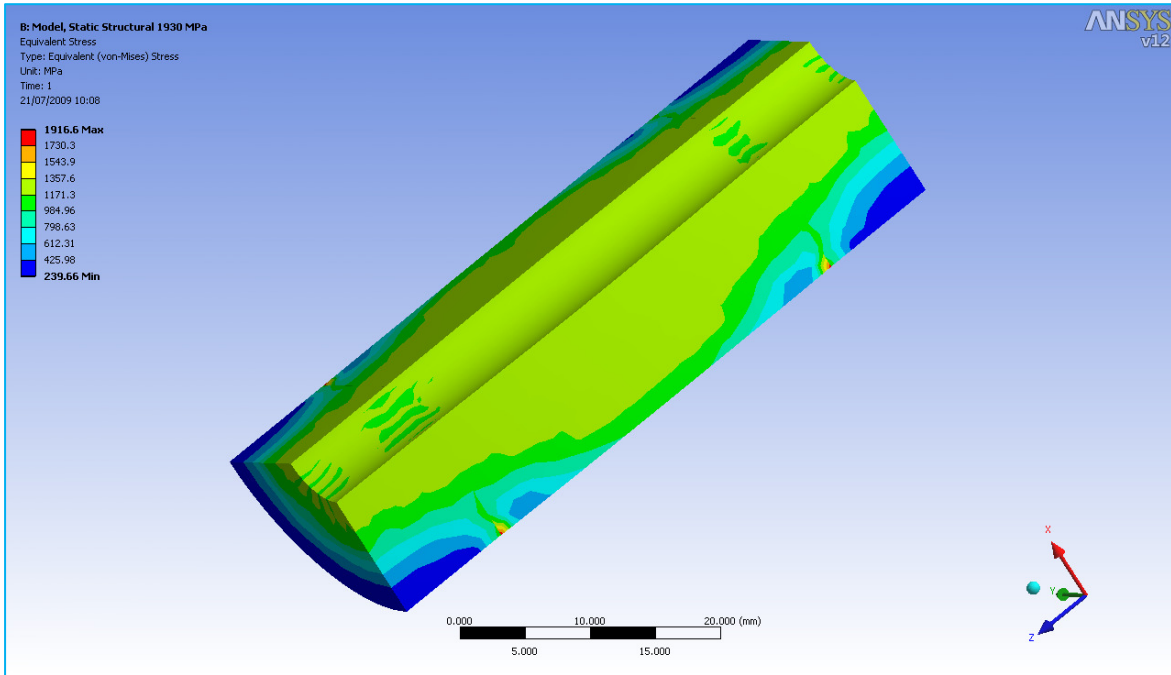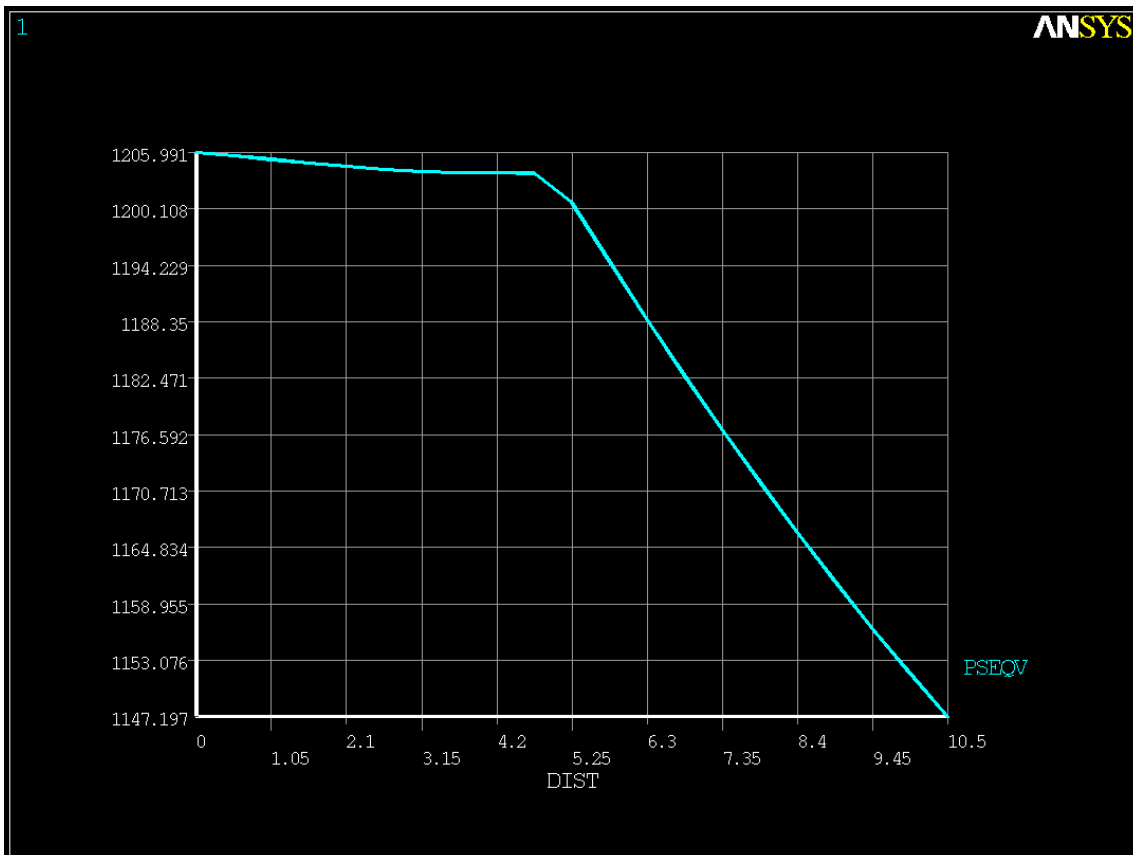**(a) The contour plot of equivalent stress**

(b) The stress variation along wall thickness

Figure 16 Equivalent stress of plain cylinder (L=100mm) under pressure level of 1300MPa (13000bar)

(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

Figure 17 Equivalent stress of plain cylinder under (L=100mm) pressure level of 1930MPa (19300bar)
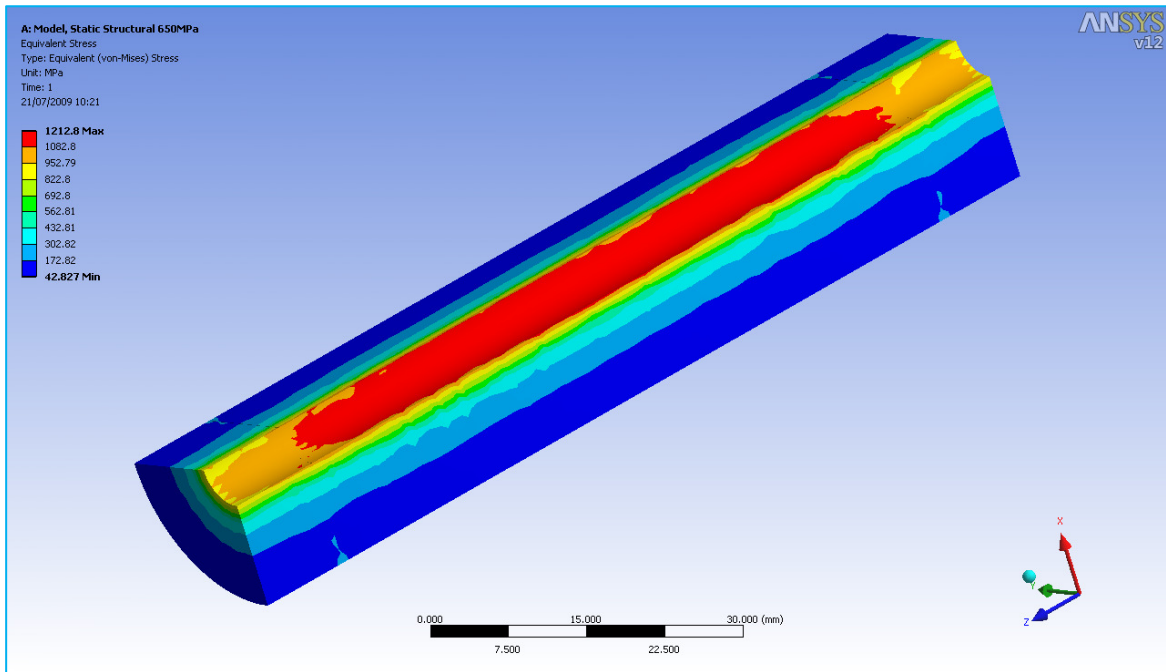
(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

Figure 18 Equivalent stress of plain cylinder (L=150mm) under pressure level of 650MPa (6500bar)

(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

Figure 4 Equivalent stress of plain cylinder under pressure level of 615MPa (6150bar)
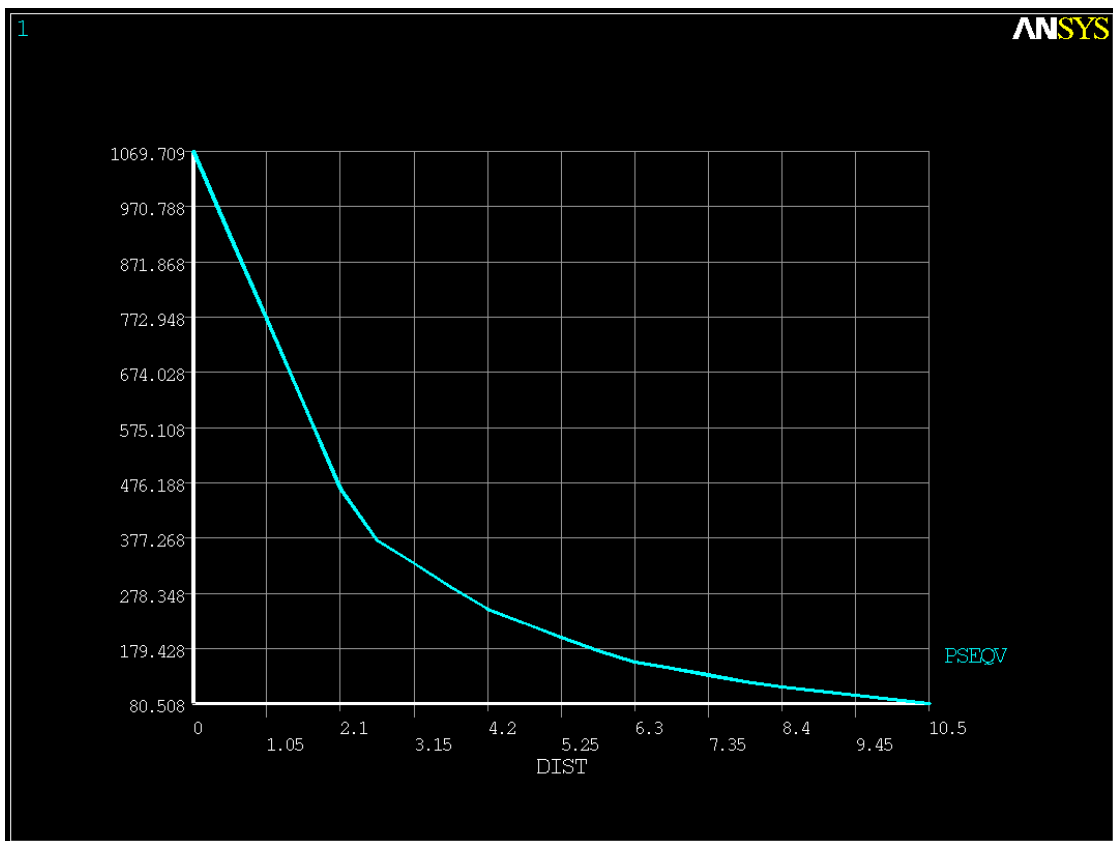
(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

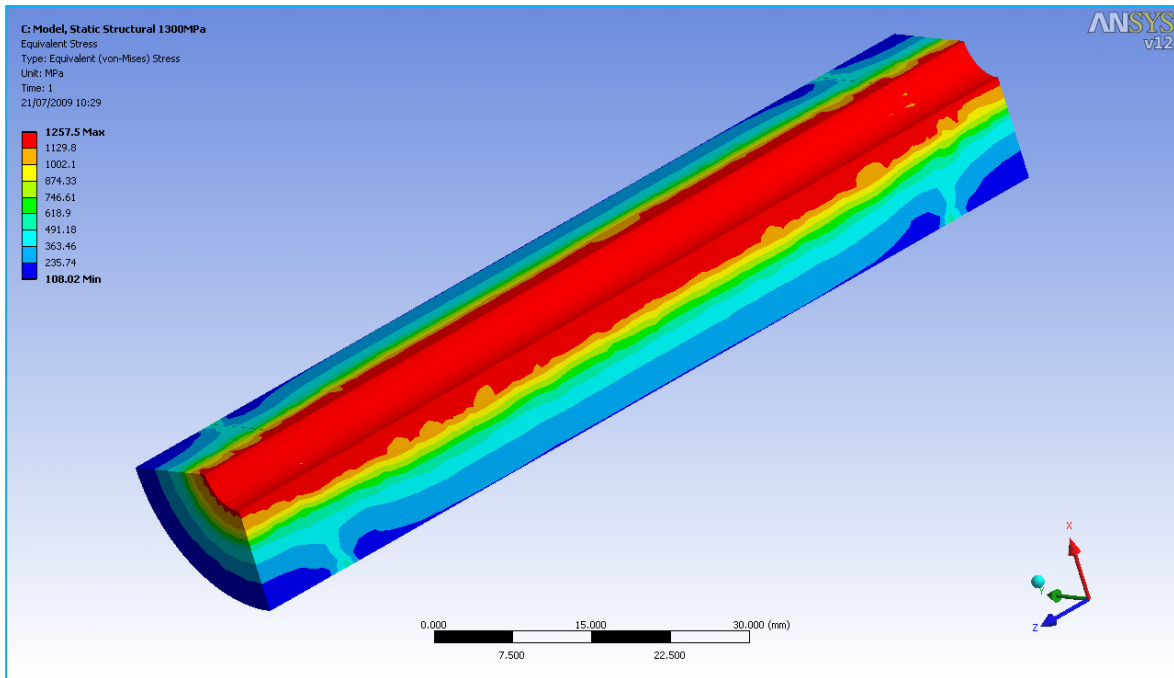**Figure 20 Equivalent stress of plain cylinder (L=150mm) under pressure level of 1930MPa (19300bar)**

(a) The contour plot of equivalent stress

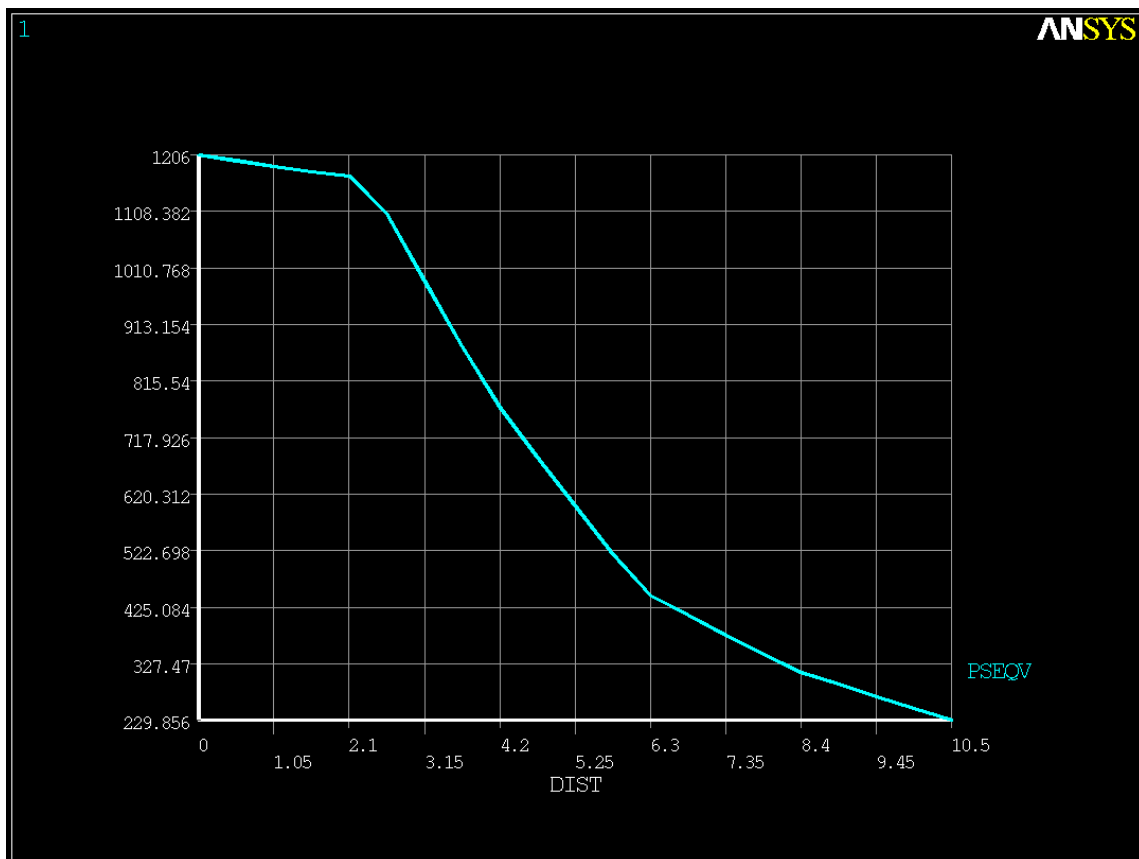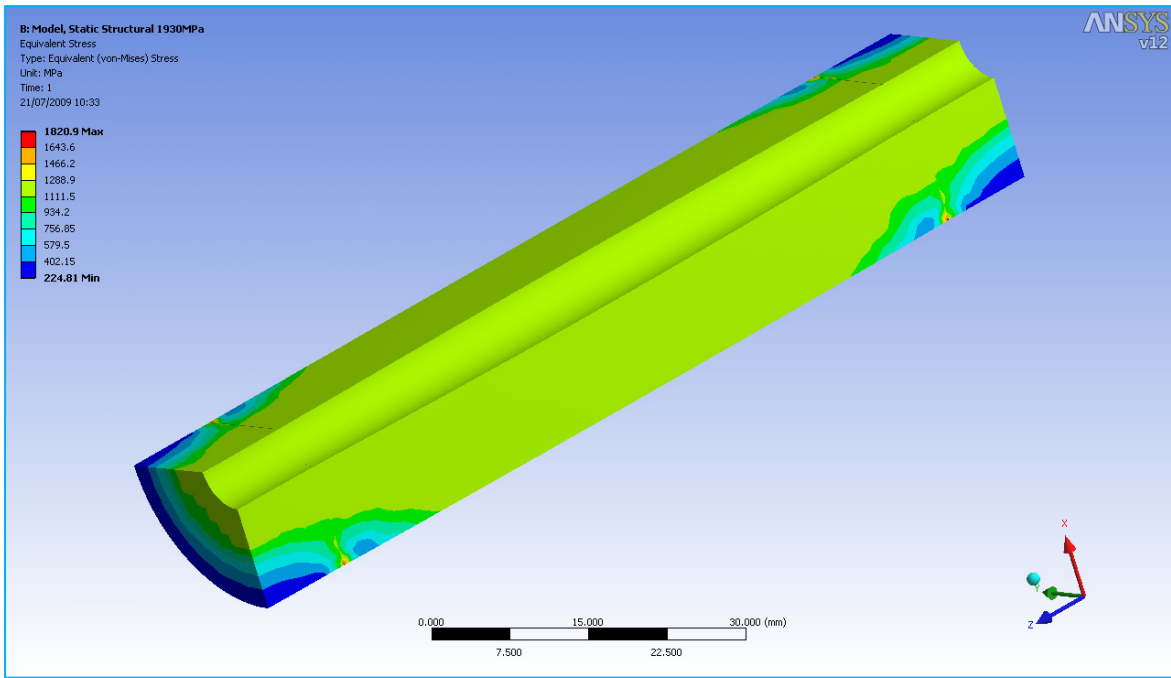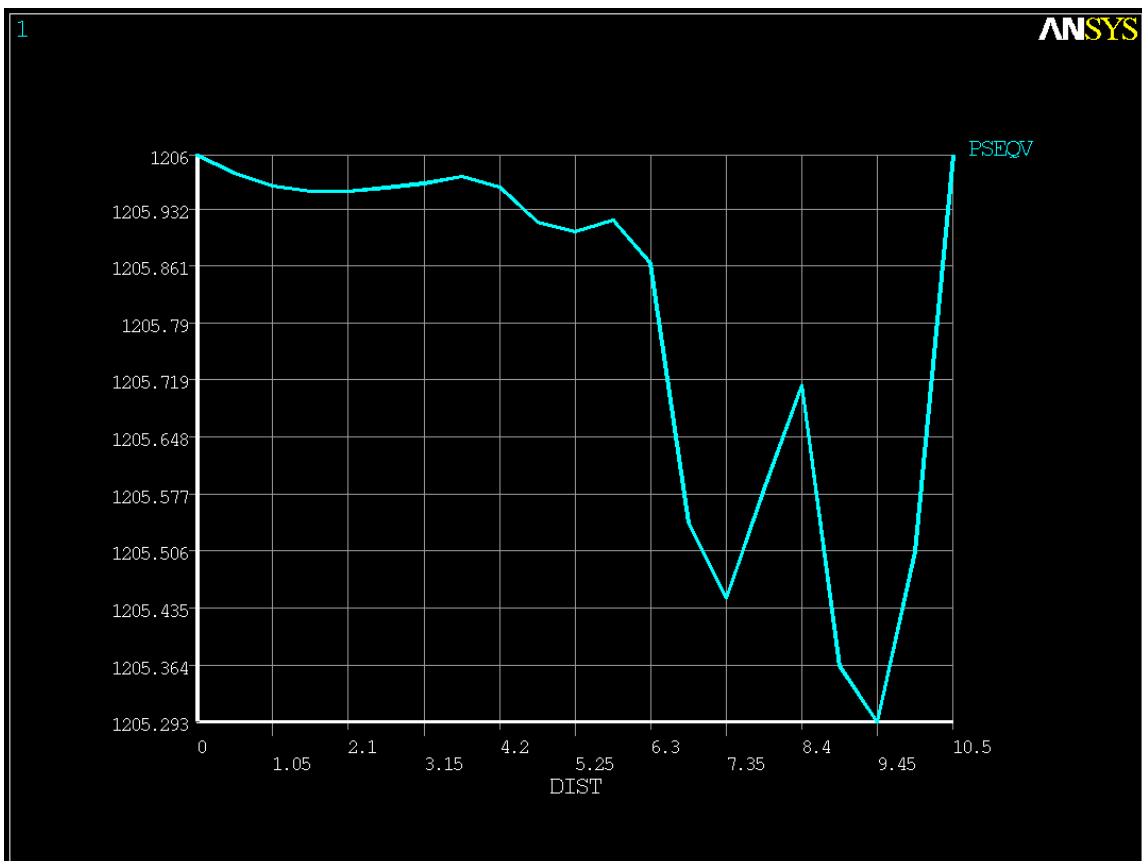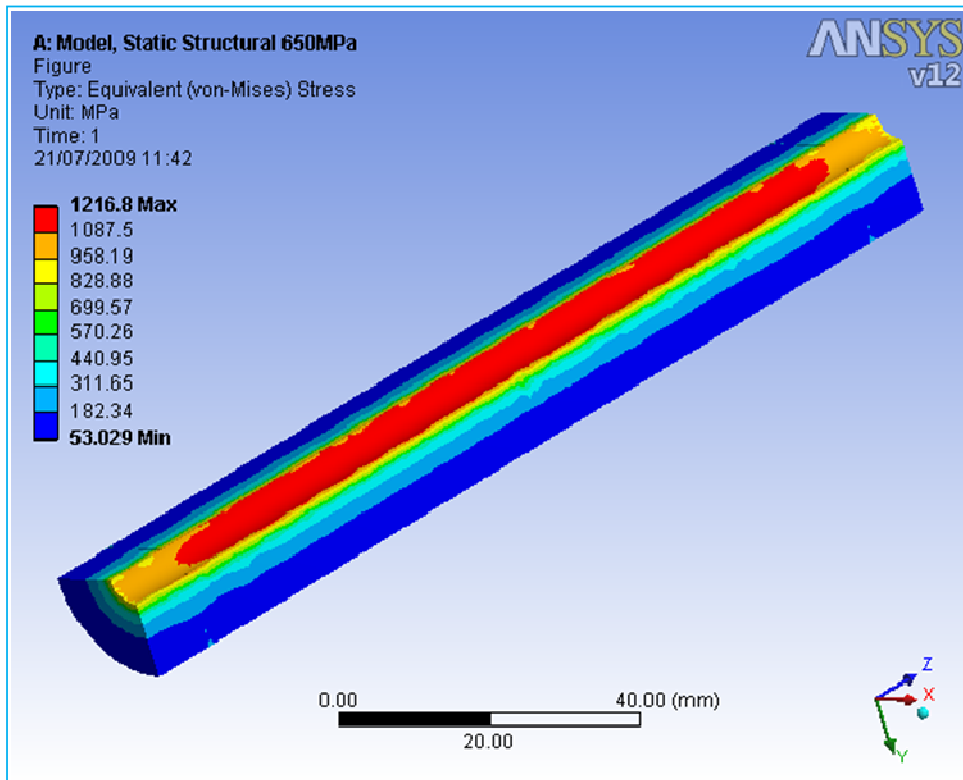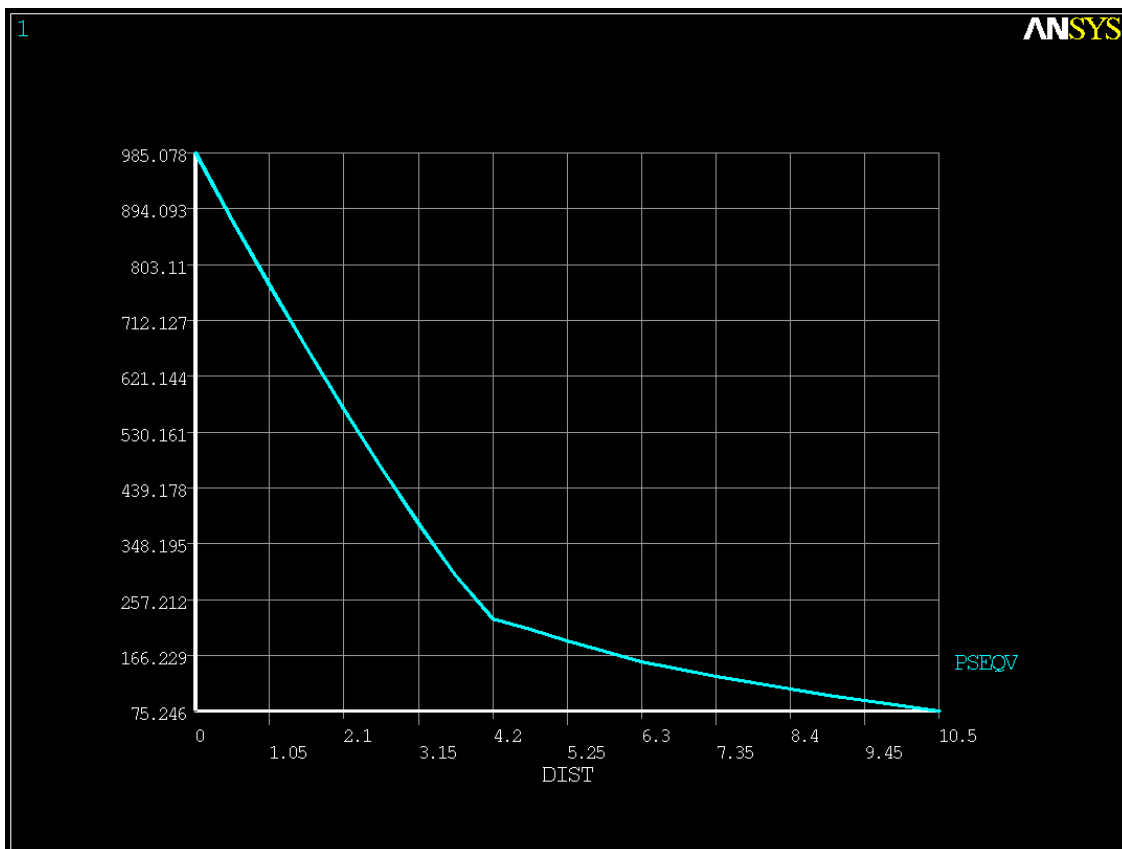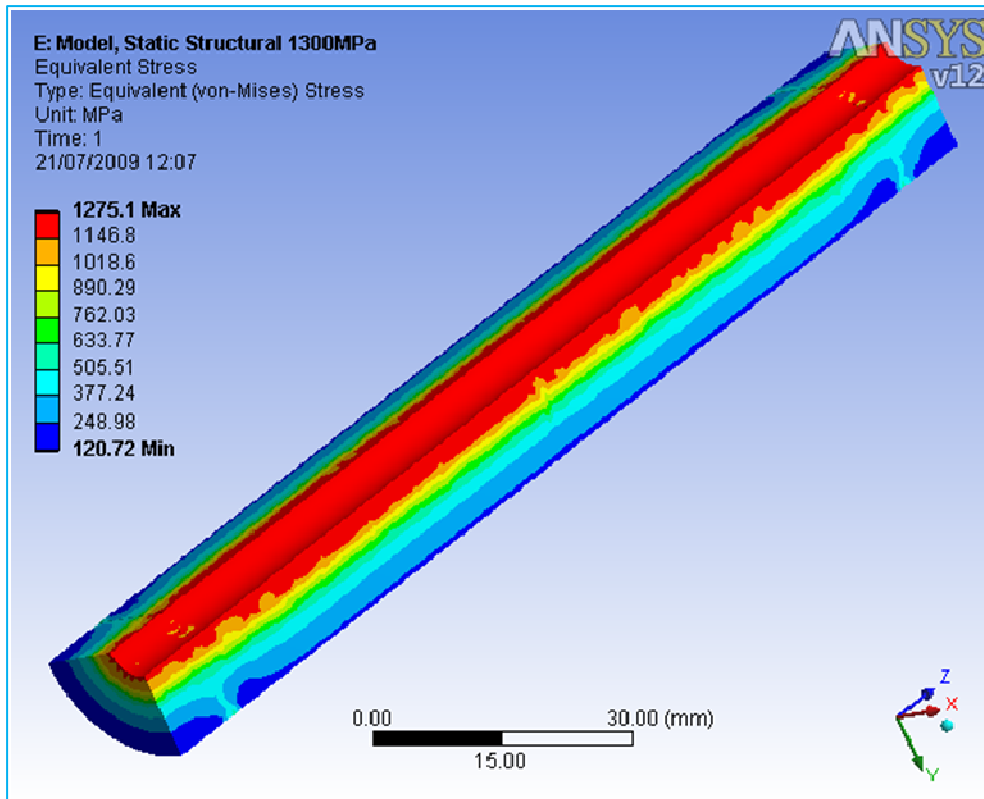**(b) The stress variation along wall thickness**

Figure 21 Equivalent stress of plain cylinder (K=4 L=100mm) under pressure level of 1288.19MPa (12881.9bar), which was used in the previous FEA

Table 4 Sub-summary of FEA results for K value 4

Loading levels and Comparison items	650 MPa/6500Bar		1300MPa/13000Bar		1930 MPa/19300Bar	
	SEQVmax (MPa)	PDDmax (mm)	SEQVmax (MPa)	PDDmax (mm)	SEQVmax (MPa)	PDDmax (mm)
ID7OD28L55mm	1002.395-84.131	0	1205.964-252.987	1.1	1205.991-1147.197	5.775
ID7OD28L100mm	1069.709-80.508	0	1206-229.856	2	1206 - 1205.293	10.5
ID7OD28L150mm	985.078-75.246	0	1205.007-213.962	1	1205.217-1203.813	10.5
UPPL for ID7OD28L100mm	Coarse mesh (UPPL=1288.19MPa)		1205.998-226.021	2		

Note: 1. PDDmax stands for maximum plastic deformation depth from the inner surface of the plain cylinder; 2. UPPL stands for using previous pressure level of 1288.19MPa.

3.2.3 FEA results for K=5

Three different pressure levels of 665MPa/6650bar, 1330MPa/13300bar and 2230MPa/22300bar, for plain cylinder with K value 5, were considered one after another, where the three different plain cylinder lengths of 55mm, 100mm and 150mm were also included, respectively, as listed in Table 2 and section 2.2. It is not exceptional here, i.e., the expected results are: (1) when the plain cylinder internally pressurised to the pressure level of 665MPa/6650bar, the inner surface of the cylinder should just start yielding; (2) when the plain cylinder internally pressurised to the pressure level of 1330MPa/13300bar, (i.e. the autofrettage pressure) the inner surface of the cylinder should be yield to a depth 5.95mm (from the inner surface, i.e. to a $\emptyset_A$=18.9mm); (3) when the plain cylinder further internally pressurised to the pressure level of 2230MPa/22300bar, the entire wall thickness of the cylinder should be yield.

Again, to validate these hand calculation results with FEA results, two points, one is on the inner surface and another is on the outer surface of the plain cylinder, as shown in Figure 2 (points A and B) , were created within the FEA model for the same purpose of plotting the variation of the equivalent stress along the entire wall thickness of 14mm here for K value 5.

Similarly, following these exercises, FEA results for K value 5 are presented in Figures 22 to 24, 25 to 27 and 28 to 30, for the three different pressure levels of 665MPa, 1330MPa and 2230MPa, respectively. Results for three plain cylinder lengths of 55mm, 100mm and 150mm, are also similarly shown there.
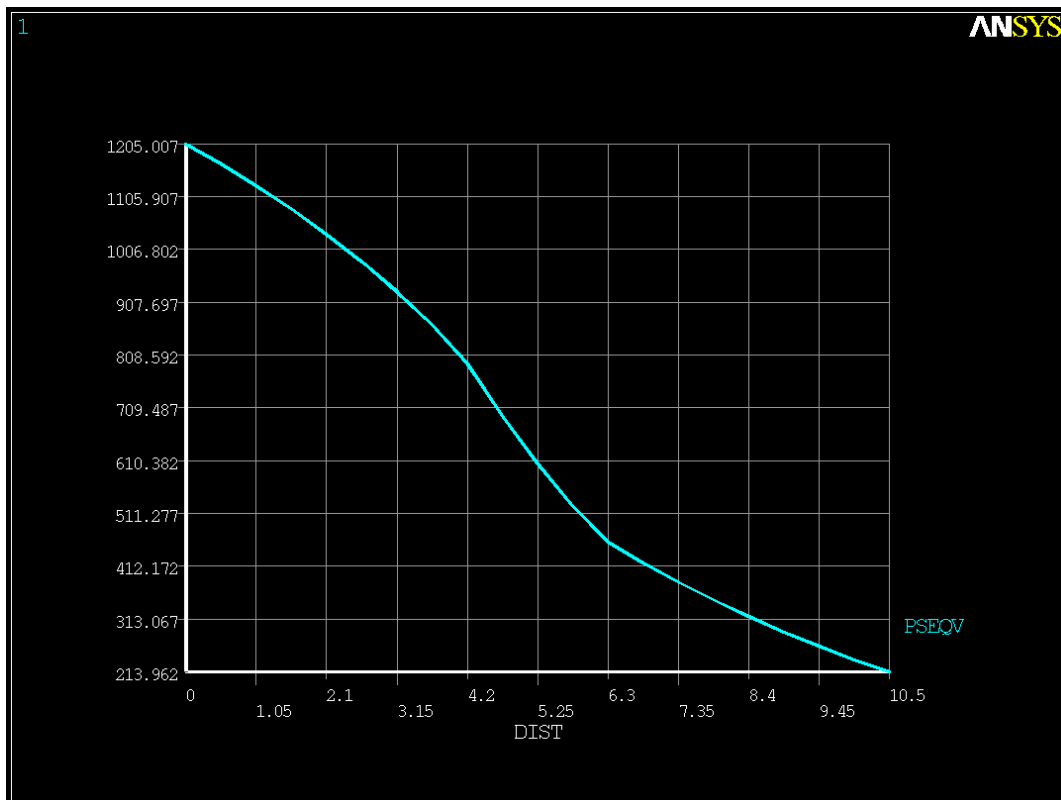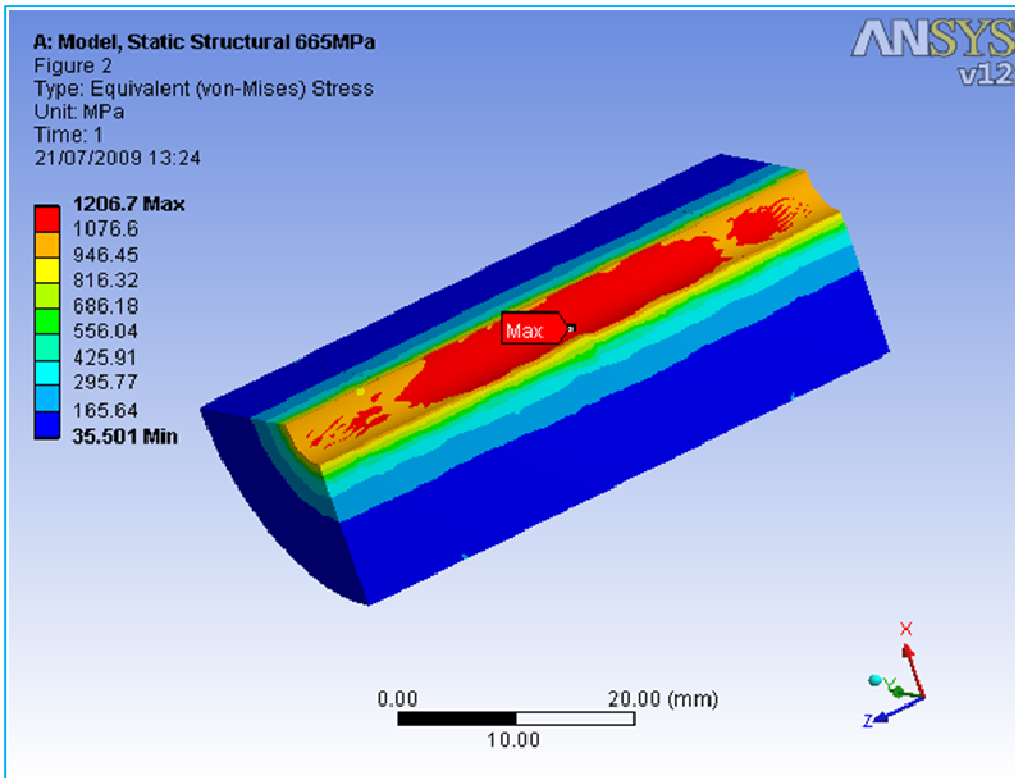
(a) The contour plot of equivalent stress
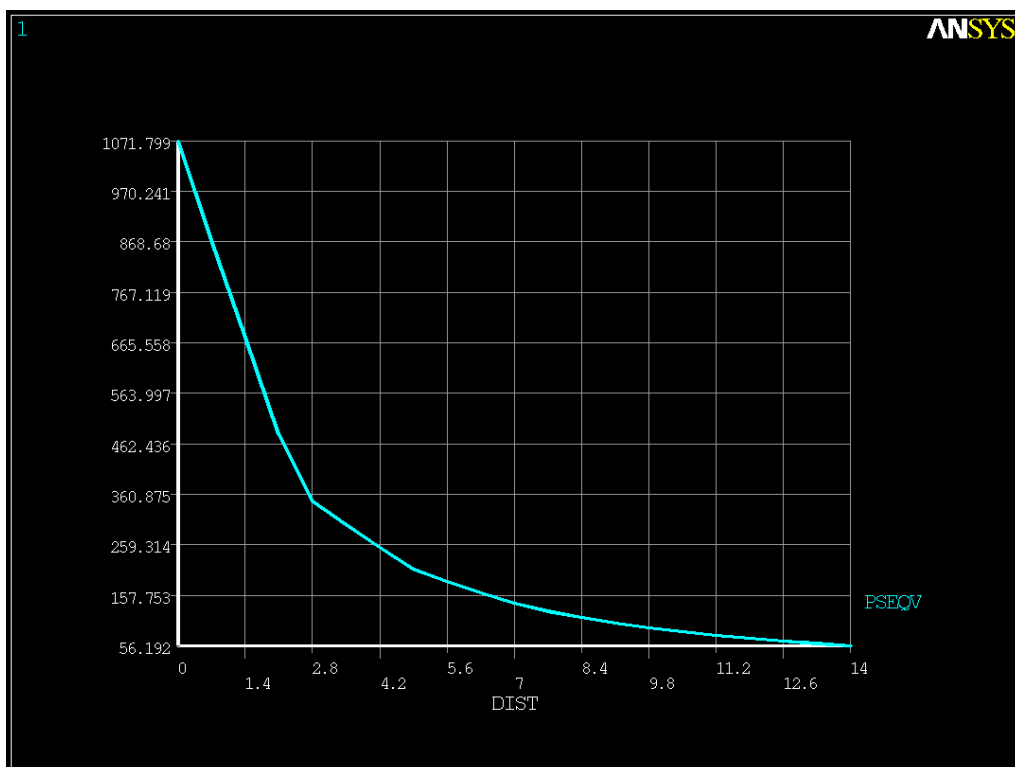
(b) The stress variation along wall thickness

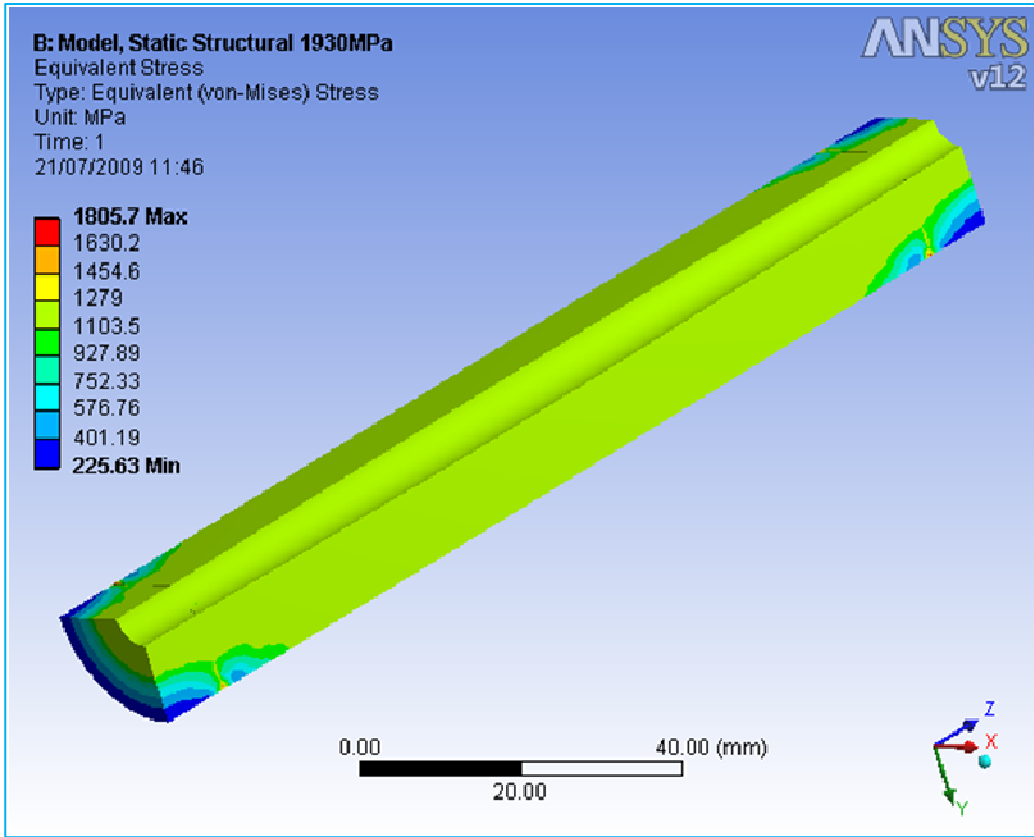**Figure 22 Equivalent stress of plain cylinder (L=55mm) under pressure level of 665MPa (6650bar)**

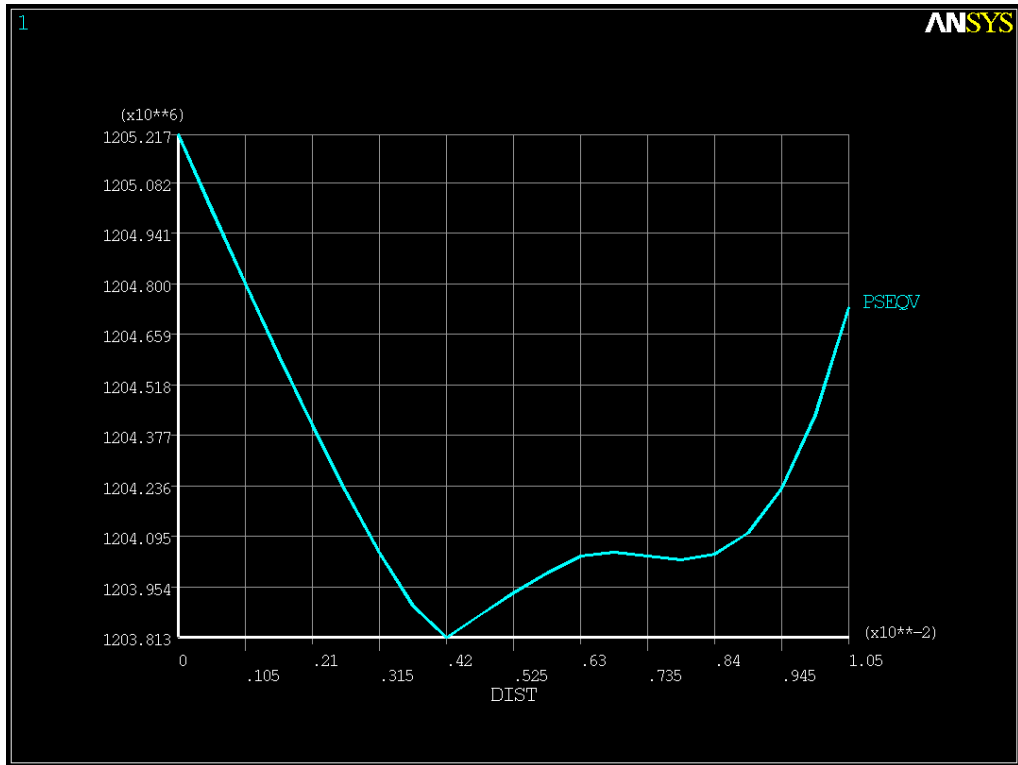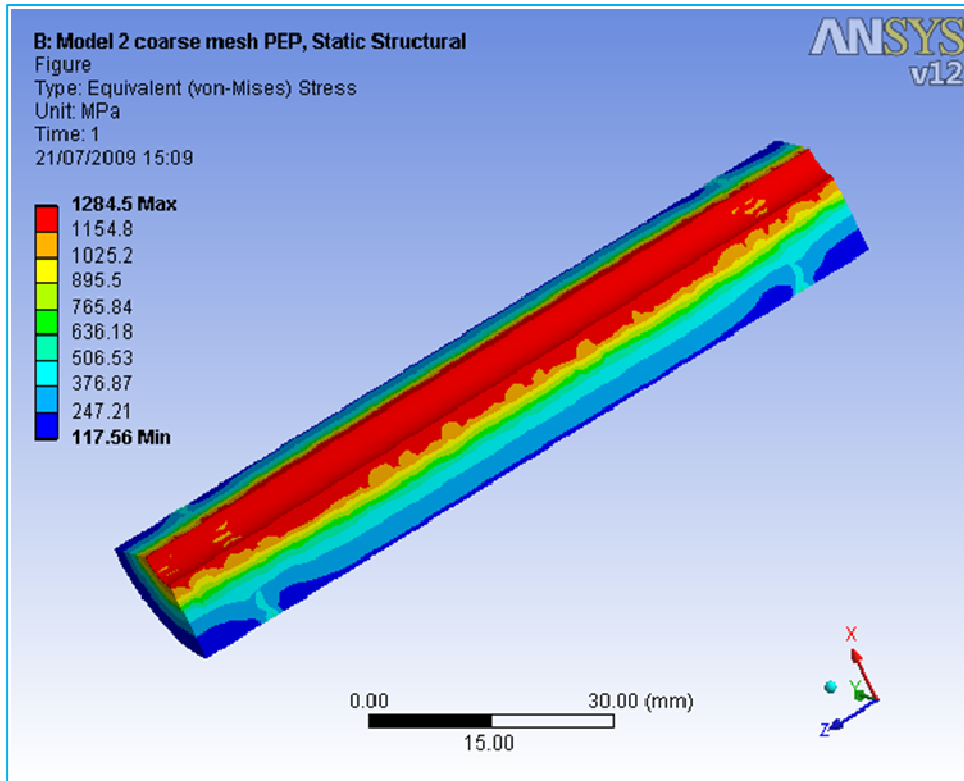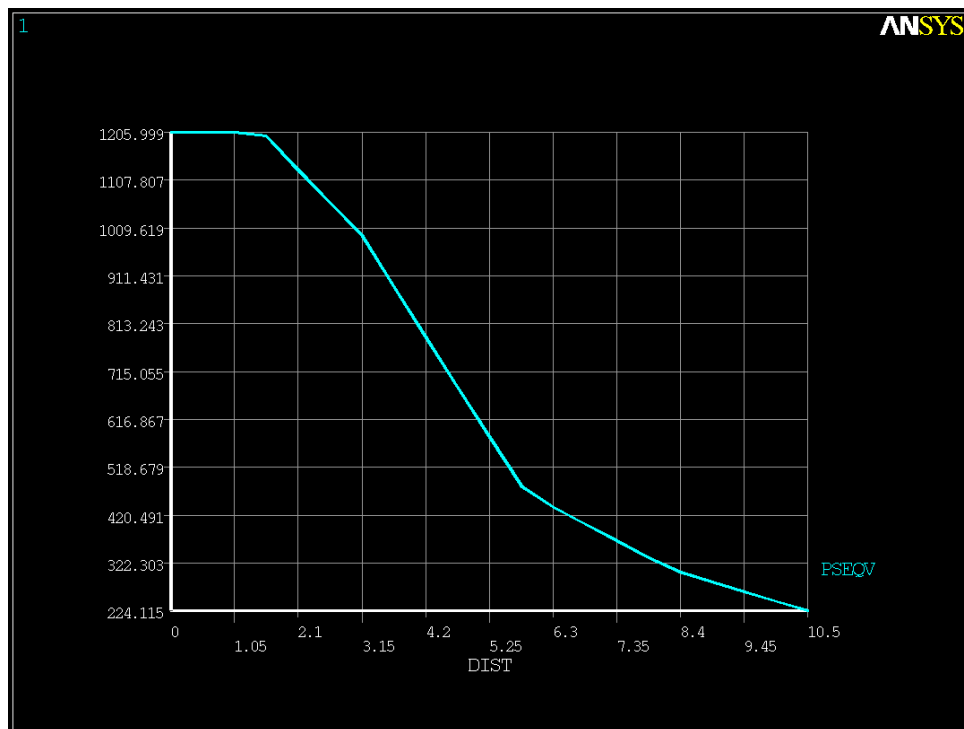**(a) The contour plot of equivalent stress**

(b) The stress variation along wall thickness

Figure 23 Equivalent stress of plain cylinder (L=55mm) under pressure level of
1330MPa (13300bar)

(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

Figure 24 Equivalent stress of plain cylinder (L=55mm) under pressure level of 2230MPa (22300bar)

(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

Figure 25 Equivalent stress of plain cylinder (L=100mm) under pressure level of 665MPa (6650bar)

(a) The contour plot of equivalent stress

(b) The stress variation along wall thickness

Figure 26 Equivalent stress of plain cylinder (L=100mm) under pressure level of 1330MPa (13300bar)

(a) The contour plot of equivalent stress
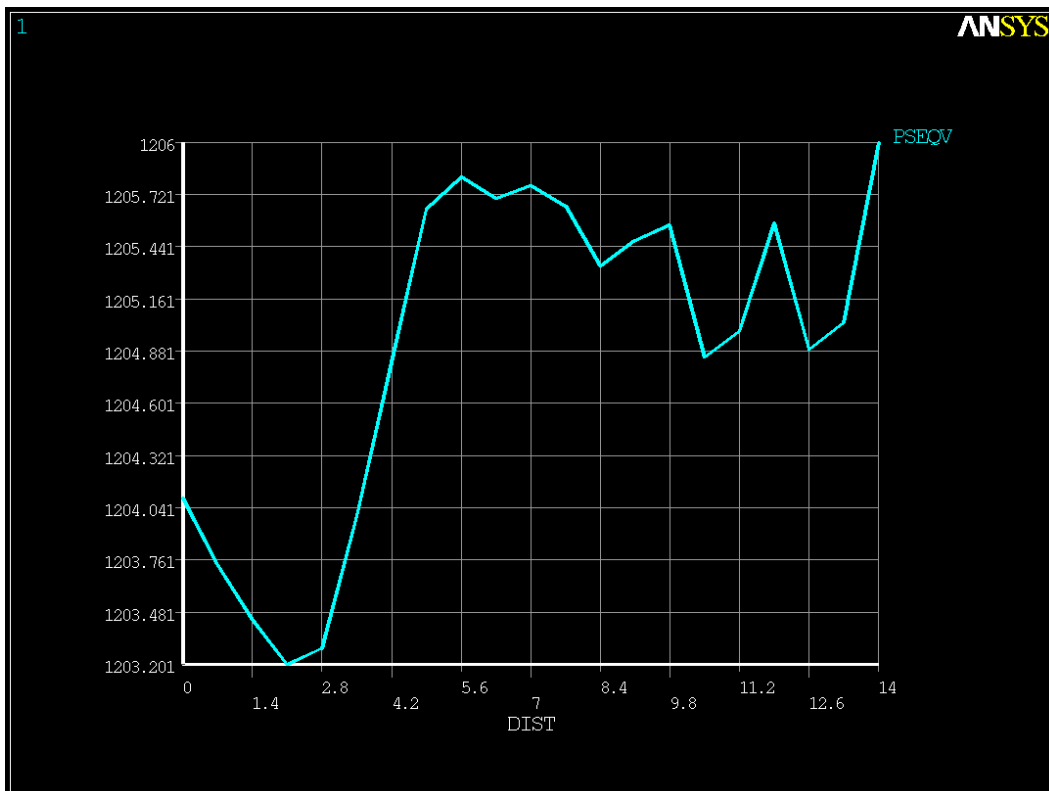
(b) The stress variation along wall thickness

Figure 27 Equivalent stress of plain cylinder (L=100mm) under pressure level
of 2230MPa (22300bar)
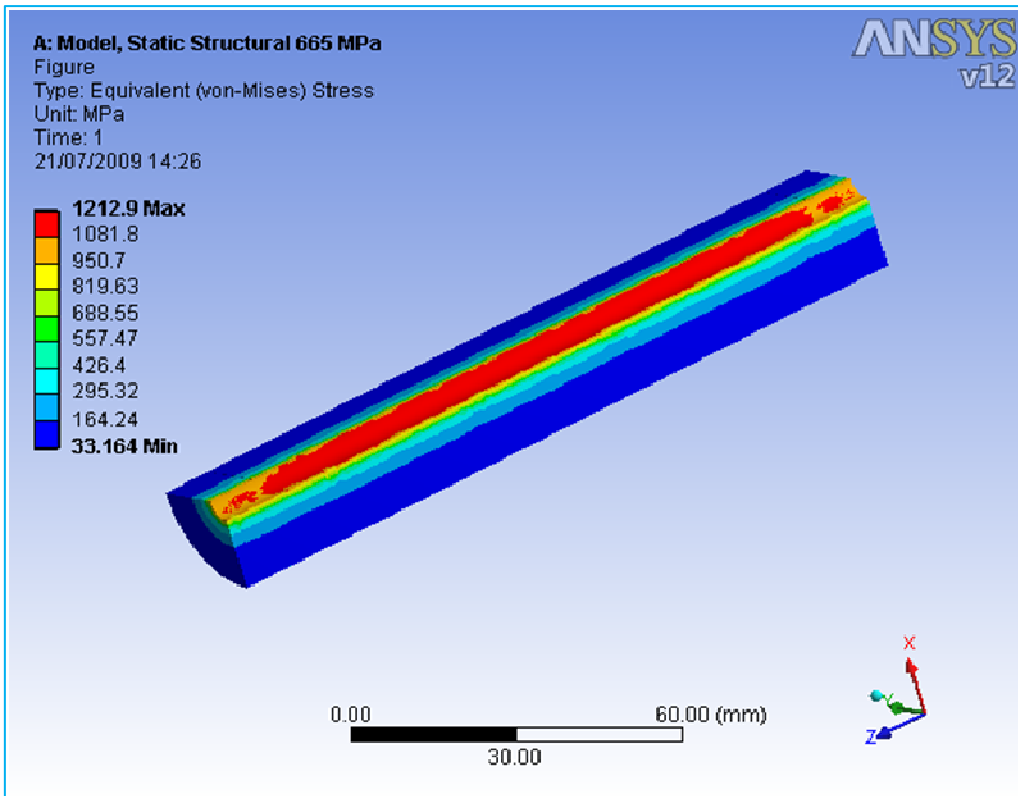
(a) The contour plot of equivalent stress
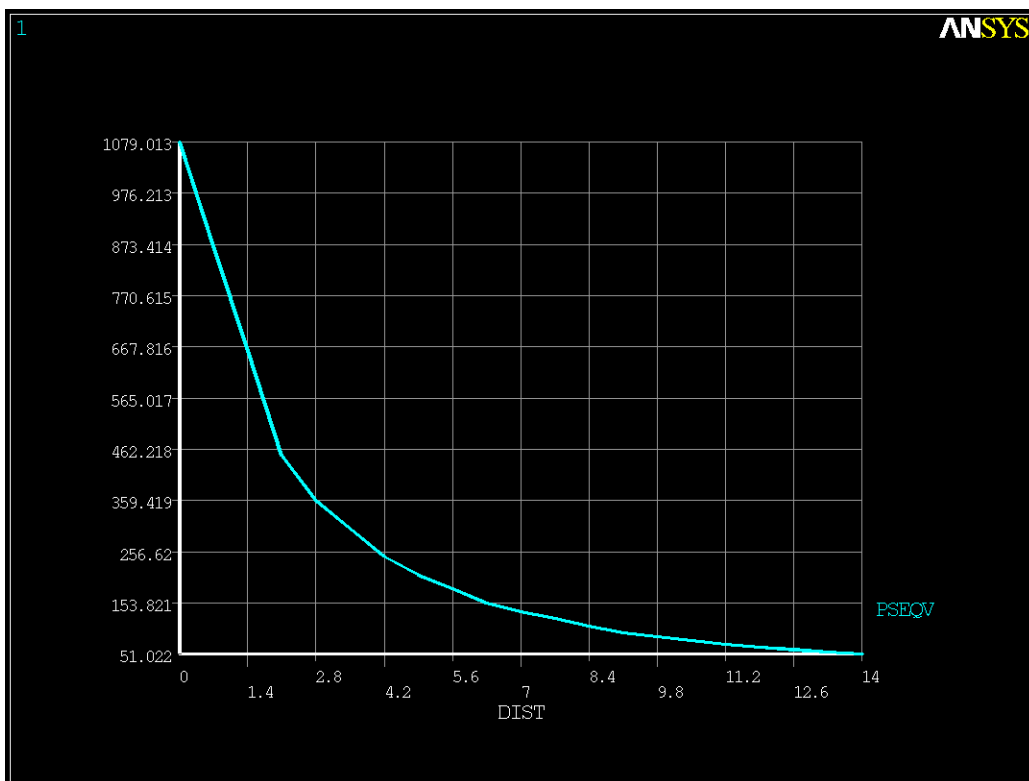
(b) The stress variation along wall thickness

Figure 28 Equivalent stress of plain cylinder (L=150mm) under pressure level of 665MPa (6650bar)
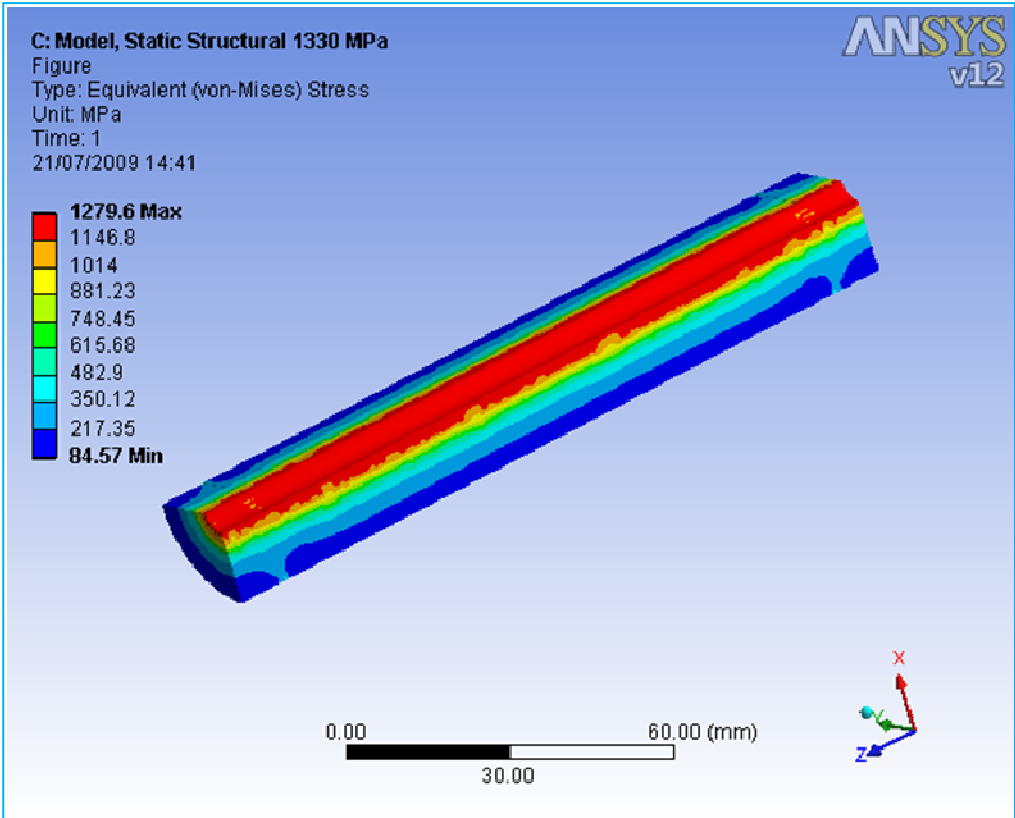
(a) The contour plot of equivalent stress
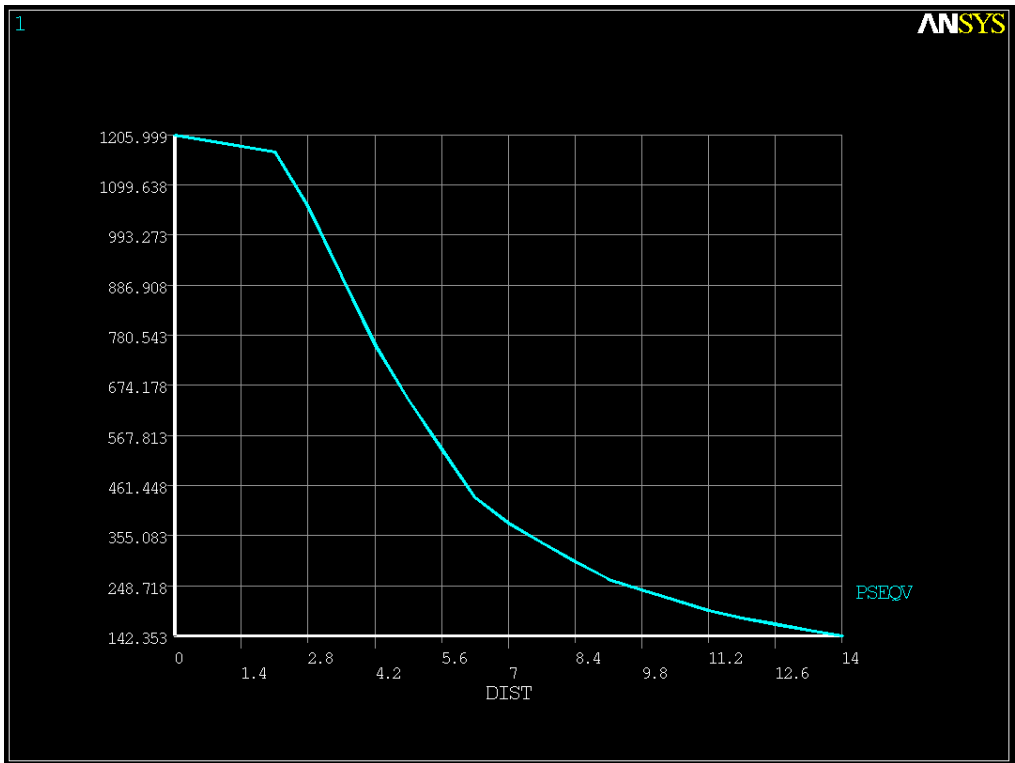
(b) The stress variation along wall thickness

Figure 29 Equivalent stress of plain cylinder (L=150mm) under pressure level of 1330MPa (13300bar)
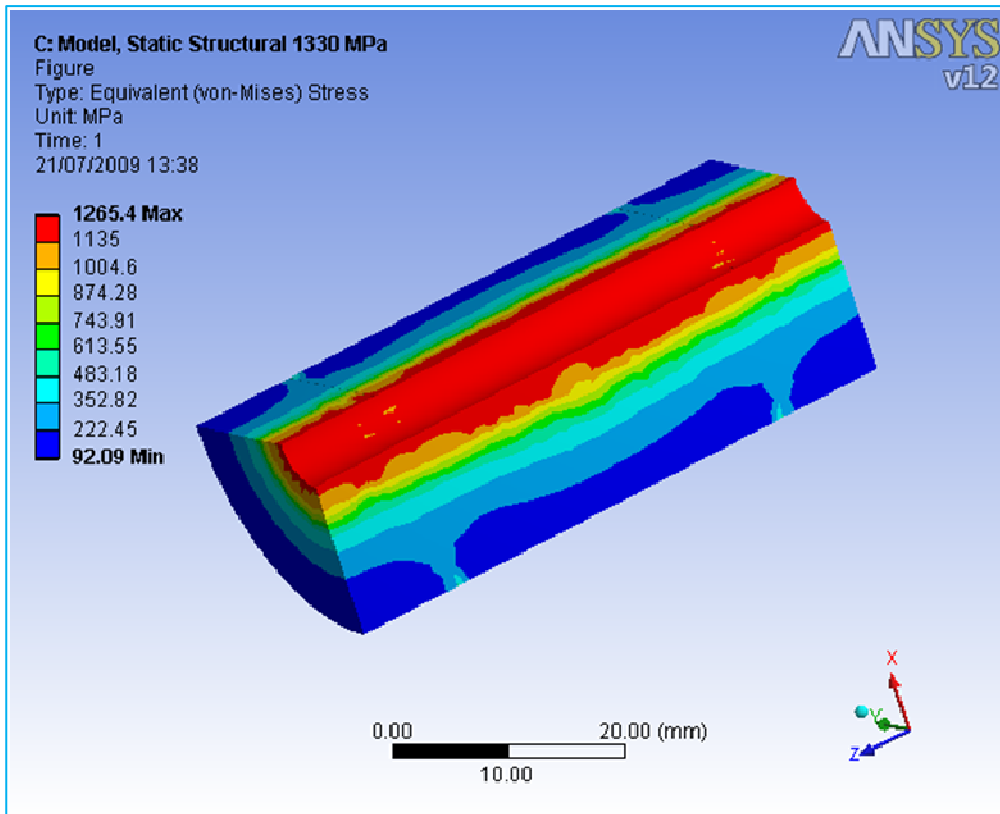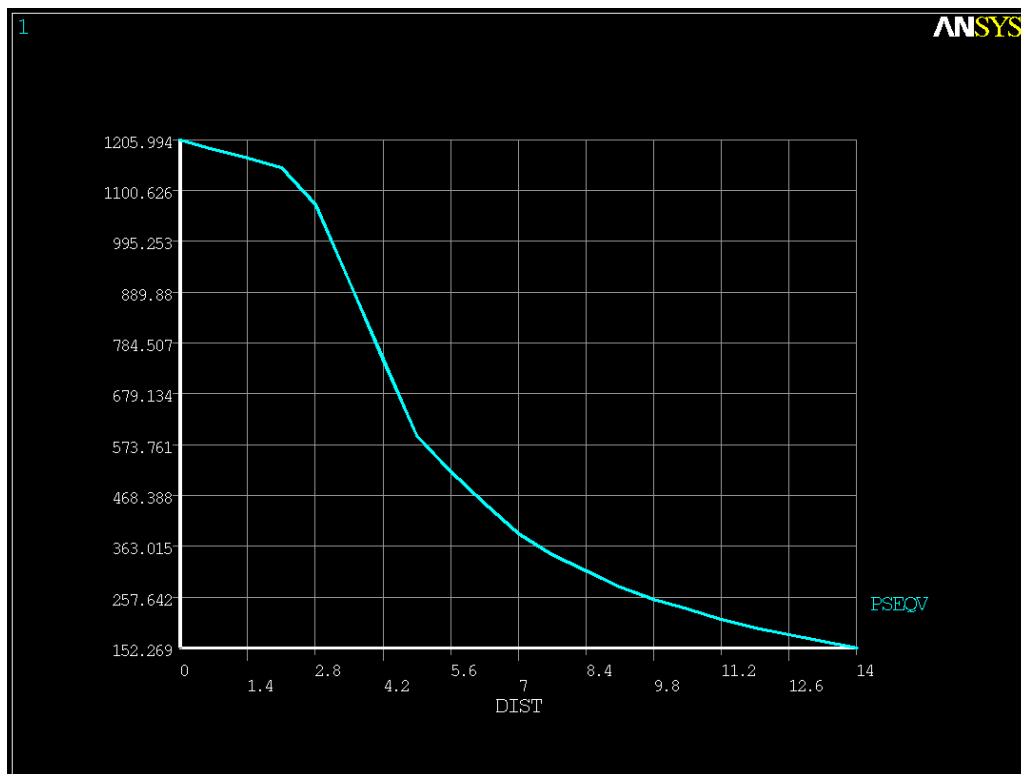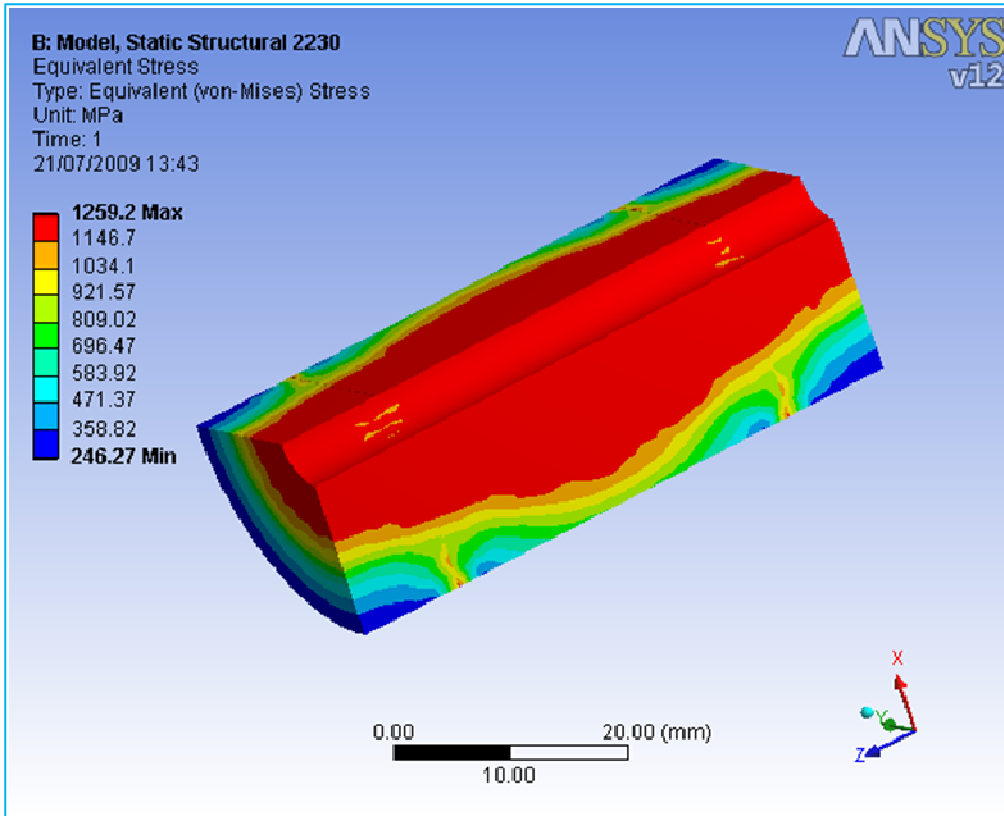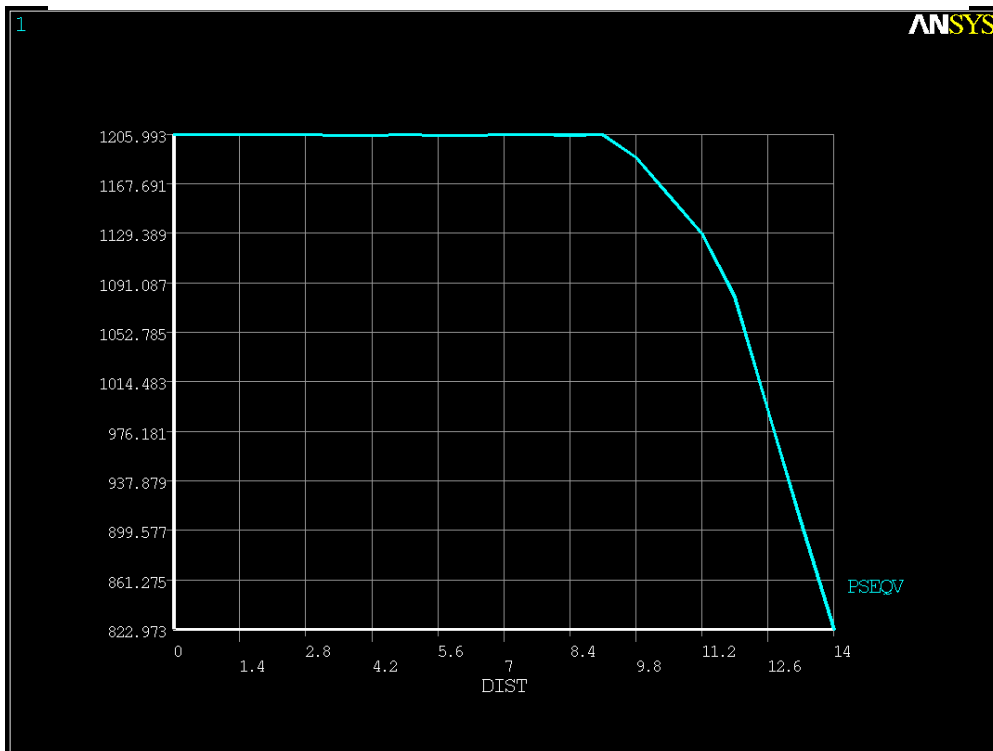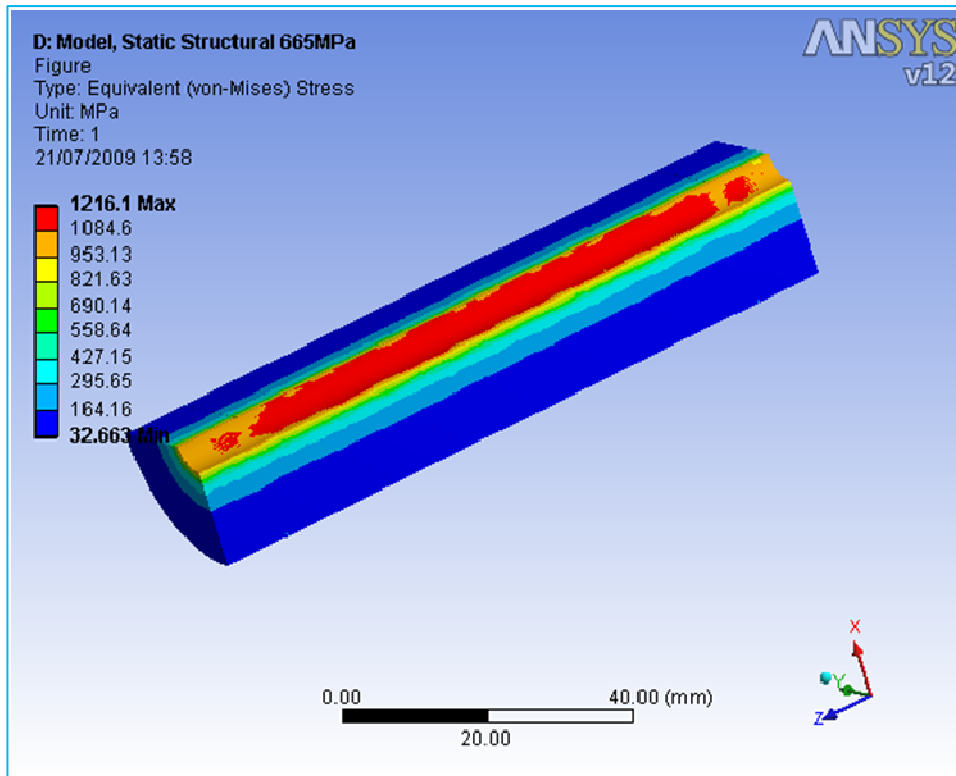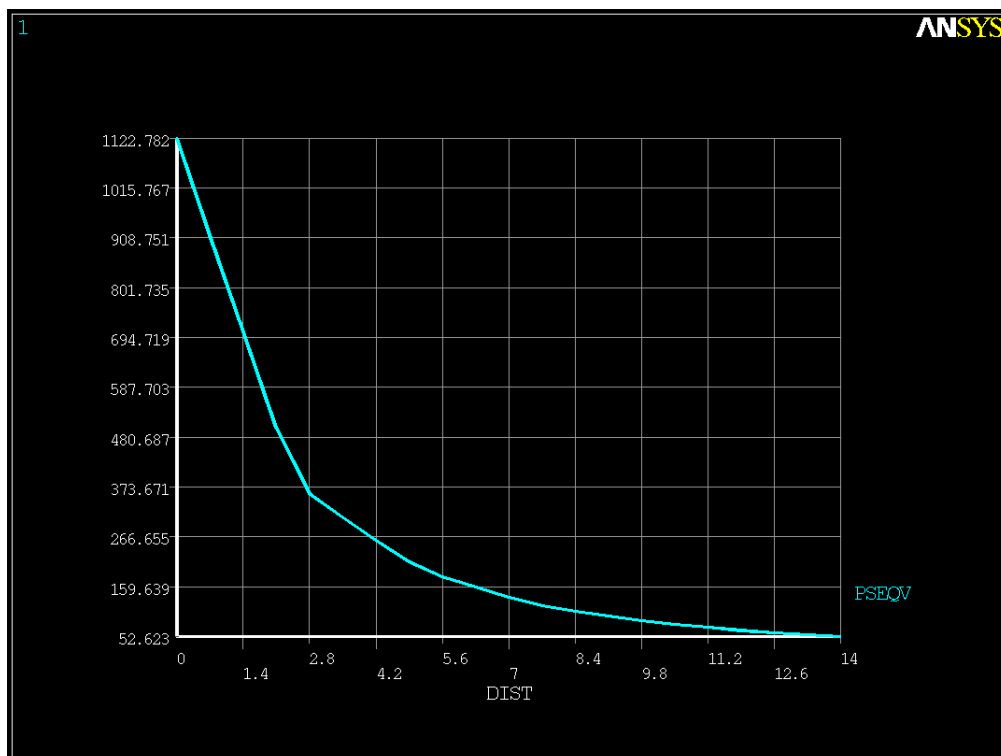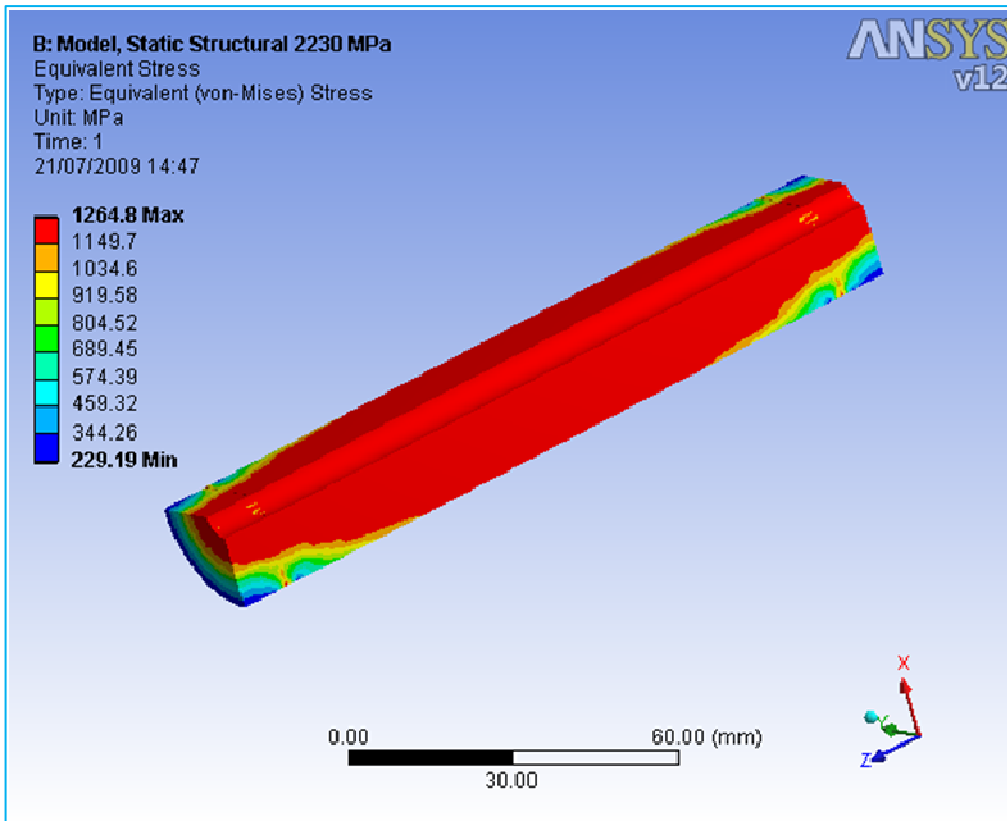
(a) The contour plot of equivalent stress
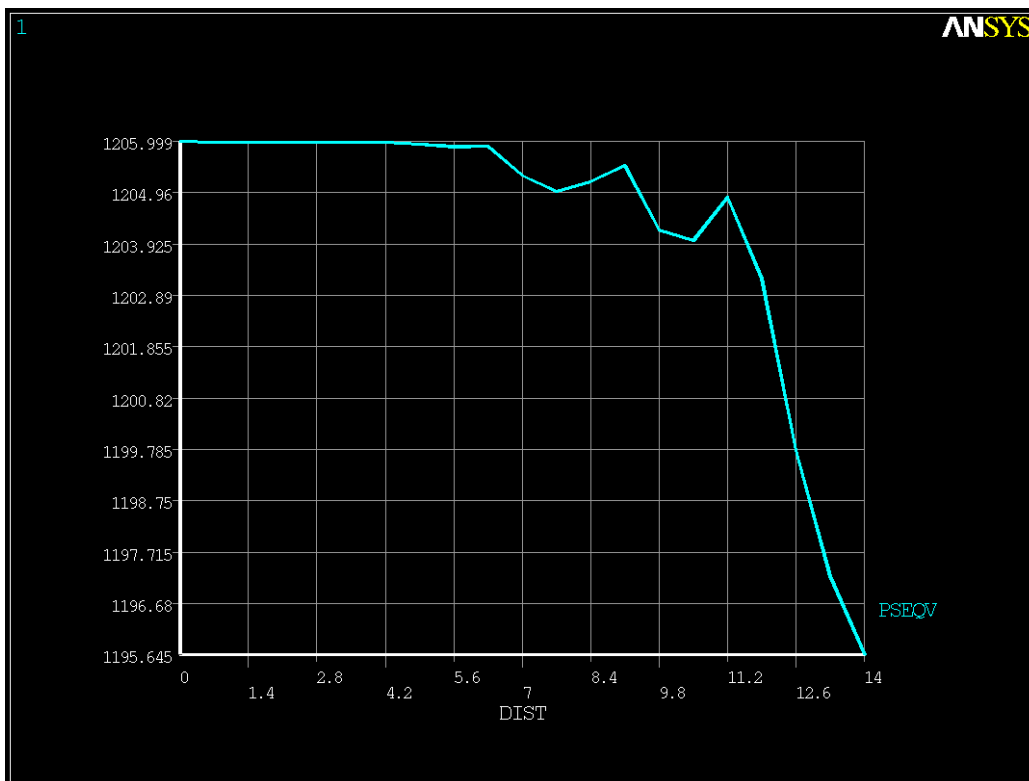
(b) The stress variation along wall thickness

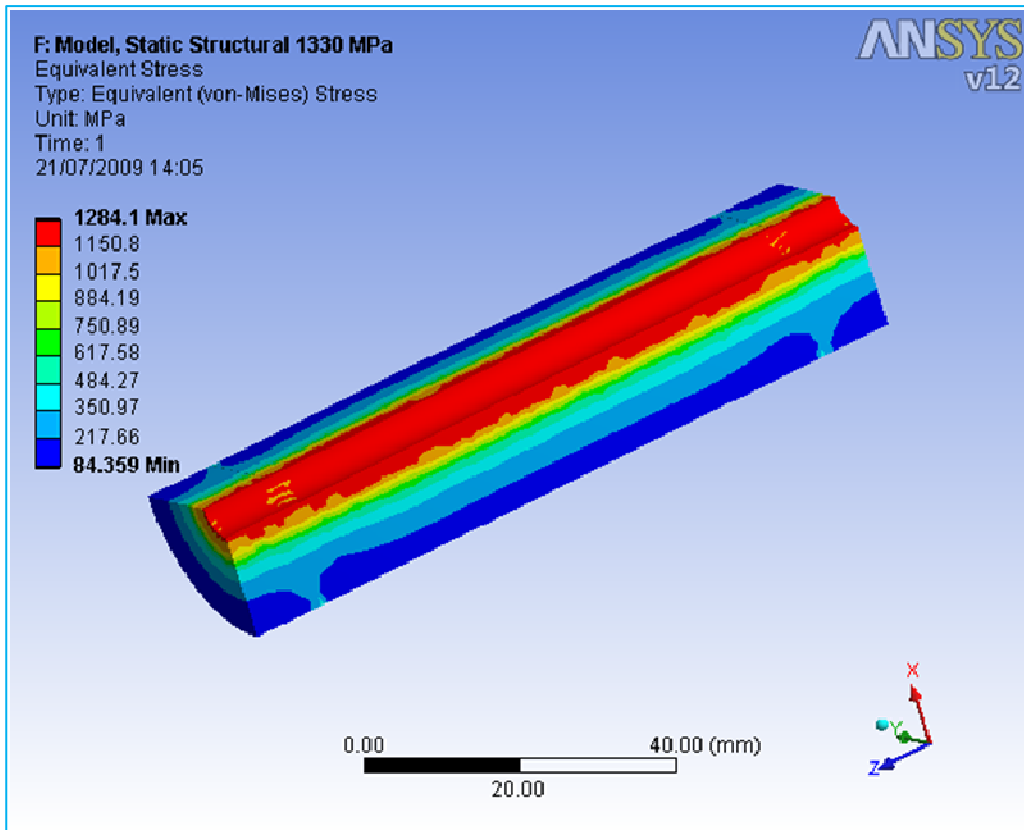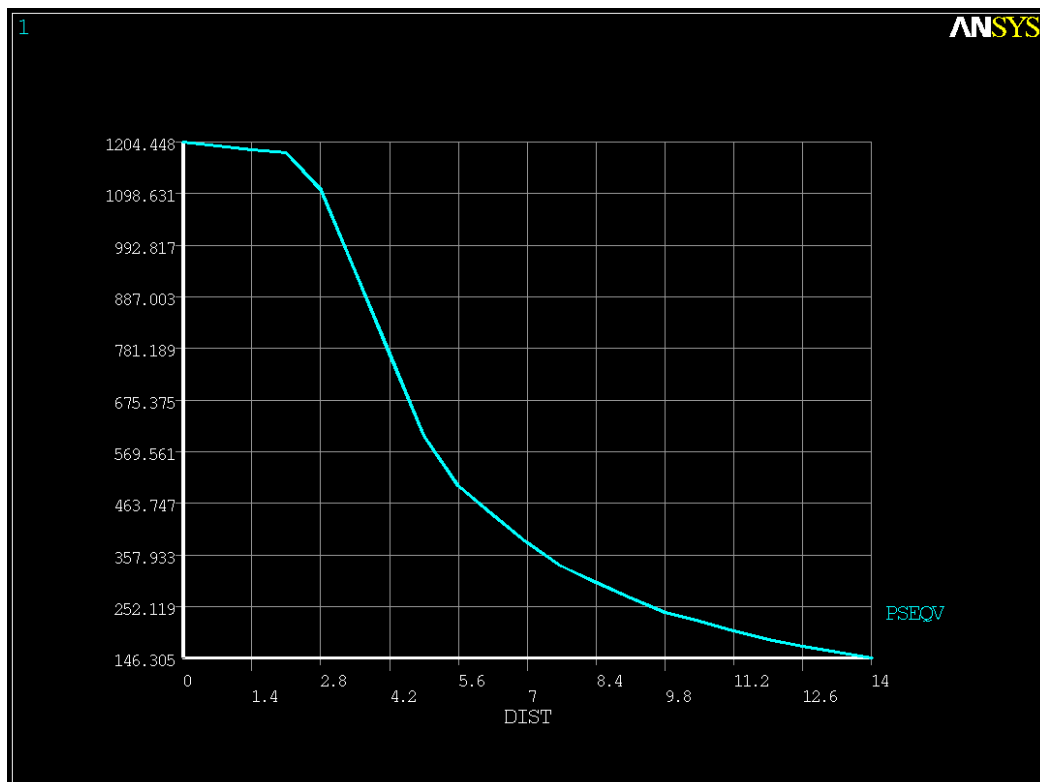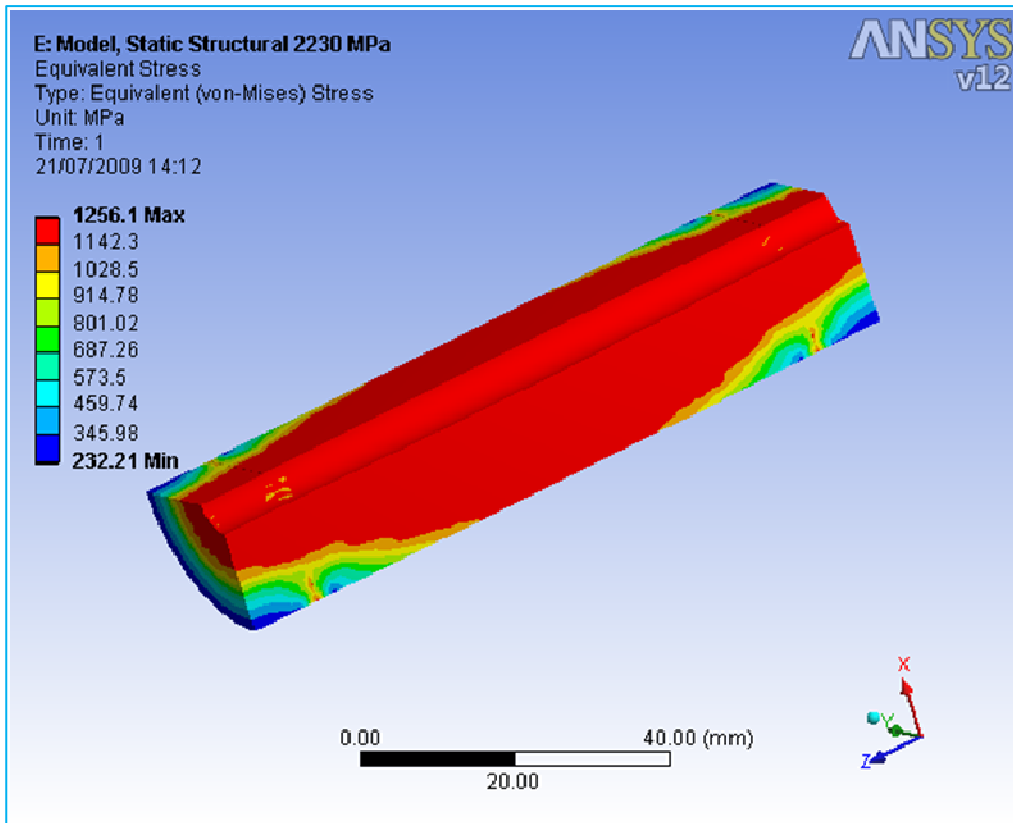**Figure 30 Equivalent stress of plain cylinder (L=150mm) under pressure level of 2230MPa (22300bar)**

Table 5 Sub-summary of FEA results for K value 5

Loading level and Comparison items	665MPa/6650Bar		1330MPa/13300Bar		2230MPa/22300Bar	
	SEQVmax (MPa)	PDDmax (mm)	SEQVmax (MPa)	PDDmax (mm)	SEQVmax (MPa)	PDDmax (mm)
ID7OD35L55mm	1071.799-56.192	0	1205.994-152.269	1.4	1205.993-822.973	10.25
ID7OD35L100mm	1122.782-52.623	0	1204.448-146.305	2	1206-1203.201	14
ID7OD35L150mm	1079.013-51.022	0	1205.999-142.353	1.9	1205.999-1195.645	13

3.3 Discussions

Results obtained via FEA simulation for the three different K values and for the three different cylinder lengths will be discussed in the following Sections 3.3.1 to 3.3.3, respectively.

3.3.1 Discussion on results for K value 3

3.3.1.1 Under initial yield pressure of 615MPa/6150Bar

As shown in Figures 3 (a), 6 (a) and 9 (a), when the internal pressure level of 615MPa was applied to the plain cylinders with the lengths of 55mm, 100mm and 150mm, respectively, the resultant maximum equivalent stresses of 1215.9MPa, 1215.4MPa and 1212.3MPa, are beyond the material yield strength of 1206MPa and the plain cylinders were therefore yield from the inner surface. Also from the above maximum stresses appearance of in the middle section and in the inner surface of the plain cylinders, it was revealed that under this lowest pressure level (615MPa) for K value 3, the boundary condition/constraint of FEA model did not affect the FEA results significantly. In addition, from stress variations along wall thickness, as shown in Figures 3 (b), 6 (b) and 9 (b), all the three maximum stress levels of 1113.655MPa (for cylinder length=55mm), 1048.207MPa (for cylinder length=100mm) and 1018.68MPa (for cylinder length=150mm) started from the inner or very near inner surface of the plain cylinders and the values are smaller than material yield strength (1206MPa). The latter group of maximum equivalent stress values reversely verified that under the issued pressure level of 615MPa/6150bar, the plain cylinder not only yield at the inner surface, but also with a zero plastic deformation depth as expected.

In another word, the FEA simulation results of under pressure level 615MPa and for plain cylinder length of 55mm, 100mm and 150mm are fully consistent with Rob's hand calculation results.

3.3.1.2 Under maximum autofrettage pressure of 1230MPa/12300Bar

When an higher internal pressure of 1230MPa was applied to the plain cylinders with the lengths of 55mm, 100mm and 150mm, respectively, the results shown in Figures 4 (a), 7 (a) and 10 (a), indicated a different picture of to those under the pressure level of 615MPa. Namely, the maximum equivalent stresses of 1256.8MPa, 1281.9MPa and 1272.3MPa under this higher pressure, are not only well beyond the material yield strength of 1200MPa/1206MPa, but also moved to near the constrained region of the FEA model. This means that the boundary conditions of the FEA model have now affected the maximum equivalent stress value fairly significantly. Results in those regions are therefore not reliable to the autofrettage process and that was why the two points for stress variation along wall thickness were created in the middle section of the plain cylinder (refer to Figure 2). Also as shown in Figures 4 (b), 7 (b) and 10 (b), under this maximum autofrettage pressure, the plain cylinder was plastically deformed from the inner surface to a depth of about 2.1mm for 55mm (length) cylinder, 1.8mm for 100mm (length) cylinder and 0.9mm for 150mm (length) cylinder respectively. In another word, under this maximum autofrettage pressure, a maximum elastic plastic interface diameter of ($\emptyset_A$ =(3.5+2.1)x2=11.2mm was formed within the 55mm long cylinder. Although this value is 32% smaller than hand calculation result of $\emptyset_A$ =16.5mm, the value of 11.2mm is correct and the value of 16.5mm is incorrect.

The reason for this is that, "In general, the autofrettage pressure should not exceed that needed to take the diameter of the plastic/elastic boundary to the geometric mean of the outside and inside diameters of the vessel." [2]. Similar requirement was also issued as "In most pressure vessel operations, it is desirable to leave the cylinder in a completely elastic state after autofrettage rather than with a reverse-yielded inner core." [1]. For ensuring the same goal, another resource also stated that: "The maximum allowable autofrettage pressure is then given as that which will produce yielding to the geometric mean radius

$$Rp = \sqrt{R1 \times R2}$$ " [3],

where R_p/$\emptyset_p$ is the geometric mean radius/diameter of the high pressure vessel (in this case plain cylinder). R_1 and R_2 are the inner and outer radius of of the plain cylinder, respectively. Hence, for K value 3, the maximum radius of plastic/elastic boundary will be $Rp = \sqrt{3.5 \times 10.5}$ =6.06mm and therefore $\emptyset_p$=2x6.06=12.12mm.

Obviously and as what I expected initially, the FEA results of under the maximum autofrettage pressure (1230MPa) for K value 3 indicated the formation of diameter of plastic elastic interface of $\emptyset_p$=11.2mm, which is smaller than the limiting geometric mean diameter for this specific plain cylinder (K=3) and the plain cylinder was autofrettaged properly. In another word, the value of $\emptyset_A$ =16.5mm listed in Table 2 and Section 2.2 is well beyond the limiting geometric mean diameter for this specific plain cylinder (K=3) and it is

not allowed to autofrettage this plain cylinder to such a plastic/elastic interface diameter as restricted by the above regulations.

However, the error between hand calculation and FEA results for this parameter ($\emptyset_A/\emptyset_p$) is not considered to be a big concern, as the hand calculation used the correct maximum atufrettage pressure with the well known formula (1) [1]. The error between $\emptyset_A$ and $\emptyset_p$ was due to a simplified linear approximation and the values for $\emptyset_A$ could therefore be ignored. Also this error might reversely indicate that a good validation result has been obtained.

3.3.1.3 Under burst pressure 1520MPa/15200Bar

With the internal pressure further increased from the maximum autofrettage pressure (1230MPa) to burst pressure (1520MPa), as expected, the boundary conditions of FEA model further influenced the maximum equivalent stress to a value of 1556.2MPa for 55mm long cylinder, 1467.7MPa for 100mm long cylinder and 1494.5MPa for 150mm long cylinder, respectively, as shown in Figures 5 (a), 8 (a) and 11 (a). These values could be ignored and attention will therefore be paid to the middle section of the plain cylinders. As depicted in Figure 5 (b), along the entire wall thickness, the equivalent stress variation is between 1205.19MPa and 1203.409MPa, i.e. the stress at every point of the defined path within the cylinder was yield. Similar results were also observed in Figure 8 (b) with a stress variation along the wall thickness of between 1205.998 and 1202.934MPa and Figure 11 (b) with a stress variation along the wall thickness of between 1205.969 and 1186.962MPa. Although the minimum stress levels in these two latter cases are slightly lower than the former one, the entire cylinder was completely plastically deformed/yield.

In other words, under the burst pressure, the FEA results are again agreed well with the hand calculation results regardless of the different cylinder lengths.

3.3.2 Discussion on results for K value 4

3.3.2.1 Under initial yield pressure of 650MPa/6500Bar

The FEA results, as shown in Figures 12 (a), 15 (a) and 18 (a), under the new value of initial yield pressure of 650MPa for K value 4 cylinder, indicated that the maximum equivalent stresses of 1215.8MPa, 1212.8MPa and 1216.8MPa for cylinder length of 55mm, 100mm and 150mm, respectively, were observed. They are again beyond the material yield strength of 1206MPa and the plain cylinders were therefore yield from the inner surface. On the other hand, from the results for stress variations along wall thickness, as shown in Figures 12 (b), 15 (b) and 18 (b). All the equivalent stresses of 1002.395MPa for 55mm cylinder, 1069.709MPa for 100mm cylinder and 985.078MPa for 150mm cylinder are all lower than the material yield strength. This again verified that under such a pressure for the cylinders with K value 4, the plain cylinders were plastically deformed but with a zero plastic

deformation depth from the inner surface in the regions of inner surface or near inner surface.

Therefore, the FEA results of under pressure level 650MPa and for plain cylinder length of 55mm, 100mm and 150mm are also fully consistent with Rob's hand calculation results.

3.3.2.2 Under maximum autofrettage pressure of 1300MPa/13000Bar

Similarly, for the K value 4, when an higher internal pressure of 1300MPa was applied to the plain cylinders with the lengths of 55mm, 100mm and 150mm, respectively, the results shown in Figures 13 (a), 16 (a) and 19 (a), indicated a different picture to those under the pressure level of 650MPa, i.e., the maximum equivalent stresses of 1268.4MPa, 1257.5MPa and 1275.1MPa under this higher pressure, are again not only well beyond the material yield strength of 1200MPa/1206MPa, but also moved to the constrained region of the FEA model, similar to those results for K value 3. Also as shown in Figures 13 (b), 16 (b) and 19 (b), under this maximum autofrettage pressure, the plain cylinder was plastically deformed from the inner surface to a depth of about 1.1mm for 55mm (length) cylinder, 2mm for 100mm (length) cylinder and 1mm for 150mm (length) cylinder respectively.

In another word, under this maximum autofrettage pressure, a maximum elastic plastic interface diameter of ($\emptyset_A$ =(3.5+2)x2=11.0mm was formed within the 100mm long cylinder. Again, although this value is 37.7% smaller than hand calculation result of $\emptyset_A$ =17.66mm, the value of 11.0mm is acceptable and the value of 17.66mm is not right due to the same reason described in the above Section 3.3.1.2.

Specifically speaking，when the plain cylinder with a K value 4, the limiting geometric mean radius correspondingly will be: $R_p = \sqrt{R_1 \times R_2} = \sqrt{3.5 \times 14} = 7mm$, therefore $\emptyset_p$=14mm in this case.

It is evident that the diameter of plastic/elastic interface of $\emptyset_A$=11mm under the maximum autofrettage pressure is smaller than the maximum limiting geometric mean diameter of $\emptyset_p$=14mm. It therefore fully meet the requirement of application of autofrettage technology into high pressure vessels, but not the value of $\emptyset_A$=17.66mm expected in Rob's hand calculation as it is well over the maximum limiting geometric mean diameter of $\emptyset_p$=14mm. The reason for this again is due to a simplified linear approximation and this value should not be used.

In addition and as mentioned previously, FEA simulation was also conducted on the plain cylinder of K=4 and length=100mm but with the previous maximum autofrettage pressure of 1288.18MPa applied. The value of 1288.18MPa was derived via the following formula:

$$P_p = \frac{\sigma_y}{2}(\frac{K^2 - m^2}{K^2}) + \sigma_y \ln m \ \ [2],$$

where m=(elastic plastic interface diameter)/(bore diameter) or via

$$P_a = \sigma_y \left[\ln \frac{R_1}{R_p} - \frac{1}{2R_2^2} \left(R_2^2 - R_p^2 \right) \right]$$ [3]

The results are shown in Figure 21 and compared in Table 4.

As you can see, the equivalent stress variation along wall thickness under both pressure levels of 1300MPa and 1288.18MPa are very similar, i.e. a change of 1206MPa to 229.856 for 1300MPa pressure and a change of 1205.998MPa to 226.021MPa for 1288.19MPa pressure (refer to Figures 16 (b) and 21 (b)). Particularly, the plastic deformation depths (2mm as shown in Table 4) are almost exactly same. This further verified that the previously used (before end of April 09) autofrettage pressure level of 1288.19MPa can equally ensure a proper application of autofrettage technology to our high pressure cell.

3.3.2.3 Under burst pressure 1930MPa/19300Bar

Similar to the K value 3, with the internal pressure further increased from the maximum autofrettage pressure (1300MPa) to burst pressure (1930MPa), as expected, the boundary conditions of FEA model further influenced the maximum equivalent stress to a value of 1916.6MPa for 55mm long cylinder, 1820.9MPa for 100mm long cylinder and 1805.7MPa for 150mm long cylinder, respectively, as shown in Figures 14 (a), 17 (a) and 20 (a). These values again should be ignored and attention will therefore be paid to the middle section of the plain cylinders. As presented in Figure 14 (b), along the entire wall thickness, the equivalent stress variation is between 1205.991MPa and 1147.197MPa, i.e. the stress at every point of the defined path from the inner surface to a depth of about 5.775mm within the cylinder was yield but not the entire wall thickness as the minimum equivalent stress level is lower than the material yield strength. This is different from those results for K value 3, where the entire wall thickness was plastically deformed. This difference may be explained by the following two factors: (1) the increased wall thickness from 7mm to 10.5mm; (2) the boundary condition of the FEA model, particularly, when the cylinder is short like this 55mm one.

However, under the same burst pressure, the entire wall thickness was plastically deformed when the plain cylinder is with 100mm (or more) length, as shown in Figures 17 (b) and 20 (b), where the equivalent stresses of 1205.293MPa to 1206MPa for 100mm cylinder and 1203.813MPa to 1205.217MPa for 150mm are all beyond the material yield strength.

Hence, a good agreement between the FEA results and the hand calculation was found for the longer cylinders (100mm and 150mm), but not for the short one (55mm one). The above results are also summarised in Table 4.

3.3.3 Discussion on results for K value 5

3.3.3.1 Under initial yield pressure of 665MPa/6650Bar

As shown in Figures 22 (a), 25 (a) and 28 (a), when the internal pressure level of 665MPa was applied to the plain cylinders with the lengths of 55mm, 100mm and 150mm, respectively, similar to those results for K values 3 and 4, the maximum equivalent stresses of 1206.7MPa, 1216.1MPa and 1212.9MPa, are higher than the material yield strength of 1206MPa and the plain cylinders were therefore yield from the inner surface. Also from the above maximum stresses appearance of in the middle section (one example in Figure 22 (a)) and in the inner surface of the plain cylinders, it was   revealed that under this lowest pressure level (665MPa) for K value 5, the boundary condition/constraint of FEA model did not affect the FEA results significantly. Again, from stress variations along wall thickness, as shown in Figures 22 (b), 25 (b) and 28 (b), all the three maximum stress levels of 1071.799MPa (for cylinder length=55mm), 1122.782MPa (for cylinder length=100mm) and 1079.013MPa (for cylinder length=150mm) started from the inner or very near inner surface of the plain cylinders and the values are smaller than material yield strength (1206MPa). The latter group of maximum equivalent stress values therefore reversely verified that under the issued pressure level of 665MPa/6650bar, the plain cylinder not only yield at the inner surface, but also with a zero plastic deformation depth, as sub-summarised in Table 5.

In another word, the FEA simulation results of under pressure level 665MPa and for plain cylinder length of 55mm, 100mm and 150mm are again agreed well with Rob's hand calculation results.

3.3.3.2 Under maximum autofrettage pressure of 1330MPa/13300Bar

When an higher internal pressure of 1330MPa was applied to the plain cylinders with the lengths of 55mm, 100mm and 150mm, respectively, the results shown in Figures 23 (a), 26 (a) and 29 (a), indicated a different picture of to those under the pressure level of 615MPa for K=3, but similar to the results for K value 4. Namely, the maximum equivalent stresses of 1265.4MPa, 1284.1MPa and 1279.6MPa under this higher pressure, are again higher than the material yield strength of 1200MPa/1206MPa, and the maximum value also moved to the constrained region of the FEA model. These values should therefore be excluded. In addition, as shown in Figures 23 (b), 26 (b) and 29 (b), under this maximum autofrettage pressure, the plain cylinder was plastically deformed from the inner surface to a depth of about 1.4mm for 55mm (length) cylinder, 2.0mm for 100mm (length) cylinder and 1.9mm for 150mm (length) cylinder, respectively.

In another word, under this maximum autofrettage pressure, a maximum elastic plastic interface diameter of ($\emptyset_A$ =(3.5+2.0)x2=11.0mm was similarly formed within the 100mm long cylinder. Again, although this value is 41.7% smaller than hand calculation result of $\emptyset_A$ =18.9mm, the value of 11.0mm is acceptable and the value of 18.9mm is incorrect due to the same reason described in the above Section 3.3.1.2.

Again here specifically speaking，when the plain cylinder with a K value 5, the limiting geometric mean radius correspondingly will be: $R_p = \sqrt{R_1 \times R_2} = \sqrt{3.5 \times 17.5} = 7.83mm$, therefore $\emptyset_p$=15.65mm in this case. It is evident that the diameter of plastic/elastic interface of $\emptyset_A$=11mm under the maximum autofrettage pressure is smaller than the maximum limiting geometric mean diameter of $\emptyset_p$=15.65mm. It therefore again fully meet the requirement of application of autofrettage technology into high pressure vessels, but not the value of $\emptyset_A$=18.9mm as expected in Rob's hand calculation as it is over the maximum limiting geometric mean diameter of $\emptyset_p$=15.65mm. The reason for this again is due to a simplified linear approximation and this value should not be used.

3.3.3.3 Under burst pressure 2230MPa/22300Bar

With the internal pressure further increased from the maximum autofrettage pressure (1330MPa) to burst pressure (2230MPa), as expected, the boundary conditions of FEA model again influenced the maximum equivalent stress to a value of 1259.2MPa for 55mm long cylinder, 1256.1MPa for 100mm long cylinder and 1264.8MPa for 150mm long cylinder, respectively, as shown in Figures 24 (a), 27 (a) and 30 (a). As depicted in Figure 24 (b), along the entire wall thickness, the equivalent stress variation is between 1205.991MPa and 822.973MPa, i.e. the stress at every point of the defined path from the inner surface to a depth of about 10.25mm within the cylinder was yield but not the entire wall thickness as the minimum equivalent stress level is lower than the material yield strength. This is different from those results for K value 3, where the entire wall thickness was plastically deformed but similar to those results for K value 4. The difference here again may be explained by the following two factors: (1) the increased wall thickness from 7mm to 14mm; (2) the boundary condition of the FEA model, particularly, when the cylinder is short like this 55mm one.

However, under the same burst pressure, the entire wall thickness was almost plastically deformed when the plain cylinder is with 100mm (or more) length, as shown in Figures 27 (b) and 30 (b), where the equivalent stresses of 1203.201MPa to 1206MPa for 100mm cylinder and 1195.645MPa to 1205.999MPa for 150mm are all almost reached the material yield strength.

Hence, a good agreement between the FEA results and the hand calculation was found for the longer cylinders (100mm and 150mm), but not for the short one (55mm one). The above results are also summarised in Table 5.

The overall FEA simulation results are now be compared and tabulated in Table 6.

Table 6 Summary of the overall FEA results

LOAD LEVELS, ITEMS for K=3	615MPa/6150Bar		1230MPa/12300Bar			1520MPa/15200Bar	
	HCEPDD (mm)	FEEPDD (mm)	HCEPDD (mm)	MLGMRD (mm)	FEEPDD (mm)	HCEPDD (mm)	FEEPDD (mm)
55mm	0	0	4.75	>2.56	>2.1	7	7
100mm	0	0	4.75	>2.56	>1.8	7	7
150mm	0	0	4.75	>2.56	>0.9	7	7
Results	Fully agreed		unsafe	criterion	safe	Fully agreed	
LOAD LEVELS, ITEMS for K=4	650MPa/6500Bar		1300MPa/13000Bar			1930MPa/19300Bar	
	HCEPDD (mm)	FEEPDD (mm)	HCEPDD (mm)	MLGMRD (mm)	FEEPDD (mm)	HCEPDD (mm)	FEEPDD (mm)
55mm	0	0	5.33	>3.5	>1.1	10.5	5.775
100mm	0	0	5.33	>3.5	>2	10.5	10.5
150mm	0	0	5.33	>3.5	>1	10.5	10.5
100mm	Use 1288.18 MPa		5.33	>3.5	>2	NA	NA
Results	Fully agreed		unsafe	criterion	safe	80% agreed	
LOAD LEVELS, ITEMS for K=5	665MPa/6650Bar		1330MPa/13300Bar			2230MPa/22300Bar	
	HCEPDD (mm)	FEEPDD (mm)	HCEPDD (mm)	MLGMRD (mm)	FEEPDD (mm)	HCEPDD (mm)	FEEPDD (mm)
55mm	0	0	5.95	>4.33	>1.4	14	10.25
100mm	0	0	5.95	>4.33	>2	14	14
150mm	0	0	5.95	>4.33	>1.9	14	14
Results	Fully agreed		unsafe	criterion	safe	90% agreed	

Note: HCEPDD-hand calculation expected plastic deformation depth (starting from inner surface);
FEEPDD- FEA expected plastic deformation depth (starting from inner surface);
MLGMR-maximum geometric mean radius difference (starting from inner surface).

4. Additional information-Two loading paths used in FEA simulations

In order to facilitate the conclusion to be made in the next section, two loading paths, i.e. one was used in the FEA simulations conducted here within this report and another was used in the FEA simulation conducted previously (before end of April 2009) are schematically shown in Figures 31 and 32, respectively.
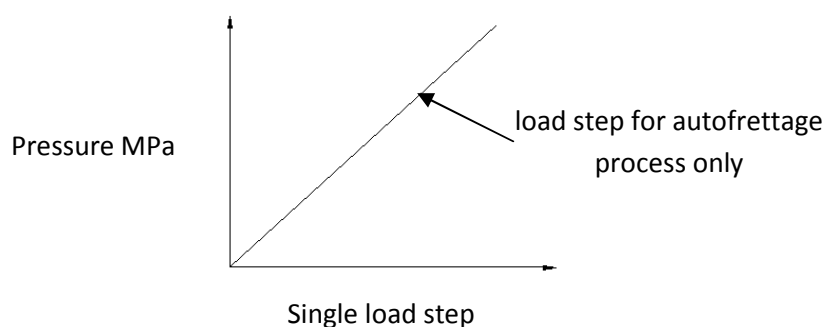

Figure 31 Loading path used in this report

Based on the FEA requirement described in Section 2.2, loading path shown in Figure 31 was used throughout the FEA simulations conducted within this report. In other words, only one load step, which was used to apply the initial yield pressure, or to apply the maximum autofrettage pressure, or to apply the final burst pressure, was used, respectively, in different analysis. The influence of unloading (e.g. autofrettage pressure) and further applied working pressure to the maximum equivalent stress therefore was not considered at all throughout this report.
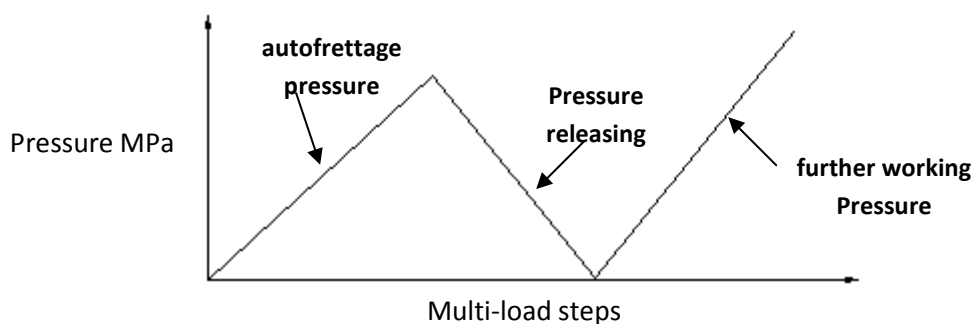

Figure 32 Loading path used in the previous report

However, for the successful use of an autofrettaged high pressure vessel, not only a correct maximum autofrettage pressure level and the limiting geometric mean radius need to be defined initially to ensure the autofrettage process to be conducted properly, but also the effects of the subsequent autofrettage pressure releasing and particularly the further applied working pressure on the equivalent stress need to be examined. That was why the loading path shown in Figure 32 was used in the previously FEA simulation (before April 2009), where the influence of three load steps of applying atufrettage pressure, releasing autofrettage pressure and further applying working pressure on equivalent stress were totally considered there.

It also should be noted that although FEA simulations were conducted on all three pressure level of initial yield pressure, autofrettage pressure and burst pressure, as requested in Section 2.2, most interesting/valuable information should only be found from those results under the maximum autofrettage pressure, as both initial yield pressure and burst pressure are not very relevant to our application of the autofrettage technology here. In another word, we will not autofrettage a high pressure vessel with either a zero plastic deformation depth or a entire wall thickness plastic deformation depth. That also was why only maximum autofrettage pressure was included in the previous FEA simulations.

5. Conclusions

Based on the exercises carried out in this report and in the previous report, following conclusions may be drawn:

(1). Under initial yield pressures of 615MPa/6150bar for K=3, 650MPa/6500bar for K=4 and 665MPa/6650bar for K=5, the FEA results are fully agreed well with hand calculation results (as shown in Table 6 but it is not very important for our needs);

(2). Under burst pressures of 1520MPa/15200bar for K=3, 1930MPa/19300bar for K=4 and 2230MPa/22300bar for K=5, the FEA results are 80% agreed well with hand calculation results (as shown in Table 6 and again it is not important for our needs);

(3). Under maximum autofrettage pressure of 1230MPa/12300bar for K=3, 1300MPa/13000bar for K=4 and 1330MPa/13300bar for K=5, the FEA results are, to some degree, agreed well with the hand calculation, i.e. the autofrettage pressure levels are correct, but the expecting diameters of $\emptyset_A$=16.5mm for K=3, $\emptyset_A$=17.66mm for K=4 and $\emptyset_A$=18.9mm for K=5 are incorrect due to a simplified linear approximation error in hand calculation (these values and the corresponding FEA results will need to be further examined experimentally, as these information is important to our needs as shown in Table 6);

(4). FEA results of under the autofrettage pressure of 1288.19MPa, which was used in previous report and FEA results under the autofrettage pressure of 1300MPa used in this report are fully agreed well;

(5). Based on the FEA simulations conducted in the previous report, where multiple load steps were considered, the maximum load bearing capacity for our high pressure cell of with a K=4 is still at about 11kbar and definitely no more than 12kbars.

(6). Unlike the FEA simulations conducted in previous report, where multiple load steps were considered, the FEA simulations conducted here in this report, a single load step for autofrettage process was considered only as requested in Section 2.2;

(7). I am therefore not in a position to comment on the maximum load bearing capacities of these plain cylinders just now, as where multiple load steps need to be considered.

6. References

[1] High pressure technology – volume 1 – chapter 7 – high pressure containment in cylindrical monobloc vessels, Marcel Dekker, 1977, p229-251.

[2] High Pressure Technology Association-High Pressure Safety Code, 1977, p29-30.

[3] E. J. Hearn, Mechanics of Materials, Int. Series on Materials and Technology, Vol. 19, 1985, pp. 516-519.