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DIRECT CALCULATION OF MAGNETIC FIELDS IN THE PRESENCE OF IRON, AS APPLIED TO THE COMPUTER PROGRAM GFUN

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ABSTRACT

A description of the method of solution of the integral equations used in the GFUN program in two-dimensional, three dimensional and axisymmetric versions.

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CONTENTS

1. INTRODUCTION

This note describes the methods used by the program GFUN to calculate the fields of magnets, in particular magnets which contain iron. GFUN has been applied to two-dimensional, axisymmetric, and three-dimensional magnet designs.[1,2]

Both the field from the currents, and the field from the iron are found by direct calculation. First the magnetisation of the iron is found by solving the integral equation it obeys. Then the field anywhere can be found from the known currents and previously determined magnetisation.

The direct method described here has two important advantages over other methods, eg. relaxation and finite-element techniques, which require the solution of partial differential equations. First, because the direct calculation is performed only over the iron, fewer elements are required. Second, difficulties with boundary conditions do not arise.

2. THE METHOD. GENERAL CONSIDERATIONS

In the presence of current-carrying conductors and magnetic materials, the magnetic field $\vec{H}$ can be written as the sum of a field $\vec{H}_c$ due to the currents and a field $\vec{H}_m$ due to the magnetisation of the iron. In SI units, the field due to the current is given by the volume integral

$$\vec{H}_c = \frac{1}{4\pi} \int \frac{\vec{J} \times \vec{r}}{r^3} \, dV \qquad (1)$$

where $\vec{r}$ is the vector from the source point to the field point. In practice the current density $\vec{J}$ is specified, and the evaluation of $\vec{H}_c$ is straightforward.

A. Finding $\vec{H}_m$ from $\vec{M}$

The field $\vec{H}_m$ can be found from the scalar potential V_m:

$$\vec{H}_m = -\text{grad } V_m$$

The scalar potential Vm is given by the volume integral:

$$V_m = \frac{1}{4\pi} \int \frac{\vec{M} \cdot \vec{r}}{r^3} \, dV \qquad (2)$$

and thus:

$$\vec{H}_m = \frac{-1}{4\pi} \text{ grad} \int \frac{\vec{M} \cdot \vec{r}}{r^3} \, dV \tag{3}$$

Let the field point have coordinates (x, y, z); and the source point (x', y', z'). Then Equation (3) has x component:

$$H_{mx} = -\frac{1}{4\pi}\frac{\partial}{\partial x}\iiint \frac{M'_x(x-x') + M'_y(y-y') + M'_z(z-z') \, dx'dy'dz'}{r^3} \tag{4}$$

with similar expressions for H_{my} and H_{mz}. In Equation (4)

$$r^2 = (x-x')^2 + (y-y')^2 + (z-z')^2.$$

Carrying out the differentiation in Eqn(4) yields:

$$H_{mx} = \frac{1}{4\pi}\iiint [M'_x \{3(x-x')^2/r^5 - 1/r^3\} + 3M'_y(x-x')(y-y')/r^5$$
$$+ 3M'_z(x-x')(z-z')/r^5] \, dx'dy'dz' \tag{5}$$

The integration is carried out over the iron region. Consider this region divided into N elements, over each of which the magnetisation is taken as constant. Also denote the field point by a. Then Eqn (5) becomes:

$$H_{mx}(a) = \sum_{b=1}^{N} C_{ax,bx} M_x(b) + C_{ax,by} M_y(b) + C_{ax,bz} M_z(b) \tag{6}$$

with

$$C_{ax,bx} = \frac{1}{4\pi}\iiint_{Vb} \{3(x-x')^2/r^5 - 1/r^3\} \, dx'dy'dz' \tag{7a}$$

$$C_{ax,by} = \frac{1}{4\pi}\iiint_{Vb} 3(x-x')(y-y')/r^5 \, dx'dy'dz \tag{7b}$$

$$C_{ax,bz} = \frac{1}{4\pi}\iiint_{Vb} 3(x-x')(z-z')/r^5 \, dx'dy'dz \tag{7c}$$

and analogous expressions for H_{my} and H_{mz}

Using the notation $x_1=x$, $x_2=y$, $x_3=z$, we can write the general expression for the $C_{ai,bj}$:

$$C_{ai,bj} = \frac{1}{4\pi}\iiint_{Vb} \{3(x_i-x_i')(x_j-x_j')/r^5 - \delta_{ij}/r^3\} \; dV' \tag{7d}$$

The C's are purely geometrical factors; their evaluation is described below. If the magnetisation is known for each element, then Eqn(6) will give the field H_m at any point a.

B. Determination of $\vec{M}$

Before Equation (6) can be evaluated, the magnetisation of each element must be known. We shall see that Eqn (6) itself suggests a way the magnetisation can be determined.

Consider the field point a to be the center of the element a. The total field at a is the sum of $\vec{H}_{ca}$ and $\vec{H}_{ma}$. Also the magnetisation is related to the total field $\vec{H}$ through the magnetic susceptibility.

$$\vec{M} = \chi H, \text{with } \chi(H) = \mu(H)/\mu_o - 1$$

Combining these considerations yields:

$$H_x(a) = H_{cx}(a) + \sum_{b=1}^{N} C_{ax,bx}\chi_b H_x(b) + C_{ax,by}\chi_b H_y(b) + C_{ax,bz}\chi_b H_z(b)$$

which can be rearranged to read:

$$\sum_{b=1}^{N} (C_{ax,bx}\chi_b - \delta_{ab})H_x(b) + C_{ax,by}\chi_b H_y(b) + C_{ax,bz}\chi_b H_z(b) = -H_{cx}(a) \tag{8a}$$

similarly

$$\sum_{b=1}^{N} C_{ay,bx}\chi_b H_x(b) + (C_{ay,by}\chi_b - {}_{ab})H_y(b) + C_{ay,bz}\chi_b H_x(b) = -H_{cy}(a) \tag{8b}$$

and

$$\sum_{b=1}^{N} C_{az,bx}\chi_b H_x(b) + C_{az,by}\chi_b H_y(b) + (C_{az,bz}\chi_b - \delta_{ab})H_z(b) = H_{cz}(a) \tag{8c}$$

If the χ_b were known, Equations (8) would be a system of 3N simultaneous linear equations, which could be solved for the $\vec{H}_b$. In practice we solve Eqns(8) using some initial values for χ_b, next find the values of χ_b corresponding to the solution of $\vec{H}_b$, and then iterate until the solution converges.

C. General remarks about the coefficients

In the sections that follow, we shall discuss the $C_{ai,bj}$ coefficients (subscripts i and j refer to x, y or z), but here we list some general relations they obey. First from the form of Eqn (7) we can write:

$$C_{aj,bi} = C_{ai,bj} \tag{9a}$$

$$C_{ax,bx} + C_{ay,by} + C_{az,bz} = 0 \text{ if } a \neq b \tag{9b}$$

or in general if the point a is outside the element b.

$$C_{ax,bx} + C_{ay,by} + C_{az,bz} = -1 \text{ if } a = b \tag{9c}$$

or in general if the point a is inside the element b.

D. General remarks about symmetry

If the magnet being calculated has a plane of symmetry, then only half the current and iron elements need enter the calculation. Likewise, if the magnet has two or three planes of symmetry, the calculation need include only the elements in one quadrant or octant. The coefficients are calculated by Eqn(7) for both the direct and reflected elements; and the results are added or subtracted depending on whether the magnetisation component of the reflected elements has the same or opposite sign as the component of the direct element.

Because the x, y and z components will have different reflection behaviour, the combined coefficients will not obey Eqn(9).

E. Point dipoles: a simple example.

In the following sections, we treat solutions of Eqns(7) and (8) that are useful in solving problems. But first we look at an oversimplification that has not proved useful in calculating fields, but which may give some insight into the way the method works. If we assume the magnetic moment of each element is concentrated at its center, Eqn(7d) can be re-written as;

$$C_{ai,bj} = \frac{1}{4\pi} \; V_b \; [3(x_{ai}-x_{bi})(x_{aj}-x_{bj})/r^5 - \delta_{ij}/r^3] \tag{10}$$

Eqn(10) is easy to evaluate, but cannot be used when r = 0, ie. for the self field coefficients $C_{ai,aj}$. For the case a = b, we could find the coefficients directly from Eqn(9c) if there is sufficient symmetry and otherwise use one of the expressions developed below. However even using better expressions for the self-field coefficients does not produce accurate calculations; Eqn(10) is too crude an expression for neighbouring elements as well. Probably Eqn(10) is useful only for checking the results of other expressions for widely separated elements.

3. TWO-DIMENSIONAL CALCULATIONS

Much of the testing of the method has been done in two-dimensional calculations because calculation by other methods are available for comparison. The general Equations (7) and (8) can be specialised to two-dimensions and solved. Many of the results have a simpler form when expressed in complex variables; the results are here expressed in both real and complex variables.

A. The general coefficients

We find the required equations by limiting i and j to x and y in Eqns (7) and (8), and integrating Eqn(7) over z between minus and plus infinity. Equations(7) become:

$$C_{ax,bx} = \frac{1}{2\pi}\iint_{A_b} [(x-x')^2 - (y-y')^2]/r^4 \, dx'dy' \tag{11a}$$

$$C_{ax,by} = \frac{1}{2\pi}\iint_{A_b} 2(x-x')(y-y')/r^4 \, dx'dy' \tag{11b}$$

$$C_{ay,bx} = C_{ax,by} \tag{11c}$$

$$C_{ay,by} = -C_{ax,bx} \tag{11d}$$

and Eqns(8) become:

$$\sum_{b=1}^{N} (C_{ax,bx}\chi_b - \delta_{ab})H_x(b) + C_{ax,by}\chi_b H_y(b) = -H_{cx}(a) \tag{12a}$$

$$\sum_{b=1}^{N} C_{ay,bx}\chi_b H_x(b) + (C_{ay,by}\chi_b - \delta_{ab})H_y(b) = -H_{cy}(a) \tag{12b}$$

If we carry out the integration of Eqn(11) for an element b which is in cross section a polygon of n sides, we obtain:

$$C_{ax,bx} = \frac{1}{2\pi} \sum_{k=1}^{n} \sin^2\phi_k [\tan^{-1} (y'_k-y)/(x'_k-x) \tan^{-1}(y'_{k+1}-y)/(x'_{k+1}-x)] + \sin\phi_k \cos\phi_k [\ln r_k - \ln r_{k+1}] \quad (13a)$$

$$C_{ax,by} = \frac{1}{2\pi} \sum_{k=1}^{n} -\sin\phi_k \cos\phi_k [\tan^{-1}(y'_k-y)/(x'_k-x) - \tan^{-1}(y'_{k+1}-y)/(x'_{k+1}-x)] - \cos^2\phi_k [\ln r_k - \ln r_{k+1}] \quad (13b)$$

where k labels the vertices in anti-clockwise order and

$$\phi_k = \tan^{-1} (y'_{k+1}-y'_k)/(x'_{k+1}-y'_k) \quad (14)$$

Problems can arise because the arctangent is not a single-valued function. If we adopt the usual convention:

$$-\pi < \tan^{-1} y/x < \pi$$

then the calculations should obey the rule

RULE 1. An element should not be intersected by the half line $y' = y, \quad x' < x$

If the location of the source element and field point causes this rule to be violated, the substitution $x \to -x$, $y \to -y$, $x' \to -x'$, $y' \to -y'$, should be made before Eqns(13) are calculated. This substitution is allowed because it leaves Eqns(11) unchanged.

B. Self-field coefficients

The self-field coefficients present particular difficulties. Clearly if the field point is inside the element, Rule 1 cannot be obeyed. The self-field coefficients should be found from Eqn(13) and then modified as follows:

Replace C_{xx} with $C_{xx}-\sin^2 \phi_\ell$ (15a)

Replace C_{xy} and C_{yx} with $C_{xy}+\sin\phi_\ell \cos\phi_\ell$ (15b)

Replace C_{yy} with $C_{yy}-\cos^2\phi_\ell$ (15c)

where ϕ_ℓ is the angle from the half line y'=y, x' x to the side of the polygon that the half line cuts.

C. Coefficients, expressed in complex variables

If we let z=x+iy, and following Beth[2] write

$$\alpha_k = e^{-i\phi k} \sin\phi_k \tag{16}$$

with ϕ_k as defined above, then we can find a complex coefficient

$$C = \frac{1}{2\pi} \sum_{k=1}^{n} (\alpha_{k+1}-\alpha_k) \ln (z_k'-z) \tag{17}$$

from which we can find the coefficients needed in Eqn(12).

$$C_{xx} = \text{Re}(C) \tag{18a}$$

$$C_{yy} = -\text{Re}(C) \tag{18b}$$

$$C_{xy} = C_{yx} = -\text{IM}(C) \tag{18c}$$

if Rule 1 is obeyed. For self field coefficients, Eqns (17) and (18) are replaced by

$$C = [\frac{1}{2\pi} \sum_{k=1}^{n} (\alpha_{k+1}-\alpha_k) \ln (z_k'-z)] - i\alpha_\ell \tag{19}$$

$$C_{xx} = \text{Re } (C) \tag{20a}$$

$$C_{yy} = \text{Re } (C) - 1 \tag{20b}$$

$$C_{xy} = C_{yx} = - \text{Im}(C) \tag{20c}$$

D. Symmetry Considerations

If the magnet being calculated has a plane of symmetry, then only half the current and iron elements need enter the calculation.

For example, let us consider a two-dimensional magnet design in which the x-z plane is a plane of symmetry. That is, if there is iron at (x, y), there is iron at (x, -y); and the same for the currents. We specify that the current densities obey:

$$J\ (x,\ -y) = J\ (x,y) \tag{21a}$$

From the symmetry we have assumed, it follows that:

$$H_x(x,\ -y) = -H_x\ (x,y) \tag{21b}$$

$$H_y(x,\ -y) = H_y\ (x,y), \tag{21c}$$

and the magnetisation obeys similar equations. These equations can be used to reduce the number of unknown field components, and also the number of linear equations that need be solved to find them, by a factor of two. For if we used Eqn(21) to eliminate on Eqn(12) all the field components of elements in the lower half plane, we would need only half as many equations to find the remaining components.

Suppose that element d is the mirror image at element b, and that neither is element a. Then initially Eqn(12) would have terms:

$$\ldots + G_{ai,bi}\chi_b H_{bx} + G_{ai,b2}\chi_b H_{by} + \ldots + G_{ai,d1}\chi_d H_x + G_{ai.d2}\chi_d H_{dy} + \ldots = -H_{cai}$$

Substituting Eqn(24) yields:

$$\ldots + G_{ai,b1}\chi_b H_{bx} + G_{ai,b2}\chi_b H_{by} + \ldots G_{ai,d1}\chi_b(-H_{bx}) + G_{ai,d2}\chi_b(H_{by}) + \ldots - H_{cai}$$

or

$$\ldots + (G_{ai,b1} - G_{ai,d1})\chi_b H_{bx} + (G_{ai,b2} + G_{ai,d2})\chi_b H_{by} + \ldots = -H_{cai}$$

Finally, we will redefine the $G_{ai,bj}$ to include contributions from the image elements.

We write below the expressions for the $G_{a,b}$ for several symmetry configurations, using complex notation for simplicity. First we define a general expression for each quadrant:

$$C_1 = \frac{1}{2\pi} \Sigma \ (\alpha_{k+1}-\alpha_k) \ln(z_k'-z_o)$$

or

$$C_1 = \frac{1}{2\pi} \Sigma \ (\alpha_{k+1}-\alpha_k) \ln(z-z_k')$$

depending on which obeys Rule 1

$$C_2 = \frac{1}{2\pi} \Sigma \ (\alpha^*_{k+1}-\alpha^*) \ln(z-z_k')$$

$$C_3 = \frac{1}{2\pi} \Sigma \ (\alpha_{k+1}-\alpha_k) \ln(z-z_k')$$

$$C_4 = \frac{1}{2\pi} \Sigma \ (\alpha^*_{k+1}-\alpha^*_k) \ln(z-z_k') \qquad (22)$$

The self-field modifications of Section B affect only C_1.

For single plane dipole symmetry, described above,

$$J(x, -y) = J(x,y), H_x(x,-y) = -H_x(x,y), H_y(x,-y) = H_y(x,y)$$

$$C_{xx} = \mathrm{Re}\ (c_1-C_4)$$

$$C_{xy} = \mathrm{Im}\ (-C_1-C_4)$$

$$C_{yx} = \mathrm{Im}\ (-C_1+C_4)$$

$$C_{yy} = \mathrm{Re}\ (-C_1-C_4) \qquad (23)$$

A symmetrical dipole magnet has, in addition to the condition of Eqn(21) the conditions $J(-x,y) = -J(x,y), H_x(-x,y)$, and $H_y(-x,y) = H_y(x,y)$ and has elements:

$$C_{xx} = Re(C_1-C_2+C_3-C_4)$$

$$C_{xy} = Im(-C_1-C_2-C_3-C_4)$$

$$Cy = Im(-C_1+C_2-C_3+C_4)$$

$$C_{yy} = Re(-C_1-C_2-C_3-C_4) \quad (24)$$

A quadrupole magnet obeys the conditions:

$$J(-x,y) = J(x,y) = J(x,-y)$$

$$H_x(-x,y) = H_x(x,y) = -H_x(x,-y)$$

$$-H_y(-x,y) = H_y(x,y) = H_y(x,-y) \quad (25)$$

and has elements:

$$C_{xx} = Re(C_1+C_2-C_3-C_4)$$

$$C_{xy} = Im(-C_1+C_2+C_3-C_4)$$

$$C_{yx} = Im(-C_1-C_2+C_3+C_4)$$

$$Cyy = Re(-C_1+C_2+C_3-C_4) \quad (26)$$

4. THREE DIMENSIONAL CALCULATIONS

In all three-dimensional calculations attempted so far the iron elements have been right polygonal prisms oriented in the z direction, and the current elements have been either infinite conductors carrying currents in the z direction or coils about the z axis.

A. The general coefficients

Equations (7) can be integrated over a prism between z_1 and Z_2 and of polygonal cross section. Introducing the expressions:

$$T_{ijk} = \tan^{-1} \frac{(z_i'-z)[(y_j'-y)\sin\phi_k + (x_j'-x)\cos\phi_k]}{r_{ij}[(x_j'-x)\sin\phi_k - (y_j'-y)\cos\phi_k]} \quad (27a)$$

$$L_{ij} = \tfrac{1}{2} \ln (r_{ij}+z_i'-z)/(r_{ij}-z_i+z) \quad (27b)$$

$$V_{ijk} = \tfrac{1}{2} \ln \{(y_j' - y)\sin\phi_k + (x_j' - x)\cos\phi_k + r_{ij}\} \tag{27c}$$

we can write the coefficients:

$$C_{xx} = \frac{1}{2\pi} \sum_{i=1}^{2} \sum_{k=1}^{3} (-1)^i \{\sin^2\phi_k (T_{ikk} - T_{i,k+1,k}) - \sin\phi_k \cos\phi_k (L_{ik} - L_{i,k+1})\} \tag{28a}$$

$$C_{yy} = \frac{1}{2\pi} \sum_{i=1}^{2} \sum_{k=1}^{3} (-1)^i \cos^2\phi_k (T_{ikk} - T_{i,k+1,k}) + \sin\phi_k \cos\phi_k (L_{ik} - L_{i,k+1}) \tag{28b}$$

$$C_{xy} = \frac{1}{2\pi} \sum_{i=1}^{2} \sum_{k=1}^{3} (-1)^i \sin\phi_k \cos\phi_k (T_{ikk} - T_{i,k+1,k}) + \cos^2\phi_k (L_{ik} - L_{i,k+1}) \tag{28c}$$

$$C_{xz} = \frac{1}{2\pi} \sum_{i=1}^{2} \sum_{k=1}^{3} -(-1)^i \sin\phi_k (V_{ikk} - V_{i,k+1,k}) \tag{28d}$$

$$C_{yz} = \frac{1}{2\pi} \sum_{i=1}^{2} \sum_{k=1}^{3} (-1)^i \cos\phi_k (V_{ikk} - V_{i,k+1,k}) \tag{28e}$$

Values for C_{yx}, C_{zx}, C_{zy} and C_{zz} can then be found by Eqn(9).

Those coefficients which include the arctangent term T_{ijk} are subject to problems because the arctangent is not single-valued. The correct quadrant of $T_{ikk,k+1,k}$ can be found by finding the correct quadrant for the corresponding two dimensional situation whan $z_i' - z$ goes to infinity.

B. Self field coefficients

The corrections needed for the self-field coefficients are exactly those listed as Eqn(15). None are needed for C_{xz}, C_{zx}, C_{yz}, C_{zy} or C_{zz}.

5. AXISYMMETRIC CALCULATIONS

Only iron elements of rectangular cross section have been used heretofore. The coefficients are given by:

$$C_{zz} = \frac{1}{4\pi} \int_{\pi}^{\pi} d\theta \int_{r_1}^{r_2} rdr \int_{z_1}^{z_2} dz(3z^2/\rho^5 - 1/\rho^3) \tag{29a}$$

$$C_{zr} = \frac{1}{4\pi} \iiint d\theta \; rdrdz \; 3z \; (r - x_o \cos\theta)/\rho^5 \tag{29b}$$

$$C_{rz} = \frac{1}{4\pi} \iiint d\theta \; rdrdz \; 3z \; (r \cos\theta - x_o)/\rho^5 \tag{29c}$$

$$C_{rr} = \frac{1}{4\pi} \iiint d\theta \; rdrdz \; [3(r \cos\theta - x_o)(r - x_o \cos\theta)/\rho^5 - \cos\theta/\rho^3] \tag{29d}$$

where for convenience, but without lack of generality, the field point has been taken as the point with cartesian coordinates (x_o, o, o). We are also denoting the source coordinates by (x,y,z) or by (ρ,θ,z):

$$r^2 = x^2+y^2$$

$$\theta = \tan^{-1} y/x$$

$$\rho^2 = z^2+x_o^2+r^2-2rx_o \cos$$

N J Diserens has evaluated the Eqn(29).

The integration over z can be performed analytically; the integration over θ yields elliptic integrals, which can then be integrated numerically over r.

$$C_{zz} = \frac{1}{\pi} \int_{r_1}^{r_2} \left[- \frac{rz}{(z^2+(x_o+r)^2)^{3/2}} \prod \left(\frac{\pi}{2}, \frac{-4rx_o}{z^2+(x_o+r)^2}, \frac{4rx_o}{z^2+(x_o+r)^2}\right) \right]_{z_1}^{z_2} dr$$

(30a)

$$C_{zr} = \frac{1}{2\pi} \int_{r_1}^{r_2} \left[\frac{x_o^2+z^2-r^2}{(z^2+(x_o+r)^2)^{3/2}} \Pi\left(\frac{\pi}{2}, \frac{-4rx_o}{z^2+(x_o+r)^2}, \sqrt{\frac{4rx_o}{z^2+(x_o+r)^2}}\right) \right.$$

$$\left. - \frac{1}{\sqrt{z^2+(x_o+r)^2}} F\left(\frac{\pi}{2}, \sqrt{\frac{4rx_o}{z^2+(x_o+r)^2}}\right) \right]_{z_1}^{z_2} dr \qquad (30b)$$

$$C_{rz} = \frac{1}{2\pi} \int_{r_1}^{r_2} \frac{r}{x_o} \left[\frac{x_o^2-r^2-z^2}{(z^2+(x_o+r)^2)^{3/2}} \Pi\left(\frac{\pi}{2}, \frac{-4rx_o}{z^2+(x_o+r)^2}, \sqrt{\frac{4rx_o}{z^2+(x_o+r)^2}}\right) \right.$$

$$\left. + \frac{1}{\sqrt{z^2+(x.+r)^2}} F\left(\frac{\pi}{2}, \sqrt{\frac{4rx_o}{z^2+(x_o+r)^2}}\right) \right]_{z_1}^{z_2} dr \qquad (30c)$$

$$C_{rr} = \frac{1}{4\pi} \left\{ [r\phi]_{r_1}^{r_2} - \int_{r_1}^{r_2} \phi dr \right\} \qquad (30d)$$

with

$$\phi = \left[\frac{2z}{x_o} \frac{1}{\sqrt{z^2+(x_o+r)^2}} \left\{ \frac{x_o-r}{x_o+r} \Pi\left(\frac{\pi}{2}, \frac{-4rx_o}{(xo+r)^2}, \sqrt{\frac{4rx_o}{z^2+(x_o+r)^2}}\right) \right. \right.$$

$$\left. \left. + F\left(\frac{\pi}{2}, \sqrt{\frac{4rx_o}{z^2+(x_o+r)^2}}\right) \right\} \right]_{z_1}^{z_2} \qquad (30e)$$

Iron elements of triangular cross section could be calculated by Eqn(30), but with z_1 and z_2 linear functions of r.

6. CONCLUSIONS

These equations should be useful to any GFUN user who wishes to understand the program or extend it.

7. ACKNOWLEDGEMENTS

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